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EOQ model for non- decreasing time dependent deterioration and Decaying demand under non-increasing time shortages

R. P. Tripathi[a*] and Manjit Kaur[b]

[a]*Department of Mathematics, Graphic Era University, Dehradun (UK), India*
[b]*Department of Mathematics, Banasthali University, Rajasthan India*

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A B S T R A C T

Deterioration of items is a phenomenon which cannot be neglected as it may provide absurd result. In high-tech business market, deterioration is not always constant but it is time dependent. This paper presents inventory model for time dependent deterioration and time dependent demand under shortages. The mathematical model is provided to optimize the cycle time by minimizing the total cost. Numerical results are discussed to validate the proposed model. The variation of the optimal solution for different parameters is discussed for some instances.

1. Introduction

In the classical economic order quantity (EOQ) models demand rate is considered as constant, but in actual practice it may not be always constant; it is in dynamic stage, inventory-dependent, dependents on selling price etc. Several factors are involved in the study of economic order quantity (EOQ) models. Out of several factors, we cannot ignore the presence of deterioration. Almost all items in the universe diminish in quality, due to dryness, changing technological situation, vaporization, spoilage etc. It means that items cannot be used for customer's point of view. Most of the inventory modelers have considered that deterioration is constant, but it is not always true. In this paper, deterioration is considered to be linearly time dependent. During the past few decades, several researchers have established different inventory system incorporating the phenomena of deterioration. Ghare and Schrader (1963) developed an economic order quantity (EOQ) model for deteriorating items by considering exponential decay. This model was generalized by Covert and Philip (1973) by considering Weibull distribution deterioration. Dave and Patel (1981) developed an optimal order quantity model for linearly increasing demand under deterioration. Hariga (1996) established an inventory model for

* Corresponding author Tel.: +09837239545
E-mail address: tripathi_rp0231@rediffmail.com (R. P. Tripathi)

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deteriorating items with time-varying demand. Teng et al. (1999) presented an EOQ models with fluctuating demand and shortages. Skouri et al. (2009) established inventory models with ramp type demand rate and Weibull deterioration rate. Mahata (2012) established an EOQ model for deteriorating item under retailer partial trade credit policy. Many researchers like Hariga and Benkherouf (1994), Sachan (1984), Raafat et al. (1991), Chung and Ting (1993), Tripathi (2014), Tripathi and Kumar (2014) have considered inventory models for deteriorating items.

It is well known that in real market, the demand rate of almost all items is always in fluctuating state. Some research papers have been published for time dependent demand. The demand increases with time for high-tech products during the growth stage. Teng et al. (2012) presented Economic Order Quantity (EOQ) model for linearly non-decreasing demand function of time. Donaldon (1997) was the first researcher who developed inventory replenishment policy by considering linearly time dependent demand. Silver and Meal (1969) established an EOQ model for the case of a varying demand. Khanra et al. (2011) proposed an EOQ model for a deteriorating item having time dependent demand rate when delay in payment is permissible. Sicilia et al. (2014) analyzed an inventory system in which demand rate changes with time. Ritchie (1994) established an EOQ model with non-decreasing demand. Mitra et al. (1984) developed a linear optimal procedure for adjusting the economic order quantity model for cases of increasing or decreasing linear trend demand pattern. Chakrabarti and Chaudhuri (1997) established EOQ model that constructed on linear dependent demand. A linear trend in demand dependent EOQ model was developed by Wen-Yang et al. (2002).

At present scenario, a shortage of stock situation may arise in most of business transactions due to uncertainty of customer's preferences. Stock-outs are costly for seller due to unavailability of the products. Large number of EOQ models have been established with the condition that shortages are allowed. In real life, shortage decreases with waiting time for high tech products and fashionable products with short life cycles. Thus, the partial backlogged shortages are more practical consideration for better marketing performance in today's business condition. Jaggi et al. (2016) developed an inventory model for a retailer dealing with deteriorating items under inflationary conditions over a fixed planning horizon under partially backlogged shortages. Abad (1996) established an inventory model for a time dependent deterioration and partial backlogging. Khanra et al. (2016) presented an inventory model for a retailer dealing with imperfect quality deteriorating items under permissible delay in payments and shortages. Chang and Dye (1999) established an EOQ model in which backlogging is inversely proportional to the waiting time. Dye et al. (2007) extended Abad (1996) model by considering the backorder cost and lost sale. Teng and Yang (2007) discussed the EOQ model to allow for time-varying purchase cost. Yang (2012) presented two warehouse partial backlogging inventory models with three-parameter Weibull distribution deterioration under inflation. Taleizadeh et al. (2013) discussed an inventory model for perishable product with special sale under shortages. Wu (2002) presented an inventory model with time- dependent demand for deteriorating item and shortages. Sicilia et al. (2014) developed a deterministic inventory system for an item with a constant deterioration rate under shortages. Many related articles can be found in Lee and Wu (2002), Dye (2004), Singh et al. (2009), Mishra and Singh (2010), Bhaula and Kumar (2014), Tripathi (2013) and their citations.

The demand pattern is known as uniform demand pattern. But demand of products is variable according to need and situation. In this paper demand is taken as linearly time dependent. The linearly time dependent demand is more realistic with respect to other types of demand pattern. Here D(t) = a- bt , or $\frac{d\{D(t)\}}{dt} = -b$. It means that demand rate decreases uniformly throughout the period. In this paper, we analyze an EOQ model by considering deterioration and demand both are linearly time dependent under shortages. The remaining part of the manuscript is designed as followed: Section 2 provides assumptions and notations. In the third section, the mathematical formulation that explains the inventory system is developed. Section 4 shows that the optimal cycle time is applicable which minimizes total cost. Section 5 presents numerical example and sensitivity analysis in order to illustrate the model and obtain managerial phenomena. Finally, in section 6 we provide the conclusion and future research directions.

2. Assumption and Notations

1. The following assumptions are made throughout the manuscript:
2. The demand rate is known and is an exponential time dependent.
3. The deterioration rate Θ is constant and, $0 < \Theta < 1$.
4. The inventory system is assumed for a finite time horizon.
5. Lead time is negligible.
6. Shortages are allowed and completely backlogged.

In addition, following notations are used:

A_o	: The fixed ordering cost per order,
C_d	: The cost of each deteriorated unit,
h	: The inventory holding cost per unit,
s	: The unit shortage cost,
$\theta(t) = \alpha + \beta t$	: Time – dependent deterioration where $0 < \alpha << 1$, $\beta > 0$,
D(t) = a -bt	: Time- dependent demand rate , a > 0 , 0 < b < 1,
T	: The length of cycle time,
t_1	: Time to finish positive inventory,
Q	: Order quantity,
I (t)	: Inventory at time t, $0 < t < t_1$,
$Z(T, t_1)$	: Total average cost of the system,
IHC	: Holding cost/cycle,
OC	: Fixed ordering cost/ order,
SC	: Shortage cost / cycle,
DC	: Deterioration cost / cycle,
T^*	: Optimal length of cycle time,
$t_1{}^*$	: Optimal time to finish positive invents,
Q^*	: Optimal order quantity,
$Z^*(T, t_1)$	: Optimal total average cost of the system.

3. Mathematical Formulations

The inventory level *I(t)* decreases over time due to (i) linearly time dependent deterioration during $[0, t_1]$ and (ii) time dependent demand during $[0, T]$. The differential equation of these states at *I (t)* during the time interval $[0, t_1]$ and $[t_1, T]$ are given by

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -(a - bt), \qquad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI(t)}{dt} = -(a - bt), \qquad t_1 \le t \le T \tag{2}$$

Under the boundary condition $I(t) = 0$. Solution of differential Eq. (1) and Eq. (2) are

$$I(t)=\Bigg(at_1-at+\frac{a\alpha t_1^2}{2}-\frac{a\alpha t^2}{2}+\frac{\beta at^3}{6}-\frac{\beta at_1^3}{6}+\frac{bt^2}{2}-\frac{bt_1^2}{2}+\frac{b\alpha t^3}{3}-\frac{b\alpha t_1^3}{3}+\frac{\beta bt_1^4}{8} \tag{3}$$

$$-\frac{\beta bt^4}{8}-a\alpha tt_1+\mathrm{a}\alpha\mathrm{t}^2-\frac{a\alpha^2 tt_1^2}{2}+\frac{a\alpha^2 t^3}{2}-\frac{a\alpha\beta t^4}{6}+\frac{a\alpha\beta t_1^3 t}{6}-\frac{b\alpha t^3}{2}+\frac{b\alpha tt_1^2}{2}-\frac{b\alpha^2 t^4}{3}+$$

$$\frac{b\alpha^2 tt_1^3}{3}-\frac{\alpha\beta btt_1^4}{8}+\frac{\alpha\beta bt^5}{8}+\frac{\beta \mathrm{a}t_1 t^2}{2}-\frac{\beta at^3}{2}+\frac{a\alpha\beta t_1^2 t^2}{4}-\frac{a\alpha\beta t^4}{4}+\frac{a\beta^2 t^5}{12}-\frac{a\beta^2 t^2 t_1^3}{12}$$

$$+\frac{b\beta t^4}{4}-\frac{b\beta t^2 t_1^2}{4}+\frac{b\alpha\beta t^5}{6}-\frac{b\alpha\beta t^2 t_1^3}{6}+\frac{\beta^2 bt_1^4 t^2}{16}-\frac{\beta^2 bt^6}{16}\Bigg),$$

and

$$I(t)=-a(t-t_1)+\frac{b}{2}(t^2-t_1^2). \tag{4}$$

The maximum inventory level is *I (0) = Q*. From Eq. (3), we get

$$I(0)=Q=t_1\left(a-\frac{bt_1}{2}+\frac{a\alpha t_1}{2}-\frac{\beta at_1^2}{6}-\frac{b\alpha t_1^2}{3}+\frac{\beta bt_1^3}{8}\right). \tag{5}$$

The ordering cost is $OC = A_o$. (6)

The total demand during [*0, T*] is

$$\int_0^T D(t)dt=T\left(a-\frac{bT}{2}\right). \tag{7}$$

The deteriorated number of units is

$$I(0)-\int_0^T D(t)dt = t_1\left(a+\frac{a\alpha t_1}{2}-\frac{\beta at_1^2}{6}-\frac{bt_1}{2}-\frac{b\alpha t_1^2}{3}+\frac{\beta bt_1^3}{8}\right)-T\left(a-\frac{bT}{2}\right). \tag{8}$$

The *DC* (deteriorated cost) during [*0, T*] = $C_d \times$ (the number of deteriorated units)

$$DC=C_d\left\{t_1\left(a+\frac{a\alpha t_1}{2}-\frac{\beta at_1^2}{6}-\frac{bt_1}{2}-\frac{b\alpha t_1^2}{3}+\frac{\beta bt_1^3}{8}\right)-T\left(a-\frac{bT}{2}\right)\right\} \tag{9}$$

The total holding cost (*IHC)* during [*0, t₁*] is

$$=h\int_0^{t_1} I(t)dt \tag{10}$$

$$=ht_1^2\left(\frac{a}{2}+\frac{a\alpha t_1}{6}-\frac{\beta at_1^2}{12}-\frac{bt_1}{3}-\frac{b\alpha t_1^2}{8}+\frac{\beta bt_1^3}{15}-\frac{a\alpha^2 t_1^2}{8}+\frac{a\alpha\beta t_1^3}{12}+\frac{b\alpha^2 t_1^3}{10}-\frac{5b\alpha\beta t_1^4}{72}-\frac{a\beta^2 t_1^4}{72}+\frac{b\beta^2 t_1^5}{84}\right).$$

Expected shortage cost *SC* during $[t_1,T]$ is

$$SC=s\int_{t_1}^{T}\{-I(t)\}\,dt=-s\left(at_1T-\frac{aT^2}{2}+\frac{bT^3}{6}-\frac{bt_1^2 T}{2}-\frac{at_1^2}{2}+\frac{bt_1^3}{3}\right). \tag{11}$$

Total average cost = $\frac{1}{T}\left(\text{OC} + \text{DC} + \text{IHC} + \text{SC}\right)$

$$= \frac{1}{T}\left[A_o + C_d \left\{ t_1 \left(a + \frac{a\alpha t_1}{2} - \frac{\beta a t_1^2}{6} - \frac{bt_1}{2} - \frac{b\alpha t_1^2}{3} + \frac{\beta b t_1^2}{8} \right) - T\left(a - \frac{bT}{2} \right) \right\} + ht_1^2 \left(\frac{a}{2} + \frac{a\alpha t_1}{6} \right.\right. \tag{12}$$

$$\left. - \frac{\beta a t_1^2}{12} - \frac{bt_1}{3} - \frac{b\alpha t_1^2}{8} + \frac{\beta b t_1^3}{15} - \frac{a\alpha^2 t_1^4}{8} + \frac{a\alpha\beta t_1^3}{12} + \frac{b\alpha^2 t_1^3}{10} - \frac{5b\alpha\beta t_1^4}{72} - \frac{a\beta^2 t_1^4}{72} + \frac{b\beta^2 t_1^5}{84} \right)$$

$$\left. - s\left(at_1 T - \frac{aT^2}{2} + \frac{bT^3}{6} - \frac{bt_1^2 T}{2} - \frac{at_1^2}{2} + \frac{at_1^3}{3} \right) \right] .$$

4. Determination of the optimal solution

Taking the first and the second partial derivative of Eq. (13) w.r.t. T and t_1 respectively, we get

$$\frac{\partial Z(T,t_1)}{\partial T} = -\frac{A_o}{T^2} - \frac{C_d}{T^2}\left(at_1 + \frac{a\alpha t_1^2}{2} - \frac{\beta a t_1^3}{6} - \frac{bt_1^2}{2} - \frac{b\alpha t_1^3}{3} + \frac{\beta b t_1^4}{8} \right) + \frac{C_d b}{2} - \frac{ht_1^2}{T^2}\left(\frac{a}{2} + \frac{a\alpha t_1}{6} \right.$$

$$\left. - \frac{\beta a t_1^2}{12} - \frac{bt_1}{3} - \frac{b\alpha t_1^2}{8} + \frac{\beta b t_1^3}{15} - \frac{a\alpha^2 t_1^2}{8} + \frac{a\alpha\beta t_1^3}{12} + \frac{b\alpha^2 t_1^3}{10} - \frac{5b\alpha\beta t_1^4}{72} - \frac{a\beta^2 t_1^4}{72} + \frac{b\beta^2 t_1^5}{84} \right)$$

$$+ \text{s}\left(\frac{\text{a}}{2} - \frac{bT}{3} - \frac{at_1^2}{2T^2} + \frac{bt_1^3}{3T^2} \right) . \tag{13}$$

$$\frac{\partial Z(T,t_1)}{\partial t_1} = \frac{1}{T}\left\{ C_d \left(a + a\alpha t_1 - bt_1 - \frac{\beta a t_1^2}{2} - b\alpha t_1^2 + \frac{\beta b t_1^3}{2} \right) + h\left(at_1 + \frac{a\alpha t_1^2}{2} - \frac{\beta a t_1^3}{3} - bt_1^2 - \frac{b\alpha t_1^3}{2} \right.\right. \tag{14}$$

$$\left.\left. + \frac{\beta b t_1^4}{3} - \frac{a\alpha^2 t_1^3}{2} + \frac{5a\alpha\beta t_1^4}{12} + \frac{b\alpha^2 t_1^4}{2} - \frac{5b\alpha\beta t_1^5}{12} - \frac{a\beta^2 t_1^5}{12} + \frac{b\beta^2 t_1^6}{12} \right) - s(aT - bt_1 T - at_1 + bt_1^2) \right\}$$

$$\frac{\partial^2 Z(T,t_1)}{\partial T^2} = \frac{2A_o}{T^3} + \frac{2C_d}{T^3}\left(at_1 + \frac{a\alpha t_1^2}{2} - \frac{\beta a t_1^3}{6} - \frac{bt_1^2}{2} - \frac{b\alpha t_1^3}{3} + \frac{\beta b t_1^4}{8} \right) + \frac{2ht_1^2}{T^3}\left(\frac{a}{2} + \frac{a\alpha t_1}{6} \right.$$

$$\left. - \frac{\beta a t_1^2}{12} - \frac{bt_1}{3} - \frac{b\alpha t_1^2}{8} + \frac{\beta b t_1^3}{15} - \frac{a\alpha^2 t_1^2}{8} + \frac{a\alpha\beta t_1^3}{12} + \frac{b\alpha^2 t_1^3}{10} - \frac{5b\alpha\beta t_1^4}{72} - \frac{a\beta^2 t_1^4}{72} + \frac{b\beta^2 t_1^5}{84} \right)$$

$$+ \text{s}\left(-\frac{b}{3} + \frac{at_1^2}{T^3} - \frac{2bt_1^3}{3T^3} \right) > 0 \tag{15}$$

$$\frac{\partial^2 Z(T,t_1)}{\partial t_1^2} = \frac{1}{T}\left\{ C_d \left(a\alpha - b - \beta a t_1 - 2b\alpha t_1 + \frac{3\beta b t_1^2}{2} \right) + h\left(a + a\alpha t_1 - \beta a t_1^2 - 2bt_1 - \frac{3b\alpha t_1^2}{2} + \right.\right.$$

$$\left.\left. \frac{4\beta b t_1^3}{3} + \frac{5a\alpha\beta t_1^3}{3} + 2b\alpha^2 t_1^3 - \frac{25b\alpha\beta t_1^4}{12} - \frac{5a\beta^2 t_1^4}{12} + \frac{b\beta^2 t_1^5}{2} \right) - s(-bT - a + 2bt_1) \right\} > 0 . \tag{16}$$

$$\frac{\partial^2 Z(T,t_1)}{\partial T \partial t_1} = -\frac{1}{T^2}\left(a + a\alpha t_1 - bt_1 - \frac{\beta a t_1^2}{2} - b\alpha t_1^2 + \frac{\beta b t_1^3}{2} \right) - s(a - bt_1) < 0. \tag{17}$$

Putting

$\frac{\partial Z(T,t_1)}{\partial T} = 0$ and $\frac{\partial Z(T,t_1)}{\partial t_1} = 0$, we get

$$A_0 + C_d\left(at_1 + \frac{a\alpha t_1^2}{2} - \frac{\beta a t_1^3}{6} - \frac{bt_1^2}{2} - \frac{b\alpha t_1^3}{3} + \frac{\beta b t_1^4}{8}\right) - \frac{C_d bT^2}{2} + ht_1^2\left(\frac{a}{2} + \frac{a\alpha t_1}{6} - \right. \tag{18}$$

$$\left. \frac{\beta a t_1^2}{12} - \frac{bt_1}{3} - \frac{b\alpha t_1^2}{8} + \frac{\beta b t_1^3}{15} - \frac{a\alpha^2 t_1^2}{8} + \frac{a\alpha\beta t_1^3}{12} + \frac{b\alpha^2 t_1^3}{10} - \frac{5b\alpha\beta t_1^4}{72} - \frac{a\beta^2 t_1^4}{72} + \frac{b\beta^2 t_1^5}{84}\right)$$

$$\text{-s}\left(\frac{\mathrm{a}T^2}{2} - \frac{bT^3}{3} - \frac{at_1^2}{2} + \frac{bt_1^3}{3}\right) = 0.$$

and

$$C_d\left(a + a\alpha t_1 - bt_1 - \frac{\beta a t_1^2}{2} - b\alpha t_1^2 + \frac{\beta b t_1^3}{2}\right) + ht_1\left(a + \frac{a\alpha t_1}{2} - \frac{\beta a t_1^2}{3} - bt_1 - \frac{b\alpha t_1^2}{2} + \frac{\beta b t_1^3}{3} - \frac{a\alpha^2 t_1^2}{2}\right. \tag{19}$$

$$\left. + \frac{5a\alpha\beta t_1^3}{12} + \frac{b\alpha^2 t_1^3}{2} - \frac{5b\alpha\beta t_1^4}{12} - \frac{a\beta^2 t_1^4}{12} + \frac{b\beta^2 t_1^5}{12}\right) - s(aT - bt_1 T - at_1 + bt_1^2) = 0.$$

The sufficient conditions $\frac{\partial^2 Z(T,t_1)}{\partial T^2} > 0, \frac{\partial^2 Z(T,t_1)}{\partial t_1^2} > 0$, and

$$\frac{\partial^2 Z(T,t_1)}{\partial T^2} \cdot \frac{\partial^2 Z(T,t_1)}{\partial t_1^2} - \left(\frac{\partial^2 Z(T,t_1)}{\partial T \partial t_1}\right)^2 > 0.$$

From the above discussion we obtain the following result:

Result: Optimal T is a non-decreasing function of optimal t_1.

Differentiating Eq. (19) with respect to t_1, we get

$$\frac{dT^*}{dt_1^*} = \frac{\left\{C_d\left(a + a\alpha t_1 - \frac{\beta a t_1}{2} - bt_1 - b\alpha t_1^2 + \frac{\beta b t_1^3}{2}\right) + hA + +st_1\left(a - bt_1\right)\right\}}{T\left\{C_d b + s(a - bT)\right\}} > 0. \tag{20}$$

where

$$A = \left(at_1 + \frac{a\alpha t_1^2}{2} - \frac{\beta a t_1^3}{3} - bt_1^2 - \frac{b\alpha t_1^2}{2} + \frac{\beta b t_1^4}{3} - \frac{a\alpha^2 t_1^3}{2} + \frac{5a\alpha\beta t_1^4}{12} + \frac{b\alpha^2 t_1^4}{2}\right.$$

$$\left. - \frac{5b\alpha\beta t_1^5}{12} - \frac{a\beta^2 t_1^5}{12} + \frac{b\beta^2 t_1^6}{12}\right)$$

Again differentiating (20) with respect to t_1, we get

From (21) and (22) it clear that $\frac{dT^*}{dt_1^*} > 0$, thus optimal T is increasing function of optimal t_1 (i.e. t_1*).

5. Numerical examples and sensitivity analysis

Example 1:

Let us take the parameter values in the inventory system A_o = 600, a = 200, b = 0.5, h = 6, C_d =1, α = 0.06, β = 0.4, s = 2, in appropriate units .Substituting these values in Eq. (18) and Eq. (19), we get the optimum solutions for T* = 1.98551 years, t_1*= 0.374103 year, Q* = 74.9272 units and optimum total cost Z* = \$ 440.069. For the sensitivity analysis considering the same values as in above numerical example 1, we discuss the changes on the optimal solution with respect to key parameters, fixed ordering cost per order A_0, initial demand a (at t = 0) , constant b, holding cost per unit time h , C_d , initial deterioration (at t = 0) α and β in the appropriate units. The computational results are shown in Table 1 as follows,

Table 1

Sensitivity analysis on optimal solution with respect to A_0, a, b, h, C_d, α and β

Changing parameter	Change	T^*	t_1^*	Q^*	$Z^*(T,t_1)$
A_0	450	1.71086	0.303468	60.8504	360.231
	500	1.80681	0.328038	65.7556	388.234
	550	1.89822	0.351529	70.4371	414.781
	650	2.06892	0.395887	79.2513	464.252
	700	2.14988	0.416982	83.4295	487.46
	750	2.22772	0.437473	87.4788	509.791
a	125	2.53943	0.519157	64.6723	372.195
	150	2.30405	0.45777	68.597	398.426
	175	2.10658	0.411136	71.9832	420.779
	225	1.86786	0.343745	77.5009	456.853
	250	1.76866	0.318252	79.76	471.543
	275	1.68334	0.29644	81.7464	484.442
b	0.2	1.98375	0.373701	74.8683	440.162
	0.3	1.98427	0.373835	74.8879	440.131
	0.4	1.98479	0.373969	74.8796	440.1
	0.6	--------	-----------	--------------	------------
	0.7	1.98676	0.374373	74.9668	440.004
	0.8	1.9871	0.374508	74.9866	439.97
h	4.5	2.06698	0.491716	98.1485	422.39
	5	2.02622	0.444649	88.8946	429.572
	5.5	2.23031	0.406195	81.2941	440.201
	6.5	1.96634	0.346868	69.5089	444.03
	7	1.95028	0.323434	64.8371	447.406
	7.5	1.93628	0.30304	60.7496	450.322
C_d	0.7	1.99795	0.416136	83.2621	486.715
	0.8	1.99434	0.402225	80.5076	471.325
	0.9	1.98991	0.388213	77.7291	455.776
	1.1	1.98022	0.359898	72.1028	424.202
	1.2	1.9749	0.345597	69.2557	408.176
	1.3	1.96869	0.331205	66.3873	391.99
α	0.03	1.98866	0.378051	75.2831	439.584
	0.04	2.89079	0.376714	75.1623	497.088
	0.05	1.98634	0.375398	75.0437	439.909
	0.07	1.98607	0.372828	74.8128	440.229
	0.08	1.98336	0.371572	74.7002	440.382
	0.09	1.98242	0.370334	74.5893	440.537
β	0.1	1.92597	0.365962	73.7987	441.009
	0.2	1.97223	0.36856	73.5597	440.566
	0.3	---------	----------	---------	---------
	0.5	---------	----------	---------	---------
	0.6	1.12869	0.380189	75.7701	567.923
	0.7	1.99324	0.383472	76.2246	439.238
	0.8	1.99616	0.386939	76.7044	438.939
	0.9	1.99934	0.390612	77.2125	438.626
s	1.7	2.10996	0.33765	67.6722	399.77
	1.8	2.06464	0.350288	70.19	413.864
	1.9	2.02326	0.362426	72.6057	427.282
	2.1	1.9505	0.385352	77.1613	452.264
	2.2	1.91843	0.396201	79.3136	463.904
	2.3	1.88811	0.406677	81.3896	475.019

Note: The dotted lines show the infeasible results.

The results obtained in above Table 1 can be summarized as follows:

(**i**). Increase of fixed ordering cost A_0 will cause increase in T^*, t_1^*, Q^* and $Z^*(T, t_1)$.

(**ii**).Increase of initial demand a (at t = 0) will cause decrease in T^* and t_1^*.

(**iii**).Increase of b leads insignificant variation in T^*, t_1^*, Q^* and $Z^*(T, t_1)$.

(**iv**).Increase of unit holding cost 'h' leads decrease in T^* (except at h = 5.5), t_1^*,Q^* and increase in $Z^*(T, t_1)$.

(**v**). Increase of the cost of each deteriorated units C_d causes decrease in T^*, t_1^*, Q^* and $Z^*(T,t_1)$.

(**vi**).Increase of α causes, decrease in T^* (except at α = 0.04), t_1^*, Q^* (slightly) and increase in $Z^*(T,t_1)$(slightly except α = 0.04) .

(**vii**).Increase of β causes, increase in T^* (except at β = 0.6) , t_1^*, Q^* and decrease in $Z^*(T,t_1)$ (Except at β = 0.6).

(**viii**).Increase of unit shortage costs result, decrease in T^*, increase in t_1^*,Q^* and $Z^*(T,t_1)$.

6. Conclusion and Future Research

In real life, demand and deterioration, both, are not constant, these are in dynamic stage. We have developed EOQ model for linearly time varying deterioration and linearly time varying, non-decreasing demand. Shortages were allowed. The main goal of the work was to obtain cycle time and time to finish positive inventory, which minimizes the total average cost $Z^*(T,t_1)$. We have also shown that T^* in increasing function of t_1^*. Next, we have provided the numerical example to validate the model proposed in this paper. Sensitivity analysis has been discussed with the change in several key parameters. From the sensitivity analysis the variations are quite sensitive with respect A_0, a, h, C_d, α, s and insignificant with respect to b and β.

The research work discussed in this model may be generalized in different ways. We may generalize the model for quadratic time varying demand and Weibull distribution deterioration. We may also extend the model for inflation and advertisement costs etc.

References

Abad, P.L. (1996). Optimal pricing and lot sizing under conditions of perishability and partial backordering. *Management Sciences*, *42*(8), 1093-1104.

Bhaula, B., & Kumar, M. R. (2014). An economic order quantity model for Weibull deteriorating items with stock dependent consumption rate and shortages under inflation. *International Journal of Computing and Technology*, *1*(5), 196-204.

Chung, K. J., & Ting, P. S. (1993). A heuristic for replenishment of deteriorating items with a linear trend in demand. *Journal of the Operational Research Society*, *44*(12), 1235-1241.

Covert, R. P., & Philip, G. C. (1973). An EOQ model for items with Weibull distribution deterioration. *AIIE transactions*, *5*(4), 323-326.

Chakrabarti, T., & Chaudhuri, K. S. (1997). An EOQ model for deteriorating items with a linear trend in demand and shortages in all cycles. *International Journal of Production Economics*, *49*(3), 205-213.

Chang, H.J., & Dye, C.Y. (1999). An EOQ model for deteriorating items with time varying demand and partial backlogging. *Journal of Operational Research Society*, *50*(11), 1176-1182.

Dave, U., & Patel, L.K. (1981) (T, Si) policy inventory model for deteriorating items with time proportional demand. *Journal of Operational Research Society*, *32*(2), 137-142.

Dye, C. Y., Ouyang, L. Y., & Hsieh, T. P. (2007). Deterministic inventory model for deteriorating items with capacity constraint and time-proportional backlogging rate. *European Journal of Operational Research*, *178*(3), 789-807.

Dye, C.Y. (2004). A note on "An EOQ model for items with Weibull distributed deterioration, shortage and power demand pattern. *International journal of Information and Management Sciences, 15*(2), 81-84.

Ghare, P.M., & Schrader, G.P. (1963). A model for an exponentially decaying inventory. *International Journal of Industrial Engineering*, *14*, 238-243.

Hariga, M.A. (1996). Optimal EOQ models for deteriorating items with time – varying demand. *Journal of the Operational Research Society*, *47*(10), 1228-1246.

Hariga, M.A., & Benkherouf, L. (1994). Optimal and heuristic inventory replenishment models for deteriorating with exponential time varying demand. *European Journal of Operational Research, 79*, 123-137.

Khanra, S., Ghosh, S. K., & Chaudhuri, K. S. (2011). An EOQ model for a deteriorating item with time dependent quadratic demand under permissible delay in payment. *Applied Mathematics and Computation*, *218*(1), 1-9.

Jaggi, C., Khanna, A., & Nidhi, N. (2016). Effects of inflation and time value of money on an inventory system with deteriorating items and partially backlogged shortages. *International Journal of Industrial Engineering Computations*, *7*(2), 267-282.

Khanna, A., Mittal, M., Gautam, P., & Jaggi, C. (2016). Credit financing for deteriorating imperfect quality items with allowable shortages. *Decision Science Letters*, *5*(1), 45-60.

Lee, W.C., & Wu, J.W. (2002). An EOQ model for items with Weibull distribution deterioration, shortages and power demand. *International Journal of information and Management Sciences, 13*(2), 19-34.

Mahata, G. C. (2012). An EPQ-based inventory model for exponentially deteriorating items under retailer partial trade credit policy in supply chain. *Expert systems with Applications*, *39*(3), 3537-3550.

Mishra, V.K., & Singh, L.S. (2010) Determining inventory model with time dependent demand and partial backlogging. *Applied Mathematical Sciences*, *4*(72), 3611-3619.

Mitra, A., Cox, J. F., & Jesse Jr, R. R. (1984). A note on determining order quantities with a linear trend in demand. *Journal of the Operational Research Society*, *35*(2), 141-144.

Raafat, F. (1991). Survey of literature on continuously deteriorating inventory model. *Journal of Operational Research Society, 42*(1), 27-37.

Ritche, E. (1984). The EOQ for linear increasing demand: a simple optimal solution. *Journal of Operational Research Society*, *35*(10), 949-952.

Sachan, R.S. (1984). On (T, Si) policy inventory model for deteriorating items with time proportional demand. *Journal of Operational Research Society*, *25*(11), 1013-1019.

Skouri, K., Konstantaras, I., Papachristos, S., & Ganas, I. (2009). Inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate. *European Journal of Operational Research, 192*(1), 79-92.

Sicilia, J., Rosa, M.G., De-la, Acosta, J.F., & Pablo, D.A.L.D. (2014). An inventory model for deteriorating items with shortages and time – varying demand. *International Journal of Production Economics, 155*, 155-162.

Sicilia, J., González-De-la-Rosa, M., Febles-Acosta, J., & Alcaide-López-de-Pablo, D. (2014). Optimal policy for an inventory system with power demand, backlogged shortages and production rate proportional to demand rate. *International Journal of Production Economics*, *155*, 163-171.

Silver, E. A., & Meal, H. C. (1969). A simple modification of the EOQ for the case of a varying demand rate. *Production and Inventory Management*, *10*(4), 52-65.

Singh, T. J., Singh, S. R., & Dutt, R. (2009). An EOQ model for perishable items with power demand and partial backlogging. *International Journal of Production Economics*, *15*(1), 65-72.

Taleizadeh, A. A., Mohammadi, B., Cárdenas-Barrón, L. E., & Samimi, H. (2013). An EOQ model for perishable product with special sale and shortage. *International Journal of Production Economics*, *145*(1), 318-338.

Teng, J. T., Chern, M. S., Yang, H. L., & Wang, Y. J. (1999). Deterministic lot-size inventory models with shortages and deterioration for fluctuating demand. *Operations Research Letters*, *24*(1), 65-72.

Teng, J., & Yang, H. (2007). Deterministic inventory lot-size models with time-varying demand and cost under generalized holding costs. *International journal of information and management sciences*, *18*(2), 113.

Teng, J. T., Min, J., & Pan, Q. (2012). Economic order quantity model with trade credit financing for non-decreasing demand. *Omega*, *40*(3), 328-335.

Tripathi, R., & Kumar, M. (2014). A new model for deteriorating items with inflation under permissible delay in payments. *International Journal of Industrial Engineering Computations*, *5*(3), 365-374.

Tripathi, R. P. (2013). EOQ model with stock level dependent demand rate and inflation under trade credits. *International Journal of Management Science and Engineering Management*, *8*(2), 102-108.

Tripathi, R.P. (2014). Economic order quantity (EOQ) model for deteriorating items with non-instantaneous receipt under trade credits. *International Journal of Operations Research*, *12*(2), 47-56.

Wu, K.S. (2002).Deterministic inventory model for item with time-varying demand, Weibull distribution deterioration and shortages. *Yugoslav Journal of Operational Research*, *12*(1), 67-71.

Yang, H.L. (2012). Two ware-house partial backlogging inventory models with three parameter Weibull distribution deterioration under inflation. *International Journal of Production Economics, 138*(1), 107-116.