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A multi-production inventory model for deteriorating items considering penalty and environmental pollution cost with failure rework

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A B S T R A C T

Many researchers focused on economic production quantity (EPQ) model for deteriorating items with rework of defective items in a single production run. Few of them proposed the model for multi-production inventory model under the assumptions (i) all the defective items can be reproduced as serviceable items and (ii) all the deteriorating items can be detached from the inventory through screening process. However, in real life manufacturing systems, due to controllable or uncontrollable factors, the defective items may not be reproduced as serviceable items and hence the generation of scrap (disposable) items is potential. It is possible that the internally deteriorating items like fruits, vegetables, eggs, pharmaceutical drugs, etc. pass on to customers which will make negative impact on corporate image in the global market. This paper presents a multi-production run inventory model for deteriorating items with scrap, defective items, failure rework and penalty cost for selling the deteriorating items to customers under finite planning horizon. Our objective is to determine the optimal production run, optimal finite planning horizon by minimizing the total cost of the system. A solution procedure with an algorithm is proposed and a numerical example is provided to demonstrate the applicability of the model. Finally, sensitivity analysis is provided to provide managerial phenomena of the system.

1. Introduction

Nowadays, the facts like deterioration, shortages and remanufacturing of imperfect items are the frequently used terms in the global market. Another major issue, in the worldwide market, is how to reduce environmental pollution. Rework is one of the major issues in reverse logistic and green supply chain because it can decrease production cost and environmental problem. Many researchers focused on developing EPQ model for deteriorating items but few of them developed model for remanufacturing of imperfect items. In this paper, we develop an EPQ model for multi- production runs considering penalty cost for selling the deteriorating items to the customers and remanufacturing of imperfect items to reduce environmental pollution. In reality, production processes are often imperfect so that the imperfect items are generated. For economic and ecological reasons, imperfect quality items are remanufactured to become serviceable items again. Due to inappropriate inventory condition or other causes, the remaining good quality items, stored in an inventory, are deteriorating. In order to provide good service to

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customers, inspection is carried out to screen out imperfect and deteriorating items. However, such inspection may not be perfect (Zhou et al., 2015) and only part of imperfect items manufactured are not fruitfully screened out internally during the production process and passed on to customers, consequently causing defect sales returns and reverse logistics from customers back to the producer. One common source of inspection error is from human factors (Drury, 1978; Drury & Prabhu, 1994). The principal operation strategies and goals of most manufacturing firms have to seek a high satisfaction to customer's demands and also to become a low-cost producer. To reach these goals, the company should be able to effectively utilize the resources and minimize the costs. Rework is very common in semiconductor, pharmaceutical, chemical and food industries. The products are considered as deteriorating items since their utility is lost with storage time due to price reduction, product useful life expiration, decay and spoilage. In our lot sizing model for deteriorated items with rework, both perfect and imperfect items are deteriorating with time.

The remainder of this paper is organized as follows. In section 2, we give a literature review. In section 3, assumptions and notations are given. The mathematical formulation for this model is given in section 4. Solution method of this model is given in section 5. Numerical example and sensitivity analysis are given in section 6, and conclusion is drawn in section 7.

2. Literature Review

Economic Production Quantity (EPQ) model is a prominent research topic in manufacturing, inventory control and management. By using EPQ model, optimal quantity of items and optimal production run time can be obtained. Classical EPQ model was developed under various assumptions. Thereafter, researchers have extended the model by relaxing one or more of the assumptions. It was assumed that the items produced are of perfect quality items in the classical EPQ model. However, in reality, imperfect quality items may be produced. Kumar et al. (2011) presented Economic Production Lot Size (EPLS) model with stochastic demand and shortage with partial backlogging rate under imperfect quality items, in which stochastic imperfect production was assumed. Yassine et al. (2012) considered disaggregating the shipments of imperfect quality items in single production run and aggregating the shipments of imperfect items over multi production runs. Pal et al. (2014) scheduled an EPQ inventory model for ramp type demand with Weibull deterioration under inflation in finite time horizon in crisp and fuzzy environment. It was further extended by Pal et al. (2015) allowing shortages. Mukhopadhyay et al. (2015) investigated an economic production quantity model for three types of imperfect items with rework. Singh et al. (2015) presented a mathematical production inventory model for deteriorating items with time dependent demand rate including the effect of inflation and shortages. Shukla et al. (2016) presented an economic production quantity model with defective products for deteriorating products. Jaggi et al. (2016b) investigated an inventory model for non-instantaneous deteriorating items under inflationary conditions with partially backlogged shortages.

Rezaei et al. (2012) discussed an economic production quantity and purchasing price for items with imperfect quality when inspection shifts from buyer to supplier. Mishra et al. (2013) considered an inventory model for deteriorating items with time-dependent demand and time dependent holding cost under partial backlogging. Barketau et al. (2015) presented a modified EPQ model with deteriorating production system and deteriorating product where rework process was considered at the end of production setup. Chandra et al. (2015) introduced the effect of deterioration on two-warehouse inventory model with imperfect quality items. Kumar et al. (2016) have developed a general inventory model for deteriorating items with probabilistic deterioration rate and ramp type demand under stock dependent consumption rate. Jaggi et al. (2016a) studied an inventory model for a retailer dealing by deteriorating items under inflationary conditions over a fixed planning horizon. A two-warehouse inventory model for deteriorating items with price dependent demand under partial backlogging was discussed by Rastogi et al. (2017).

Remanufacturing process is also an important issue in reverse logistics where used products are remanufactured to reduce total inventory cost, waste and ecological problem. The earliest research that focused on remanufacturing process was done by Schrady (1967). Since then, researches on rework have attracted many researchers. Khouja (2000) planed direct rework for Economic lot Sizing and Delivery Scheduling Problem (ELDSP). Yoo et al. (2009) presented an EPQ model with imperfect production, imperfect inspection and rework. Widyadana et al. (2012) developed an EPQ model for deteriorating items with rework wherein the rework was performed after *m* production setups. Tai (2013) proposed an EPQ model for deteriorating/imperfect products with rework wherein the rework was performed at the end of production setup. Sarkar et al. (2014) assumed rework for single stage production system. Hsu et al. (2014) considered an EPQ model under an imperfect production process with shortages backordered. An EPQ model based on the retailer's stock level was proposed by Kaliraman et al. (2015) wherein it is prescribed that the rate of deterioration is Weibull distribution and the production cost is consisting of raw material cost, labor cost, wear and tear cost and environmental cost. Singh et al. (2014) proposed an economic production model for time dependent demand with rework and multiple production setups where production is depending on the demand. Due to learning effect of employees, holding cost partially constant and partially decreasing in each cycle which is studied in Sangal et al. (2016). Khanna (2017) formulated a strategic production model to study the combined effects of imperfect quality items, inspection error and remanufacturing process under two level trade credits. We notice that not many studies considered a model with multi-manufacturing setups including defective items considering penalty cost, scrap cost and rework. In this paper, we intend at providing analytic results to solve the said above issues.

3. Assumptions and Notations

The inventory model is developed with the following assumptions and notations:

3. 1. Assumptions

1. Production and demand rate are constants.
2. Rework and deterioration rate are constants.
3. Deterioration starts as soon as it comes to the inventory.
4. There is a replacement for deteriorated items.
5. This model is considered under finite time horizon.
6. The lead time is assumed as negligible.
7. The production rate of serviceable items and rework must be greater than the demand rate.
8. No machine breakdown occurs during production run and rework process.
9. The inspection cost is incorporated in the unit production cost.
10. Shortages are not permitted.

3.2. Notations

d	Demand rate (units/unit time).
p	Production rate (units/unit time).
P_r	Rework process rate (units/unit time).
θ	Deterioration rate (units/unit time).
α	Percentage of good quality items.
α_r	Percentage of defective items recovered
γ	Percentage of deteriorated items screened out from the inventory
m	The number of replenishment cycle over $[0, T]$.
p_c	Penalty cost of selling the deteriorated items to the customers ($/unit).

K_s Production setup cost (\$/setup).

r Raw material cost (\$/unit).

l Labor cost (\$/unit).

x Environmental pollution cost (\$/unit).

K_p Production cost (\$/unit) where $K_p = r + \frac{l}{p} + x\sqrt{p}$.

K_r Rework setup cost (\$/setup).

h_s Holding cost of serviceable items (\$/unit/unit time).

h_r Holding cost of imperfect quality items (\$/unit/unit time).

D_c Deteriorating cost (\$/unit/unit time).

s_c Scrap cost (\$/unit).

I_1 Inventory level of serviceable items in a production run.

I_2 Inventory level of serviceable items in a rework-production run.

I_3 Inventory level of serviceable items in a non -production run.

I_s Maximum inventory level of serviceable items at the end of production run.

I_{ms} Maximum inventory level of serviceable items at the end of rework-production run.

I_{mr} Maximum inventory level of imperfect items at the end production run.

T_1 Production run time period (i.e. production uptime), the decision variable.

T_2 Rework-production run time period.

T_3 Regular non-production run time period.

T The planning horizon (decision variable).

TC Total cost of the system per unit time.

4. Formulation of the model

The behavior of the inventory level of serviceable items with m production runs is studied as illustrated in Fig. 1. The production run is performed during T_1 time period. When production run is established, $(1-\alpha)p$ defect products are produced per unit time. T_2 is the rework production run time period. T_3 is non-production run time period.

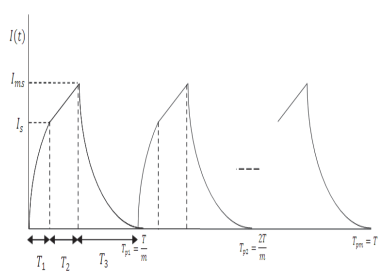

Fig. 1. Graphical representation of the EPQ inventory system

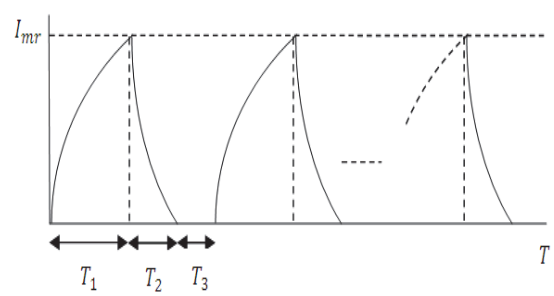

Fig. 2. Graphical representation of the rework-production runs

The inventory level of serviceable items in a production period can be formulated as:

$$\frac{dI_1(t_1)}{dt_1} + \gamma\theta I_1(t_1) = \alpha p - d \qquad 0 \le t_1 \le T_1 \tag{1}$$

Since $I_1(0) = 0$, the inventory level of serviceable items in a production period is:

$$I_1(t_1) = \frac{\alpha p - d}{\gamma\theta}\left[1 - e^{-\gamma\theta t_1}\right] \qquad 0 \le t_1 \le T_1 \tag{2}$$

The inventory level of serviceable items in a rework process is represented as:

$$\frac{dI_2(t_2)}{dt_2} + \gamma\theta I_2(t_{2)} = (\alpha_r p_r - d) \qquad 0 \le t_2 \le T_2 \tag{3}$$

Since $I_2(0) = I_s$, the inventory level of serviceable items in a production run time period is:

$$I_2(t_2) = \left(I_s - \frac{(\alpha_r p_r - d)}{\gamma\theta}\right) e^{-\gamma\theta t_2} + \frac{(\alpha_r p_r - d)}{\gamma\theta} \qquad 0 \le t_2 \le T_2 \tag{4}$$

The inventory level of serviceable item in a non-production period is represented as:

$$\frac{dI_3(t_3)}{dt_3} + \gamma\theta I_3(t_{3)} = -d \qquad 0 \le t_2 \le T_2 \tag{5}$$

Since $I_3(T_3) = 0$, the inventory level of serviceable item in a non-production run time period is:

$$I_3(t_3) = \frac{d}{\gamma\theta}\left[e^{\gamma\theta(T_3 - t_3)} - 1\right] \tag{6}$$

It can be deduced from Eq. (2) that the inventory level of serviceable items when $t_1 = T_1$ is:

$$I_s = \frac{\alpha p - d}{\gamma\theta}\left[1 - e^{-\gamma\theta T_1}\right] \tag{7}$$

The maximum inventory level of serviceable items, when $t_2 = T_2$, from Eq. (4), is

$$I_{ms} = I_s e^{-\gamma\theta T_2} + \frac{(1 - e^{-\gamma\theta T_2})(\alpha_r p_r - d)}{\gamma\theta} \tag{8}$$

The maximum inventory level of imperfect quality items in a production run is given by

$$I_{mr} = (1 - \alpha)pT_1 = p_r T_2 \qquad \text{(See Fig. 2)} \tag{9}$$

Using Eq. (9), we can derive the rework-production run time in terms of T_1:

$$T_2 = \frac{(1 - \alpha)p}{p_r} T_1 \tag{10}$$

The relation between T_i $(i = 1, 2, 3)$ and T is $T = T_1 + T_2 + T_3$. From the relation we can easily find out the non-production run time period (T_3) in terms of T_1 which is given by:

$$T_3 = \frac{p_r T - (p_r + (1 - \alpha)p)}{p_r} T_1 \tag{11}$$

The total cost function for *m* replenishment cycle consists of the deteriorating cost, penalty cost of selling deteriorated items to customers, holding costs of serviceable and imperfect quality items, setup cost, production cost, rework cost and scrap cost. Our objective is to minimize the total cost function per unit time which can be stated as:

$$TC = \left[\begin{array}{c} \frac{Dc}{T/m}\{m(\alpha p - d)T_1 - mI_s\} + \frac{Dc}{T/m}\{m(\alpha_r p_r - d)T_2 - m(I_{ms} - I_s)\} + \frac{Dc}{T/m}\{m(I_{ms} - dT_3)\} \\ + \frac{(1-\gamma)}{\gamma}\left\{\frac{p_c}{T/m}[m(\alpha p - d)T_1 - mI_s] + \frac{p_c}{T/m}[m(\alpha_r p_r - d)T_2 - m(I_{ms} - I_s)] + \frac{p_c}{T/m}[m(I_{ms} - dT_3)]\right\} \\ + \frac{h_s}{T/m}\left\{m\int_0^{T_1} I_1(t_1)dt_1 + m\int_0^{T_2} I_2(t_2)dt_2 + m\int_0^{T_3} I_3(t_3)dt_3\right\} + \frac{h_r}{T/m}\left\{\frac{m(T_1 + T_2)p_r T_2}{2}\right\} + \frac{mk_s}{T/m} \\ + \frac{mk_p}{T/m}pT_1 + \frac{mp_r T_2}{T/m}k_r + \frac{ms_c}{T/m}(1 - \alpha_r)p_r T_2 \end{array}\right] \tag{12}$$

Simplifying the first and second term of TC in Eq. (12), it reduces to

$$\left(\frac{m^2D_c\gamma+m^2(1-\gamma)p_c}{\gamma}\right)\left((\alpha p-d)T_1+(\alpha_r p_r-d)T_2-dT_3\right) \tag{13}$$

Using Eq. (10) and (11) in Eq. (13) which results

$$\frac{\mu(\alpha p-d)T_1}{T}+\frac{\mu(\alpha_r p_r-d)(1-\alpha)pT_1}{Tp_r}+\frac{\mu d(p_r+(1-\alpha)p)T_1}{Tp_r}-\frac{\mu d}{m} \tag{14}$$

where $\mu=\frac{m^2D_c\gamma+m^2(1-\gamma)p_c}{\gamma}$

With Eq. (14), the first and second term of TC in Eq. (12) then becomes

$$TC=\left[\begin{array}{c}\frac{\mu(\alpha p-d)T_1}{T}+\frac{\mu(\alpha_r p_r-d)(1-\alpha)pT_1}{Tp_r}+\frac{\mu d(p_r+(1-\alpha)p)T_1}{Tp_r}-\frac{\mu d}{m}\\ +\frac{h_s}{T/m}\left\{m\int_0^{T_1}\mathrm{I}_1(\mathrm{t}_1)dt_1+m\int_0^{T_2}\mathrm{I}_2(\mathrm{t}_2)dt_2+m\int_0^{T_3}\mathrm{I}_3(\mathrm{t}_3)dt_3\right\}+\frac{h_r}{T/m}\left\{\frac{m(T_1+T_2)p_rT_2}{2}\right\}\\ +\frac{mk_s}{T/m}+\frac{mk_p}{T/m}pT_1+\frac{mp_rT_2}{T/m}k_r+\frac{ms_c}{T/m}(1-\alpha_r)p_rT_2\end{array}\right] \tag{15}$$

It can be shown that the second term of TC in Eq. (15) as

$$\frac{m^2h_s}{T}\left\{\frac{(\alpha p-d)}{2}T_1^2+(\alpha p-d)T_1T_2+\frac{(\alpha_r p_r-d)}{2}T_2^2+\frac{d}{2}T_3^2\right\}+\frac{h_r}{T}\left\{\frac{m^2(T_1+T_2)p_rT_2}{2}\right\}$$

The derivation of the above term can be found in the Appendix.

Using Eq. (10) and Eq. (11) in the above equation which gives:

$$\left[\begin{array}{c}\frac{m^2h_s(\alpha p-d)T_1^2}{2T}+\frac{m^2h_s(\alpha p-d)(1-\alpha)pT_1^2}{Tp_r}+\frac{m^2h_s(\alpha_r p_r-d)(1-\alpha)^2p^2T_1^2}{2Tp_r^2}+\frac{h_sdT}{2}\\ +\frac{m^2h_sd(p_r+(1-\alpha)p)^2T_1^2}{2Tp_r^2}-\frac{mh_sd(p_r+(1-\alpha)p)T_1}{p_r}+\frac{m^2h_r(1-\alpha)pT_1^2}{2T}+\frac{m^2h_r(1-\alpha)^2p^2T_1^2}{2Tp_r}\end{array}\right] \tag{16}$$

Finally, we can express $TC(T_1,\ T)$ in Eq. (15) in terms of T and T_1 only:

$$TC(T_1,T)=\left[\begin{array}{c}\frac{\mu(\alpha p-d)T_1}{T}+\frac{\mu(\alpha_r p_r-d)(1-\alpha)pT_1}{Tp_r}+\frac{\mu d(p_r+(1-\alpha)p)T_1}{Tp_r}-\frac{\mu d}{m}\\ \frac{m^2h_s(\alpha p-d)T_1^2}{2T}+\frac{m^2h_s(\alpha p-d)(1-\alpha)pT_1^2}{Tp_r}+\frac{m^2h_s(\alpha_r p_r-d)(1-\alpha)^2p^2T_1^2}{2Tp_r^2}+\frac{h_sdT}{2}\\ +\frac{m^2h_sd(p_r+(1-\alpha)p)^2T_1^2}{2Tp_r^2}+\frac{mh_sd(p_r+(\alpha-1)p)T_1}{p_r}+\frac{m^2h_r(1-\alpha)pT_1^2}{2T}+\frac{m^2h_r(1-\alpha)^2p^2T_1^2}{2Tp_r}\\ \frac{m^2k_s}{T}+\frac{pm^2k_pT_1}{T}+\frac{m^2k_r(1-\alpha)pT_1}{T}+\frac{m^2s_c(1-\alpha)(1-\alpha_r)pT_1}{T}\end{array}\right] \tag{17}$$

To make our solution procedure trouble-free, we write the above Eq. (17) as:

$$TC(T_1,T)=AT+\ BT_1+\frac{C}{T}+D\frac{T_1}{T}+E\frac{T_1^2}{T}+F. \tag{18}$$

where $A=\frac{h_sd}{2}>0$,

$B=\frac{mh_sd(p_r+(\alpha-1)p)}{p_r}<0$,

$C=m^2k_s>0$

$$D = \begin{bmatrix} \mu(\alpha p - d) + \frac{\mu(\alpha_r p_r - d)(1-\alpha)p}{p_r} + \frac{\mu d(p_r + (1-\alpha)p)}{p_r} + pm^2 k_p \\ +m^2 k_r (1-\alpha)p + m^2 s_c (1-\alpha)(1-\alpha_r)p \end{bmatrix} > 0$$

$$E = \begin{bmatrix} \frac{m^2 h_s(\alpha p - d)}{2} + \frac{m^2 h_s(\alpha p - d)(1-\alpha)p}{p_r} + \frac{m^2 h_s(\alpha_r p_r - d)(1-\alpha)^2 p^2}{2p_r^2} \\ + \frac{m^2 h_s d(p_r + (1-\alpha)p)^2}{2p_r^2} + \frac{m^2 h_r (1-\alpha)p}{2} + \frac{m^2 h_r (1-\alpha)^2 p^2}{2p_r} \end{bmatrix} > 0 \text{ and}$$

$$F = -\frac{\mu d}{T} < 0$$

5. Solution method

In order to obtain the optimal solution of the model, a proof of the convexity of the objective function $TC(T_1, T)$ is provided. An optimization technique using partial derivatives is carried out to derive the optimal solutions.

Theorem 1. The objective function $TC(T_1, T)$ in (17) is strictly convex.

Proof: (See Appendix)

The optimal pair exists if $4CE > D^2$, $4AE > D^2$, $B < 0$ and $B\sqrt{4CE - D^2} > D\sqrt{4AE - D^2}$ and is given by

$$(T_1^*, T^*) = \left(\frac{-1}{2E}\left(B\sqrt{\frac{4CE - D^2}{4AE - B^2}} + D \right), \sqrt{\frac{4CE - D^2}{4AE - B^2}} \right) \tag{19}$$

Algorithm

Step 1: Start.

Step 2: From the Eq. (18), find the values of A, B, C, D, E and F by initialising the values of the parameters $p, \alpha, \theta, d, r, x, k_r, s_c, \alpha_r, k_s, D_c, \gamma, p_c, h_s, h_r, l$ and p_r.

Step 3: From step 2, choose one set of values of A, B, C, D and E satifying the condition: $4CE > D^2$, $4AE > D^2$, $B < 0$ and $B\sqrt{4CE - D^2} > D\sqrt{4AE - D^2}$.

Step 4: Substitute the values of A, B, C, D and E, obtained in step 3, in Eq. (19) and calculate T_1 and T.

Step 5: Substitute the values of A, B, C, D, E, F, T_1 and T, obtained in step 2 and 4, in equation (18) and calculate the total inventory cost (TC).

Step 6: If the values of C, D and E, obtained in step 2, satisfy the conditions $\frac{2E}{T} > 0$ and $\frac{1}{T^4}(2EC - 2DET_1 - 2E^2T_1^2 - D^2) > 0$, then the corresponding values of T_1, $T$and TC obtained in step 4 and 5, are taken as the optimal production run time (denoted by T_1^*) , optimal finite planing horizon time (denoted by T^*) and minimum total inventory cost (denoted by TC^*) respectively. Otherwise go to setp 2 and choose another set of values of A, B, C, D and E.

Step 6: Repeat the steps 2 to 6 until we get T_1^*, T^*, and TC^*.

Step 7: End.

6. Numerical example and sensitivity analysis

In this section, a numerical example and sensitivity analysis are given to illustrate the model.

6.1 Numerical example

Consider an inventory model with the following data $p = 4000$, $\alpha = 0.8$, $\theta = 0.1$, $d = 1545$, $r = \$\,0.001$, $x = \$\,0.00035$, $k_r = \$\,0.5$, $s_c = \$\,2$, $\alpha_r = 0.9$, $k_s = \$\,150$, $D_c = \$\,10$, $\gamma = 0.7$, $p_c = \$\,3$, $h_s = \$\,100$, $h_r = \$\,2$, $l = \$\,100$ and $p_r = 2000$.
We follow from Eq. (18) that the optimal pair is given by

$(T_1^*, T^*) = (0.0209, 0.3107)$

From Eq. (10) and (11) we can obtain rework process time and non-production time which is given by

$T_2^* = 0.0083$.

$T_3^* = 0.0329$.

From Eq. (7), the maximum inventory level of serviceable items during production run period:

$I_s \approx 35$ units

From Eq. (8), we get the maximum inventory level of serviceable items:

$I_{ms} \approx 51$ units

From Eq. (9), we get the maximum inventory level of recoverable items during production run period:

$I_{mr} \approx 17$ units.

The economic production quantities per production run:

$Q^* = pT_1^* \approx 83$ units.

The optimal total inventory cost per unit time is found to be

$TC^* = \$\,25394.76$

6.2. Sensitivity analyzes

In this section, we discuss the sensitivity analysis with the variation of different parameters. We now study about the effects of changes in the values of the parameters k_r, s_c, h_s and k_s on the optimal production time, rework process time, non-production time, total inventory cost and economic production quantity based on our numerical results. We change one parameter at a time keeping the other parameters unchanged. The results of sensitivity analysis are summarized in Table 1 to 4. The following managerial phenomena can be made from the results obtained.

(i) When rework setup cost k_r increases/decreases, the optimal run time values of T_1, T_2 and T decreases/increases, the optimal time value of T_3 increases/decreases. The economic order quantity is reduced when the parameter k_r increases. The total inventory cost of the manufacturer is increased when the parameter k_r is increased. It is observed that as the length of the cycle time decreases (as the cycle time is reduced) the order quantity is also decreased with respect to the setup cost.

(ii) If the scrap cost is increased, then the optimal run time values of T_1, T_2 and T are moderately sensitive but order quantity is decreased while total inventory cost per unit time of the manufacturer is increased. It is obvious that total average cost increases when setup cost increases.

(iii) The holding cost of serviceable items is highly sensitive to the production run time, rework process time and cycle length. When the parameter h_s decreases, T_1, T_2, T_3, T and optimum quantity Q increases while the total inventory cost is decreased. It is obvious that total cost per unit time is decreased/increased when holding cost of the serviceable items is decreased/ increased.

(iv) When the setup cost is increasing/decreasing, the production run time, rework production run time, non-production time, replenishment cycle length, economic order quantity and total inventory cost are increasing/decreasing.

Table 1
Sensitivity analysis of rework setup cost k_r on optimal values of T_1, T_2, T_3, T, Q and TC

k_r	T_1	T_2	T_3	T	Q	TC
0.5	0.02087	0.00835	0.03293	0.31072	83	25394.76
1	0.02016	0.00806	0.03379	0.31005	81	26055.67
1.5	0.01943	0.00777	0.03461	0.30910	78	26695.16
2	0.01869	0.00748	0.03541	0.30788	75	27313.03
2.5	0.01792	0.00717	0.03618	0.30637	72	27909.02
3	0.01713	0.00685	0.03693	0.30458	69	28482.82

Table 2
Sensitivity analysis of scrap cost s_c on optimal values of T_1, T_2, T_3, T, Q and TC

s_c	T_1	T_2	T_3	T	Q	TC
2	0.02087	0.00835	0.03293	0.31072	83	25394.76
4	0.02073	0.00829	0.03310	0.31060	83	25528.65
6	0.02030	0.00812	0.03362	0.31020	81	25925.19
8	0.02002	0.00801	0.03395	0.30988	80	26185.29
10	0.01973	0.00789	0.03429	0.30951	79	26441.94
12	0.01943	0.00777	0.03461	0.30910	78	26695.16

Table 3
Sensitivity analysis of holding cost h_s on optimal values of T_1, T_2, T_3, T, Q and TC.

h_s	T_1	T_2	T_3	T	Q	TC
100	0.02087	0.00835	0.03293	0.31072	83	25394.76
90	0.02190	0.00876	0.03481	0.32740	88	24160.08
80	0.02312	0.00925	0.03705	0.34709	92	22854.61
70	0.02456	0.00983	0.03977	0.37082	98	21464.52
60	0.02633	0.01053	0.04318	0.40021	105	19970.87
50	0.02855	0.01142	0.04761	0.43790	114	18346.31

Table 4
Sensitivity analysis of setup cost k_s on optimal values of T_1, T_2, T_3, T, Q and TC.

k_s	T_1	T_2	T_3	T	Q	TC
150	0.02087	0.00835	0.03293	0.31072	83	25394.76
140	0.02011	0.00805	0.03187	0.30015	80	24576.24
130	0.01933	0.00773	0.03077	0.28919	77	23727.83
120	0.01852	0.00741	0.02963	0.27780	74	22845.97
110	0.01767	0.00707	0.02844	0.26592	71	21926.38
100	0.01679	0.00672	0.02719	0.25349	67	20963.75

The following results are made from Fig. 3 to 6. The optimal production period is sensitive to changes in k_r, h_s and moderately sensitive with other parameters. The optimal total cost per unit time tends to increase when the value of the parameters increase. The rework production run time increases when h_s decreases, decreases when k_r and s_c increase while k_s decreases. The non-production run time T_3 is increasing when holding cost, scrap cost and rework setup cost increased. Hence the manufacturer has to keep the inventory having small quantities. The economic quantity is highly sensitive to k_r, k_s

and h_s and moderately sensitive with s_c. The order quantity increases when h_s decrease and decrease when k_r and k_s increase while k_s decrease. The total inventory cost per unit time increases when k_s and s_c increase and decreases when h_s and k_s decrease.

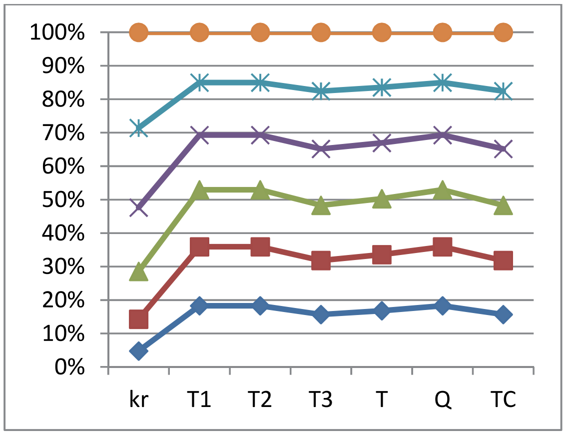

Fig. 3. Changes in optimal values in percentage with respect to k_r

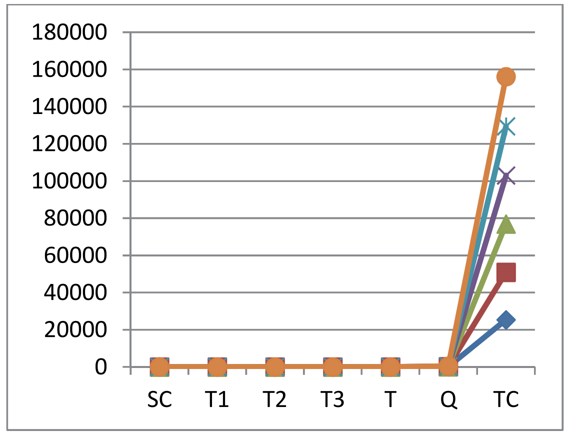

Fig. 4. Total cost with respect to optimal values and s_c

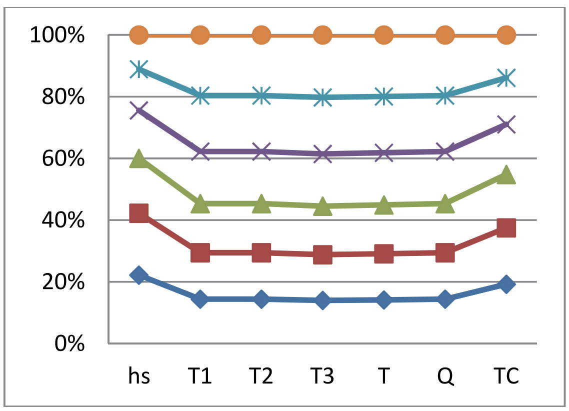

Fig. 5. Changes in optimal values in percentage with respect to h_s

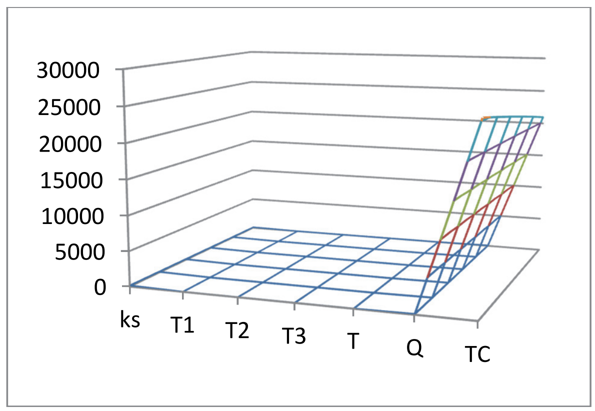

Fig. 6. Total cost with respect to optimal values and k_s

7. Conclusion

In this model, we have studied the inventory production system for defective and deteriorating items in which the production process contains multi-production setups and in each production setup, rework production setup is developed. Production setup cost consists of raw material cost, labor cost and environmental pollution cost. Since the production process is imperfect, the defective items are produced. The defective items are reworked as original quality items and the items are passed to customers. This EPQ inventory model investigates optimal replenishment quantity, optimal number of replenishment , optimal production run time and optimal finite planing horizon time. An algorithm is presented for deriving the optimal values. The sensitivity analysis shows that the production cost, setup cost and holding cost of serviceable items are much affected in the proposed model. This paper presents an inventory model of direct application to the venture that consider the fact that the storage item is deteriorated during storage periods and defective items are produced during production period.

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Appendix

Proof of Theorem 1.

Differentiating $TC(T_1, T) = AT + BT_1 + \frac{C}{T} + D\frac{T_1}{T} + E\frac{T_1^2}{T} + F$ with respect to T_1 and T we get

$$\frac{\partial TC}{\partial T_1} = B + \frac{D}{T} + \frac{2ET_1}{T}$$

$$\frac{\partial TC}{\partial T} = A - \frac{C}{T^2} - \frac{DT_1}{T^2} - E\frac{T_1^2}{T^2}$$

Solving $\frac{\partial TC}{\partial T_1} = 0$ and $\frac{\partial TC}{\partial T} = 0$ simultaneously, one can get the optimal pair (T_1, T) which is given by

$$(T_1^*, T^*) = \left(\frac{-1}{2E}\left(B\sqrt{\frac{4CE - D^2}{4AE - B^2}} + D\right), \sqrt{\frac{4CE - D^2}{4AE - B^2}}\right) \tag{19}$$

Since we must have $T_1^* > 0$ and $T^* > 0$, we get

$4CE > D^2$, $4AE > D^2$, $B < 0$ and $B\sqrt{4CE - D^2} > D\sqrt{4AE - D^2}$.

The Hessian matrix is given by

$$H = \begin{pmatrix} \frac{2E}{T} & -\left(\frac{D}{T^2} + \frac{2ET_1}{T^2}\right) \\ -\left(\frac{D}{T^2} + \frac{2ET_1}{T^2}\right) & \frac{C}{T^3} + \frac{DT_1}{T^3} + \frac{ET_1^2}{T^3} \end{pmatrix}$$

Since $\det(H_1) = \frac{2E}{T} > 0$ and $\det(H_2) = \frac{1}{T^4}(2EC - 2DET_1 - 2E^2T_1^2 - D^2) > 0$, H is positive definite and hence the objective function $TC(T_1, T)$ is strictly convex.

Derivation of second term of TC in Eq. (15).

$$\frac{h_s}{T/m}\left\{m\int_0^{T_1} \mathrm{I}_1(\mathrm{t}_1)dt_1 + m\int_0^{T_2} \mathrm{I}_2(\mathrm{t}_2)dt_2 + m\int_0^{T_3} \mathrm{I}_3(\mathrm{t}_3)dt_3\right\} + \frac{h_r}{T/m}\left\{\frac{m(T_1+T_2)p_rT_2}{2}\right\}$$

Using the equations (2), (4) and (6) in the above term, one can obtain the following:

$$= \frac{m^2h_s}{T}\left[\begin{array}{c} \int_0^{T_1} \frac{(\alpha p-d)}{\gamma\theta}\left[1 - e^{-\gamma\theta t_1}\right]dt_1 \\ + \int_0^{T_2}\left[\left(I_s - \frac{(\alpha_r p_r - d)}{\gamma\theta}\right)e^{-\gamma\theta t_2} + \frac{(\alpha_r p_r - d)}{\gamma\theta}\right]dt_2 \\ + \int_0^{T_3}\frac{d}{\gamma\theta}\left[e^{\gamma\theta(T_3 - t_3)} - 1\right]dt_3 \end{array}\right] + \frac{m^2h_r}{T}\left\{\frac{(T_1+T_2)p_rT_2}{2}\right\}$$

$$= \frac{m^2h_s}{T}\left[\begin{array}{l} \frac{(\alpha p-d)}{\gamma\theta}\int_0^{T_1}\left[1 - e^{-\gamma\theta t_1}\right]dt_1 \\ + \left(I_s - \frac{(\alpha_r p_r - d)}{\gamma\theta}\right)\int_0^{T_2} e^{-\gamma\theta t_2}\,dt_2 + \frac{(\alpha_r p_r - d)}{\gamma\theta}\int_0^{T_2} dt_2 \\ + \frac{d}{\gamma\theta}\int_0^{T_3}\left[e^{\gamma\theta(T_3 - t_3)} - 1\right]dt_3 \end{array}\right] + \frac{m^2h_r}{T}\left\{\frac{(T_1+T_2)p_rT_2}{2}\right\}$$

$$= \frac{m^2h_s}{T}\left[\begin{array}{l} \frac{(\alpha p-d)}{\gamma\theta}\left[t_1 + \frac{e^{-\gamma\theta t_1}}{\gamma\theta}\right]_0^{T_1} \\ + \left(I_s - \frac{(\alpha_r p_r - d)}{\gamma\theta}\right)\left[\frac{e^{-\gamma\theta t_2}}{-\gamma\theta}\right]_0^{T_2} + \frac{(\alpha_r p_r - d)}{\gamma\theta}T_2 \\ - \frac{d}{\gamma\theta}\left[\frac{e^{\gamma\theta T_3}\,e^{-\gamma\theta t_3}}{\gamma\theta} + t_3\right]_0^{T_3} \end{array}\right] + \frac{m^2h_r}{T}\left\{\frac{(T_1+T_2)p_rT_2}{2}\right\}$$

$$= \frac{m^2h_s}{T}\left[\begin{array}{l} \frac{(\alpha p-d)}{\gamma\theta}\left[T_1 + \frac{e^{-\gamma\theta T_1}}{\gamma\theta} - \frac{1}{\gamma\theta}\right] \\ + \left(I_s - \frac{(\alpha_r p_r - d)}{\gamma\theta}\right)\left[\frac{e^{-\gamma\theta T_2}}{-\gamma\theta} + \frac{1}{\gamma\theta}\right] + \frac{(\alpha_r p_r - d)}{\gamma\theta}T_2 \\ - \frac{d}{\gamma\theta}\left[\frac{1}{\gamma\theta} + T_3 - \frac{e^{\gamma\theta T_3}}{\gamma\theta}\right] \end{array}\right] + \frac{m^2h_r}{T}\left\{\frac{(T_1+T_2)p_rT_2}{2}\right\}$$

We use the Taylor series approximation under the assumptions that $\gamma\theta T_1, \gamma\theta T_2$ and $\gamma\theta T_3$ are small. We espouse that $e^x \approx 1 + x + \frac{x^2}{2}$. (see Tai (2013))

$$= \frac{m^2 h_s}{T}\left[\begin{array}{c} \frac{(\alpha p-d)}{\gamma\theta}\left[T_1+\frac{\left(1-\gamma\theta T_1+\frac{(\gamma\theta T_1)^2}{2}\right)}{\gamma\theta}-\frac{1}{\gamma\theta}\right] \\ +\left(I_s-\frac{(\alpha_r p_r-d)}{\gamma\theta}\right)\left[\frac{\left(1-\gamma\theta T_2+\frac{(\gamma\theta T_2)^2}{2}\right)}{-\gamma\theta}+\frac{1}{\gamma\theta}\right]+\frac{(\alpha_r p_r-d)}{\gamma\theta}T_2 \\ -\frac{d}{\gamma\theta}\left[\frac{1}{\gamma\theta}+T_3-\frac{\left(1+\gamma\theta T_3+\frac{(\gamma\theta T_3)^2}{2}\right)}{\gamma\theta}\right] \end{array}\right]+\frac{m^2 h_r}{T}\left\{\frac{(T_1+T_2)p_r T_2}{2}\right\}$$

$$= \frac{m^2 h_s}{T}\left[\frac{(\alpha p-d)T_1^2}{2}+\left(I_s-\frac{(\alpha_r p_r-d)}{\gamma\theta}\right)\left[T_2-\frac{\gamma\theta T_2^2}{2}\right]+\frac{(\alpha_r p_r-d)}{\gamma\theta}T_2+\frac{dT_3^2}{2}\right]+\frac{m^2 h_r}{T}\left\{\frac{(T_1+T_2)p_r T_2}{2}\right\}$$

Using $e^x \approx 1 + x + \frac{x^2}{2}$ in Eq. (7), we can get $I_s = (\alpha p - d)\left(T_1 - \frac{\gamma\theta T_1^2}{2}\right)$

$$= \frac{m^2 h_s}{T}\left[\frac{(\alpha p-d)T_1^2}{2}+\left((\alpha p-d)\left(T_1-\frac{\gamma\theta T_1^2}{2}\right)-\frac{(\alpha_r p_r-d)}{\gamma\theta}\right)\left[T_2-\frac{\gamma\theta T_2^2}{2}\right]+\frac{(\alpha_r p_r-d)}{\gamma\theta}T_2+\frac{dT_3^2}{2}\right]+\frac{m^2 h_r}{T}\left\{\frac{(T_1+T_2)p_r T_2}{2}\right\}$$

$$= \frac{m^2 h_s}{T}\left[\frac{(\alpha p-d)T_1^2}{2}+(\alpha p-d)\left(T_1-\frac{\gamma\theta T_1^2}{2}\right)\left(T_2-\frac{\gamma\theta T_2^2}{2}\right)+\frac{(\alpha_r p_r-d)T_2^2}{2}+\frac{dT_3^2}{2}\right]+\frac{m^2 h_r}{T}\left\{\frac{(T_1+T_2)p_r T_2}{2}\right\}$$

Since $\left(1-\frac{\gamma\theta T_1}{2}\right)\left(1-\frac{\gamma\theta T_2}{2}\right) \approx 1$, the above term can be simplified as:

$$= \frac{m^2 h_s}{T}\left[\frac{(\alpha p-d)T_1^2}{2}+(\alpha p-d)T_1 T_2+\frac{(\alpha_r p_r-d)T_2^2}{2}+\frac{dT_3^2}{2}\right]+\frac{m^2 h_r}{T}\left\{\frac{(T_1+T_2)p_r T_2}{2}\right\} \blacksquare$$