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Optimal ordering policy for an integrated inventory model with stock dependent demand and order linked trade credits for twin ware house system

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A B S T R A C T

Many entrepreneurs use different ways to offer credit to enhance long term profit as well as relations with their customers. Order-size dependent credit period is one of them that encourage customers to order large lots to grab more credits in payments. Since display of items play positive role in boosting demand, stock dependent demand is assumed in the proposed model. Ordering large lots can create space issue so the proposed model presents a rented warehouse along with owned one. Rented warehouse is used only when owned warehouse is utilized, completely. Here we propose an integrated inventory model with capacity utilization dependent holding cost to optimize joint profit of supplier and retailer. An algorithm is developed to determine the optimal replenishment policies in order to enhance total profit of supply chain under different ordering schemes. Total joint profit for supplier and retailer is optimized using MATLAB 2015. Numerical examples are presented to illustrate the solution procedure and the results. Sensitivity analysis for some key parameters is carried out to demonstrate the influence of different parameters on over-all profit and cycle time. The proposed model is applicable to fast moving consumer goods (FMCG) and home textile industry.

1. Introduction

A traditional EOQ model has a basic assumption that the retailer will pay the full amount to the supplier when the products are received. However, in real market scenario, supplier usually allow delay in payment to retailer that is well known as trade credit. For suppliers, offering delay in payment may attract retailers and increase in sales and, at the same time, it decreases supplier's inventory carrying cost. For retailers, delay in payment not only reduces the opportunity cost of capital, but also allows them to earn interest on the revenue generated during the permissible delay period. Hence, trade credit policy is a win – win situation for both supplier and retailer. Goyal (1985) was the first to develop an EOQ model with fixed credit period. Later on Agrawal and Jaggi (1995) and Chang et al. (2008) extended the same idea for deterioration items. The trade credit policy that the supplier offers could be of different types in real life situations. Some researchers assumed that both retailers and customers can buy with trade credit which is called a two-level trade credit policy. There are several relevant papers

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related to the two-level trade credit policy such as Huang (2003), Teng and Chang (2009), Min et al. (2010) and Kumar et al. (2011). Moreover, some researchers assumed that the supplier provides the delay payment and cash discount simultaneously, i.e., the supplier offers a cash discount to the retailer to encourage him to settle the account earlier. The available results adopting this assumption can be found in the work of Ouyang et al. (2002), Ouyang et al. (2005), Chang (2002) and Sana and Chaudhuri (2008). All of the above mentioned papers discussed the issue from the perspective of the supplier or the retailer, and just focused on one sided optimal strategies. Shah et al. (2010) presented a complete review of trade credit in inventory models.

Businesses must integrate their supply chain to enhance the operational efficiency, satisfy customers more efficiently and lowering the inventory cost. Since the motive of players is different and some of the time conflicting in a non-integrated supply chain, it leads to loss of competitive advantage. Goyal (1976) first developed an integrated inventory model to determine the optimal joint inventory policy for a single supplier and a single retailer. Abad and Jaggi (2003) combined the concept of an integrated inventory model and trade credit policy, and established a supplier–retailer integrated system in which the supplier offers trade credit to the retailer. Afterward, few integrated inventory model with various trade credit policies can be found in Su et al. (2007) and Ho et al. (2008). An inventory model in which two level trade credit with an agreement of profit sharing is found in Shah (2015). Shah (2015) formulated an integrated inventory model for three players dealing with deteriorating item and quadratic demand rate. Shah and Chaudhari (2015) also formulated an integrated inventory model for three players dealing with deteriorating item with fixed lifetime and demand rate quadratically decreasing and credit period dependent.

The above papers assumed that the warehouse owned by the retailer is adequate to store the entire procured inventory. Practically, retailer may order more items when faced an attractive price discount or when the ordering cost is relatively higher than the holding cost. If the space in the own warehouse (OW) is not sufficient to store all the purchased units, an external rented warehouse (RW) is used to hold the excess units. Generally, the unit holding cost in RW is higher than the OW. Therefore, purchased units are kept in the OW first and the excess units are kept in the RW. In addition, the units at the RW should be exhausted before units from the OW are retrieved. An inventory model with two levels of storage, attracted the attention of many researchers. Some related studies in this area include Goyal and Giri (2001) who used it for deteriorating inventory, Zhou and Yang (2005) and Panda et al. (2010) who considered stock dependent demand. However, the impact of trade credit policy on the optimal solution is not considered in these papers. Huang (2006) first established an EOQ model with two warehouses and a two-level trade credit policy. Ouyang et al. (2007) proposed an EOQ model with limited storage capacity and trade credit. Later, Liang and Zhou (2011) developed a two-warehouse inventory model with trade credit policy for deteriorating items, which assumed that the RW has a higher deterioration rate than the OW. These papers discuse two warehouses determined the optimal inventory policy only from the retailer’s perspective.

Deterministic inventory models usually consider the demand rate to be either constant or time-varying but independent of level of inventory i.e. stock dependent. However, it has been noted, especially in the retail market, that these inventory models are unsuitable for representations of the reality of inventory control situation in the retail field. Levin et al. (1972) quoted that “large piles of goods attract more customers” this is termed as stock-dependent demand. Urban (2005) computed optimal order quantity when demand is stock-dependent. Liang and Zhou (2011) formulated two warehouse inventory model with stock dependent demand under conditional permissible delay payment. This implies that holding higher inventory level will probably make the retailer sell more items. Under this situation, the demand rate should depend on the level of inventory.

Trade credit policy is a common business practice nowadays and it is important to explore the impact of trade credit policy on an integrated inventory problem. Chang, Ho et al. (2009), Shah et al. (2014) formulated an integrated inventory model with stock dependent demand where trade credit is linked to order quantity where threshold for trade credit is carefully decided benefiting both the players. Liao et al. (2012) formulated an integrated inventory model for deteriorating items when trade credit is order size dependent. Liao et al. (2013) established an integrated inventory model for deteriorating items with fixed trade credit period and two warehouses system. When the length of the delayed payment is linked to order quantity rather than a fixed parameter, the retailer is encouraged to order large quantities with a longer delayed payment period so that the order quantity might exceed the OW capacity. Ouyang, Ho et al. (2015) formulated an integrated inventory problem with an order-size dependent trade credit and limited storage capacity.

Literature review clearly shows that integrated inventory model is still not studied and analyzed for capacity utilization dependent holding cost for supplier and stock dependent demand for retailer. This paper deals with an integrated inventory model with different permissible delay periods based on ordered quantity. Model considers owned as well as rented ware house system to manage stock dependent demand and trade credit offers. This model is realistic as it studies joint profit scenario of supply chain instead of optimizing retailer or supplier individual profit. An algorithm is developed to determine the optimal policy for the supplier and the retailer. Model as well as algorithm is validated with numerical examples. Sensitivity analysis for important inventory parameters is carried out.

2. Notations and Assumptions

The proposed model is based on the following notations and assumptions.

2.1 Notations

D: Demand rate, $\alpha + \beta\, I(t)$where $\alpha > 0$ denotes scale demand, $0 < \beta < 1$ is stock dependent parameter.

P: Supplier production rate, $P > D$

A_r: Retailer's ordering cost per order

A_s: Supplier's setup cost per order

r_{R1}: Retailer's holding cost for items in own warehouse (OW), excluding interest charges

r_{R2}: Retailer's holding cost for items in rented warehouse (RW), excluding interest charges, $r_{R2} > r_{R1}$

r_s: Supplier's holding cost, excluding interest charges

f_0: Fixed transportation cost per shipment

f_1: Unit transportation cost

$c(P)$: Supplier unit production cost which is a convex function of P

v: Retailer's unit purchasing cost, $v > c$

s: Retailer's unit retail price, $s > v$

ρ: Supplier capacity utilization, i.e. ratio of demand to production rate. $\rho < 1$

w: Maximum storage capacity of retailer's OW

I_{sp}: Supplier's capital opportunity loss per \$ per unit time.

I_{rp}: Retailer's capital opportunity loss per \$ per unit time.

I_{re}: Retailer's interest earned per \$ per unit time.

m: Number of shipments from supplier to the retailer.

T: Retailer's replenishment cycle time (a decision variable).

T_w: Time for depletion of maximum storage capacity in OW.

$I(t)$: Inventory level at any instant of time$t$, $0 \leq t \leq T$, $I(t) = \begin{cases} I_1(t), & T \leq T_w \\ I_2(t), & T > T_w \end{cases}$

Q: Retailer's ordering quantity (a decision variable) $Q(t) = \begin{cases} Q_1(t), & T \leq T_w \\ Q_2(t), & T > T_w \end{cases}$

$\pi_i(m, T)$: Joint total profit per unit time.

2.2 Assumptions

1) Inventory system consists of single retailer and single supplier. The retailer orders Q units in each order. The supplier manufactures mQ units in each production run to reduce the setup cost and delivers Q units to the retailer.
2) The unit production cost $c(P)$ is a convex function of production rate P and is given by $c(P) = c_0 + \frac{1}{P^{c_1}} + P^{c_2}$, where c_0, c_1 and c_2 are non-negative real numbers. The fixed cost c_0 can be regarded as the material cost. The cost component $\frac{1}{P^{c_1}}$ decreases as the production rate increases, representing costs such as labor cost or energy cost. The third term P^{c_2} denotes a cost component that increases with the production rate such as additional tool or die wear at high production rate. For notational simplicity, $c(P)$ and c are used interchangeably in this paper.
3) The trade credit offered by the supplier is related to the retailer's order quantity$Q$, and is given as

i	Q	M
1	$q_1 \leq Q < q_2$	M_1
2	$q_2 \leq Q < q_3$	M_2
:	:	:
k	$q_k \leq Q < q_{k+1} = \infty$	M_k

where $0 \leq M_1 < M_2 < \cdots < M_k$ and $0 \leq q_1 < q_2 < \cdots < q_k < \infty$ is a sequence of quantities at which a specific credit period is offered. That is, M_i denotes the trade credit applicable to lot size falling in the range $[q_i, q_{i+1})$. This is done to attract the retailer to order more quantities.
4) During the credit period, the retailer sells the items and uses the sales revenue to earn interest at the rate of I_{re}. At the end of permissible delay period, the retailer pays purchase cost to the supplier and incurs a capital opportunity cost at the rate of I_{rp} for the items in stock.
5) By offering trade credit to the retailer, supplier bears opportunity cost at the rate of I_{sp} for the offered credit period.
6) To obtain longer credit period, retailer may order more items than his own warehouse (OW) capacity, the excess quantities are then stored in rented warehouse (RW). The OW has limited capacity, and the RW has an unlimited capacity. The holding cost of RW is more than OW, therefore items in RW are sold first and then the items in OW. When opting for rented warehouse, the units from RW are displayed until the stock depletes to zero and then the items in OW are displayed. Moreover both the warehouses are at the same place.
7) Lead time is zero. Shortages are not allowed.

3. Mathematical Model

In the proposed, model inventory level of retailer depletes due to stock dependent demand at any instant of time. Hence, inventory is governed by following differential equations at any instant of time t, depending on the two cases.

<u>Case 1</u> - $T \leq Tw$

Here in this case the cycle time T is less than the time taken for depletion of maximum storage capacity of OW. For this case retailer will not need a RW and the rate of change in inventory level is given by

$$\frac{dI_1(t)}{dt} = -(\alpha + \beta\, I_1(t)), \qquad 0 \le t \le T \tag{1}$$

with boundary conditions $Q_1 = I(0)$ and $I_1(T) = 0$

The solution of the differential equation (1) is

$$I_1(t) = \frac{\alpha}{\beta}\left(e^{\beta(T-t)} - 1\right), \qquad 0 \le t \le T \tag{2}$$

Therefore, the optimal order quantity in each shipment if $T \le T_w$ is

$$Q_1 = I(0) = \frac{\alpha}{\beta}\left(e^{\beta(T)} - 1\right) \tag{3}$$

3.1 Supplier's total profit per unit time

Supplier produces mQ_1 units in each production run, so the production cycle length for supplier is mT. The supplier total profit per unit time consist of sales revenue, production cost, setup cost, holding cost and opportunity cost.

1) Sales revenue, $ssr_1 = vQ_1$
2) Production cost, $spc_1 = cQ_1$
3) Setup cost, $ssc_1 = \frac{A_s}{m}$
4) Holding cost, $shc_1 = c(r_s + I_{sp})((m-1)(1-\rho) + \rho)\left[\int_0^T I_1(t)\, dt\right]$ (using Joglekar (1988))
5) Offering a credit period M_i to the retailer the opportunity cost is $soc_1 = \frac{vI_{sp}mQ_1M_i}{m} = vI_{sp}Q_1M_i$

Consequently, when the supplier offers a credit period $M_i, i = 1,2, \dots, k$ to the retailer the total profit per unit time denoted by $STP1_i(m,T)$ is a function of m and T can be expressed as

$$STP1_i(m,T) = \frac{1}{T}(\, ssr_1 - spc_1 - ssc_1 - shc_1 - soc_1) \tag{4}$$

3.2 Retailer's total profit per unit time

The retailer's total profit per unit time consists of sales revenue, purchase cost, ordering cost, transportation cost, holding cost, interest earned and opportunity cost.

1) Sales revenue, $rsr_1 = sQ_1$
2) Purchase cost, $rpc_1 = vQ_1$
3) Ordering cost, $roc_1 = A_r$
4) Transportation cost, $rtc_1 = f_0 + f_1Q_1$
5) Holding cost, $rhc_1 = r_{R1}v\left[\int_0^T I_1(t)\, dt\right]$
6) Opportunity cost and interest earned: The credit period $M_i, i = 1,2, \dots, k$ offered by the supplier is based on the retailer's order quantity. Based on the values of M_i and T, we have the following two cases: (a) $T \le M_i$ and (b) $T > M_i$.

Case (a) - $(T \le M_i)$

When $(T \le T_w \le M_i)$ and $(T \le M_i \le T_w)$ the opportunity cost and interest earned will be the same. Here the cycle time T gets over before the permissible credit period M_i as shown in Fig. 1, so irrespective of the value of T_w opportunity cost is zero.

So opportunity cost is,

$$roc_{1a} = 0 \tag{5}$$

Retailer uses sales revenue to earn interest, so the interest earned is given by,

$$rie_{1a} = sI_{re}\left[\int_0^T (\alpha + \beta\, I_1(t))t\, dt + (M_i - T)Q_1\right] \tag{6}$$

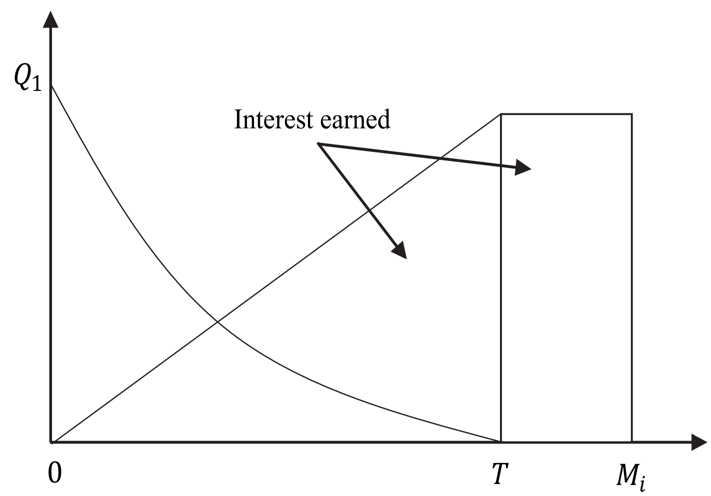

Fig. 1. Interest earned by the retailer when $T \leq M_i$

Case (b) - $(T > M_i)$ i.e. $(M_i < T \leq T_w)$

In this case, the cycle time T gets over after the permissible credit period M_i as shown in Fig. 2. So the retailer capital opportunity cost is given as,

$$roc_{1b} = vI_{rp}\left[\int_{M_i}^T I_1(t)dt\right] \tag{7}$$

Retailer uses sales revenue to earn interest till M_i, so the interest earned is given by,

$$rie_{1b} = sI_{re}\left[\int_0^{M_i} (\alpha + \beta\, I_1(t))t\, dt\right] \tag{8}$$

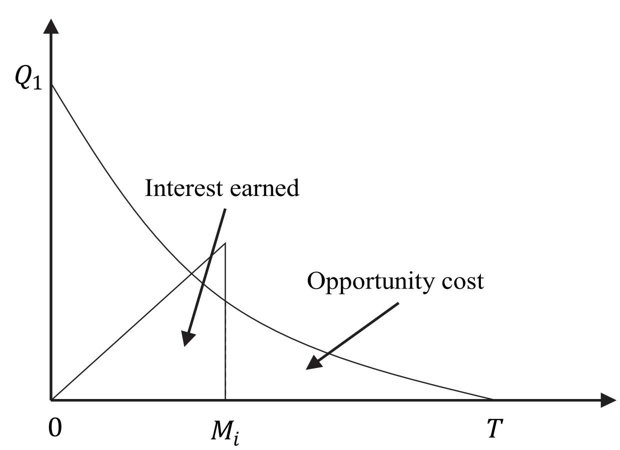

Fig. 2. Opportunity cost and Interest earned by the retailer when $T > M_i$

Hence, the total profit per unit time for the retailer is,

$$RTP1_i(T) = \begin{cases} RTP1_i^a(T), & T \le M_i \\ RTP1_i^b(T), & T > M_i \end{cases} \tag{9}$$

where,

$$RTP1_i^a(T) = \frac{1}{T}(rsr_1 - rpc_1 - roc_1 - rtc_1 - rhc_1 - roc_{1a} + rie_{1a}) \tag{10}$$

$$RTP1_i^b(T) = \frac{1}{T}(rsr_1 - rpc_1 - roc_1 - rtc_1 - rhc_1 - roc_{1b} + rie_{1b}) \tag{11}$$

<u>Case 2</u> - $T > Tw$

Here in this case the cycle time T is more than the time taken for depletion of maximum storage capacity of OW. This means the ordered quantity is more than w which is the maximum storage capacity of OW, for this case retailer will need a RW.

As per the assumption T_w is the time for depletion of maximum storage capacity of OW and the units from RW are sold first. So for $T > T_w$, the inventory level in OW remains constant during the period $[0, T - T_w]$ and depletes from $[T - T_w, T]$. In RW, the inventory level becomes 0 at time $T - T_w$.

The rate of change of inventory level in RW is given by

$$\frac{dI_r(t)}{dt} = -\big(\alpha + \beta\, I_r(t)\big), \qquad 0 \le t \le T - T_w \tag{12}$$

With the conditions $I_r(T - T_w) = 0$

The solution of the differential Eq. (12) is

$$I_r(t) = \frac{\alpha}{\beta}\big(e^{\beta(T - T_w - t)} - 1\big), \qquad 0 \le t \le T - T_w \tag{13}$$

In OW, the rate of change is given by

$$I_o(t) = w, \qquad 0 \le t \le T - T_w \tag{14}$$

$$\frac{dI_o(t)}{dt} = -\big(\alpha + \beta\, I_o(t)\big), \qquad T - T_w \le t \le T \tag{15}$$

With $I_o(T) = 0$

The solution of the differential Eq. (15) is,

$$I_o(t) = \frac{\alpha}{\beta}\big(e^{\beta(T - t)} - 1\big), \qquad T - T_w \le t \le T \tag{16}$$

Hence the inventory level in OW is given by Eq. (17)

$$I_o(t) = \begin{cases} w, & 0 \le t \le T - T_w \\ \frac{\alpha}{\beta}\big(e^{\beta(T - t)} - 1\big), & T - T_w \le t \le T \end{cases} \tag{17}$$

From the continuity of $I_o(t)$ at $t = T - T_w$, we have $w = \frac{\alpha}{\beta}\big(e^{\beta(T_w)} - 1\big)$

$$\Rightarrow T_w = \frac{1}{\beta}\ln\left(1 + \frac{\beta w}{\alpha}\right) \tag{18}$$

Therefore inventory level at any instance of time when $T > T_w$ is given by,

$$I_2(t) = \begin{cases} w + \frac{\alpha}{\beta}\left(e^{\beta(T-T_w-t)} - 1\right), & 0 \le t \le T - T_w \\ \frac{\alpha}{\beta}\left(e^{\beta(T-t)} - 1\right), & T - T_w \le t \le T \end{cases} \tag{19}$$

Moreover the optimal order quantity when $T > T_w$ is given by$Q_2 = I_2(0)$

$$Q_2 = w + \frac{\alpha}{\beta}\left(e^{\beta(T-T_w)} - 1\right) \tag{20}$$

3.3 Supplier's total profit per unit time

Supplier produces mQ_2 units in each production run, so the production cycle length for supplier is mT. The supplier total profit per unit time consists of sales revenue, production cost, setup cost, holding cost and opportunity cost.

1) Sales revenue, $ssr_2 = vQ_2$
2) Production cost, $spc_2 = cQ_2$
3) Setup cost, $ssc_2 = \frac{A_s}{m}$
4) Holding cost, $shc_2 = c(r_s + I_{sp})((m-1)(1-\rho) + \rho)\left[\int_0^T I_2(t)\,dt\right]$(using Joglekar (1988))
5) Offering a credit period M_i to the retailer the opportunity cost is $soc_2 = \frac{vI_{sp}mQ_2M_i}{m} = vI_{sp}Q_2M_i$

Consequently, when the supplier offers a credit period $M_i, i = 1,2,\dots,k$ to the retailer the total profit per unit time denoted by $STP2_i(m,T)$ is a function of m and T can be expressed as

$$STP2_i(m,T) = \frac{1}{T}(\,ssr_2 - spc_2 - ssc_2 - shc_2 - soc_2) \tag{21}$$

3.4 Retailer's total profit per unit time

The retailer's total profit per unit time consists of sales revenue, purchase cost, ordering cost, transportation cost, holding cost, interest earned and opportunity cost.

1) Sales revenue, $rsr_2 = sQ_2$
2) Purchase cost, $rpc_2 = vQ_2$
3) Ordering cost, $roc_2 = A_r$
4) Transportation cost, $rtc_2 = f_0 + f_1Q_2$
5) Holding cost, $rhc_2 = r_{R2}v\left[\int_0^{T-T_w} I_r(t)\,dt\right] + r_{R1}v\left[\int_0^T I_o(t)\,dt\right]$
6) Opportunity cost and interest earned, the credit period$M_i, i = 1,2,\dots,k$ offered by the supplier is based on the retailer's order quantity. Based on the values of M_i and T, we have the following two cases: (a) $T \le M_i$ and (b) $T > M_i$.

Case (a) - ($T \le M_i$) i.e. ($T_w < T \le M_i$)

Here the cycle time T gets over before the permissible credit period M_i as shown in Fig. 3, so irrespective of the value of T_w opportunity cost is zero.

So opportunity cost is,

$$roc_{2a} = 0 \tag{22}$$

Retailer uses sales revenue to earn interest, so the interest earned is given by,

$$rie_{2a} = sI_{re}\left[\int_0^T (\alpha + \beta\, I_2(t))t\,dt + (M_i - T)Q_2\right] \tag{23}$$

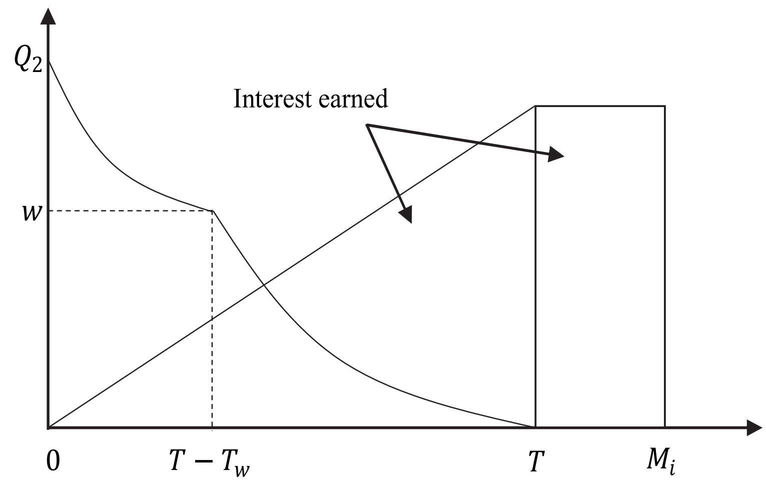

Fig. 3. Interest earned by the retailer when $T_w < T \leq M_i$

Case (b)- $(T > M_i)$

When $(T_w \leq M_i < T)$ and $(M_i \leq T_w < T)$ the opportunity cost and interest earned will be given by the same formula. In this case, depending on the value of T_w and M_i capital opportunity cost and interest earned will be calculated.

Subcase (I) - $(T - T_w \leq M_i)$

The cycle time lasts more than the available credit period. So the retailer bares the opportunity cost on inventory held in OW is shown in Fig. 4(a) and is given by,

$$roc_{2bI} = vI_{rp}\left[\int_{M_i}^{T} \frac{\alpha}{\beta}\left(e^{\beta(T-t)} - 1\right)dt\right] \tag{24}$$

Retailer uses sales revenue to earn interest till M_i, so the interest earned is shown in 4 (a) and given by,

$$rie_{2bI} = sI_{re}\left[\int_{0}^{M_i} (\alpha + \beta\, I_2(t))t\, dt\right] \tag{25}$$

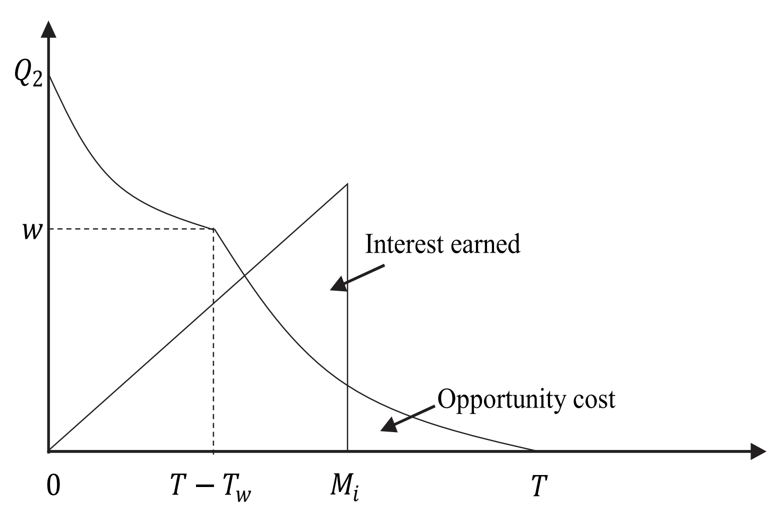

Fig. 4(a). Opportunity cost and Interest earned by the retailer when $T > M_i$ and $T - T_w \leq M_i$

Subcase (II) - $(M_i < T - T_w)$

Opportunity cost bared by the retailer on inventory held in both OW and RW is shown in figure 4(b) is given by,

$$roc_{2bII} = vI_{rp}\left[\int_{M_i}^{T} I_2(t)dt\right] \tag{26}$$

Retailer uses sales revenue to earn interest till M_i, so the interest earned is shown in figure 4(b) and given by,

$$rie_{2bII} = sI_{re}\left[\int_{0}^{M_i} (\alpha + \beta\ (w + \frac{\alpha}{\beta}\left(e^{\beta(T-T_w-t)} - 1\right)))t\ dt\right] \tag{27}$$

Hence, the total profit per unit time for the retailer is,

$$RTP2_i(T) = \begin{cases} RTP2_i^{a}(T), & T_w < T \le M_i \\ RTP2_i^{bI}(T), & T > M_i, T - T_w \le M_i \\ RTP2_i^{bII}(T), & T > M_i, M_i < T - T_w \end{cases} \tag{28}$$

where,

$$RTP2_i^{a}(T) = \frac{1}{T}(rsr_2 - rpc_2 - roc_2 - rtc_2 - rhc_2 - roc_{2a} + rie_{2a}) \tag{29}$$

$$RTP2_i^{bI}(T) = \frac{1}{T}(rsr_2 - rpc_2 - roc_2 - rtc_2 - rhc_2 - roc_{2bI} + rie_{2bI}) \tag{30}$$

$$RTP2_i^{bII}(T) = \frac{1}{T}(rsr_2 - rpc_2 - roc_2 - rtc_2 - rhc_2 - roc_{2bII} + rie_{2bII}) \tag{31}$$

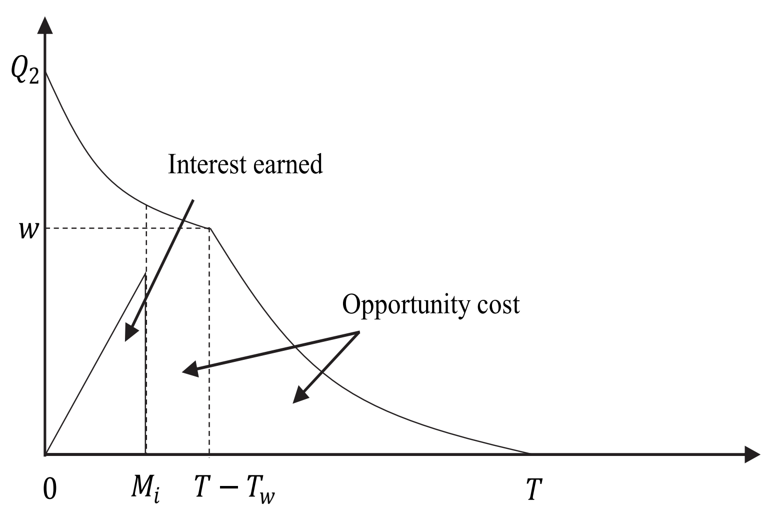

Fig. 4(b). Opportunity cost and Interest earned by the retailer when $T > M_i$ and $M_i < T - T_w$

For given $M_i, i = 1,2 \dots n$ based on the values of T_w and M_i, the total profit per unit time (denoted by $RTP_i(T)$) for the retailer in either case is given by,

$RTP_i(T)$ = Sales revenue – purchasing cost – ordering cost – transportation cost – opportunity cost + interest earned

$$RTP_i(T) = \begin{cases} RTP_i^1(T), & T_w \le M_i \\ RTP_i^2(T), & T_w > M_i \end{cases} \tag{32}$$

where,

$$RTP_i^1(T) = \begin{cases} RTP_{1i}, & 0 < T \le T_w \le M_i \\ RTP_{2i}, & T_w < T \le M_i \\ RTP_{3i}, & T_w \le M_i < T\ , T - T_w \le M_i \\ RTP_{4i}, & T_w \le M_i < T\ , M_i < T - T_w \end{cases} \tag{33}$$

$$RTP_i^2(T) = \begin{cases} RTP_{5i}, & 0 < T \le M_i < T_w \\ RTP_{6i}, & M_i < T \le T_w \\ RTP_{7i}, & M_i < T_w < T\ , T - T_w \le M_i \\ RTP_{8i}, & M_i < T_w < T\ , M_i < T - T_w \end{cases} \tag{34}$$

The detailed explanation for Eq. (33) and Eq. (34) are as follows. For $0 < T \le T_w \le M_i$, it indicates that the retailer does not need to opt for a RW, also he pays no opportunity cost and uses sales revenue to generate interest. Therefore,

$$RTP_{1i} = RTP1_i^a(T) = \frac{1}{T}(rsr_1 - rpc_1 - roc_1 - rtc_1 - rhc_1 - roc_{1a} + rie_{1a}) \tag{35}$$

Similarly, for $T_w < T \le M_i$ retailer pays no opportunity cost and uses sales revenue to generate interest. But here he needs an additional warehouse i.e. RW.

Hence,

$$RTP_{2i} = RTP2_i^a(T) = \frac{1}{T}(rsr_2 - rpc_2 - roc_2 - rtc_2 - rhc_2 - roc_{2a} + rie_{2a}) \tag{36}$$

For $T_w \le M_i < T$, the retailer uses revenue to generate interest and pays opportunity cost, moreover he needs an additional warehouse. Depending on the values of T_w and M_i this case is divided in two parts:

$(a)\ T - T_w \le M_i$, For this case the inventory level in RW finishes before the trade credit and then the units from OW are sold. Hence the retailer total profit is given by

$$RTP_{3i} = RTP2_i^{bI}(T) = \frac{1}{T}(rsr_2 - rpc_2 - roc_2 - rtc_2 - rhc_2 - roc_{2bI} + rie_{2bI}) \tag{37}$$

$(b) M_i < T - T_w$, For this case the inventory level in RW finishes after the trade credit offered and then the units from OW are sold. Hence the retailer total profit is given by

$$RTP_{4i} = RTP2_i^{bII}(T) = \frac{1}{T}(rsr_2 - rpc_2 - roc_2 - rtc_2 - rhc_2 - roc_{2bII} + rie_{2bII}) \tag{38}$$

For $0 < T \leq M_i \leq T_w$, it indicates that the retailer does not need to opt for a RW, also he pays no opportunity cost and uses sales revenue to generate interest, which is same as RTP_{1i}. Therefore,

$$RTP_{5i} = RTP1_i^a(T) = \frac{1}{T}(rsr_1 - rpc_1 - roc_1 - rtc_1 - rhc_1 - roc_{1a} + rie_{1a}) \tag{39}$$

Similarly, for $M_i < T \leq T_w$ retailer pays opportunity cost and uses sales revenue to generate interest. Also cycle time is less thanT_w, so an additional warehouse (i.e. RW) is not required.

Hence total profit is given by,

$$RTP_{6i} = RTP1_i^b(T) = \frac{1}{T}(rsr_1 - rpc_1 - roc_1 - rtc_1 - rhc_1 - roc_{1b} + rie_{1b}) \tag{40}$$

For$M_i < T_w < T$, the retailer uses revenue to generate interest and pays opportunity cost, moreover he needs an additional warehouse. Depending on the values of T_w and M_i this case is divided in two parts:

$(a)\ T - T_w \leq M_i$, The result is same as RTP_{3i} and is given by

$$RTP_{7i} = RTP2_i^{bI}(T) = \frac{1}{T}(rsr_2 - rpc_2 - roc_2 - rtc_2 - rhc_2 - roc_{2bI} + rie_{2bI}) \tag{41}$$

$(b) M_i < T - T_w$, The result is same as RTP_{4i} and is given by

$$RTP_{8i} = RTP2_i^{bII}(T) = \frac{1}{T}(rsr_2 - rpc_2 - roc_2 - rtc_2 - rhc_2 - roc_{2bII} + rie_{2bII} \tag{42}$$

3.5 The joint total profit per unit time

The supplier and retailer have decided to share their resources to undertake mutually beneficial cooperation, the joint total profit per unit time which is a function of m and T is the sum of retailer and supplier's total profit per unit time. By the given arguments, for given$M_i, i = 1,2,\dots,k$, the joint total profit per unit time is,

$$\pi_i(m,T) = \begin{cases} \pi_i^{(1)}(m,T), & T_w \leq M_i \\ \pi_i^{(2)}(m,T), & T_w > M_i \end{cases} \tag{43}$$

where,

$$\pi_i^{(1)}(m,T) = \begin{cases} \pi_{1i}(m,T) = STP1_i(m,T) + RTP_{1i}(T), & 0 < T \leq T_w \leq M_i \\ \pi_{2i}(m,T) = STP2_i(m,T) + RTP_{2i}(T), & T_w < T \leq M_i \\ \pi_{3i}(m,T) = STP2_i(m,T) + RTP_{3i}(T), & T_w \leq M_i < T, T - T_w \leq M_i \\ \pi_{4i}(m,T) = STP2_i(m,T) + RTP_{4i}(T), & T_w \leq M_i < T, M_i < T - T_w \end{cases} \tag{44}$$

$$\pi_i^{(2)}(m,T) = \begin{cases} \pi_{5i}(m,T) = STP1_i(m,T) + RTP_{5i}(T), & 0 < T \leq M_i \leq T_w \\ \pi_{6i}(m,T) = STP1_i(m,T) + RTP_{6i}(T), & M_i < T \leq T_w \\ \pi_{7i}(m,T) = STP2_i(m,T) + RTP_{7i}(T), & M_i < T_w < T, T - T_w \leq M_i \\ \pi_{8i}(m,T) = STP2_i(m,T) + RTP_{8i}(T), & M_i < T_w < T, M_i < T - T_w \end{cases} \tag{45}$$

Now the problem is to determine optimal cycle length T^*, and the optimal number of shipment per production run $m^*$from supplier to retailer, which maximizes the joint total profit per unit time, $\pi_i(m^*, T^*)$, for given $M_i, i = 1,2,\dots,k$. Once the optimal solution is obtained the buyers ordering quantity Q can be determined by Eq. (46).

$$Q = \begin{cases} Q_1, & T \leq T_w \\ Q_2, & T > T_w \end{cases} \tag{46}$$

4. Solution Procedure

We follow the below procedure to find optimal value of shipments m and cycle time T.

$Step$ 1: Set the parameters value and compute T_w using (18).

$Step$ 2: Set $m = 1$

$Step$ 3: For each $i = 1,2, \ldots k$ perform $Step$ 4 or $Step$ 5

$Step$ 4: If $T_w \leq M_i$ then,

$Step$ 4.1: Find T_j which maximizes the profit π_{ji} for $j = 1,2,3,4$ in the corresponding domain of π_{ji}.

$Step$ 4.2: Compute $Q_j = \begin{cases} Q_1, & T_j \leq T_w \\ Q_2, & T_j > T_w \end{cases}$ for $j = 1,2,3,4$.

If $Q_j \geq q_{i+1}$ then set $\pi_{ji}(m, T_j) = 0$.

If $q_i \leq Q_j < q_{i+1}$ then $\pi_{ji}(m, T) = \pi_{ji}(m, T_j)$.

If $Q_j < q_i$then set $Q_j = q_i$ and find T_j using $Q_j = q_i$ then $\pi_{ji}(m, T) = \pi_{ji}(m, T_j)$.

$Step$ 4.3: Set $P_i(m, T) = \max_{j=1,2,3,4} \{\pi_{ji}(m, T)\}$

$Step$ 5: If $T_w > M_i$ then,

$Step$ 5.1:Find T_j which maximizes the profit π_{ji} for $j = 5,6,7,8$ in the corresponding domain.

$Step$ 5.2: Compute $Q_j = \begin{cases} Q_1, & T_j \leq T_w \\ Q_2, & T_j > T_w \end{cases}$for $j = 5,6,7,8$.

If $Q_j \geq q_{i+1}$ then set $\pi_{ji}(m, T_j) = 0$.

If $q_i \leq Q_j < q_{i+1}$ then $\pi_{ji}(m, T) = \pi_{ji}(m, T_j)$.

If $Q_j < q_i$ then set $Q_j = q_i$ and find T_j using $Q_j = q_i$ then $\pi_{ji}(m, T) = \pi_{ji}(m, T_j)$.

$Step$ 5.3: Set $P_i(m, T) = \max_{j=5,6,7,8} \{\pi_{ji}(m, T)\}$

$Step$ 6: Now $\pi(m, T_{(m)}) = \max_{i=1,2 \ldots k} \{P_i(m, T)\}$

$Step$ 7: Increase m by 1 and go to $step$ 3

$Step$ 8: Repeat the above till $\pi(m-1, T_{(m-1)}) < \pi(m, T_{(m)}) > \pi(m+1, T_{(m+1)})$

$Step$ 9: The optimal solution $(m^*, T^*) = (m, T_{(m)})$ is the optimal solution.

5. Numerical Examples

Example 1. To illustrate the solution procedure we consider an inventory system with following data:

$\alpha = 7500$ units, $\beta = 15\%$, $P = 10000$ units/year, $A_r = \$800/\text{order}$, $A_s = \$1500/\text{order}$, $r_{R1} = 3\%$ per unit per annum, $r_{R2} = 5\%$ per unit per annum, $r_s = 1\%$ per unit per annum, $f_0 = \$75/\text{shipment}$, $f_1 = \$0.25$/unit, $c_0 = \$10$, $c_1 = \$2.5 * 10^4$, $c_2 = \$2.5 * 10^{-5}$, $v = \$15$/unit, $s = \$20$/unit, $\rho = 0.5$, $w = 1500$ units, $I_{sp} = 10\%$ per annum, $I_{rp} = 15\%$ per annum, $I_{re} = 20\%$ per annum. The trade credit terms offered by the supplier is listed in the Table 1 as follows,

Table 1
Credit terms for Example 1

Order Quantity (units)	Credit period (years)
$q_1 = 0 \le Q < q_2 = 2500$	$M_1 = 15/365$
$q_2 = 2500 \le Q < q_3 = 4000$	$M_2 = 30/365$
$q_3 = 4000 \le Q$	$M_3 = 45/365$

As shown in Table 2, $T_w = 0.1970$, now apply the solution procedure mentioned above the optimal solution is $(m^*, T^*) = (3, 0.3291)$. As $T > T_w$ the retailer's ordering quantity is $Q^* = Q_2(T^*) = 2500$, the supplier's production quantity is $m^*Q^* = 7500$ and the joint total profit is $\pi^* = \$57210$. Because the optimal order quantity is more than 2500 units the retailer gets a credit period of 30 days. Moreover the quantity ordered is more than the capacity of OW, hence a RW will be required.

Table 2
Solution procedure

m	M_i (Years)	T (Years)	Q Quantity	Profit	
1	$\frac{45}{365}$	0.5223	4000	56576	
2	$\frac{30}{365}$	0.3351	2546	57206	
3	$\frac{30}{365}$	0.3291	2500	57210	max ←
4	$\frac{15}{365}$	0.2393	1818	56873	

Example 2. To see the impact of trade credit period and OW capacity, we keep the values of parameters same as in example 1 except for the values of w and (M_1, M_2, M_3). Table 3 below shows the optimal solution for each value of $w \in \{500, 1000, 1500, 2000, 2500\}$ and $(M_1, M_2, M_3) \in \{(15,30,45), (20,40,60), (30,60,90)\}$. Increasing the OW capacity helps retailer to save his expenses made for RW, resulting increase in joint total profit.

Table 3
Sensitivity analysis on (M_1, M_2, M_3) and w.

(M_1, M_2, M_3) days	w	m^*	T^* Years	Q^*	π^*	RW	Credit Period
(15,30,45)	500	2	0.3721	2846	57359	Yes	M_2
	1000	2	0.3593	2734	57172	Yes	M_2
	1500	3	0.3290	2500	57209	Yes	M_2
	2000	3	0.3278	2500	57503	Yes	M_2
	2500	3	0.3252	2500	58040	No	M_2
(20,40,60)	500	2	0.5173	4000	57642	Yes	M_3
	1000	2	0.3552	2702	57430	Yes	M_2
	1500	3	0.3290	2500	57477	Yes	M_2
	2000	3	0.3278	2500	57772	Yes	M_2
	2500	3	0.3252	2500	58306	No	M_2
(30,60,90)	500	2	0.5173	4000	58563	Yes	M_3
	1000	2	0.5204	4000	58278	Yes	M_3
	1500	2	0.5223	4000	58130	Yes	M_3
	2000	3	0.3278	2500	58397	Yes	M_2
	2500	3	0.3252	2500	58930	No	M_2

6. Sensitivity Analysis

Fig. 5 (a) shows change in cycle time with respect to different inventory parameters by -20%, -10%, 0%, 10% and 20% increase in retailer ordering cost (A_r), supplier holding cost (r_s) and retailer unit selling price (s) increases the cycle time. Whereas increase in scale demand (α), stock dependent parameter (β), retailer holding cost in rented warehouse (r_{R2}), retailer unit purchase cost (v), supplier capacity utilization (ρ), maximum warehouse capacity in own warehouse (w) and retailer capital opportunity loss $\left(I_{rp}\right)$ decreases the cycle time.

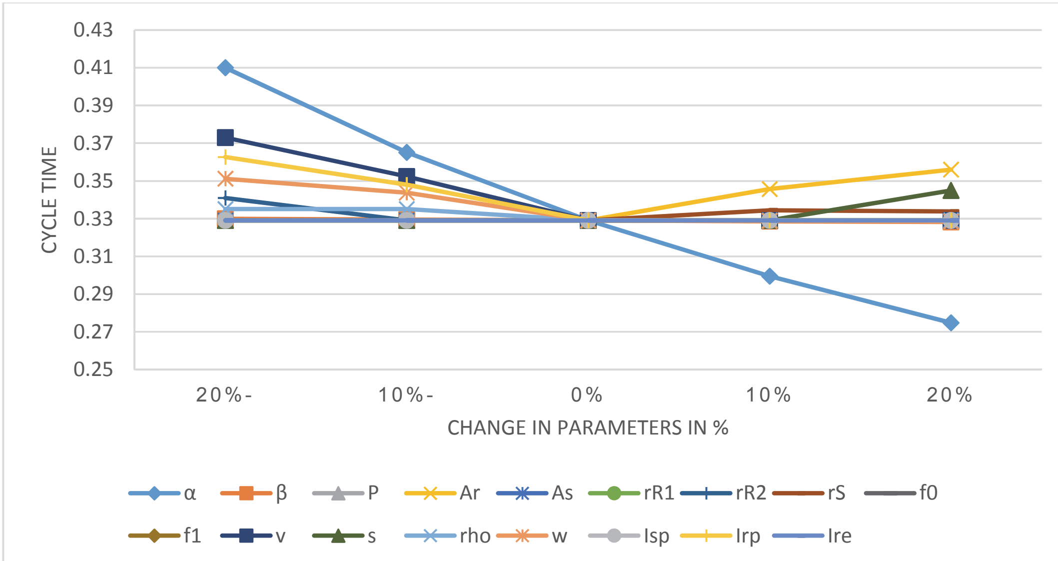

Fig. 5(a). Sensitivity analysis of inventory parameters with respect to cycle time

It is observed that the joint total profit increases with increase in scale demand (α), retailer unit selling price (s). Whereas other parameters show very slight change in total cost as shown in Fig. 5(b).

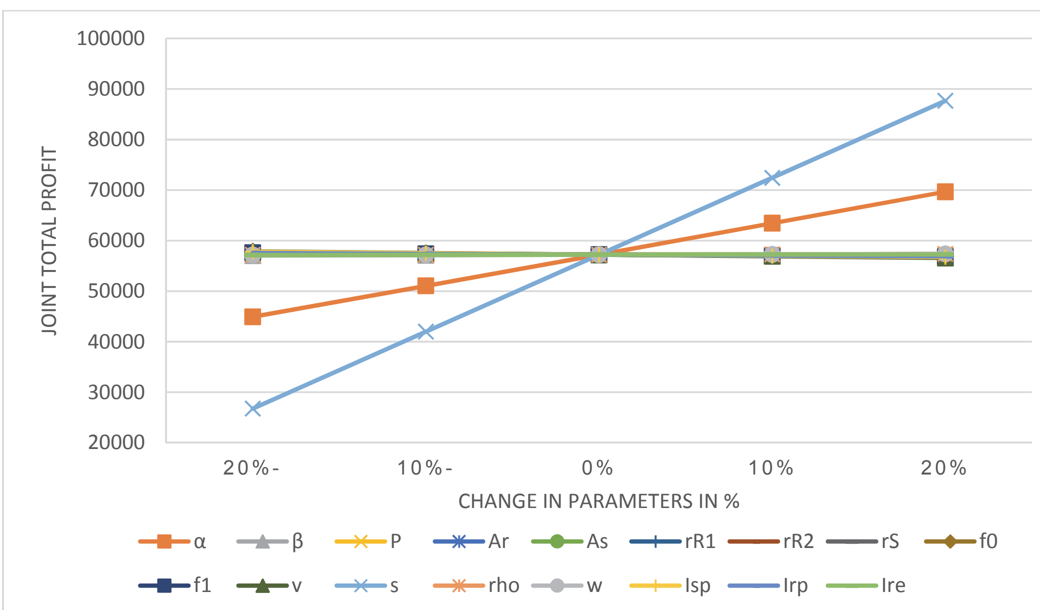

Fig. 5(b) Sensitivity analysis of inventory parameters with respect to joint profit

Fig. 6(a), Fig. 6(b) and Fig. 6(c) show the bar graph for retailer – supplier individual profit as well as joint profit in all three scenarios of credit offered as (15, 30, 45), (20, 40, 60) and (30, 60, 90) respectively.

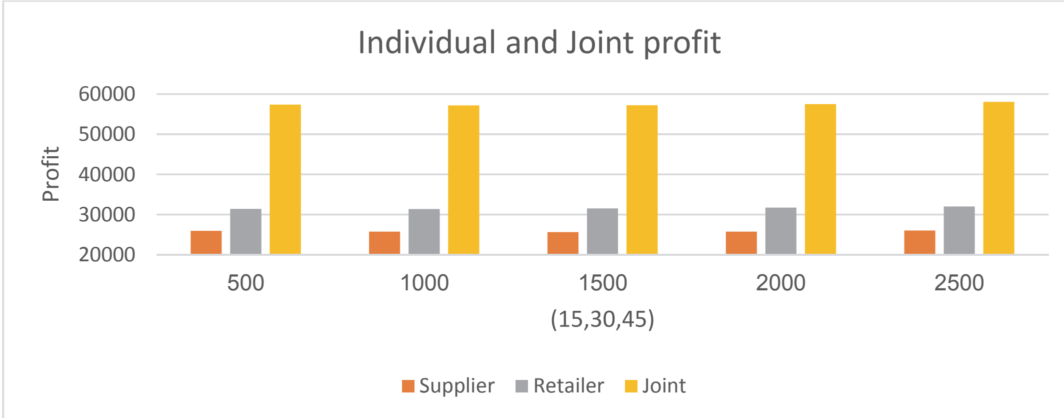

Fig. 6(a) Comparison of individual and joint profit for credit period offered is 15, 30, 45

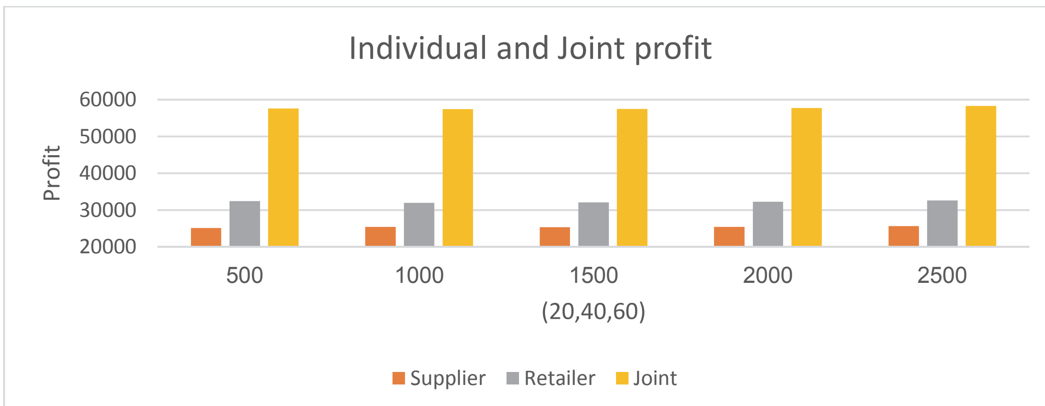

Fig. 6(b) Comparison of individual and joint profit for credit period offered is 20, 40, 60

Fig. 6(c) Comparison of individual and joint profit for credit period offered is 30, 60, 90

7. Conclusion

This paper was an attempt to study the ordering strategies for an integrated inventory model with capacity constraint and order size dependent trade credit. Stock dependent demand and twin ware house system have made this model realistic and applicable to a wide range of items. We have suggested an algorithm to determine optimal order quantity and number of shipments from supplier during the production run which will be benefit both retailer and supplier. Policies were framed in such a way that joint total profit per unit time was maximized with respect to optimal number of shipments and cycle time for which the retailer stock depletes to zero. Order size trade credit works as an effective tool to increase the ordering quantity or to decrease the number of shipments from the supplier. The supplier may consider extending the credit limit or the retailer may expands his warehouse capacity by having a RW, in order to increase the joint total profit. The retailer may order more quantities to have extended credit period which results in reducing number of shipments from the supplier. Further research can be carried out by extending this model with some more realistic form of demand, for imperfect products produced, deteriorating products and two level trade credit.

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