A production inventory model for single vendor single buyer integrated demand with multiple production setups and rework

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C H R O N I C L E	A B S T R A C T
	In this paper, an imperfect production inventory system with multi-production run for single vendor and single buyer is studied where the defective items are reworked after several production processes and there is no break down during the rework process. The model is considered for deteriorating items under two delivery policies in order to determine the delivery policy leading to the minimum holding cost of system. The objective of this model is to determine the optimal production setup, number of deliveries and lot size by minimizing the total relevant cost of the proposed vendor-buyer integrated system. To minimize the manufacturer's and buyer's holding cost, the supply of finished products are made in small quantities. The total relevant cost of the vendor and the buyer is derived and a numerical example is presented to demonstrate the utility of the model.

1. Introduction

In the global competition market, it is not easy to produce high quality products. But serving high quality products and providing good service can always attract customers and make them to come back. Many researchers have discussed on Economic Production Quantity (EPQ) model for multi-production setups. But very few of them have discussed an inventory model for single vendor and single buyer integrated decreasing demand with multiple production setups. Rework process is also one important issue in reverse logistics where used products are reworked to reduce waste and environmental pollution. Rework is common in Semiconductor, Pharmaceutical, Chemical, Food industries, Textile industries, Paper industries, Glass industries, Metal processing industries and Plastic industries (Buscher & Lindner, 2007; Chiu et al., 2007; Barketau et al., 2008). Cárdenas-Barrón (2008) discussed on rework with single stage production system. Singh et al. (2014) discussed on rework with multi-production setup. Recently, rework process have attracted considerable attention because of the reduction of the natural resources and the rise of the cost of raw material. Rework process play an important role in eliminating waste and effectively controlling the cost of manufacturing in a production system. Therefore, determining optimal lot size in a system that allows

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rework is a useful objective to minimize the total inventory cost. Rework process reduces energy use and saves more natural resources for the future generations. Therefore, the companies are contributing to sustainable development.

In this paper, we consider a multi-production setups and one rework setup. By this EPQ model, it is determined that optimal production setup and optimal number of equal interval so that the total relevant cost is minimized. Since production process in each production setup is imperfect because of human mistakes, (Drury, 1978; Drury & Prabhu, 1994), non-perfect technology or many other factors. In order to provide good service to customers, after each production setup, inspection is carried out to screen out the defective items and the deteriorating items. The defective items found are stocked separately in an inventory until reach the optimal production setup. The rework process starts immediately after determined production setups end. The serviceable items, produced in each production setup, is stored separately in an inventory and sold to customers immediately. Both serviceable and defective items are considered as deteriorating items because their values go down with time. After determined production setup ends, the defective items are sent to rework. When the waiting time of the defective items exceeds the deterioration time limit, they cannot be recovered and must be disposed. The rework process manufactures all defective items as perfect quality items. We assume that all defective items after rework are considered as new. These reworked items are sold to customers to satisfy the demand immediately. There are m production setups and n shipment intervals and lot sizes (q). At the time of m production process, the $n-$ equal shipment lot sizes are exported to buyer at equal shipment interval. An economic order quantity (EOQ) model is considered at buyer's side. In each cycle, buyer receives equal lot size (q) for every equal interval. Here we consider two level integrated demands one is demand at buyer's side which is linearly decreasing demand with time and another one is demand at manufacturer's side which is exponentially decreasing demand with time. The production process is considered as exponentially increasing. We consider that the items stored in manufacturer and buyer's inventory are deteriorating.

The remainder of this paper is organized as follows. In section 2, we give a literature review. In section 3, assumptions and notations are given. The mathematical formulation for this model is given in section 4. Numerical example and sensitivity analysis are given in section 5, and conclusions are drawn in section 6.

2. Literature Review

Economic Production Quantity (EPQ) model is one of the prominent research topics in production, inventory control and management. By using EPQ model, optimal quantity of items and optimal production time can be obtained. Classical EPQ model was developed under various assumptions. Since then, researchers have extended the model by relaxing one or more of its assumptions. It was assumed that the items produced are of perfect quality items in the classical EPQ model. However, imperfect quality items may be produced in reality. Salameh and Jaber (2000) presented an EPQ model for imperfect items wherein the imperfect items are screened out and sold at lower price. Thereafter, the paper was re-examined by Papachristos and Konstantaras (2006) wherein the imperfect items are withdrawn at the end of a production cycle. Chung et al. (2009) proposed a two-warehouse inventory model with imperfect items. Kumar et al. (2011) presented Economic Production lot size (EPLS) model with stochastic demand and partial backlogging shortage under imperfect quality items in which stochastic imperfect production was assumed. Yassine et al. (2012) considered disaggregating the shipments of imperfect quality items in a single production run and aggregating the shipments of imperfect items over multiple production runs. Rezaei and Salimi (2012) discussed an economic production quantity and purchasing price for items with imperfect quality when inspection shifts from buyer to supplier. Mukhopadhyay and Goswami (2015) investigated an economic production quantity model for three types of imperfect items with rework. Singh et al. (2015) presented a mathematical production inventory model for deteriorating items with time dependent demand rate under the effect

of inflation and shortages. Recently, the economic production quantity (EPQ) model problem for single item under the assumptions of imperfect production and perfect rework has been carried out by Khanna et al. (2017).

Since deterioration affects the inventory production system, the effect of deterioration should be taken into account. The deterioration has already been studied in various papers (for example: Wagner and Whitin (1958); Covert and Philip (1973); Jolai et al. (2006)). Mishra et al. (2013) considered an inventory model for deteriorating items with time-dependent demand and time varying holding cost under partial backlogging shortage. Li et al. (2015) presented a modified EPQ model with deteriorating production system and deteriorating product where rework process was considered at the end of production setup. Pal et al. (2015) proposed a production inventory model for deteriorating item with ramp type demand allowing inflation and shortages under fuzziness in which multi-production setup was considered without rework. Jaggi et al. (2015) introduced the effect of deterioration on two-warehouse inventory model with imperfect quality items. Taleizadeh et al. (2015a) developed the economic production and inventory model in a three-layer supply chain including one distributor, one manufacturer and one retailer for a single-product and general demand functions under three scenarios. Taleizadeh et al. (2015b) developed a Vendor Managed Inventory (VMI) model for a two-level supply chain comprised of one vendor and several non-competing retailers in which both the raw material and the finished product have different deterioration rates. An EOQ model for retailer wherein deterioration rate is time dependent having their expiration dates, demand rate is linearly time dependent and non-increasing function of time under trade credits is studied by Kaur et al. (2016). Dhandapani and Uthayakumar (2016) presented an EOQ model for fresh fruits with preservation technology investment for variable deterioration rate. Jawla and Singh (2016) introduced the preservation technology investment to reduce the deterioration rate of products for their EPQ model with imperfect items and rework. Singh et al. (2016) studied that the retailers invest in preservation technology to reduce the rate of product deterioration. A two-warehouse inventory model for deteriorating items with price dependent demand under partial backlogging was discussed by Rastogi et al. (2017).

Revamp process is likewise one critical issue backward coordination where utilized items are modified to reduce total inventory cost, waste and environmental pollution. The earliest research that focused on rework and remanufacturing process was done by Schrady (1967). Since then, researchers on rework have attracted many researchers. Koh et al. (2002) discussed on production inventory models where provider can fill the request in two options: either arranges new items externally or recovers defective items are reworked in the same cycle; and in the second policy, rework is completed after N cycles. Inderfuth et al. (2005) considered an EPQ model with rework and deteriorating recoverable products. Yoo et al. (2009) developed an EPQ model with imperfect production, imperfect inspection and rework. Widyadana and Wee (2012) proposed an EPQ model for deteriorating items with rework which was performed after m production setups. Chiu et al. (2012) improved shipment policy for a vendor-buyer system with rework and an improving delivery plan. Tai (2013) proposed an EPQ model for deteriorating/imperfect product with rework which was performed as soon as production setup ends. Sarkar et al. (2014) assumed rework for single stage production system. Hsu and Hsu (2014) considered an EPQ model under an imperfect production process with shortages backordered. Chiu et al. (2014) developed an EPQ model for multi-items with scrap, rework and multi-delivery using common cycle policy. Ullah and Kang (2014) presented a more realistic approach for the modelling of optimum lot size and total cost with a focus on the work in process inventory and the effect of rework, rejects and inspection on work in process inventory.

Goyal (1977) introduced the idea of integrating a single vendor with a single buyer as component of two entity supply line system. Banerjee (1986) presented an EPQ model where the vendor manufactures product at a finite rate and that follows a lot-for-lot policy. Later, many modifications are reported in the literature. For example, the two-level vendor and buyer problem and its related issues can be studied in Goyal (1995), Hill (1977, 1999), Valentini and Zavanella (2003), Zanoni and Grubbstrom (2004)

and Hill and Omar (2006). Benkherouf and Omar (2010) presented a method for finding the optimal replenishment run and production plan for a single-vendor single-buyer inventory model. Hsu and Hsu (2012) proposed an integrated single-vendor single-buyer production-inventory model for items with imperfect quality and inspection errors. Yang et al (2013) discussed a vendor-buyers integrated inventory model involving quality improvement investment in a supply chain. Taleizadeh et al. (2015c) presented a model which deals with the problem of the joint determination of selling price, replenishment lot size and the number of shipments for an economic production quantity (EPQ) model with rework of defective items where multi-shipment policy is used.

For a three stage supply chain, Banerjee and Kim (1995) extended an integrated just-in-time inventory model where the demand rate, production rate, and delivery time are constant and deterministic. Munson and Rosenblatt (2001) introduced a three-level supply chain with quantity discount model. Lee and Moon (2006) proposed a coordinated inventory model with compensation policy in a three-level supply chain. Jaber and Goyal (2008) also considered a similar problem consisting of a single raw material supplier, a single vendor, and a single buyer. Omar et al. (2013) developed an integrated just-in-time inventory model where the demand rate is linearly decreasing with time, production rate is finite, and delivery time is constant and deterministic. Tavakoli and Mirzaee (2014) proposed a coordination of a three-level supply chain under disruption using profit sharing and return policy contracts. Kundu and Chakrabarti (2015) presented the supply chain structure consists of a single manufacturer with multi-buyer where manufacturer orders a fixed quantity of raw materials from outside suppliers, process the materials and delivers the finished products in unequal shipments to each customer.

3. Assumptions and Notations

3.1. Assumptions

1. Shortages are not allowed and the rate of producing good quality items and rework must be greater than the demand rate.
2. No machine breakdown occurs in the production run time and rework period.
3. Rework rate is constant.
4. There is a replacement for a deteriorated item.
5. Deteriorating rate is constant.
6. Defective items are generated only during production period. Rework process results in only good quality items.
7. Multiple production setup inventory system is considered.
8. The demand rate at the buyer's end is $f(t)$ at any time t in $(0, T)$ and assumed to be linearly decreasing with time.
9. Production rate at manufacturer's end is assumed to be exponentially increasing and demand rate is exponentially decreasing with time.

3.2. Notations

$D(t)$	Demand rate (unit/year)
$P(t)$	Production rate (unit/year)
P_r	Rework process rate (unit/year)
θ_1	Deterioration rate at manufacturer side (unit/year)
θ_2	Deterioration rate at buyer side (unit/year) ($\theta_2 > \theta_1$)
α	Percentage of good quality items
m	Number of production setup in one cycle
D_i	Total deteriorating units (unit)
K_s	Production setup cost ($/setup)

K_r	Rework setup cost (\$/setup)
h_s	Serviceable items holding cost (\$/unit/year)
h_r	Recoverable items holding cost (\$/unit/year)
D_c	Deteriorating cost (\$/unit)
I_1	Serviceable inventory level in a production period
I_2	Serviceable inventory level in a non -production period
I_{r1}	Recoverable inventory level in a production period
I_{r2}	Recoverable inventory level in a non - production period
I_{r1}	Recoverable inventory level in a rework production period
I_{t1}	Total serviceable inventory in a production period
I_{t2}	Total serviceable inventory in a non - production period
I_{t3}	Total serviceable inventory in a rework production period
I_{t4}	Total serviceable inventory in a rework non - production period
I_{v1}	Total recoverable inventory in m production period
I_{v2}	Total recoverable inventory in non - production period
I_{v3}	Total recoverable inventory in a rework production period
TTI_1	Total recoverable inventory in a production period
TTI_2	Total recoverable inventory in a non-production period
TRI	Total recoverable inventory
IM_r	Maximum inventory level of recoverable items in production setups
IE_r	Maximum inventory level of recoverable items when rework process start
T_1	Production period
T_2	Non production period
T_3	Rework process period
T_4	Non rework process period
TCT	Total cost per unit time
n	Number of shipments
H_b	Inventory holding cost per unit time at the buyer's side
S_c	Shipment cost at buyer's side

4. Mathematical formulation of the model

We frame the model as 3-production and 4-shipment levels which are depicted in Fig.1 and Fig. 2. Here m production batch is delivered in n equal interval shipments and equal shipment lot size(q) to the buyer as shown in the figure given below.
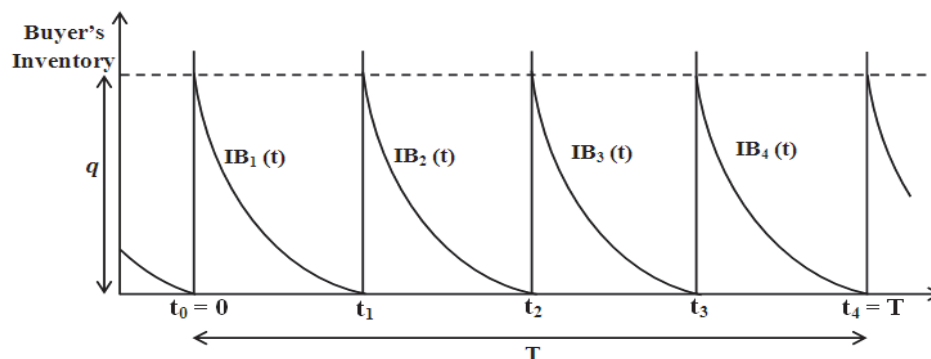

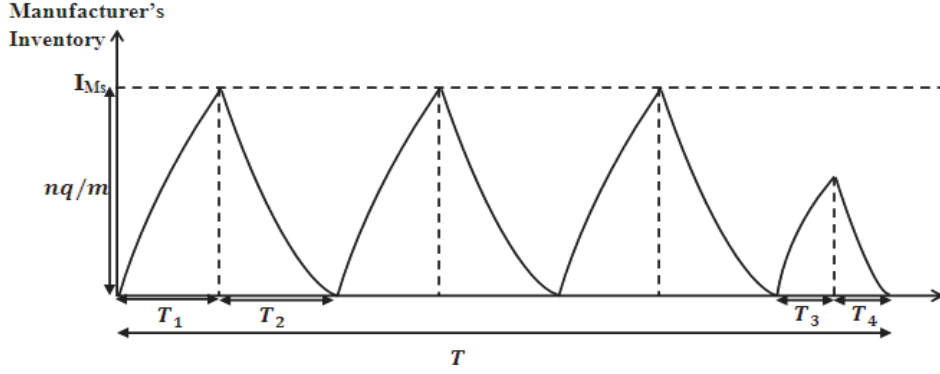**Fig. 1.** Inventory level of equal lots shipments size q with n = 4.

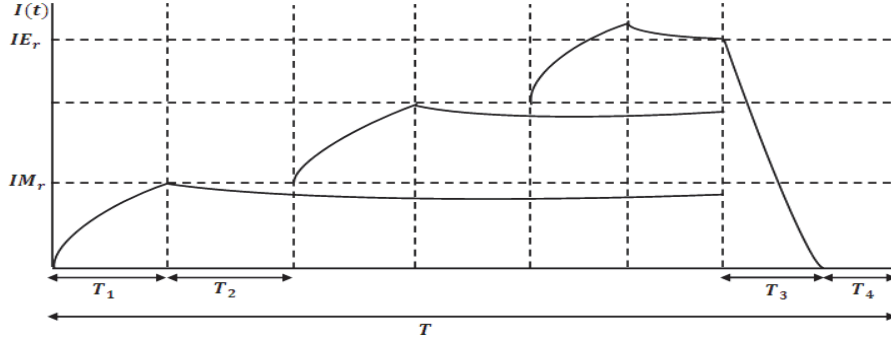**Fig. 2.** Serviceable inventory level of 3 production setups and 1 rework setup.

Fig. 3. Recoverable inventory level of 3 production setups and 1 rework setup.

4.1. Buyer's total cost

The amount of stock holding at the buyer's side during the time period T is calculated by assuming that $t_0 = 0$. From Fig. 1, the buyer stock level, IB_i in any period $i = 0, 1, 2, ..., n\text{-}1$ is modeled as

$$\frac{dI_B(t)}{dt} + \theta_2 I_B(t) = -D(t) \qquad\qquad t_i \leq t \leq t_{i+1} \qquad\qquad (1)$$

where $D(t) = a - bt$ where a is the intercept and b is the slope of the linearly decreasing demand function.

$$I_B(t) = Ae^{-\theta_2 t} + \frac{bt}{\theta_2} - \frac{a}{\theta_2} - \frac{b}{\theta_2^2} \qquad\qquad (2)$$

Since $I_B(t_{i+1}) = 0$, we have

$$I_B(t) = \left[\frac{a}{\theta_2} + \frac{b}{\theta_2^2} - \frac{bt_{i+1}}{\theta_2}\right] e^{\theta_2(t_{i+1} - t_i)} + \frac{bt}{\theta_2} - \frac{a}{\theta_2} - \frac{b}{\theta_2^2} \qquad\qquad (3)$$

Total inventory at buyer's side between t_i to t_{i+1} is

$$TI_B(t) = \int_{t_i}^{t_{i+1}} \left\{\left[\frac{a}{\theta_2} + \frac{b}{\theta_2^2} - \frac{bt_{i+1}}{\theta_2}\right] e^{\theta_2(t_{i+1} - t_i)} + \frac{bt}{\theta_2} - \frac{a}{\theta_2} - \frac{b}{\theta_2^2}\right\} dt$$

Integrating and using Taylor series expansion and neglecting higher orders, we get

$$TI_B(t) = \left(\frac{a}{2} - \frac{bt_{i+1}}{2}\right)(t_i - t_{i+1})^2 - \frac{b}{2}\left(t_i^2 - \frac{t_{i+1}^2}{\theta_2}\right) \qquad\qquad (4)$$

where (t_i, t_{i+1}) is the cycle for $(i + 1)$th shipment. In this paper, we consider two possible policies. In the first policy we assume equal shipments interval where $t_i = i\left(\frac{T}{n}\right)$. For the second policy we assume equal shipments lot size where for n-shipments we have $q = \frac{1}{n}\left(\int_0^T D(t)dt\right)$.

For a linearly decreasing demand rate, $D(t) = a - bt$, we have $t_i = \left(\frac{1}{b}\right)\left\{a - \sqrt{\frac{1}{n^2}[(an)^2 - 2abinT + (ib)^2 T^2]}\right\}$

Total cost per unit time at the buyer's side, TCB, is

$$TCB = \frac{1}{T}[nS_c + H_bTI_B(t)]$$
(5)

For the first policy, $t_i = i\left(\frac{T}{n}\right)$, TCB becomes

$$TCB_t = \frac{1}{T}\left[nS_c + \frac{H_b}{24\theta_2 n^2}\{[12an\theta_2 - 2b(\theta_2 - 1)(2n^3 - 3n^2 + n) - 12bn^2]T^2 - b(n+1)T^3\}\right]$$
(6)

For the second policy, TCB becomes

$$TCB_q = \frac{1}{T}\left[nS_c + H_b\left(\frac{a}{2} - \frac{bt_{i+1}}{2}\right)(t_i - t_{i+1})^2 - \frac{b}{2}\left(t_i^2 - \frac{t_{i+1}^2}{\theta_2}\right)\right]$$
(7)

where $t_i = \left(\frac{1}{b}\right)\left\{a - \sqrt{\frac{1}{n^2}[(an)^2 - 2abinT + (ib)^2 T^2]}\right\}$

Hence we conclude that the TCB is a function of single variable T for both policies.

4. 2. Manufacturer's total cost

Fig. 2 shows the manufacturing inventory level during the period T. Production is performed during T_1 time period. When production process, αP good quality items is produced and $(1 - \alpha)P$ defective items are screened out. T_2 is the non-production period and T_3 is the rework process period. The rework process starts immediately after m the production setups end. The manufacturer's inventory level in a production period is formulated as:

$$\frac{dI_1(t_1)}{dt_1} + \theta_1 I_1(t_1) = \alpha P(t_1) - D(t_1) \qquad\qquad 0 \le t_1 \le T_1$$
(8)

where $P(t) = \lambda e^{kt}$ and $D(t) = \lambda e^{-kt}$, $\lambda, k > 0$.

Since $I_1(0) = 0$, the inventory level in a production period is:

$$I_1(t_1) = \frac{\lambda}{k^2 - \theta_1^2}\{(k\alpha - \theta_1\alpha)(e^{kt_1} - e^{-\theta_1 t_1}) + (k + \theta_1)(e^{-kt_1} - e^{-\theta_1 t_1})\}$$
(9)

The total inventory in a production up time can be modeled as:

$$I_{t1}(t_1) = \int_0^{T_1} \frac{\lambda}{k^2 - \theta_1^2}\{(k\alpha - \theta_1\alpha)(e^{kt_1} - e^{-\theta_1 t_1}) + (k + \theta_1)(e^{-kt_1} - e^{-\theta_1 t_1})\}\, dt$$

$$I_{t1} = \frac{\lambda}{k\theta_1(k^2 - \theta_1^2)}\{(k\alpha - \theta_1\alpha)(\theta_1 e^{kT} + ke^{-\theta_1 T_1}) - (k + \theta_1)(\theta_1 e^{-kT_1} - ke^{-\theta_1 T_1}) - (\alpha + 1)(k^2 - \theta_1^2)\}$$

For small value of $\theta_1 T_1$ and using Taylor series expansion, we have

$$I_{t1} = \frac{(\alpha - 1)\lambda T_1^2}{2}$$

Since $I_1(T_1) = I_{Ms}$, equation (9) becomes

$$I_{Ms} = \frac{\lambda}{k^2 - \theta_1^2}\left\{(\alpha - 1)(k^2 - \theta_1{}^2)T_1 + (k^3 + (\alpha + \theta_1)k^2 - (\alpha + \alpha\theta_1 + \theta_1)\theta_1 k + (\alpha - 1)\theta_1^3)\frac{T_1^2}{2}\right\}$$

The inventory level in a non-production period is represented by

$$\frac{dI_2(t_2)}{dt_2} + \theta_1 I_2(t_2) = -D(t_1) \qquad\qquad 0 \le t_2 \le T_2$$
(10)

Since $I_2(T_2) = 0$ and we get the total inventory in a non-production period can be represented as

$$I_{t2} = \frac{-\lambda}{k\theta_1(k - \theta_1)}\{e^{-kT_2}(\theta_1 - k + ke^{\theta_1 T_2}) - \theta_1\}$$

$$I_{t2} = \frac{\lambda}{2}T_2^2. \tag{11}$$

Since $I_1 = I_2$ when $t_1 = T_1$ and $t_2 = 0$, we get

$$\frac{\lambda}{k^2 - \theta_1^2}\{(k\alpha - \theta_1\alpha)(e^{kT_1} - e^{-\theta_1 T_1}) + (k + \theta_1)(e^{-kT_1} - e^{-\theta_1 T_1})\} = \frac{a}{k-\theta_1}\{1 - e^{(\theta_1-k)T_2}\}$$

Using Taylor series expansion and $\frac{(\theta_1-k)^2 T_2^2}{2}$ is very small, we approximate T_2 as:

$$T_2 \cong (1 - \alpha)T_1 - \frac{[k(\alpha+1)+ \theta_1(1-\alpha)]}{2} T_1^2. \tag{12}$$

The inventory level in a rework production period is represented by

$$\frac{dI_3(t_3)}{dt_3} + \theta_1 I_3(t_3) = P_r(t_3) - D(t_3) \qquad\qquad 0 \le t_3 \le T_3 \tag{13}$$

$$I_3(t_3) = \frac{\lambda}{\theta_1 - k}[e^{-\theta_1 t_3} - e^{-kt_3}] + \frac{P_r}{\theta_1}[1 - e^{-\theta_1 t_3}] \qquad 0 \le t_3 \le T_3 \tag{14}$$

The inventory level in a rework non-production period is represented by

$$\frac{dI_4(t_4)}{dt_4} + \theta_1 I_4(t_4) = -D(t_4) \qquad\qquad 0 \le t_4 \le T_4 \tag{15}$$

$$I_4(t_4) = \frac{\lambda}{k-\theta_1}[e^{-kt_4} - e^{(\theta_1-k)T_4}\, e^{-\theta_1 t_4}] \qquad 0 \le t_4 \le T_4 \tag{16}$$

Using similar steps as above, the total serviceable inventory in a rework production period and the total serviceable inventory in a rework non-production period time are derived as follows:

$$I_{t3} = \frac{(P_r - \lambda)\, T_3^2}{2} - \frac{P_r}{\theta_1}T_3 \tag{17}$$

$$I_{t4} = \frac{\lambda\, T_4^2}{2} \tag{18}$$

Since $I_3 = I_4$ when $t_3 = T_3$ and $t_4 = 0$, we get

$$T_4 \cong \frac{2(P_r-\lambda)\, T_3 + [\lambda(k+ \theta_1)- \theta_1 P_r]T_3^2}{2\lambda} \tag{19}$$

The inventory level of recoverable items is shown in Fig. 3. The inventory level of recoverable items in a production period can be modeled as

$$\frac{dI_{r1}(t_{r1})}{dt_{r1}} + \theta_1 I_{r1}(t_{r1}) = (1 - \alpha)P \qquad\qquad 0 \le t_{r1} \le T_1 \tag{20}$$

Since $I_{r1}(0) = 0$, the inventory level of recoverable items in a production period is

$$I_{r1}(t_{r1}) = \frac{\lambda(1- \alpha)}{(k+ \theta_1)}[e^{kt_{r1}} - e^{-\theta_1 t_{r1}}] \qquad\qquad 0 \le t_{r1} \le T_1 \tag{21}$$

Using Taylor series approximation, the total recoverable inventory in a production up time in one setup is

$$TTI_1 = \frac{\lambda(1- \alpha)T_1^2}{2} \tag{22}$$

Since there are m production setups in one cycle, the total inventory for recoverable items in one cycle is

$$I_{v1} = \frac{m\lambda(1- \alpha)\, T_1^2}{2} \tag{23}$$

The initial recoverable inventory level in each production setup is equal to I_{Mr} and it can be represented as:

$$I_{Mr} = \frac{\lambda(1- \alpha)}{(k+ \theta_1)}[e^{kT_1} - e^{-\theta_1 T_1}]$$

Using Taylor series approximation, we get

$$I_{Mr} = \lambda(1 - \alpha)\left[T_1 + (k - \theta_1)\frac{T_1^2}{2}\right] \tag{24}$$

The inventory level of recoverable items in a non-production period is:

$$\frac{dI_{r2}(t_{r2})}{dt_{r2}} + \theta_1 I_{r2}(t_{r2}) = 0 \qquad\qquad 0 \le t_{r2} \le (m - 1)T_1 + mT_2 \tag{25}$$

Since the inventory level $I_{r2}(0) = I_{Mr}$, the inventory level of recoverable items in a non-production time for each production setup can be modeled as:

$$I_{r2}(t_{r2}) = I_{Mr} e^{-\theta_1 t_{r2}} \qquad 0 \leq t_{r2} \leq (m-1)T_1 + mT_2 \qquad (26)$$

Using Taylor series expansion, the total inventory of recoverable items in a production down time in one setup is:

$$TTI_2 = \sum_{k=1}^{m} I_{Mr} \left\{ [(k-1)T_1 + kT_2] - \frac{\theta_1 [(k-1)T_1 + kT_2]^2}{2} \right\} \qquad (27)$$

The total inventory of recoverable items in m production periods can be modeled as follows:

$$I_{v2} = \sum_{k=1}^{m} I_{Mr} \left\{ [(k-1)T_1 + kT_2] - \frac{\theta_1 [(k-1)T_1 + kT_2]^2}{2} \right\} \qquad (28)$$

Inventory level of recoverable item in the end of production cycle is equal to maximum inventory level of recoverable items in a production setup reduced by deteriorating rate during production up time and down time. The inventory level can be formulated as follows:

$$I_{Er} = \sum_{k=1}^{m} I_{Mr} e^{-\theta_1 [(k-1)T_1 + kT_2]} \qquad (29)$$

Using Taylor series expansion and then substituting I_{Mr} in Eq. (29) we get

$$I_{Er} = \sum_{k=1}^{m} \left\{ \lambda(1-\alpha)(T_1 + (k-\theta_1)\frac{T_1^2}{2} \right\} \left(\begin{array}{c} 1 - \theta_1[(k-1)T_1 + kT_2] \\ + \frac{\{\theta_1[(k-1)T_1 + kT_2]\}^2}{2} \end{array} \right) \qquad (30)$$

The inventory level of recoverable items in a rework period can be formulated as:

$$\frac{dI_{r3}(t_{r3})}{dt_{r3}} + \theta_1 I_{r3}(t_{r3}) = -P_r \qquad 0 \leq t_{r3} \leq T_3 \qquad (31)$$

$$I_{r3}(t_{r3}) = \frac{P_r}{\theta_1} \left[e^{\theta_1(T_3 - T_{r3})} - 1 \right] \qquad 0 \leq t_{r3} \leq T_3 \qquad (32)$$

The total inventory of recoverable items in a rework period is:

$$I_{r3}(t_{r3}) = \int_{t_{r3}=0}^{T_3} \frac{P_r}{\theta_1} \left[e^{\theta_1(T_3 - T_{r3})} - 1 \right] dt_{r3}$$

Using Taylor series expansion, we get

$$I_{v3} = \frac{P_r T_3^2}{2} \qquad (33)$$

When $t_{r3} = 0$, $I_{r3}(t_{r3}) = I_{Er}$.

$$I_{Er} = \frac{P_r}{\theta_1} \left[e^{\theta_1 T_3} - 1 \right]$$

Since $\theta_1 T_3 \ll 1$ and using Taylor series expansion we have:

$$T_3 = \frac{I_{Er}}{P_r} \qquad (34)$$

Substitute I_{Er} in equation (34) we have

$$T_3 = \frac{1}{P_r} \sum_{k=1}^{m} \left\{ \lambda(1-\alpha)(T_1 + (k-\theta_1)\frac{T_1^2}{2} \right\} \left(\begin{array}{c} 1 - \theta_1[(k-1)T_1 + kT_2] \\ + \frac{\{\theta_1[(k-1)T_1 + kT_2]\}^2}{2} \end{array} \right) \qquad (35)$$

The total recoverable inventory is

$$TRI = I_{v1} + I_{v2} + I_{v3}$$

$$TRI = \frac{m\lambda(1-\alpha)T_1^2}{2} + \sum_{k=1}^{m} I_{Mr} \left\{ [(k-1)T_1 + kT_2] - \frac{\theta_1 [(k-1)T_1 + kT_2]^2}{2} \right\} + \frac{P_r T_3^2}{2} \qquad (36)$$

The number of deteriorating item is equal to the number of items produced minus the number of total demands. The total deteriorating units can be modeled as:

$$D_i = (m\alpha P T_1 f(t) + P_r T_3) - D(t)[m(T_1 + T_2) + T_3 + T_4] \qquad (37)$$

The total inventory cost consists of production setup cost, rework setup cost, serviceable inventory cost, recoverable inventory cost and deteriorating cost. The total inventory cost per unit time can be modeled as follows:

$$TCM(m, T_1) = \frac{mk_s + k_r + h_s\{[m(I_{t1}+I_{t2})+I_{t3}+I_{t4}]-TI_B(t)\}+h_r(TRI)+D_cD_i}{[m(T_1+T_2)+T_3+T_4]} \tag{38}$$

It follows from Eq. (5) and Eq. (38), the total relevant cost for the system per unit time, TRC, is

$$TRC = TCM + TCB, \quad TRC = \frac{\frac{1}{T}nS_c + mk_s + k_r + h_s[m(I_{t1}+I_{t2})+I_{t3}+I_{t4}]+h_r(TRI)+D_cD_i+\left(\frac{H_b}{T}-h_s\right)TI_B(t)}{m(T_1+T_2)+T_3+T_4} \tag{39}$$

For our first policy, $t_i = i\frac{T}{n}$ the TRC is, denoted by TRC_t,

$$TRC_t = \frac{1}{T}nS_c + \left[\frac{H_b}{T} - \frac{h_s}{m(T_1+T_2)+T_3+T_4}\right]\left[\frac{1}{24\theta n^2}\{[12an\theta - 2b(\theta-1)(2n^3-3n^2+n)-12bn^2]T^2 - \right.$$
$$\left. b(n+1)T^3\}\right] + \frac{mk_s+k_r+h_s\left[m(\alpha-1)\lambda T_1^2+\frac{m\lambda}{2}T_2^2-\frac{P_r}{\theta}T_3+(P_r-\lambda)\frac{T_3^2}{2}+\frac{\lambda}{2}T_4^2\right]+h_r(TRI)+D_cD_i}{m(T_1+T_2)+T_3+T_4} \tag{40}$$

For our second policy,

$$TRC_t = \frac{1}{T}nS_c + \left[\frac{H_b}{T} - \frac{h_s}{m(T_1+T_2)+T_3+T_4}\right]\left[\frac{1}{24\theta n^2}\{[12an\theta - 2b(\theta-1)(2n^3-3n^2+n)- \right.$$
$$12bn^2]T^2 - b(n+1)T^3\}\right] +$$
$$\frac{mk_s+k_r+h_s\left[m(\alpha-1)\lambda T_1^2+\frac{m\lambda}{2}T_2^2-\frac{P_r}{\theta}T_3+(P_r-\lambda)\frac{T_3^2}{2}+\frac{\lambda}{2}T_4^2\right]+h_r(TRI)+D_cD_i}{m(T_1+T_2)+T_3+T_4} \tag{41}$$

$$TRC_q = \frac{1}{T}nS_c + \left[\frac{H_b}{T} - \frac{h_s}{m(T_1+T_2)+T_3+T_4}\right]TI_B(t)$$
$$+ \frac{mk_s+k_r+h_s\left[m(\alpha-1)\lambda T_1^2+\frac{m\lambda}{2}T_2^2-\frac{P_r}{\theta}T_3+(P_r-\lambda)\frac{T_3^2}{2}+\frac{\lambda}{2}T_4^2\right]+h_r(TRI)+D_cD_i}{m(T_1+T_2)+T_3+T_4} \tag{42}$$

where $t_i = \frac{1}{b}\left[a - \sqrt{\frac{1}{n^2}(a^2n^2 - 2abinT + (ib)^2T^2)}\right]$.

The optimal solution must satisfy the following conditions:

$$\frac{\partial[TRC(m, T_1)]}{\partial T_1} = 0 \tag{43}$$

and $\dfrac{\partial[TRC(n, T_1)]}{\partial T_1} = 0$ \hfill (44)

Also the optimal solution of m and n, denoted by m^* and n^*, must satisfy the following condition:
$TCR(m^* - 1, T_1) \geq TCR(m^*, T_1) \leq TCR(m^* + 1, T_1)$ and
$TCR(n^* - 1, T_1) \geq TCR(n^*, T_1) \leq TCR(n^* + 1, T_1)$

Since the cost function Eq. (41) and Eq. (42) are nonlinear equations and the second derivatives of Eq. (41) and Eq. (42) with respect to T_1 is extremely complicated, closed form solution of Eq. (41) and Eq. (42) cannot be derived. This means that the optimality solution cannot be guaranteed. However, by means of empirical experiments, one can conclude that Eq. (41) and Eq. (42) are convex for a small value of T_1. The optimal T_1 , m and n values can be obtained using maple mathematical software.

5. Numerical example and sensitivity analysis

Figs. (4–6) give that the convexity behavior of the total relevant cost for both policies against m, n and T_1. Fig. 4 shows that the total cost per unit time is convex for small values of T_1. The best solution for each policy is the number of production (m^*) should be 4 and the number of shipments (n^*) should be 3. The optimal relevant total cost for the system per unit time for each policy is $TRC_t = 244.3815$ and $TRC_q = 247.6221$ when production time $(T_1^*) = 0.03306$. The lot size for the equal

shipment policy is 27 units with a total production size of 82 and the lot sizes for the equal shipment interval policy is 26 units with total production size of 78.

The sensitivity analysis has been performed by changing each of the parameters by -60%, -40%,   -20%, +20%, +40% and +60%. One parameter has been taken at a time and the remaining parameters have been kept unchanged. Table 1 shows that the the total relevant cost TRC_t, equal shipment interval, is sensitive to the parameters $k_s, k_r, p_r, a, b, \lambda, h_s, h_r, D_c, \theta_1, \theta_2, H_b$ and s_c. The total relevant cost (TRC_t) increases when the parameters $k_s, k_r, p_r, \lambda, h_s, h_r, \theta_1, \theta_2, H_b$ and s_c increase and TRC_t decreases when the parameters b, and D_c increase. Table 2 shows that the the total relevant cost TRC_q, equal shipment lot size, is sensitive to the parameters $k_s, k_r, p_r, a, b, \lambda, h_s, h_r, D_c, \theta_1, \theta_2, H_b$ and s_c. The total relevant cost TRC_q increases when the parameters $k_s, k_r, p_r, b, \lambda, h_s, h_r, \theta_1, H_b$ and s_c increase and decrease when the parameters D_c, and θ_2 increases. The total relevant costs TRC_t and TRC_q are insensitive with the parameter k and fluctuation sensitive with the parameter a. The optimal total relevant cost TRC_t for varying parameters is shown in Fig. 7. The figure shows that the optimal total relevant cost TRC_t is sensitive to changes in h_s, h_r, λ, s_c and moderately sensitive to changes in a, H_b and D_c and insensitive to changes in the other parameters. The optimal total relevant cost TRC_q for varying parameters is shown in Fig. 8. The figure shows that the optimal total relevant cost TRC_q is sensitive to changes in h_s, h_r, s_c and moderately sensitive to changes in a, θ_1 and D_c and insensitive to changes in the other parameters.
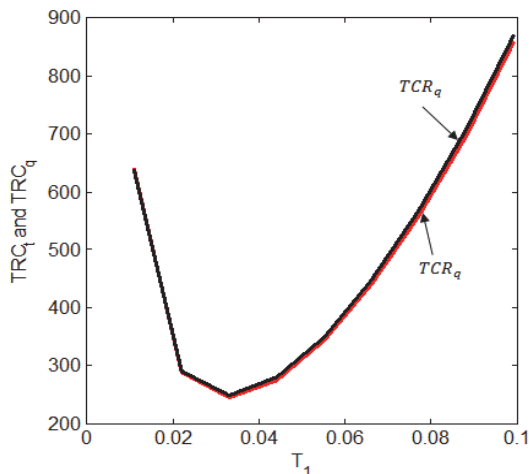

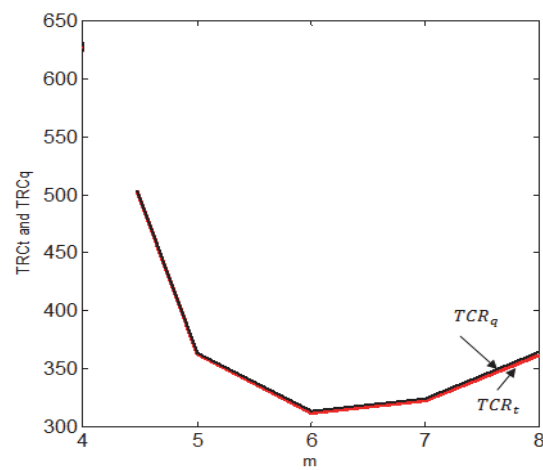**Fig. 4.** Total relevant cost for both policies when $m = 4$ and $n = 3$ against T_1

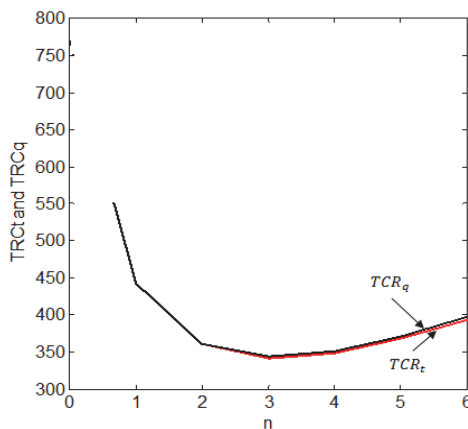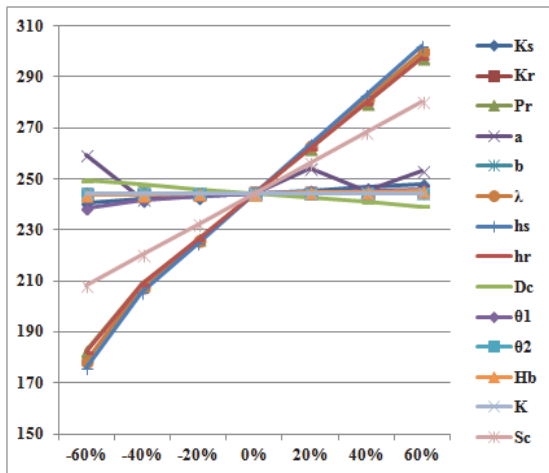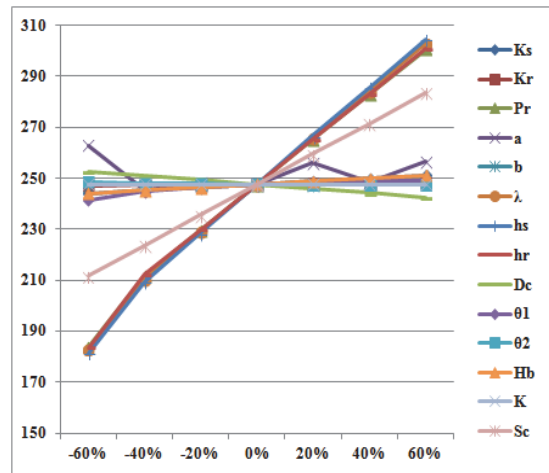**Fig. 5**. Total relevant cost for both policies when $n = 2$ against m

Fig. 6. Total relevant cost for both policies   when $m = 3$ against n

Table 1

Sensitivity analysis of TRC_t

Parameters	-60%	-40%	-20%	20%	40%	60%
	TRC_t	TRC_t	TRC_t	TRC_t	TRC_t	TRC_t
K_s	240.8402	242.0207	243.2011	245.5619	246.7423	247.9228
K_r	243.9388	244.0864	244.2340	244.5291	244.6766	244.8242
P_r	182.6555	208.7126	226.6158	262.0785	279.7362	297.3693
a	259.1780	242.0132	244.4957	254.0289	245.5206	253.0852
b	244.4156	244.4042	244.3929	244.3701	244.3588	244.3474
λ	178.3733	207.5012	225.9413	262.8217	281.2618	299.7020
h_s	175.9143	205.7751	225.0783	263.6847	282.9879	302.2911
h_r	182.2138	208.9491	226.6653	262.0977	279.8139	297.5301
D_c	249.4964	247.7914	246.0865	242.6766	240.9716	239.2666
θ_1	238.3595	241.7062	243.3787	245.0495	245.5261	245.8831
θ_2	244.3102	244.3509	244.3705	244.3883	244.3927	244.3956
H_b	243.6798	243.9137	244.1476	244.6154	244.8493	245.0832
K	244.3815	244.3815	244.3815	244.3815	244.3815	244.3815
S_c	208.3815	220.3815	232.3815	256.3815	268.3815	280.3815

Table 2

Sensitivity analysis of TRC_q

Parameters	-60%	-40%	-20%	20%	40%	60%
	TRC_q	TRC_q	TRC_q	TRC_q	TRC_q	TRC_q
K_s	244.0808	245.2612	246.4416	248.8025	249.9829	251.1633
K_r	247.1794	247.3270	247.4745	247.7696	247.9172	248.0647
P_r	183.9122	211.9456	229.8535	265.3209	282.9800	300.6141
a	262.8846	245.3968	248.1553	256.1710	248.4088	256.6511
b	247.2414	247.3683	247.4952	247.7489	247.8758	248.0027
λ	183.3507	210.7417	229.1819	266.0622	284.5024	302.9426
h_s	181.5286	209.5583	228.5902	266.6539	285.6858	304.7177
h_r	183.4942	212.1897	229.9059	265.3383	283.0544	300.7706
D_c	252.7369	251.0320	249.3270	245.9171	244.2122	242.5072
θ_1	241.6010	244.9474	246.6196	248.2897	248.7660	249.1227
θ_2	248.6153	248.0648	247.7886	247.5104	247.4301	247.3694
H_b	244.1621	245.3154	246.4687	248.7754	249.9287	251.0821
K	247.6221	247.6221	247.6221	247.6221	247.6221	247.6221
S_c	211.6221	223.6221	235.6221	259.6221	271.6221	283.6221

Fig. 7. TRC_t sensitivity

Fig. 8. TRC_q sensitivity

6. Conclusion

In this paper, we have developed a two-stage production-inventory system considering multiple production setups with deteriorating items and rework process in each cycle where the manufacturer must deliver the finished products in small quantities to minimize the buyer's holding cost. The model helps to management to determine optimal number of production setups (m^*), number of deliveries (n^*) and lot size (q^*) of each deliveries by minimizing the total relevant cost under two (equal shipment interval and equal shipment lot size) policies. We also study the reworking of imperfect quality items produced on the production-inventory model. The sensitivity analysis shows that the total relevant cost under the equal shipment interval policy is increasing when production setup cost, rework setup cost, rework process rate, serviceable items holding cost, recoverable items holding cost, inventory holding cost, shipment cost increase and decrease when deteriorating cost and demand rate decrease. The total relevant cost under the equal shipment lot size policy is increasing when production setup cost, rework setup cost, rework process rate, serviceable items holding cost, inventory holding cost, shipment cost increase and decrease when deteriorating cost increase. Our numerical result shows that the equal shipment interval policy is always superior to the equal shipment lot size policy. In future, this paper can be extended as three stage production-inventory systems in which a solitary maker acquires crude materials from a solitary provider, handle them to create completed items, and then deliver the products to a single buyer where deterioration rate and holding cost are time dependent.

Acknowledgement

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