An imperfect production inventory model with setup cost reduction and carbon emission for an integrated supply chain

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C H R O N I C L E	A B S T R A C T
	The present paper develops a sustainable framework under a supply chain environment by considering a vendor and a buyer. The vendor governs the manufacturing process to serve the demand of the buyer. The manufacturing system is imperfect which ends up producing some fraction of defectives. For the sake of maintaining the goodwill of the organization, it is imperative to maintain the quality standards. Thus, the buyer employs the screening process to bifurcate the perfect items from the imperfect ones. The imperfect items secluded after screening are reverted back to the vendor, for which a warranty cost is incurred. Further, in an attempt to achieve sustainability the model considers the carbon-emissions costs during the transportation process. A one-time discrete investment is also considered for minimizing the setup cost of the vendor for subsequent cycles. The models are developed for the case of the individual decision and for the joint verdict of the players under consideration. Moreover, it has been proved that the integrated model performs proficiently over the individual model. The objective is to jointly optimize the costs sustained by the two players. Numerical analysis and sensitivity analysis is done for imparting the validity and robustness of the model.

1. Introduction

Efficient management of production and inventory decisions is the backbone for maintaining smooth drive of the supply-chain system. The fundamental of any inventory modeling revolves around the products. Therefore, it becomes essential to cogitate about products practicality in the supply chain structure. Thus, it is quite unrealistic to consider the products to be of perfect quality, as due to numerous reasons (faulty production process, mishandling during transportation etc.) the end product may not withstand with the quality standards set by the manufacturer. Consequently, the firm has to deal with imperfect quality items. The researchers who initiated working in this area were Porteus (1986), Rosenblatt and Lee (1986), Lee and Rosenblatt (1987), Kim and Hong (1999), Later, Ben-Daya and Hariga (2000), Salameh and Jaber (2000), Cardenas Barron (2000), Yeh et al. (2000), Huang (2002), Chung and Hou (2003), Ben-Daya and Rahim (2003), Huang (2004), Ha and Kim (1997), Hsieh and Lee (2005), Chen and Lo (2006), Ben-Daya and Al-Nassar (2008), Sana (2011), Lu et al. (2016) and many others came up with the pioneer research in the field of imperfect quality items.

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To achieve sustainability, spherical growth is required rather than individual growth. The success of a single entity will not pay in the long run until the overall supply chain system is benefitted. Thus, the act of working in collaboration is the best supply chain strategy for today's business enterprises. Accordingly, the emphasis is on optimizing the inventory and production decisions mutually. The area of integrated inventory modeling is explored by plenty of researchers. The preliminary research in this area was carried out by Goyal (1977), in which he presented a problem of single vendor and single customer. Afterwards, Banerjee (1986), Goyal (1988), Goyal and Gupta (1989), Aderohunmu et al. (1995), Lu (1995), Ha and Kim (1997), Viswanathan (1988), Hill (1997, 1999), Hoque and Goyal (2000), Goyal and Szendrovits (1986), Goyal and Nebebe (2000), Yang and Wee (2000), Wu and Ouyang (2003), Chung (2008), Shah and Shah (2012),Hsu and Hsu (2012, 2013) and several others carried forward the research work in the field of integrated modeling and presented the decision makers with integrated policies, that can be adopted to maximize benefits for all the participating players.

Further, in the vendor-buyer inventory models, it is tacitly presumed that the vendor handles the manufacturing department and the buyer governs the sales department. Also, the buyer places the orders and the vendor needs to fulfill the request. The flow of products between the two players is done at some transportation cost which is incurred by either the vendor or the buyer. During transportation, carbon-emissions take place which is the foremost reason behind environmental degradation. The organizations nowadays operate in a global fishbowl, so, any shortcomings are on a full display. Thus, to maintain the goodwill it is mandatory to look towards sustainable business which involves the consideration of carbon-emissions. The researchers who contributed in this area are Shi and Meier (2012), Shi et al. (2012), Zhang et al. (2013), Hammami et al. (2015), Tang et al. (2015), Sarkar et al. (2016), Tang et al. (2016), Cheng et al. (2016). These researchers considered the carbon-emissions in their inventory modeling.

While talking about the manufacturing sector it is crucial to building a reliable setup for the smooth operation of the manufacturing process. Also, investment in the setup process is quite beneficial as it aids in minimizing the costs for the subsequent cycles. Recently, researchers incorporated scenario of investment in setup costs which aids in cost reduction at manufacturing end (Denizel et al., 1997; Diaby, 2000; Nyea et al., 2001; Freimer et al., 2006; Annadurai & Uthayakumar, 2010; Huang et al., 2011; Sarkar & Majumder, 2013; Sarkar et al., 2015; Sarkar et al., 2016). Moreover, the uncertainty in supply and demand leads to shortages which is a real phenomenon to incorporate in inventory and supply chain modeling. The presence of imperfect quality items may result in shortages as a fraction of on-hand inventory is salvaged/removed from the stock. Thus, it becomes pragmatic to include shortages. Planned backorders are usually adopted by numerous industries viz. furniture showrooms, automobile, and mobile industries, etc. Researchers who incorporated shortages in their modeling were, Wee et al. (2007), Eroglu and Ozdemir (2007), Chang and Ho (2010), Cardenas-Barron (2009), Hsu and Hsu (2012, 2013), Sarkar and Moon (2014), Khanna et al. (2016), etc.

The present paper highlights the significance of collaborative strategies in today's competition inherited marketing world. A supply chain problem with a single vendor and buyer is discussed. An imperfect production scenario at the vendor's end is considered which ends up producing defectives. The defectives are secluded with the help of a screening process employed by the buyer so as to satisfy the end demand with good quality products. In order to manage the defectives efficiently, the buyer returns it to the vendor for further treatment which incurs a warranty cost. Climate degradation is a key issue in sustainability. It may lead to dangerous increases in temperature and sea level, flooding etc. Scientists all over the world declare GHG as the main malefactor for environmental change and this compels the government to set various emission norms. "Carbon" is often used as shorthand for greenhouse gases (GHGs) as it is regarded as the main element for measuring GHG. A sound environment-friendly policy of a firm can earn it an extremely good image which will ultimately elevate sales and profitability. Thus, the paper incorporates the carbon-emissions costs while transportation so as to achieve sustainability. A discrete investment in setup cost is also considered so as to reduce the overall cost of manufacturing for subsequent production cycles. Shortages are considered with complete backlogging

at the side of the buyer. Accounting to the above-discussed scenario the present paper proposes an inventory framework under integrated environment to address a faulty manufacturing process with setup cost reductions, carbon-emissions, and shortages. Moreover, two scenarios are discussed viz. the individual decision-making strategy and the integrated strategy. The findings are compared with both the strategies and it is further suggested that the integrated model is superior by showing appropriate cost reductions. The numerical analysis is given along with the detailed sensitivity analysis to demonstrate and support the model features. The present model is extensively relevant for production firms like leather, plastic, electronics etc.

The remaining paper is presented as: *Section 2* gives the notations, *Section 3* lists the assumptions on which the present model is developed, *Section 4* describes the problem and gives mathematical model, *Section 5* gives model optimization and provides the solution procedure, *Section 6 and 7* establishes the validity and robustness by giving numerical example and model sensitivity, and the last section summarizes the paper by giving conclusion and scope for future research.

2. Notations

λ	Rate of demand (unit/year)
ψ	Rate of production (unit/year)
$\hat{o}$	Defective percentage in the ordered lot
Q	Order quantity
B	Backordering quantity
N	Number of freights
O_B	Cost of ordering ($/order)
H_B	Cost of holding inventory at buyer's end ($/unit/year)
H_V	Cost of holding inventory at vendor's end ($/unit/year)
S	Cost of screening at buyer's end ($/unit)
C_F	Fixed cost of carbon-emissions at vendor's end($/transport)
C_V	Variable cost of carbon-emissions at vendor's end($/unit)
T_F	Fixed cost of transportation at vendor's end ($/transport)
T_V	Variable cost of transportation at vendor's end ($/unit)
S_0	Initial cost of setup ($/setup)
S_v	Setup cost (vendor)after investment ($/setup)
C_B	Cost of backordering at buyer's end ($/order)
W_V	Unit warranty cost (per defective item at buyer)

3. Assumptions

  i.   Demand and production rates are considered to be known and constant. Some of the references of such type are: Khanna et al. (2016b), Chung et al. (2002), Choi et al. (2008), Wee et al. (2014), Sarkar and Saren (2015), Kaliraman et al. (2016) etc.

  ii.   Known and constant lead time is considered.

  iii.   Lot size Q carries defectives. The defective rate $\hat{o}$ follows the pdf $f(\delta)$. Moreover, it is presumed that $E[\delta] < 1 - \lambda/\psi$, so as to safeguard the manufacturing capacity to be enough to fulfill the annual demand of the buyer.

  iv.   Single-vendor, single buyer and a single type of product is considered.

 v. Shortages are fully backlogged.

 vi. Fixed and variable costs of carbon-emissions are considered at the vendor's end.

 vii. The lot undergo a complete screening test prior to the commencement of each cycle time T. Entire lot of defectives are reverted to the vendor, upon reception of the subsequent delivery. In this case fixed and variable parts of transportation costs are considered. A warranty cost W_V is incurred by the vendor for an imperfect item. The vendor sells out the defectives on a lesser amount in some subordinate market. The inspection duration is reckoned as lead-time for supply.

 viii. A discrete investment I is incorporated for minimizing the setup cost which is defined as $S_v(\xi) = S_0 e^{-k\xi}$, where k is a known parameter. It has been established that increase in investment amount lowers the resultant cost of setup.

4. Problem Definition and Mathematical Modeling

4.1. Problem Definition

The present paper attempts to bridge the research gap within an integrated supply chain system by presenting a sustainable framework for the problem of vendor and buyer. A yearly demand of λ units is considered at buyer. The buyer orders regularly in fixed batches of size Q_{prod}. The vendor takes the responsibility to cater the orders by manufacturing the products in batches (of size Q_{prod}), and plans to ship the quantities to the buyer in N number of consignments each of Q units. Since the production process is presumed as imperfect, accordingly the shipments contain items of imperfect quality, to deal with it the buyer employs an inspection process and revert the defectives to vendor. The vendor incurs warranty cost for defectives, in this manner; end demand of the customer is satisfied with perfect quality items. It has been further presumed that shortages at buyer is allowed and are assumed to be completely backlogged. The transportation process between the two players is governed by the vendor for which a fixed as well as a variable component of shipments and carbon-emissions is taken into account. The prime goal lies in minimizing the costs acquired by the two players by jointly optimizing the number of shipments, the size of shipments, back-ordering quantity and investment in setup cost. Also, to achieve sustainability by showing appropriate cost reductions in case of the integrated model while comparing with the independent model.

4.2. Vendor's Viewpoint

The Fig. 1(a) represents the scenario of the vendor over time and Fig. 1(b) represents the inventory carrying cost per cycle. The components of vendor total cost comprises of costs of setup, investment in setup, inventory carrying, transportation and carbon-emission which are evaluated below:
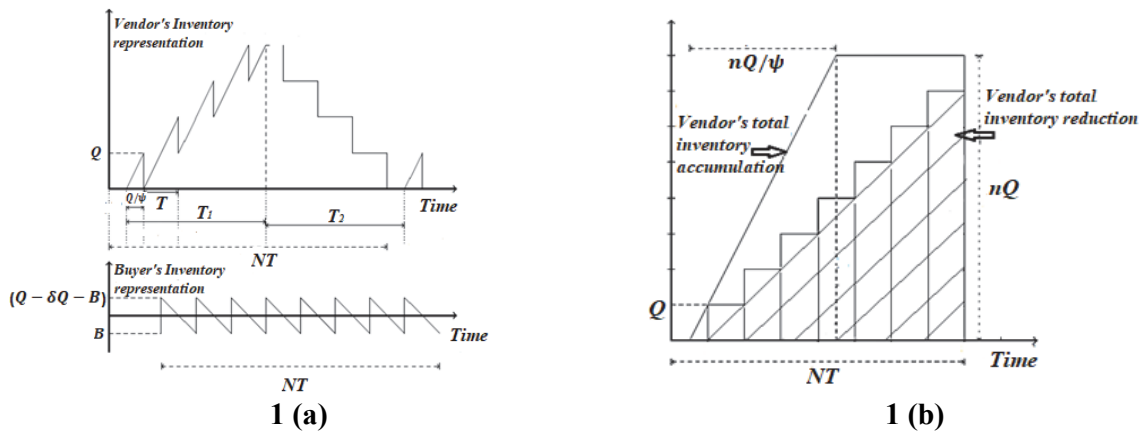
1 (a)   **1 (b)**

Fig. 1. Inventory representation over time for vendor

The model considers investment in setup cost so the setup cost is given as:

Setup cost $S_v(\xi) = S_0 e^{-k\xi}$,   (1)

Investment in setup cost $= \xi$,   (2)

The cost of holding inventory in case of vendor is obtained [Goyal et al (2003) and Huang (2004)] by subtracting the shaded area from the bold area (see Fig. 1 (b)),

Holding cost $= H_V \{BoldArea - ShadedArea\}$

$$= H_V \left\{ \left[NQ\left(\frac{Q}{\psi} + (N-1)T\right) - \frac{NQ(NQ/\psi)}{2} \right] - T\left[Q + 2Q + ... + (N-1)Q\right] \right\}$$

$$= H_V \left\{ \left[NQ\left(\frac{Q}{\psi} + (N-1)\frac{Q(1-\delta)}{\lambda}\right) - \frac{nQ(nQ/\psi)}{2} \right] - \frac{Q(1-\delta)}{\lambda}\left[Q + 2Q + ... + (N-1)Q\right] \right\}$$

$$= H_V \left\{ \frac{NQ^2}{\psi} - \frac{N^2Q^2}{2\psi} + \frac{N(N-1)Q^2(1-\delta)}{2\lambda} \right\},$$ (3)

Since the vendor is responsible for shipping the products. Also, the buyer sends back the defectives to the vendor, in view of this there are fixed and variable components associated with the transportation cost which is defined as follows:

Transportation cost $= NT_F + T_V NQE[\delta]$,   (4)

While shipping the products from the vendor to the buyer or vice-versa, fixed and variable costs of carbon-emissions is incurred this is given as:

Cost of Carbon Emission $= NC_F + C_V NQE[\delta]$,   (5)

Warranty Cost $= W_V NQE[\delta]$ (6)

Thus, total cost per cycle for vendor is:

$$TC_V(N,Q,\xi) = S_0 e^{-k\xi} + \xi + H_V \left\{ \frac{NQ^2}{\psi} - \frac{N^2Q^2}{2\psi} + \frac{N(N-1)Q^2(1-\delta)}{2\lambda} \right\} + NT_F + T_V NQE[\delta] + NC_F +$$ (7)

$$C_V NQE[\delta] + W_V NQE[\delta]$$

4.3. Buyer's Viewpoint

The buyer's situation is represented in **Figure 2.** The buyer's total cost is a sum of the costs of ordering, holding, screening and backordering. The cost components are evaluated as follows:
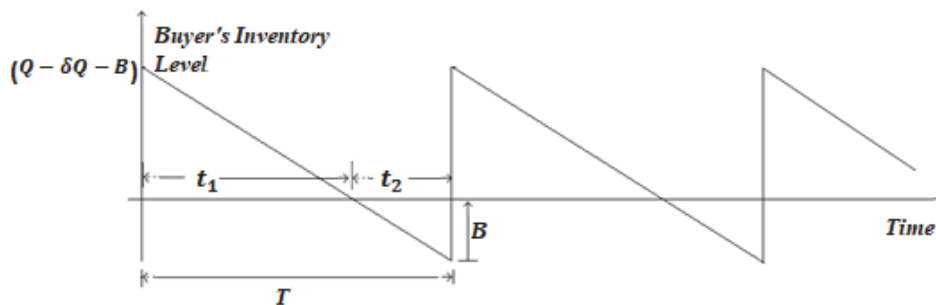
Fig. 2. Inventory representation over time for the buyer

Orders are placed by the buyer with fixed ordering cost given as:

$$\text{Ordering Cost} = O_B \tag{8}$$

Since, the received lot contains both perfect as well as imperfect items it is therefore necessary to conduct screening, so the following cost of screening is incurred by the buyer:

$$\text{Screening Cost} = SNQ \tag{9}$$

Considering the perfect quality articles the average inventory is $\frac{1}{2}\{Q(1-\delta)-B\}$. The average inventory level of the defective item is δQ and these are reverted to the vendor on accepting the subsequent lot. Thus, the time points are defined as (see Fig. 1):

The depletion of positive inventory level is governed by the following time point: $t_1 = \dfrac{(Q-\delta Q-B)}{\lambda}$, and

$$\text{The total cycle time is defined as: } T = t_1 + t_2, \tag{10}$$

$T = \dfrac{Q(1-\delta)}{\lambda}$ so, the total inventory carrying cost of the buyer is given as:

$$\text{Holding Cost} = H_B N \left\{ \frac{1}{2} \frac{(Q-\delta Q-B)^2}{\lambda} + \frac{Q^2 \delta(1-\delta)}{\lambda} \right\} \tag{11}$$

The shortage time is defined as: $t_2 = \dfrac{B}{\lambda}$, so the backordering cost of the buyer can be given as:

$$\text{Backordering Cost} = \frac{NC_B B^2}{2\lambda} \tag{12}$$

The total cost of buyer is incurred due to placing order, holding inventory, backorders and screening process. Thus, the buyer's total cost per cycle is:

$$TC_B(N,Q,B) = O_B + SNQ + H_B N \left\{ \frac{1}{2} \frac{(Q-\delta Q-B)^2}{\lambda} + \frac{Q^2 \delta(1-\delta)}{\lambda} \right\} + C_B N \frac{1}{2} \frac{B^2}{\lambda} \tag{13}$$

4.4. Joint Perspective of vendor and buyer

The joint total cost is given as:

$$TC_I(N,Q,B,\xi) = TC_B(N,Q,B) + TC_V(N,Q,\xi) \tag{14}$$

$$TC_I(N,Q,B,\xi) = O_B + SNQ + S_0 e^{-k\xi} + \xi + H_B N \left\{ \frac{1}{2} \frac{(Q-\delta Q-B)^2}{D} + \frac{Q^2 \delta(1-\delta)}{\lambda} \right\} + C_B N \frac{1}{2} \frac{B^2}{\lambda} + \tag{15}$$

$$H_V \left\{ \frac{NQ^2}{\psi} - \frac{N^2 Q^2}{2\psi} + \frac{N(N-1)Q^2(1-\delta)}{2\lambda} \right\} + NT_F + T_V NQE[\delta] + NC_F + C_V E[\delta]NQ + W_V NQE[\delta]$$

The cycle length is given as $E\left[T_{cyc}\right] = \dfrac{NQ(1-E[\delta])}{\lambda}$. $\tag{16}$

By implementing the renewal reward theorem, the joint total expected cost is:

$$ETC(N,Q,B,\xi) = \frac{ETC_I(N,Q,B,\xi)}{E[T_{cyc}]} = \frac{O_B\lambda}{NQ(1-E[\delta])} + \frac{SNQ\lambda}{NQ(1-E[\delta])} + \frac{S_0\lambda e^{-k\xi}}{NQ(1-E[\delta])} + \frac{\xi\lambda}{NQ(1-E[\delta])} + \tag{17}$$

$$H_B\left\{\frac{1}{2}\left(\frac{E\left[(1-\delta)^2\right]}{(1-E[\delta])} - 2B + \frac{B^2}{Q(1-E[\delta])}\right) + \frac{Q(E[\delta]-E[\delta^2])}{(1-E[\delta])}\right\} + C_BN\frac{1}{2}\frac{B^2\lambda}{\lambda nQ(1-E[\delta])} +$$

$$H_V\left\{\frac{Q\lambda}{P(1-E[\delta])} - \frac{NQ\lambda}{2P(1-E[\delta])} + \frac{(N-1)Q}{2}\right\} + \frac{NT_F\lambda}{NQ(1-E[\delta])} + \frac{T_V NQE[\delta]\lambda}{NQ(1-E[\delta])} + \frac{NC_F\lambda}{NQ(1-E[\delta])} +$$

$$\frac{C_V\lambda E[\delta]NQ}{NQ(1-E[\delta])} + \frac{W_V NQE[\delta]\lambda}{NQ(1-E[\delta])}$$

5. Optimality and Solution Procedure

5.1. Optimality

To get the optimum values of the order quantity, backordering quantity and investment in setup, the necessary conditions to establish convexity are given as:

$$\frac{\partial ETC(N,Q,B,\xi)}{\partial Q} = 0, \tag{18}$$

$$\frac{\partial ETC(N,Q,B,\xi)}{\partial B} = 0, \tag{19}$$

$$\frac{\partial ETC(N,Q,B,\xi)}{\partial \xi} = 0. \tag{20}$$

By using Eq. (17), Eq. (18) and Eq. (19), the values of Q, B, and ξ are obtained as:

$$Q^*(N) = \sqrt{\frac{\frac{\left(O_B + S_0e^{-k\xi} + \xi\right)\lambda}{N} + \frac{\left(H_B + C_B\right)B^2}{2} + \left(T_F + C_F\right)\lambda}{H_V\left\{\frac{\lambda}{\psi} - \frac{N\lambda}{2\psi} + \frac{(N-1)(1-E[\delta])}{2}\right\} + \left(E[\delta] - E\left[\delta^2\right]\right)}} \tag{21}$$

$$B^*(N) = \frac{H_B Q^*(N)(1 - E[\delta])}{(H_B + C_B)} \tag{22}$$

$$\xi^*(n) = \frac{1}{k}\ln S_0 k \tag{23}$$

All the first order derivatives are calculated in Appendix A.

N is a discrete variable thus to obtain the optimum value of N a solution procedure is adopted which is discussed in subsequent section, the sufficient condition for rest of the variables is in the form of hessian matrix given below:

$$\begin{bmatrix} \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial Q^2} & \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial Q\partial B} & \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial Q\partial \xi} \\ \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial B\partial Q} & \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial B^2} & \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial B\partial \xi} \\ \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial \xi\partial Q} & \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial \xi\partial B} & \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial \xi^2} \end{bmatrix} \tag{24}$$

$$D_1 = \frac{\partial^2 ETC(N,Q,B,\xi)}{\partial Q^2}, D_2 = \begin{bmatrix} \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial Q^2} & \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial Q \partial B} \\[3mm] \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial B \partial Q} & \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial B^2} \end{bmatrix},$$

$$D_3 = \det H_e = \begin{bmatrix} \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial Q^2} & \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial Q \partial B} & \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial Q \partial \xi} \\[3mm] \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial B \partial Q} & \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial B^2} & \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial B \partial \xi} \\[3mm] \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial \xi \partial Q} & \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial \xi \partial B} & \dfrac{\partial^2 ETC(N,Q,B,\xi)}{\partial \xi^2} \end{bmatrix}$$, where D_1, D_2 and

D_3 are minors of the Hessian matrix.

All the second order derivatives and optimality conditions are calculated in Appendix B and C.

The second derivative of N shows the joint total expected cost is a convex function of N. To obtain the optimum value of N a solution procedure is adopted which is given below:

5.2. Solution Procedure

Step 1: Obtain the corresponding values of the optimal order size, backordering quantity and investment in setup from the Eq. (18), Eq. (19), and Eq. (20) for a set of N values and calculate the total expected cost from Eq. (17).

Step 2: Obtain the optimum value of N so that the condition given below is fulfilled:

$$TC(N+1, Q^*(N+1), B^*(N+1), \xi^*(N+1)) \geq TC(N, Q^*(N), B^*(N), \xi^*(N)) \leq$$
$$TC(N-1, Q^*(N-1), B^*(N-1), \xi^*(N-1))$$

Step 3: After finding the optimal value of the number of shipment the optimum production batch quantity is calculated as $Q_{prod.} = N^* Q^*(N^*)$

5.3. The individual decision-making of buyer

When the traditional approach of working independently is preferred over working in collaboration, the expected total annual buyer's cost is calculated as:

$$ETC_B(Q,B) = \frac{O_B \lambda}{Q(1-E[\delta])} + \frac{SN\lambda}{(1-E[\delta])} + H_B \left\{ \frac{\frac{1}{2}\left(\frac{QE\left[(1-\delta)^2\right]}{(1-E[\delta])} - 2B + \frac{B^2}{Q(1-E[\delta])} \right) +}{\frac{Q\left(E[\delta]-E[\delta^2]\right)}{(1-E[\delta])}} \right\} + \frac{C_B B^2}{2Q(1-E[\delta])} \tag{25}$$

and the optimum values are obtained as:

$$Q_B^* = \sqrt{\frac{2O_B\lambda + C_B B^2 + H_B B^2}{H_B E\left[(1-\delta)^2\right] + 2\left(E[\delta]-E[\delta^2]\right)}} \tag{26}$$

$$B_B^* = \frac{H_B\left(1-E[\delta]\right)Q_B^*}{\left(H_B+C_B\right)} \tag{27}$$

The expected total annual cost of the buyer is:

$$ETC_V\left(Q,\xi\right) = \frac{S_0\lambda e^{-k\xi}}{Q\left(1-E[\delta]\right)} + \frac{\xi\lambda}{Q\left(1-E[\delta]\right)} + \frac{H_V Q\lambda}{2\psi\left(1-E[\delta]\right)} + \frac{n\lambda T_F}{Q\left(1-E[\delta]\right)} + \frac{T_V\lambda E[\delta]}{\left(1-E[\delta]\right)} + \tag{28}$$

$$\frac{N\lambda C_F}{Q\left(1-E[\delta]\right)} + \frac{C_V\lambda E[\delta]}{\left(1-E[\delta]\right)} + \frac{W_V\lambda E[\delta]}{\left(1-E[\delta]\right)}$$

If the decision making depends on the buyer only, then by putting $Q = Q_B^*$ and $B = B_B^*$ in Eq. (24) and $Q = Q_B^*$ in Eq. (27), correspondingly the expected total annual costs of the vendor and buyer is obtained.

6. Numerical Analysis

Example: The value of the parameters are taken as: $\psi = 160000$ units/year, $\lambda = 50000$ units/year, $O_B = \$300$ /order, $T_F = 25$ /delivery, $T_V = 0.1$ /unit, $C_F = \$5$ /delivery, $C_V = \$5$ /unit, $C_B = \$10$ /unit/year, $S = \$0.5$ /unit, $H_B = \$5$ /unit/year, $H_V = \$2$ /unit, $W_V = \$30$ /unit, $\beta = 0.04$, $S_0 = \$1000$ /setup, $k = 0.00140$ by making use of the numerical data from Sarkar et al. (2016) and Hsu and Hsu (2012).

The following results are obtained: $N^* = 9, Q^* = 1073.43$ units, $B^* = 350.65$ units, $\xi^* = \$240.33$ per production run, and the total expected integrated cost $= \$77325.83$.

The buyer's independent decision making policy leads to $Q^* = 2971.89$ units, $B^* = 970.82$ units, total cost of the buyer is \$35810.77, total cost of vendor is \$43717.02, and the reduction of cost through integrated model is \$35810.77 + \$43717.02 - \$77325.83 = \$2201.96.

7. Sensitivity Analysis

In the present section, the proposed model is tested for its robustness by changing the model parameters. Table 1 presents the model sensitivity with respect to buyer and vendor holding costs, defective fraction, fixed and variable parts of shipments and carbon-emissions. Table 2 presents the sensitivity with respect to setup cost reduction parameter.

Table 1
The model sensitivity with respect to buyer and vendor holding costs, defective fraction, fixed and variable parts of shipments and carbon-emissions

H_B	Q_B^*	B_B^*	Buyer's Cost	Vendor's Cost	N	$Q^*(N)$	$B^*(N)$	Integrated Cost	Reduction
2.5	3855.22	755.62	33450.66	43717.02	6	1601.84	313.95	76453.84	713.84
3.75	3293.45	880.25	34805.06	43717.02	8	1211.17	323.71	76962.55	1559.53
5.00	2971.90	970.82	35810.77	43717.02	9	1073.43	350.65	77325.83	2201.96
6.25	2760.08	1040.33	36601.27	43717.02	10	968.38	365.00	77603.91	2714.38
7.50	2608.49	1095.56	37245.79	43717.02	11	885.2	371.78	77828.28	3134.53
β									
0.02	2985.47	985.2	35402.66	25625.43	9	1068.75	352.68	58964.64	2063.45
0.03	2978.57	977.96	35606.02	34631.78	9	1071.07	351.67	68105.26	2132.54
0.04	2971.89	970.81	35810.77	43717.02	9	1073.43	350.65	77325.83	2201.96
0.05	2965.45	963.77	36016.93	52882.36	9	1075.82	349.64	86627.60	2271.69
0.06	2959.22	956.81	36224.54	62129.05	9	1078.24	348.63	96011.79	2341.80

H_V

1.0	2971.89	970.81	35810.77	41372.47	14	982.69	321.01	73643.14	3540.10
1.5	2971.89	970.81	35810.77	42644.58	11	1018.83	332.81	75650.33	2805.02
2.0	2971.89	970.81	35810.77	43717.02	9	1073.43	350.65	77325.83	2201.96
2.5	2971.89	970.81	35810.77	44661.85	8	1085.37	354.55	78786.91	1685.71
3.0	2971.89	970.81	35810.77	45516.05	7	1131.26	369.54	80095.22	1231.60

T_F

12.50	2971.9	970.82	35810.77	43666.04	12	807.74	263.86	76641.03	2835.80
18.75	2971.9	970.82	35810.77	43691.57	10	964.59	315.10	77007.04	2495.30
25.00	2971.9	970.82	35810.77	43717.02	9	1073.43	350.65	77325.83	2202.00
31.25	2971.9	970.82	35810.77	43742.38	8	1201.86	392.60	77615.20	1938.00
37.50	2971.9	970.82	35810.77	43767.67	8	1221.16	398.91	77878.40	1700.00

C_F

2.50	2971.89	970.81	35810.77	43706.85	10	976.64	319.03	77204.16	2313.46
3.75	2971.89	970.81	35810.77	43711.93	9	1069.46	349.35	77266.31	2256.39
5.00	2971.89	970.81	35810.77	43717.02	9	1073.43	350.65	77325.83	2201.96
6.25	2971.89	970.81	35810.77	43722.1	9	1077.38	351.94	77385.14	2147.73
7.50	2971.89	970.81	35810.77	43727.17	9	1081.32	353.23	77444.22	2093.72

T_V

0.050	2971.89	970.81	35810.77	43667.02	9	1073.43	350.65	77275.80	2201.96
0.075	2971.89	970.81	35810.77	43692.02	9	1073.43	350.65	77300.80	2201.96
0.100	2971.89	970.81	35810.77	43717.02	9	1073.43	350.65	77325.83	2201.96
0.125	2971.89	970.81	35810.77	43742.02	9	1073.43	350.65	77350.80	2201.96
0.250	2971.89	970.81	35810.77	43867.02	9	1073.43	350.65	77475.80	2201.96

C_V

2.50	2971.89	970.81	35810.77	41217.02	9	1073.43	350.65	74825.83	2201.96
3.75	2971.89	970.81	35810.77	42467.02	9	1073.43	350.65	76075.83	2201.96
5.00	2971.89	970.81	35810.77	43717.02	9	1073.43	350.65	77325.83	2201.96
6.25	2971.89	970.81	35810.77	44967.02	9	1073.43	350.65	78575.83	2201.96
7.50	2971.89	970.81	35810.77	46217.02	9	1073.43	350.65	79825.83	2201.96

Table 2
The sensitivity with respect to setup cost reduction parameter

k	Q_B^*	B_B^*	Buyer's Cost	Vendor's Cost	N	$Q^*(N)$	$B^*(N)$	ξ	Integrated Cost	Reduction
0.0007	2971.89	970.81	35810.77	43899.39	9	1089.29	355.83	0	77563.70	2146.4
0.0010	2971.89	970.81	35810.77	43899.39	9	1089.29	355.83	0	77563.70	2146.5
0.0014	2971.89	970.81	35810.77	43717.02	9	1073.43	350.65	240.33	77325.83	2202.0
0.0017	2971.89	970.81	35810.77	43493.36	9	1054.16	344.35	312.13	77036.70	2267.4
0.0021	2971.89	970.81	35810.77	43191.10	9	1028.44	335.95	353.30	76650.90	2351.0

7.1. Observations and Managerial Implications

The effective management of any production firm requires proper care of its inventory. In the present scenario (see Table 1), as the inventory carrying cost of buyer raises, the total cost of the system increases, the order size decreases, and backorders increases. With the advent of increase in holding cost, the wise decision would be to backorder the demand to the hilt so as to avoid the incurrence of higher holding costs. However, with an increase in the vendor's holding cost the total cost of the inventory system along with the order quantity rises and shipment frequency decreases which is fairly reasonable because the shipment of the items is done in bulk in an attempt to avoid the vendor's high holding cost.

The presence of defective items in the inventory system requires vigilant decision to pacify its effect. Here, with an increase in the defective percentage δ, which is assumed to be distributed uniformly over

0 and β, the economic order quantity rises so as to satisfy the end demand and the total cost rises as the items are vended at a reduced price which was supposed to be sold at the original price. The increasing number of defectives suggests that the vendor should adopt better investment policies in setup so as to avoid incurrence of loss due defectives.

The proposed model assumes that the shipments are done between vendor and buyer and this incurs a cost of carbon-emissions at the vendor. So, in this case with increasing fixed cost of transportation, the total cost and shipment quantity is increasing and the frequency of the shipments is decreasing which is fairly logical as the vendor will deliver larger quantities to the buyer so as to avoid frequent shipments and thereby prevents high transportation costs. Also, the rise in variable transportation cost implies arise in total costs of the inventory system. This cost is incurred by the vendor thereby the vendor's cost also increases. And, when the fixed cost of carbon-emissions increases the frequency of shipments is decreased so as to avoid large emissions while transporting the goods between vendor and buyer and the total cost will increase. The cost at the vendor is increasing because the cost of carbon-emissions is adhered by the vendor, however, the cost of buyer remains intact. It has been further noticed that as the variable cost of carbon emissions raises the total cost of inventory system rises along with the total cost of vendor. The cost of buyer remains insensitive to the change in the carbon-emissions costs. Furthermore, with an increase in the value of k (see Table 2), it is seen that the total cost of the inventory system is declining, however, the investment cost is increasing. The increasing value of k is beneficial for the organization. With the increase in the investment cost there is a significant decrease in the total cost, hence, investment in setup cost is recommended.

8. Concluding Remarks

8.1. Conclusion

In this paper, the problem of two players' viz. vendor and buyer has been formulated under integrated as well as individual environments. At vendor's end, the manufacturing process was presumed to be faulty and henceforward items of imperfect quality were manufactured. Serving the end customers with goods of expected standards and quality was the ultimate motto of any efficient supply chain system. Thus, the imperfect items were screened by the buyer and returned to the vendor and this incurs a warranty cost. The vendor makes a one-time discrete investment in setup costs so as to reduce the overall cost of production for the upcoming cycles. In order to control imperfect quality items, the investment in the setup process is beneficial as it helps in achieving an efficient production process that will ultimately assist in monitoring the production of defectives. The vendor administers the transportation process. The prominence of transportation from an emission viewpoint is apparent. Thus, while transporting the goods, fixed and variable cost components of carbon-emissions are considered so as to attain sustainability. The findings give cost reduction in case of the integrated model and suggest better performance of the integrated model over independent model which supports the notion of working in collaboration. Further, investment in setup process aids in minimizing the overall cost of the system which makes the present model quite applicable to real-time inventory scenarios. The model is validated with numerical analysis. Further, a comprehensive sensitivity analysis is incorporated which imparts the vital characteristics of the model. The present model caters an interesting contribution towards managing integrated supply chain and production related problems. Various production firms like plastic, leather, footwear etc. will be benefited with an enhanced approach of decision-making.

8.2.   Directions for Future Research

The present model is extensively applicable to numerous production firms and to scenarios in which a vendor and a buyer are associated, however, this could have been more pragmatic by taking into account partial backordering, price discounts to encourage backorders, inspection errors etc. The study

can be extended further for the case of multi-buyers and multi-item under green supply chain. Finally, a good contribution would be made by considering the supply disruptions.

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Appendix A

$$\frac{\partial ETC(n,Q,B,\xi)}{\partial Q} = -\frac{O_B D}{nQ^2(1-\mathrm{E}[\delta])} - \frac{S_0 D e^{-k\xi}}{nQ^2(1-\mathrm{E}[\delta])} - \frac{\xi D}{nQ^2(1-\mathrm{E}[\delta])} - \frac{H_B B^2}{2Q^2(1-\mathrm{E}[\delta])} + \frac{\left(\mathrm{E}[\delta]-\mathrm{E}[\delta^2]\right)}{(1-\mathrm{E}[\delta])} - \tag{A1}$$

$$\frac{C_B B^2}{2Q^2(1-\mathrm{E}[\delta])} + \frac{H_V D}{P(1-\mathrm{E}[\delta])} - \frac{H_V nD}{2P(1-\mathrm{E}[\delta])} + \frac{H_V(n-1)}{2} - \frac{T_F D}{Q^2(1-\mathrm{E}[\delta])} - \frac{C_F D}{Q^2(1-\mathrm{E}[\delta])}$$

$$\frac{\partial ETC(n,Q,B,\xi)}{\partial B} = -H_B + \frac{H_B B}{Q(1-\mathrm{E}[\delta])} + \frac{C_B B}{Q(1-\mathrm{E}[\delta])} \tag{A2}$$

$$\frac{\partial ETC(n,Q,B,\xi)}{\partial \xi} = \frac{D}{nQ(1-\mathrm{E}[\delta])}\left[S_0 e^{-k\delta} + 1\right] \tag{A3}$$

$$\frac{\partial ETC(n,Q,B,\xi)}{\partial n} = -\frac{O_B D}{n^2 Q(1-\mathrm{E}[\delta])} - \frac{S_0 D e^{-k\xi}}{n^2 Q(1-\mathrm{E}[\delta])} - \frac{\xi D}{n^2 Q(1-\mathrm{E}[\delta])} + H_V\left\{-\frac{QD}{2P(1-\mathrm{E}[\delta])} + \frac{Q}{2}\right\} \tag{A4}$$

Appendix B

$$\frac{\partial^2 ETC(n,Q,B,\xi)}{\partial Q^2} = \frac{2O_B D}{nQ^3(1-\mathrm{E}[\delta])} + \frac{2S_0 D e^{-k\xi}}{nQ^3(1-\mathrm{E}[\delta])} + \frac{2\xi D}{nQ^3(1-\mathrm{E}[\delta])} + \frac{H_B B^2}{Q^3(1-\mathrm{E}[\delta])} + \tag{A5}$$

$$\frac{C_B B^2}{Q^3(1-\mathrm{E}[\delta])} + \frac{2T_F D}{Q^3(1-\mathrm{E}[\delta])} + \frac{2C_F D}{Q^3(1-\mathrm{E}[\delta])}$$

$$\frac{\partial^2 ETC(n,Q,B,\xi)}{\partial B^2} = \frac{H_B + C_B}{Q(1-\mathrm{E}[\delta])} \tag{A6}$$

$$\frac{\partial^2 ETC(n,Q,B,\xi)}{\partial \xi^2} = \frac{DS_0 k^2 e^{-k\delta}}{nQ(1-\mathrm{E}[\delta])} \tag{A7}$$

$$\frac{\partial^2 ETC(n,Q,B,\xi)}{\partial n^2} = \frac{2O_B D}{n^3 Q(1-\mathrm{E}[\delta])} + \frac{2S_0 D e^{-k\xi}}{n^3 Q(1-\mathrm{E}[\delta])} + \frac{2\xi D}{n^3 Q(1-\mathrm{E}[\delta])} \tag{A8}$$

$$\frac{\partial ETC(n,Q,B,\xi)}{\partial Q \partial \xi} = \frac{D}{nQ^2(1-\mathrm{E}[\delta])}\left[S_0 k e^{-k\xi} - 1\right] \tag{A9}$$

$$\frac{\partial ETC(n,Q,B,\xi)}{\partial Q \partial B} = \frac{-B(H_B + C_B)}{Q^2(1-\mathrm{E}[\delta])} \tag{A10}$$

$$\frac{\partial ETC(n,Q,B,\xi)}{\partial B \partial Q} = \frac{-B(H_B + C_B)}{Q^2(1-\mathrm{E}[\delta])} \tag{A11}$$

$$\frac{\partial ETC(n,Q,B,\xi)}{\partial B \partial \xi} = 0 \tag{A12}$$

$$\frac{\partial ETC(n,Q,B,\xi)}{\partial \xi \partial Q} = \frac{D}{nQ^2(1-\mathrm{E}[\delta])}\left[S_0 k e^{-k\xi} - 1\right] \tag{A13}$$

$$\frac{\partial ETC(n,Q,B,\xi)}{\partial \xi \partial B} = 0 \tag{A14}$$

APPENDIX C

$$D_1 = \frac{\partial^2 ETC(n,Q,B,\xi)}{\partial Q^2} = \left(\begin{array}{c} \dfrac{2O_B D}{nQ^3(1-\mathrm{E}[\delta])} + \dfrac{2S_0 D e^{-k\xi}}{nQ^3(1-\mathrm{E}[\delta])} + \dfrac{2\xi D}{nQ^3(1-\mathrm{E}[\delta])} + \dfrac{H_B B^2}{Q^3(1-\mathrm{E}[\delta])} + \\[4mm] \dfrac{C_B B^2}{Q^3(1-\mathrm{E}[\delta])} + \dfrac{2T_F D}{Q^3(1-\mathrm{E}[\delta])} + \dfrac{2C_F D}{Q^3(1-\mathrm{E}[\delta])} \end{array}\right) > 0 \tag{A15}$$

$$D_2 = \left(\frac{\partial^2 \mathrm{E}TC(n,Q,B,\xi)}{\partial Q^2} * \frac{\partial^2 \mathrm{E}TC(n,Q,B,\xi)}{\partial B^2} \right) - \left(\frac{\partial \mathrm{E}TC(n,Q,B,\xi)}{\partial Q \partial B} \right)^2 \tag{A16}$$

$$= \left(\left(\left[\begin{array}{l} \dfrac{2O_B D}{nQ^3(1-\mathrm{E}[\delta])} + \dfrac{2S_0 D e^{-k\xi}}{nQ^3(1-\mathrm{E}[\delta])} + \dfrac{2\xi D}{nQ^3(1-\mathrm{E}[\delta])} + \dfrac{H_B B^2}{Q^3(1-\mathrm{E}[\delta])} + \\[3mm] \dfrac{C_B B^2}{Q^3(1-\mathrm{E}[\delta])} + \dfrac{2T_F D}{Q^3(1-\mathrm{E}[\delta])} + \dfrac{2C_F D}{Q^3(1-\mathrm{E}[\delta])} \end{array} \right] * \left[\dfrac{H_B + C_B}{Q(1-\mathrm{E}[\delta])} \right] \right) - \left(\left[\dfrac{-B(H_B + C_B)}{Q^2(1-\mathrm{E}[\delta])} \right]^2 \right) \right) > 0$$

$$D_3 = \left(A_1 - A_2 - A_3 \right) > 0 \tag{A17}$$

where,

$$A_1 = \left(\left[\begin{array}{l} \dfrac{2O_B D}{nQ^3(1-E[\delta])} + \dfrac{2S_0 D e^{-k\xi}}{nQ^3(1-E[\delta])} + \dfrac{2\xi D}{nQ^3(1-E[\delta])} + \dfrac{H_B B^2}{Q^3(1-E[\delta])} + \dfrac{C_B B^2}{Q^3(1-E[\delta])} + \\[3mm] \dfrac{2T_F D}{Q^3(1-E[\delta])} + \dfrac{2C_F D}{Q^3(1-E[\delta])} \end{array} \right] * \left[\dfrac{H_B + C_B}{Q(1-E[\delta])} \right]^2 * \left[\dfrac{D S_0 k^2 e^{-k\delta}}{nQ(1-E[\delta])} \right]^2 \right) \tag{A17.1}$$

$$A_2 = \left(\dfrac{-B(H_B + C_B)}{Q^2(1-E[\delta])} * \left[\dfrac{D S_0 k^2 e^{-k\delta}}{nQ(1-E[\delta])} \right]^2 * \dfrac{-B(H_B + C_B)}{Q^2(1-E[\delta])} \right) \tag{A17.2}$$

$$A_3 = \left(\left[\dfrac{D}{nQ^2(1-E[\delta])} \left[S_0 k e^{-k\xi} - 1 \right] \right] * \left[\dfrac{H_B + C_B}{Q(1-E[\delta])} \right]^2 * \left(\dfrac{D}{nQ^2(1-E[\delta])} \left[S_0 k e^{-k\xi} - 1 \right] \right) \right) \tag{A17.3}$$