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An EOQ model with quantity incentive strategy for deteriorating items and partial backlogging

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A B S T R A C T

Quantity discount is a usual term in business and has been a topic of interest for a long time, but have received very little attention. Many vendors offer different schemes to their customers to increase the existing size of order, which results in higher annual sale for the vendor and a lower purchasing price for the retailer. Therefore, the buyer adjusts his/her selling price, which influences the demand for the product. The objective of this paper is to develop an inventory model for deteriorating products with quantity discount and partial backlogging to determine the optimal ordering quantity for the retailer optimizing the total cost or profit of the associated model. A numerical example is also given to illustrate the model and its significant features.

1. Introduction

For the deterioration of inventory there are so many reasons such as the spoilage, obsolescence, damage, decrement in usefulness, etc. There are so many models available which considered the deteriorating products. Wee (1995) presented a joint pricing and replenishment inventory policy for deteriorating items with constant rate of deterioration. Wu et al. (2006) developed an inventory model for the determination of optimal replenishment policy for non-instantaneous deteriorating items and stock dependent demand. In this model, backlogging rate is considered as a function of waiting time. Singh and Singh (2009) considered a production inventory model with variable demand rate for deteriorating items under permissible delay in payments. Tayal et al. (2014) presented a multi item inventory model for deteriorating items with expiration date and allowable shortages and after this Tayal et al. (2015) introduced an EPQ model for non–instantaneous deteriorating item with time dependent holding cost and exponential demand rate. Thus we can say that the existence of literature for deteriorating inventory is vast but in most of all developed models, during stock out the occurring shortages are assumed to be completely backlogged or completely lost. Both of these conditions during stock out do not reflect the reality. In general, the occurring demand during shortages is partially backlogged/partially lost. Montgomery et al. (1973) considered an inventory model for non-perishable products and allowable shortages. They assumed the partial backlogging of occurring shortages in their

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model. Chang and Dye (1999) introduced an inventory model for deteriorating items allowing the partial backlogging of unsatisfied demand in their model. Singh and Rastogi (2013) developed an integrated inventory model with amelioration and deterioration under shortages and inflation. Singh and Singhal (2008) presented a two warehouse partial backlogging inventory model for perishable products having exponential demand rate. Tayal et al. (2014) developed a production inventory model for deteriorating items with the problem of space restriction and backlogging of occurring shortages. Tayal et al. (2015) introduced an integrated production inventory model for perishable products with trade credit period and investment in preservation technology with partial backlogging of occurring shortages.

Generally, in the development of inventory models, demand rate is assumed to be constant or the function of time. There are very few models which are developed with price sensitive demand. In real life situations selling price is a major factor affecting the demand. Hsu et al. (2007) presented an inventory model for deteriorating items with expiration date and uncertain lead time having price sensitive demand. Singh et al. (2011) considered a soft computing based inventory model with deterioration and price dependent demand. Tayal et al. (2014) introduced a two echelon supply chain model for deteriorating items with effective investment in preservation technology. In this model the demand for the products is assumed to be a function of price and season. Shastri et al (2014) developed a supply chain management for two level trade credit financing with selling price dependent demand under the effect of preservation technology. Tayal et al. (2015) introduced an inventory model for deteriorating items with seasonal products and an option of an alternative market. In this model the demand rate for the products is taken as a function of selling price and season.

Inventory models with quantity discount and price break are common in practice but got a very little attention from researchers. Wee (1999) developed an inventory model for deteriorating products with quantity discount, pricing and partial backordering. Papachristos and Skouri (2003) generalized the model of Wee (1999) by time dependent rate of backlogging. Chang et al. (2006) presented an inventory model on the single item multi-supplier system with variable lead-time, price-quantity discount, and resource constraints.

In this present paper, we combine all above mentioned factors to make this study more realistic. The model is developed for deteriorating items where rate of deterioration is a time dependent function. The demand rate is taken as a function of selling price. The shortages are allowed and partially backlogged. The different price breaks are mentioned and the retailer chooses that policy for purchasing quantity and price that maximize the unit time profit of the system. A numerical example and sensitivity analysis are shown to illustrate the model and its significant features.

2.1 Assumptions

1. Demand rate for the products is taken as a function of selling price.
2. The products considered in this model are deteriorating in nature and rate of deterioration is a linear function of time.
3. The shortages are allowed and partially backlogged.
4. The backlogging rate is considered as a constant fraction of occurring shortages.
5. During a given cycle the deteriorated items are neither repaired nor replaced.
6. Lead time is not considered in this model.
7. The model is developed with all-units quantity discount and the material cost per unit is defined as:

$$c_i = \begin{cases} c_1 & 1 \le q_1 < b_1 \\ c_2 & b_1 \le q_2 < b_2 \\ c_3 & b_2 \ge q_3 \end{cases}$$

Here $c_1 > c_2 > c_3$ and b_1, b_2, b_3 denote the incremental boundaries of quantity.

2.2 Notations

c_i	purchasing cost per unit
p	selling price per unit
q_i	ordering quantity
θ	deterioration coefficient, θ<<<1
α	positive demand coefficient
β	demand coefficient, β>1
T	cycle length
v	the time at which inventory level becomes zero
Q_1	initial inventory level at the beginning of each cycle
Q_2	backordered quantity
h	holding cost per unit
s	shortage cost per unit
l	lost sale cost per unit
O	ordering cost per order
η	rate of backlogging
R (v, p)	sales revenue per replenishment cycle

3. Mathematical Modeling

The behavior of the inventory level during a given cycle T is depicted in Fig. 1. The differential equations showing the fluctuation of inventory with time t are shown as below:

$$\frac{dI_1(t)}{dt} = -\theta t I_1(t) - \frac{\alpha}{p^\beta} \quad 0 \le t \le v\,, \tag{1}$$

$$\frac{dI_2(t)}{dt} = -\frac{\alpha}{p^\beta} \quad v \le t \le T\,. \tag{2}$$

With boundary condition:

$$I_1(v) = I_2(v) = 0 \tag{3}$$

The solutions of these equations are given by:

$$I_1(t) = \frac{\alpha}{p^\beta}[(v-t) + \frac{\theta}{6}(v^3 - t^3)]e^{-\frac{\theta t^2}{2}} \quad 0 \le t \le v \tag{4}$$

$$I_2(t) = \frac{\alpha}{p^\beta}(v-t) \qquad\qquad v \le t \le T \tag{5}$$

Now if F(v,p) is the unit time profit function then:

$$F(v,p) = \frac{1}{T}[\text{Sales revenue} - \text{purchasing cost} - \text{deterioration cost} - \text{shortage cost} \tag{6}$$
$$\text{-Lost sale cost} - \text{holding cost} - \text{ordering cost}]$$

3.1.1 Sales Revenue

Sales revenue = $(Q_1+Q_2)p$ (7)

where

$$Q_1 = \frac{\alpha}{p^\beta}(v + \frac{\theta}{6}v^3) \tag{8}$$

$$Q_2 = \int_v^T \frac{\alpha}{p^\beta}\eta\, dt \tag{9}$$

$$Q_2 = \frac{\alpha}{p^\beta}\eta(T - v)$$

$$R(v, p) = \{\frac{\alpha}{p^\beta}(v + \frac{\theta}{6}v^3) + \frac{\alpha}{p^\beta}\eta(T - v)\}p \tag{10}$$

3.1.2 Purchasing cost

The purchasing cost of the system is given by

$$P.C. = \{\frac{\alpha}{p^\beta}(v + \frac{\theta}{6}v^3) + \frac{\alpha}{p^\beta}\eta(T - v)\}c_i \tag{11}$$

3.1.3 Deterioration Cost

The cost of deterioration for the system is given by

$$D.C. = \{I_1(0) - \int_0^v \frac{\alpha}{p^\beta}dt\}c_i = \{\frac{\alpha}{p^\beta}(v + \frac{\theta}{6}v^3) - \frac{\alpha}{p^\beta}v\}c_i \tag{12}$$

3.1.4 Holding Cost

The holding cost is given by

$$H.C. = h\int_0^v I_1(t)dt = \frac{h\alpha}{p^\beta}(\frac{v^2}{2} + \frac{\theta}{12}v^4) \tag{13}$$

3.1.5 Shortage cost

The shortage cost is given by

$$S.C. = s\int_v^T \frac{\alpha}{p^\beta}dt = \frac{s\alpha}{p^\beta} \tag{14}$$

3.1.6 Lost Sale Cost

The lost sale cost for the system can be calculated as

$$L.S.C. = l\int_v^T (1-\eta)\frac{\alpha}{p^\beta}dt = \frac{l\alpha}{p^\beta}(1-\eta)(T - v) \tag{15}$$

3.1.7 Ordering Cost

The ordering cost per order

$$O.C. = O \tag{16}$$

3.2.1 Unit time Profit

The unit profit for the system can be calculated as follows

$$F(v, p) = \frac{1}{T}[R(v, p) - P.C.(v, p) - D.C.(v, p) - H.C.(v, p) - S.C.(v, p) - L.S.C.(v, p) - O.C.] \tag{17}$$

$$F(v,p)=\frac{1}{T}[\{\frac{\alpha}{p^{\beta}}(v+\frac{\theta}{6}v^{3})+\frac{\alpha}{p^{\beta}}\eta(T-v)\}(p-c_i)-\{\frac{\alpha}{p^{\beta}}(v+\frac{\theta}{6}v^{3})-\frac{\alpha}{p^{\beta}}v\}c_i$$
$$-\frac{h\alpha}{p^{\beta}}(\frac{v^2}{2}+\frac{\theta}{12}v^4)-\frac{s\alpha}{p^{\beta}}-\frac{l\alpha}{p^{\beta}}(1-\eta)(T-v)-O] \tag{18}$$

3.2.2 Solution Procedure

We know that unit time profit is a function of two variables v and p. To find out the optimal solution we obtain the partial derivatives

$$\frac{\partial F(v,p)}{\partial v}=0 \quad \frac{\partial F(v,p)}{\partial p}=0 \tag{19}$$

After solving these equations simultaneously we find the optimal values of v and p.

4. Numerical Illustration

T=50 days, θ=0.05, α=500 units, β=1.5, η=0.6, s=6 rs/unit, l=15 rs/unit, O=500 rs/order h=0.2 rs/unit

$$c_i = \begin{cases} 25\, rs/unit & 1 \le q_1 < 500 \\ 24 rs/unit & 500 \le q_2 < 1000 \\ 22 rs/unit & q_3 \ge 1000 \end{cases}$$

After solving this model corresponding to these values we have $q_2^* \cong 981$units.

But $F(v,p)_{q_2^*} < F(v,p)_{b_2}$. So $F(v,p)_{b_2}$ is the optimal value of unit time profit and b_2 will be the optimal ordering quantity.

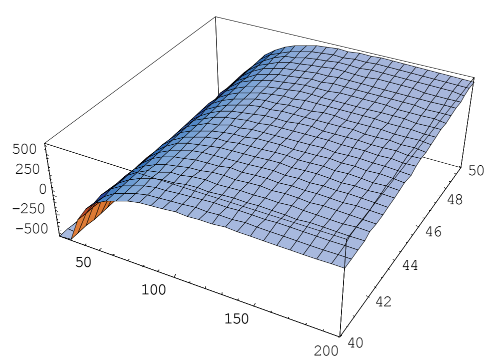

Fig. 2. Concavity of the Unit time profit function

Sensitivity Analysis

Table 1
Sensitivity analysis with the variation in θ

% variation in θ	θ	v	p	F(v,p)
-20%	0.04	44.1686	53.5055	35.1418
-15%	0.0425	44.1892	53.531	36.5359
-10%	0.045	44.2074	53.5537	37.929
-5%	0.0475	44.2238	53.5739	39.3189
0%	0.05	44.2384	53.5921	40.708
5%	0.0525	44.2517	53.6086	42.0968
10%	0.055	44.2638	53.6235	43.4827
15%	0.0575	44.2748	53.6371	44.8679
20%	0.06	44.2849	53.6496	46.2531

Table 2
Sensitivity analysis with the variation in α

% variation in α	α	v	p	F(v,p)
-20%	400	44.2384	53.5921	30.5664
-15%	425	44.2384	53.5921	33.1018
-10%	450	44.2384	53.5921	35.6372
-5%	475	44.2384	53.5921	38.1726
0%	500	44.2384	53.5921	40.708
5%	525	44.2384	53.5921	43.2434
10%	550	44.2384	53.5921	45.7788
15%	575	44.2384	53.5921	48.3142
20%	600	44.2384	53.5921	50.8496

Table 3
Sensitivity analysis with the variation in β

% variation in β	β	v	p	F(v,p)
-20%	1.2	44.2384	53.5921	157.42
-15%	1.275	44.2384	53.5921	114.201
-10%	1.35	44.2384	53.5921	82.1386
-5%	1.425	44.2384	53.5921	58.3532
0%	1.5	44.2384	53.5921	40.708
5%	1.575	44.2384	53.5921	27.6178
10%	1.65	44.2384	53.5921	17.9069
15%	1.725	44.2384	53.5921	10.7028
20%	1.8	44.2384	53.5921	5.35841

Table 4
Sensitivity analysis with the variation in h

% variation in h	h	v	p	F(v,p)
-20%	0.16	43.1567	52.2513	33.3983
-15%	0.17	43.4225	52.5807	35.1569
-10%	0.18	43.6913	52.914	36.9619
-5%	0.19	43.9633	53.2511	38.8116
0%	0.2	44.2384	53.5921	40.708
5%	0.21	44.5168	53.9371	42.6531
10%	0.22	44.7985	54.2863	44.6509
15%	0.23	45.0835	54.6395	46.6979
20%	0.24	45.3719	54.997	48.7999

Table 5
Sensitivity analysis with the variation in η

% variation in η	η	v	p	F(v,p)
-20%	0.48	44.1387	53.4685	37.6669
-15%	0.51	44.1637	53.4994	38.4286
-10%	0.54	44.1886	53.5303	39.1893
-5%	0.57	44.2135	53.5612	39.9491
0%	0.6	44.2384	53.5921	40.708
5%	0.63	44.2633	53.6229	41.4641
10%	0.66	44.2882	53.6538	42.2211
15%	0.69	44.313	53.6845	42.9735
20%	0.72	44.3379	53.7153	43.7269

Table 6
Sensitivity analysis with the variation in c

% variation in c	c	v	p	F(v,p)
-20%	17.6	32.4607	38.9941	21.2105
-15%	18.7	34.5001	41.5218	24.552
-10%	19.8	36.5314	44.0396	27.8376
-5%	20.9	38.5563	46.5494	31.1212
0%	22	40.5762	49.0529	34.4478
5%	23.1	42.5918	51.5512	37.8493
10%	24.2	44.6041	54.0453	41.354
15%	25.3	46.6135	56.5358	44.9813
20%	26.4	48.6205	59.0234	48.7514

5. Observations

1. From Table 1 we observe the changes in deterioration coefficient (θ) and other variable unchanged. It is observed that with the increment in deterioration coefficient (θ), the value of selling price per unit increases and so the unit time profit.
2. Table 2 lists the variation in demand coefficient (α) and it is observed that with the increment in demand coefficient (α), the value of critical time (v) and selling price (p) remain unchanged and the value of unit time profit increases.
3. It is shown in Table 3 that with the increment in demand parameter β the unit time profit of the system decreases.
4. From Table 4 it is observed that as the value of holding cost increases both the selling price per unit and the unit time profit of the system show the same effect.
5. Table 5 shows the sensitivity of backlogging rate (η). It is observed that as the rate of backlogging (η) increases, the unit time profit of the system also increases.
6. From Table 6 we observe that with the increment in purchasing cost (c), the selling price and unit time profit of the system increases.

6. Conclusion

This paper has presented an economic order quantity model for deteriorating items by considering a joint pricing and replenishment policy. The demand pattern considered here was dependent on selling price which is quite realistic, since selling price is a major factor affecting the demand. Here in this model the deterioration rate was taken as a linear function of time. To make this study close to reality the occurring shortages were partially backlogged. In this model, the retailer accepts the policy of quantity and price that maximizes the unit time profit. A numerical example has been shown to illustrate the model. Sensitivity analysis with respect to different associated parameters has also been shown to observe the behavior of unit time profit with the change in these parameters. This model further can be extended for different demand patterns and deterioration rate.

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