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A study on inventory model with negative exponential demand and probabilistic deterioration under backlogging

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A B S T R A C T

The present study is an attempt to develop an inventory model for deteriorating items with negative exponential demand. Shortages are allowed with partial back logging. This model is different from the existing models where deterioration is a function of time. Accordingly, three different types of probabilistic deterioration functions have been considered to find the associated decision variables and also to make comparisons among them. The optimality is illustrated with numerical values of system parameters and the graphical representations are given to depict the trend. The necessary observations in obtaining optimal values of decision variables are analyzed in the light of the practical aspect of the developed model. Finally, considering the numerical values of system parameters, sensitivity analyses are carried out to study the effect of changes in most important system parameters.

1. Introduction

Inventory problems are almost common to all spheres of industrial as well as business houses. It has become an area of research to academicians and practitioners. Considering its usefulness in real world problems, particularly in business and industry, researchers have taken an interest and extended their thoughts in obtaining optimal solutions for different decision variables under considerations for inventory problems. In the theory of inventory control, models are developed by considering the parameters like demand, deterioration, allowing shortages and without shortages. Among the developed models in inventory control, models with deterioration have significantly drawn the interest of researchers. In this regards, a number of researchers have put forward different models under different assumptions. Ghare and Schrader (1963) first discussed the effect of deterioration. Shah and Pandey (2009) developed an inventory model with time dependent deterioration when demand depends on displayed stock level and frequency of advertisement via media. Sarkar and Sarkar (2013) also developed an inventory model with time dependent deterioration. Bhunia and Shiakh (2011) discussed an inventory model for deteriorating items. In their model deterioration was considered to follow Weibull distribution. Sarkar and Sarkar (2013) discussed EMQ model for deteriorating items. In this study, deterioration was taken as probabilistic. Further Sarkar (2013) developed production inventory

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model for deteriorating items in a two-echelon supply chain management and applied algebraic approach to estimate the minimum cost. Palanivel et al. (2015) developed an inventory model for deteriorating items under permissible delay in payment and the rate of deterioration as probabilistic towards minimizing the total cost. Palanivel and Uthayakumar (2014) developed an economic production quantity model with inflation and time value of money for probabilistic deteriorating items. Singh and Pattnayak (2013) presented an inventory model with trapezoidal type demand rate and Weibull distribution deterioration. Kumar and Rajput (2016) also considered deterioration rate as probabilistic. Latha and Uthayakumar (2014) in their developed model considered different types of continuous deterioration function with shortages and partial back logging. Basu and Sinha (2007) discussed inventory model with Weibull deterioration and demand rate as a linear function of time. Dave and Patel (1981) considered deterioration as a constant fraction of on hand inventory. By taking linear trend in demand under two different assumptions on backlogging, Goswami and Choudhuri (1992) discussed inventory model. Goyal and Giri (2001) presented a review of advances in inventory modelling for deteriorating items. Wee and Law (1999) in their study used discounted cash-flow approach for deteriorating items in order to obtain optimal production and pricing policy to maximize profit. Manna and Choudhuri (2006) developed inventory model for deteriorating items with demand rate as ramp type function of time. Khurana and Chakraborty (2016) gave optimal ordering and pricing policy for deteriorating items incorporating price and stock sensitivity demand and partial back logging during shortages. A generalized dynamic programming for inventory items with Weibull distributed deterioration proposed by Chen (1998) with assumption of time proportional demand and shortages under influence of inflation and time discounting. Wang and Chen(2001) extended the model of Hariga (1995) by taking finite rates of production and deterioration, demand was allowed to very over finite planning horizon with variable replenishment cycle. Shah (1998) made an attempt to develop discrete-time probabilistic inventory model for deteriorating items to determine optimal cycle time when vendor announces price escalation from a future date. Cheng et al.(2011) proposed an optimal replenishment policy for inventory model with demand rate as trapezoidal function of time and allowing partial back logging. Chou and Julian (2015) studied the inventory model of Ouyang et al. (2013) and provided new solution procedure to show that interior minimum exist and unique and pointed out that there are two local minimums on the boundaries. Liao and Huang (2010) developed inventory model for deteriorating items to obtain optimal cycle time under permissible delay in payment and constraints on ware house capacity. Lee and Dye (2012) formulated inventory model with stock dependent demand and technology cost as one of the decision variables under the assumption of shortages and partial backlogging depending on the length of waiting time for next replenishment. The contribution of Tripati and Kumar (2014), Bhunia et al. (2015) for deteriorating items is significant. However, inventory model with probabilistic deterioration and negative exponential demand has not been reported yet.

In our present investigation through inventory modelling, the attempt has been made to study the effect on optimality when deterioration is considered as different probability functions and then to make comparisons.

2. Model formulation

To develop the proposed inventory model, the following assumptions are made:

2.1. Assumptions

i) Demand rate is exponentially declining.
ii) Shortages are allowed and partially backlogged.
iii) Holding cost is independent of time.
iv) Deterioration is time proportional.
v) There is no repair of deteriorating items.
vi) Lead time is zero.

2.2. Notations

i) $h = a \quad (a > 0)$ holding cost per unit per unit time

ii) $\theta(t) = \theta t$, θ is the rate of deterioration and $0 < \theta < 1$

iii) $D(t) = Ae^{-\alpha t}$

iv) C_0 is the ordering cost.

v) C_s is the shortage cost per unit per unit time.

vi) C is the unit cost of the item

vii) β is the backlogging rate , $0 \le \beta < 1$

viii) T is the cycle length

ix) I_0 is the maximum inventory level in $(0, T)$

x) S lost sale cost per unit.

3. Formulation and Solution of the model

Based on the above assumptions, the differential equation representing the inventory situation during the interval [0, t_1] is

$$\frac{dI_1(t)}{dt} + \theta t I_1(t) = -Ae^{-\alpha t}, \quad 0 \le t \le t_1, \text{ with } \ I_1(0) = I_{0,} \ I_1(t_1) = 0 . \tag{1}$$

We solve Eq. (1) by the series solution method. Let

$$I_1(t) = \sum_{n=0}^{\infty} d_n t^n \tag{1a}$$

Differentiating w. r. t. t,

$$\frac{dI_1(t)}{dt} = \sum_{n=1}^{\infty} n d_n t^{n-1} \tag{1b}$$

Using Eq. (1a) and Eq. (1b) in Eq. (1) we have

$$\sum_{n=1}^{\infty} n d_n t^{n-1} + \theta t \sum_{n=0}^{\infty} d_n t^n = -A \sum_{n=0}^{\infty} \frac{(-\alpha)^n t^n}{n!} \tag{1c}$$

Equating the coefficients of like powers of t,

$$d_1 = -A$$

$$d_2 = A\frac{\alpha}{2!} - \frac{\theta}{2} d_0$$

$$d_3 = -A\frac{\alpha^2}{3!} + \frac{\theta}{3} A$$

$$d_4 = A\frac{\alpha^3}{4!} - \frac{\theta}{4} A \frac{\alpha}{2!} + \frac{\theta^2}{8} d_0$$

$$d_5 = -A\frac{\alpha^4}{5!} + \frac{\theta}{5} A \frac{\alpha^2}{3!} - \frac{\theta^2}{15} A$$

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Putting d_i's in (1a) and neglecting the terms containing θ^2

$$I_1(t) = d_0 + A\left(-t + \frac{\alpha t^2}{2!} - \frac{\alpha^2 t^3}{3!} - \cdots\cdots\right) - \frac{\theta}{2} d_0 + A\left(\frac{\theta}{3} t^3 + \frac{\theta}{4}\left(\frac{\alpha}{2!}\right) t^4 + \frac{\theta}{5}\left(\frac{\alpha^2}{3!}\right) t^5 + \cdots\cdots\right) \tag{1d}$$

Since $I_1(0) = I_0$, Eq. (1d) gives

$$d_0 = \frac{2I_0}{(2-\theta)}$$

Therefore, after simplification Eq. (1d) can be written as

$$I_1(t) = I_0 + \frac{A}{\alpha}\left(e^{-\alpha t} - 1\right) + A\theta \sum_{n=3}^{\infty} \frac{\alpha^{n-3} t^n}{(n-2)!\,n}. \tag{1e}$$

The rate of change of the inventory during the shortage period [t_1, T] is governed by the differential equation

$$\frac{dI_2(t)}{dt} = -A\beta e^{-\alpha t}, t_1 \le t \le T_1, I_2(t_1) = 0 \tag{2}$$

$$I_2(t) = \frac{A\beta}{\alpha}\left(e^{-\alpha t} - e^{-\alpha t_1}\right) \tag{3}$$

Holding cost is given by

$$HC = \int_0^{t_1} h(t) I_1(t) dt = I_0 t_1 a - \frac{aA}{\alpha^2}\left(e^{-\alpha t_1} - 1 + \alpha t_1\right) + Aa\theta \sum_{n=3}^{\infty} \frac{\alpha^{n-3} t_1^{\,n+1}}{(n-2)!\,n(n+1)}. \tag{4}$$

Shortage cost during the period [t_1, T] is given by

$$SC = -C_s \int_{t_1}^{T} I_2(t) dt = -C_s \frac{A\beta}{\alpha}\left[\frac{\left(e^{-\alpha t_1} - e^{-\alpha T}\right)}{\alpha} + e^{-\alpha t_1}\left(t_1 - T\right)\right]. \tag{5}$$

Lost sale cost during the shortage period is calculated as

$$LSC = S\int_{t_1}^{T} \left(1-\beta\right) D(t) dt \quad = S\int_{t_1}^{T} \left(1-\beta\right) A e^{-\alpha t} dt \ = \frac{SA(1-\beta)}{\alpha}\left(e^{-\alpha t_1} - e^{-\alpha T}\right). \tag{6}$$

The purchase cost is

$$PC = C\left[I_0 + \int_{t_1}^{T} \beta D(t) dt\right] = C\left[I_0 + \int_{t_1}^{T} \beta A e^{-\alpha t} dt\right] = CI_0 + \frac{C\beta A}{\alpha}\left(e^{-\alpha t_1} - e^{-\alpha T}\right). \tag{7}$$

Deterioration cost is

$$DC = C\left[I_0 - \int_0^{t_1} D(t) dt\right] = CI_0 + \frac{CA}{\alpha}\left(e^{-\alpha t_1} - 1\right). \tag{8}$$

Therefore, the total inventory cost is given by

$$\begin{aligned} TC = OC + PC + HC + SC + LSC + DC = C_0 + \left[CI_0 + \frac{C\beta A}{\alpha}\left(e^{-\alpha t_1} - e^{-\alpha T}\right)\right] \\ + \left[I_0 t_1 a - \frac{aA}{\alpha^2}\left(e^{-\alpha t_1} - 1 + \alpha t_1\right) + Aa\theta \sum_{n=3}^{\infty} \frac{\alpha^{n-3} t_1^{\,n+1}}{(n-2)!\,n(n+1)}\right] \\ - C_s \frac{A\beta}{\alpha}\left[\frac{\left(e^{-\alpha t_1} - e^{-\alpha T}\right)}{\alpha} + e^{-\alpha t_1}\left(t_1 - T\right)\right] + \frac{SA(1-\beta)}{\alpha}\left(e^{-\alpha t_1} - e^{-\alpha T}\right) + \left[CI_0 + \frac{CA}{\alpha}\left(e^{-\alpha t_1} - 1\right)\right]. \end{aligned} \tag{9}$$

Our aim is to minimize total inventory cost corresponding to decision variables. In order to show the total cost is minimum, it is essential to test the convexity of the cost function defined by Eq. (9). Differentiating Eq. (9) partially w.r.t. t_1 and T, we get

$$\frac{\partial(TC)}{\partial t_1} = e^{-\alpha t_1}\left[\frac{aA}{\alpha} - C\beta A - SA(1-\beta) + C_s A\beta\left(t_1 - T\right) - CA\right] + I_0 a + \frac{aA}{\alpha} + Aa\theta\sum_{n=3}^{\infty}\frac{\alpha^{n-3}t_1^{\,n}}{(n-2)!n} \tag{10}$$

$$\frac{\partial^2(TC)}{\partial t_1^{\,2}} = e^{-\alpha t_1}\left[C\beta A\alpha - aA + SA\alpha(1-\beta) + C_s A\beta + CA\alpha\right] - C_s A\beta\alpha\left(t_1 - T\right)e^{-\alpha t_1} + Aa\theta\sum_{n=3}^{\infty}\frac{\alpha^{n-3}t_1^{\,n-1}}{(n-2)!} \tag{11}$$

$$\frac{\partial(TC)}{\partial T} = e^{-\alpha T}\left[C\beta A - \frac{C_s A\beta}{\alpha} + SA(1-\beta)\right] + \frac{C_s A\beta}{\alpha}e^{-\alpha t_1} \tag{12}$$

$$\frac{\partial^2(TC)}{\partial T^2} = e^{-\alpha T}\left[C\beta A\alpha + C_s A\beta + SA\alpha(1-\beta)\right] \tag{13}$$

$$\frac{\partial^2(TC)}{\partial T\partial t_1} = \frac{\partial^2(TC)}{\partial t_1\partial T} = -C_s A\beta e^{-\alpha t_1} \tag{14}$$

Solving Eq. (12), $\frac{\partial(TC)}{\partial T} = 0$ yields,

$$e^{-\alpha T}\left[C\beta A - \frac{C_s A\beta}{\alpha} + SA(1-\beta)\right] + \frac{C_s A\beta}{\alpha}e^{-\alpha t_1} = 0 \Rightarrow t_1 = T - \frac{1}{\alpha}\log\left\{1 - R_0\right\}$$

$\log\left\{1-R_0\right\}$ is valid for $\left|R_0\right| < 1$, Where $R_0 = \frac{\alpha}{C_s}\left\{C + \frac{S(1-\beta)}{\beta}\right\}$

Hessian matrix is

$$H(T,t_1) = \begin{bmatrix} \frac{\partial^2(TC)}{\partial t_1^{\,2}} & \frac{\partial^2(TC)}{\partial t_1\partial T} \\ \frac{\partial^2(TC)}{\partial T\partial t_1} & \frac{\partial^2(TC)}{\partial T^2} \end{bmatrix}$$

$$D_3 = \begin{vmatrix} \frac{\partial^2(TC)}{\partial t_1^{\,2}} & \frac{\partial^2(TC)}{\partial t_1\partial T} \\ \frac{\partial^2(TC)}{\partial T\partial t_1} & \frac{\partial^2(TC)}{\partial T^2} \end{vmatrix} = \left(\frac{\partial^2(TC)}{\partial t_1^{\,2}}\right)\left(\frac{\partial^2(TC)}{\partial T^2}\right) - \left(\frac{\partial^2(TC)}{\partial t_1\partial T}\right)^2$$

$$= \left[e^{-\alpha t_1}\left\{C\beta A\alpha - aA + SA\alpha(1-\beta) + C_s A\beta + CA\alpha\right\} - C_s A\beta\alpha\left(t_1 - T\right)e^{-\alpha t_1} + Aa\theta\sum_{n=3}^{\infty}\frac{\alpha^{n-3}t_1^{\,n-1}}{(n-2)!}\right] * \left[e^{-\alpha T}\left\{C\beta A\alpha + C_s A\beta + SA\alpha(1-\beta)\right\}\right] - C_s^{\,2}A^2\beta^2 e^{-2\alpha t_1}$$

Using t_1 in the above expression, we get,

$D_3 > 0$

Hence, $\frac{\partial^2(TC)}{\partial t_1^{\,2}} = D_1 > 0$, $\left(\frac{\partial^2(TC)}{\partial T^2}\right) = D_2 > 0$ and $D_3 > 0$.

As all the principal minor determinants of Hessian matrix are positive, therefore TC is a convex function and attains global minima.

4. Numerical Illustration

All the calculations are performed in MATLAB. For illustration, the following numerical values of different parameters are taken as input for numerical and graphical analysis of the model, A =100, θ =0.12, α =0.001, β= 0.3, a=2, C=50, c_s=5, S=10, C_0=100, I_0=500.

Here $|R_0| = 0.014 < 1$

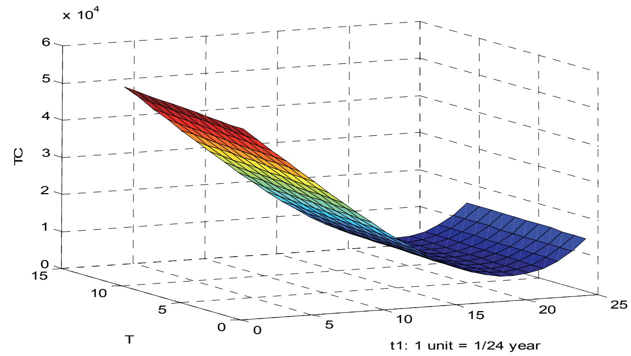

Fig. 1. The plot for the optiaml soution

From the above, we obtain optimal values ast_1=0. 833year and total cost (TC) =Rs5590

Case -I

When θ follows uniform Distribution, the following numerical values are taken for numerical and graphical representation of the model, A =100, θ_1=0.12, θ_2=0.62, α =0.001, β= 0.3, a=2, C=50, c_s =5, S=10, C_0=100, I_0=500.

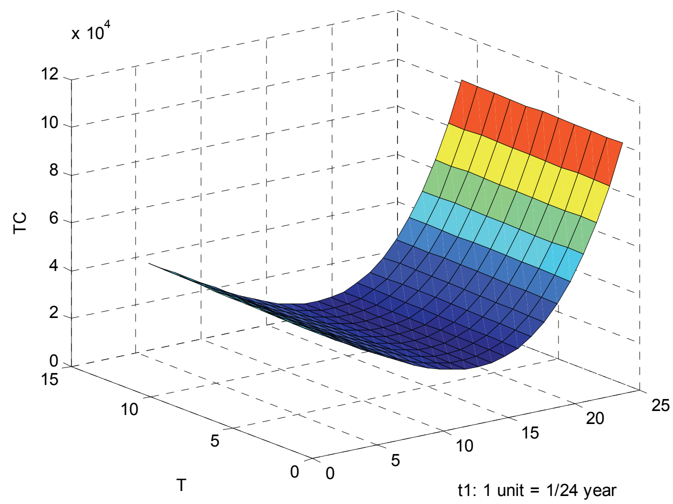

Fig. 2. The optimal solution

Here the optimal values ast_1=0. 5833 yrs and total cost (TC) =Rs 19920

Case -II

When θ follows triangular Distribution, the following numerical values are taken for numerical and graphical representation of the model, A =100, θ_1=0.12, θ_2=0.32, θ_3 =0.62, α =0.001, β= 0.3, a=2, C=50, c_s=5, S=10, C_0=100, I_0=500.

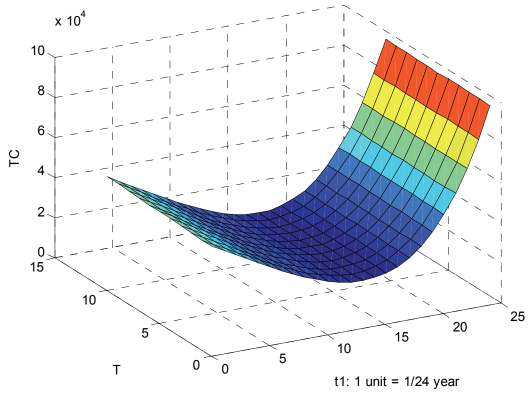

Fig. 3. The optimal solution (Case II)

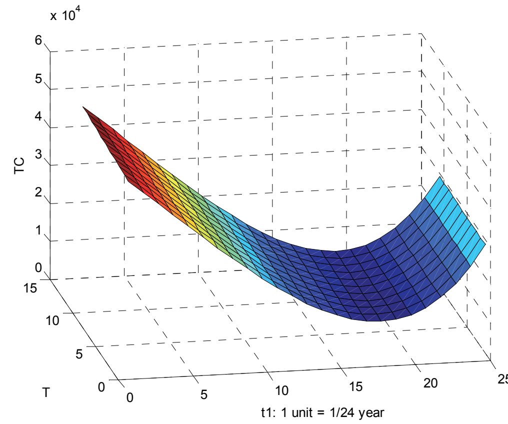

Fig. 4. The optimal solution (Case III)

The optimal values as t1=0.5833years and total cost (TC) =Rs 19420.

Case -III

When θ follows Beta Distribution, the following numerical values are taken for numerical and graphical representation of the model, A =100, α_θ=0.12, β_θ =0.62, α =0.001, β= 0.3, a=2, C=50, c_s =5, S=10, C_0=100, I_0=500.

The optimal values as t1=0. 7500 years and the total cost (TC) =Rs 9987

From the numerical examples, Figs. (1-4) show the convexity of the function. Among all the optimal solutions obtained in three different cases when deterioration is considered probabilistic, the better optimal solution is obtained in case-III.

5. Comparison between three different cases of the model under three different distributions:

The comparison among three different cases of the developed model when deterioration follows three different distributions is given in the following figure.

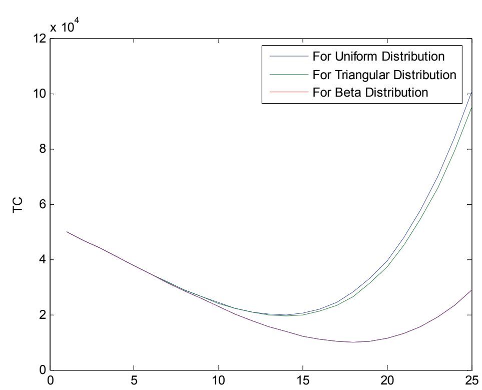

Fig. 5. Total cost with respect to different distributions

6. Sensitivity Analysis

On the basis of the results obtained in figures (2-4), we perform the sensitivity analysis of key parametersθ,α and β as below.

6.1. Sensitivity analysis for θ :

For positive change in the value of the parameter θ, the new value of θ is evaluated as $\theta = \theta + (1-\theta) * (P\%)$, where, P is the percentage of positive change and for negative change in the value of the parameter θ, the new value of θ is evaluated as $\theta = \theta - \theta * (N\%)$, where, N is the percentage of negative change.

6.1.1. Under uniform distribution for θ

A=100, α=0.001, β= 0.3, a=2, C=50, c_s=5, S=10, C_0=100, I_0=500

Table 1

The results of sensitivity analysis under uniform distribution for θ

	θ_1	θ_2	TC (Rs)	t_1(years)
+50%	0.56	0.810	25710	0.4583
+25%	0.34	0.715	23410	0.5000
0%	0.12	0.62	19920	0.5833
-25%	0.09	0.465	16770	0.6520
-50%	0.06	0.31	11780	0.7083

6.1.2. Under triangular Distribution for θ :

A=100, α=0.001, β= 0.3, a=2, C=50, c_s=5, S=10, C_0=100, I_0=500

Table 2

The results of sensitivity analysis under triangular distribution for θ

	θ_1	θ_2	θ_3	TC (Rs)	t_1(years)
+50%	0.56	0.66	0.81	25620	0.4583
+25%	0.34	0.49	0.715	23220	0.5000
0%	0.12	0.32	0.62	19420	0.5833
-25%	0.09	0.24	0.465	16270	0.6250
-50%	0.06	0.16	0.31	11210	0.7083

6.1.2. Under beta Distribution for θ

A=100, α=0.001, β= 0.3, a=2, C=50, c_s=5, S=10, C_0=100, I_0=500

Table 3

The results of sensitivity analysis under beta distribution for θ

	α_θ	β_θ	TC (Rs)	t_1(years)
+50%	0.56	0.81	20970	0.5416
+25%	0.34	0.715	18500	0.5833
0%	0.12	0.62	9987	0.7500
-25%	0.09	0.465	9987	0.7500
-50%	0.06	0.31	9987	0.7500

6.2. Sensitivity analysis for α :

Following similar procedure described in 6.1, we perform sensitivity analysis for α .

6.2.1. Under uniform distribution for α :

A=100, θ_1=0.12, θ_2=0.62, β= 0.3, a=2, C=50, c_s=5, S=10, C_0=100, I_0=500

Table 4

The results of sensitivity analysis under uniform distribution for α

	α	TC (Rs)	t_1(years)
+50%	0.0015	20000	0.5833
+25%	0.0013	19920	0.5833
0%	0.001	19920	0.5833
-25%	0.00075	19880	0.5833
-50%	0.0005	19840	0.5833

6.2.2. Under triangular distribution for :

A=100, θ_1=0.12, θ_2=0.32, θ_3 =0.62, β= 0.3, a=2, C=50, c_s=5, S=10, C_0=100, I_0=500

Table 5

The results of sensitivity analysis under triangular distribution for α

	α	TC(Rs)	t_1(years)
+50%	0.0015	19510	0.5833
+25%	0.0013	19470	0.5833
0%	0.001	19420	0.5833
-25%	0.00075	19380	0.5833
-50%	0.0005	19340	0.5833

6.2.3. Under beta distribution for

A=100, α_θ=0.12, β_θ =0.62, β= 0.3, a=2, C=50, c_s=5, S=10, C_0=100, I_0=500

Table 6

The results of sensitivity analysis under beta distribution for α

	α	TC (Rs)	t_1(years)
+50%	0.0015	10130	0.7500
+25%	0.0013	10070	0.7500
0%	0.001	9987	0.7500
-25%	0.00075	9916	0.7500
-50%	0.0005	9845	0.7500

6.3.Sensitivity analysis for β

Following the procedure mentioned in 6.1, we perform the sensitivity analysis for β.

6.3.1.Under uniform distribution for β

A=100, θ_1=0.12, θ_2=0.62, α =0.001, a=2, C=50, c_s=5, S=10, C_0=100, I_0=500

Table 7

The results of sensitivity analysis under uniform distribution for β

	β	TC (Rs)	t_1(years)
+50%	0.4500	17610	0.5833
+25%	0.3750	18770	0.5833
0%	0.3	19920	0.5833
-25%	0.2250	21080	0.5833
-50%	0.1500	22230	0.5833

6.3.2. Under triangular distribution for β :

A=100, θ_1=0.12, θ_2=0.32, θ_3 =0.62, α =0.001, a=2, C=50, c_s=5, S=10, C_0=100, I_0=500

Table 8

The results of sensitivity analysis under triangular distribution for β

	β	TC (Rs)	t_1(years)
+50%	0.4500	17110	0.5833
+25%	0.3750	18270	0.5833
0%	0.3	19420	0.5833
-25%	0.2250	20580	0.5833
-50%	0.1500	21730	0.5833

6.3.3. Under beta distribution for β :

A=100, α_θ=0.12, β_θ =0.62, α =0.001, β= 0.3, a=2, C=50, c_s=5, S=10, C_0=100, I_0=500

Table 9

The results of sensitivity analysis under beta distribution for β

	β	TC (Rs)	Time(years)
+50%	0.4500	7603	0.7500
+25%	0.3750	8795	0.7500
0%	0.3	9987	0.7500
-25%	0.2250	11180	0.7500
-50%	0.1500	12370	0.7500

Sensitivity analysis of the parameters is discussed in Tables (1-9). On the basis of the sensitivity analysis, the following observations are made.

From Table 1, when the deterioration rate follows uniform distribution and other parameters remain same, it is seen that with increase in the rate of deterioration, total inventory cost increases and occurrence shortage is earlier. The reverse event is observed with the negative change in deterioration rate. Thus, the total cost is reduced in case of negative change in the deterioration. Table 2 and Table 3, when deterioration follows triangular and beta distribution, respectively, show similar events in case of total cost and time of occurrence of shortages. From Table 4, when we make positive or negative changes in scaling parameter under the uniform distribution keeping other system parameters with the same values, total cost increases for positive changes and decreases for negative changes but there is no change in occurrence of shortages. Table 5 and Table 6 discuss sensitivity analysis for scaling parameter when deterioration follows triangular and beta distributions, respectively. It is also observed that with positive change, total cost increases and with negative change, total cost decreases with no change in occurrence of shortages. Further, it is also observed that the occurrence of shortages remains the same under triangular and uniform distributions, however Table 6 shows that the occurrence of shortage is a bit later than that of the observed in Table 4 and Table 5.

Table 7, Table 8, and Table 9 light on sensitivity analysis when backlogging rate is changed in both positive and negative directions. The positive change in the value of backlogging rate, total cost decreases and negative change in the value of backlogging rate, total cost increases. In both the cases of change in backlogging rate, there is no effect on the occurrence of shortages.

7. Conclusion

The proposed model considering probabilistic deterioration was an extension of existing models in literature where demand was not considered as negative exponential. It is seen that total cost, minimizes when deterioration follows the beta distribution. So far as author's knowledge is concerned, such type of model has not been reported in literature. The developed model can be further extended by taking complete backlogging, different type of demand like stock-dependent demand, ramp-type demand etc. and also for two warehouses.

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