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A supply chain model with shortages under inflationary environment

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C H R O N I C L E

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A B S T R A C T

This paper formulates a multi-level supply chain network with a single producer, multi distributors and multi retailers during a finite planning horizon. The demand rate is assumed to be exponential function of time; shortages are allowed and completely backlogged. The stock is assumed to undergo deterioration as soon as it is produced. The production rate is dependent on demand rate and greater than the demand rate. Optimal solution for the proposed model is derived and using a numerical example, the behavior of the model is analyzed.

1. Introduction

Inventory models normally consider different sub systems in the supply chain, independently. With the recent advances in communication and information technologies, the integration of these functions is a common phenomenon. Moreover, due to limited resources, increasing competition and market globalization, enterprises are forced to develop supply chain, which could respond quickly to customer needs with minimum stock and maximum service level. The idea of joint total cost of the supplier and the customer was first introduced by Goyal (1976). Later, Cohen and Lee (1988) put forward a model for determining material requirement for all materials at every stage in the supply chain. Pake and Cohen (1993) extended the above study to consider for stochastic sub systems to explore the supply chain system. Goyal and Nebebe (2000) considered a problem of determining economic production from a vendor to a buyer. Wee (2003) developed an integrated inventory model with constant rate of deterioration and multiple deliveries. Lee and Wu (2006) developed a study on inventory replenishment policies in a two-echelon supply chain system. Ahmed et al. (2007) coordinated a two level supply chain in which they considered production interruptions for restoring of the quality of the production process.

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Several studies have examined the inflationary effect on an inventory policy. Inflation also influences demand of certain products. Buzacott (1975), Misra (1979), Chandra and Bahner (1985) are amongst the first few who studied the concept of inflation with regard to inventory. Moon and Lee (2000) investigated the impact of inflation and unit cost. They developed their model taking both complete backlogging and without backlogging. Chang (2004) deliberated the effects of inflation on an economic order quantity model when the supplier permits a delay in payment by the retailer if the retailer orders a large quantity. Yang (2006) considered two-warehouse partial backlogging inventory models for deteriorating items under inflation. Lo et al. (2007) developed an integrated production and inventory model from the perspectives of both the manufacturer and the retailer assuming a varying rate of deterioration, partial backordering, inflation, imperfect production processes and multiple deliveries.

Although a number of studies have been performed on the supply chain system with inflation but none of the above paper considers the effect of inflation on demand in supply chain system. This is a major drawback in the studies performed till date. Hence, in our present study we undertake to study a supply chain network for a multi echelon system with a single producer, multi distributors and multi retailers. Also we have considered the rate of production to depend on the demand in the market. This way, the whole research caters to put forward some aspects of a supply chain with some real world considerations and market deliberations.

2. Assumptions and notations

The following assumptions have been used throughout the study.

1. Single item inventory is assumed.
2. Lead time is zero.
3. Production rate is demand rate dependent, and is greater than the demand rate.
4. There is a single producer, multi distributors and multi retailers.
5. Deterioration of the inventory sets in as soon as an item is produced.
6. Deterioration rate of the inventory is finite.
7. Shortages are allowed in the system for distributor and retailer.
8. Planning horizon is known and fixed.
9. Deliveries to both the distributors and the retailers are made at fixed interval.

The following notations have been used throughout the study.

$I_p(t)$	Inventory level of the producer at any time.
$I_d(t)$	Inventory level of the distributor at any time.
$I_r(t)$	Inventory level of the retailer at any time.
P	Production rate of the inventory by the producer, $P=KD$, $K>1$.
D	Annual demand rate in the whole market, $D=\lambda_0 e^{\alpha t}$ units/year, where 'λ_0' is the initial demand rate and $0\leq\alpha\leq 1$ is a constant.
θ	Deterioration rate of the inventory, $\theta>0$ and a constant.
r	Discount rate, where $r>\alpha$.
H	Planning horizon of the supply chain.
q	Number of distributors.
p	Number of retailers corresponding to each distributor.
n	Total number of cycles of the producer in the complete planning horizon.
n_d	Total number of deliveries from the producer to the distributor in one cycle of producer.
n_r	Total number of deliveries from the distributor to the retailer in one cycle of distributor.
T_{i1}	Time when the production is stopped by the producer.
T_i	Time when the inventories reduce to zero at the producer's end.
A_p	Setup cost of the producer per cycle.

A_d Ordering cost of the distributor per cycle.
A_r Ordering cost of the retailer per cycle.
U_p Unit production cost per item, \$/item.
U_d Unit purchase cost for the distributor, \$/unit item.
U_r Unit purchase cost for the retailer, \$/unit item.
C_p Holding cost of the producer, \$/unit item/unit time.
C_d Holding cost of the distributor, \$/unit item/unit time.
C_r Holding cost of the retailer, \$/unit item/unit time.
S_d Shortage cost of the distributor, \$/unit item/unit time.
S_r Shortage cost of the distributor, \$/unit item/unit time.

3. Mathematical model

The model under study has a single producer who fulfills the requirements of '*q*' distributors, and in turn every distributor satisfies '*p*' retailers and there are '*pq*' retailers. The whole planning horizon has been divided equally into *n* cycles. The cycle starts at time $t = 0$, when the production starts. We assume that there are n_d numbers of deliveries made by the producer in any cycle to his distributors, all at equal intervals of time.

3.1 Retailer's model

Assuming continuous compounding of inflation, the ordering cost, unit cost of the item, out-of-pocket inventory carrying cost and shortage cost at any time *t* are

$$A(t) = A_r e^{\alpha t},\ U(t) = U_r e^{\alpha t},\ C(t) = C_r e^{\alpha t},\ S(t) = S_r e^{\alpha t} \tag{1}$$

The planning horizon (*H*) has been divided into *n* equal cycles of length *T* (*i.e.* $T = H/n$). Let us consider the ith cycle, i.e. $t_{i-1} \le t \le t_i$, where $t_0 = 0$, $t_n = H$, $t_i - t_{i-1} = T$ and $t_i = iT$ ($i = 1, 2, \ldots, n$). At the beginning of the ith cycle, a batch of q_i units enters the inventory system from which s_i units are delivered towards backorders leaving a balance of I_{oi} units as the initial inventory level of the ith cycle, i.e. $q_i = I_{oi} + s_i$. Thereafter, as time passes, the inventory level gradually decreasing mainly due to demand and partly due to deterioration and reaches zero at time t_{i1} (Fig 1.). Further, demands during the remaining period of the cycle, i.e. from t_{i1} to t_i are backlogged and are fulfilled by a new procurement. Now $t_{i1} = t_i - kT = (i - k) H/n$, level of the ith cycle at time t ($t_{i-1} \le t \le t_i$, $i = 1, 2, \ldots, n$).

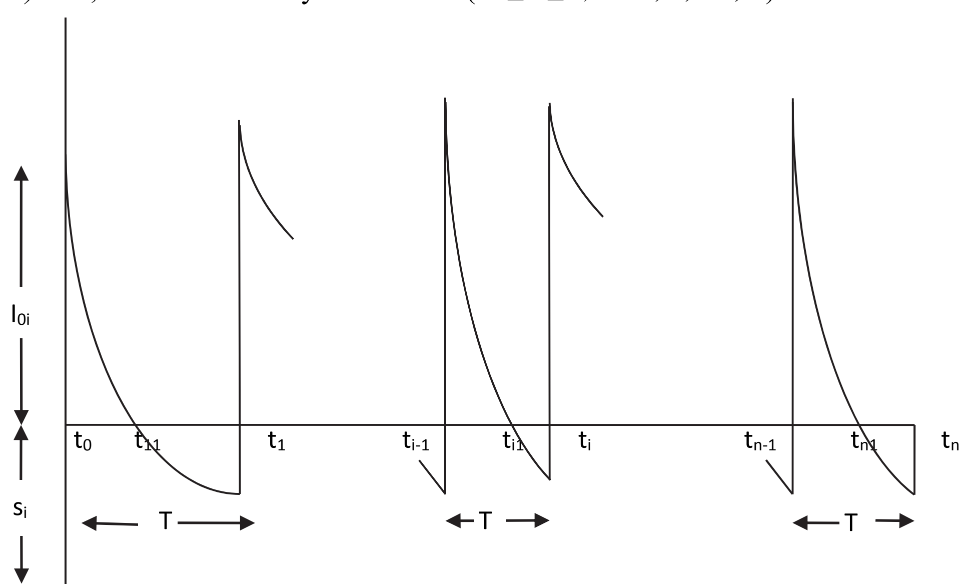

Fig.1. Graphical representation of inventory model of the retailer

The differential equations describing the instantaneous states of $I_i(t)$ over (t_{i-1}, t_i) are

$$I'_{ri}(t)+\theta I_i(t)=-\frac{\lambda_0}{pq}e^{\alpha t},\frac{t_{i-1}}{n_d n_r}\le t\le\frac{t_{i1}}{n_d n_r},\qquad i = 1, 2, \ldots, n \tag{2}$$

$$I'_{ri}(t)=-\frac{\lambda_0}{pq}e^{\alpha t},\frac{t_{i1}}{n_d n_r}\le t\le\frac{t_i}{n_d n_r},\qquad i = 1, 2, \ldots, n \tag{3}$$

The solution of the above differential equations along with the boundary conditions $I_{ri}(t_{i-1}) = I_{oi}$ and $I_{ri}(t_{i1}) = 0$ are

$$I_{ri}(t)=\begin{cases} I_{0i}e^{\theta(\frac{t_{i-1}}{n_d n_r}-t)}-\dfrac{\lambda_0 e^{-\theta t}}{pq(\alpha+\theta)}(e^{(\theta+\alpha)t}-e^{(\theta+\alpha)\frac{t_{i-1}}{n_d n_r}}) & ,\dfrac{t_{i-1}}{n_d n_r}\le t\le\dfrac{t_{i1}}{n_d n_r} \\ -\dfrac{\lambda_0}{pq\alpha}(e^{\alpha t}-e^{\alpha\frac{t_{i1}}{n_d n_r}}) & ,\dfrac{t_{i1}}{n_d n_r}\le t\le\dfrac{t_i}{n_d n_r}\end{cases} \tag{4a, 4b}$$

Since $I_i(t_{i1}) = 0$ and $I_i(t_i) = -s_i$, Eqs. (2-3) give

$$I_{0i} = \frac{\lambda_0 e^{-\theta\frac{t_{i-1}}{n_d n_r}}}{pq(\alpha+\theta)}(e^{(\theta+\alpha)\frac{t_{i1}}{n_d n_r}}-e^{(\theta+\alpha)\frac{t_{i-1}}{n_d n_r}}) \tag{5}$$

and

$$s_{ri} = \frac{\lambda_0}{pq\alpha}(e^{\alpha\frac{t_i}{n_d n_r}}-e^{\alpha\frac{t_{i1}}{n_d n_r}}) \tag{6}$$

Substituting I_{0i} from Eq. (5), Eq. (4a) becomes

$$I_{ri}(t)= \frac{\lambda_0 e^{-\theta t}}{pq(\alpha+\theta)}(e^{(\theta+\alpha)\frac{t_{i1}}{n_d n_r}}-e^{(\theta+\alpha)t}),\frac{t_{i-1}}{n_d n_r}\le t\le\frac{t_{i1}}{n_d n_r} \tag{7}$$

Further, batch size q_i for the ith cycle is $q_{ri} = I_{0i} + s_{ri}$. From Eqs. (5-6), we get

$$q_{ri} = \frac{\lambda_0 e^{-\theta\frac{t_{i-1}}{n_d n_r}}}{pq(\alpha+\theta)}(e^{(\theta+\alpha)\frac{t_{i1}}{n_d n_r}}-e^{(\theta+\alpha)\frac{t_{i-1}}{n_d n_r}})+\frac{\lambda_0}{pq\alpha}(e^{\alpha\frac{t_i}{n_d n_r}}-e^{\alpha\frac{t_{i1}}{n_d n_r}}) \tag{8}$$

Present worth of ordering cost for the ith cycle, A_i, is

$$A_{ri} = A_r e^{(\alpha-r)\frac{t_{i-1}}{n_d n_r}},\qquad i=1,2,\ldots\ldots,n \tag{9}$$

Present worth of purchase cost for the ith cycle, P_i, is

$$P_{ri} = q_{ri}U_r e^{(\alpha-r)\frac{t_{i-1}}{n_d n_r}},\qquad i=1,2,\ldots\ldots,n \tag{10}$$

Present worth of holding cost for the ith cycle, H_i, is

$$H_{ri}=C(t_{i-1})e^{-r\frac{t_{i-1}}{n_d n_r}}\int_{t_{i-1}}^{t_{i1}} I_{ri}(t)e^{-rt}dt$$

$$=\frac{\lambda_0 C_r}{pq(\alpha+\theta)}\left[\frac{(e^{-(r+\theta)\frac{t_{i-1}}{n_d n_r}}e^{(\alpha+\theta)\frac{t_{i1}}{n_d n_r}}-e^{(\alpha-r)\frac{t_{i1}}{n_d n_r}})}{(r+\theta)}-\frac{(e^{(\alpha-r)\frac{t_{i1}}{n_d n_r}}-e^{(\alpha-r)\frac{t_{i-1}}{n_d n_r}})}{(\alpha-r)}\right]e^{(\alpha-r)\frac{t_{i-1}}{n_d n_r}} \quad (11)$$

Present worth of shortage cost for the ith cycle, S_i, is

$$S_{ri}=S(t_{i-1})e^{-r\frac{t_{i-1}}{n_d n_r}}\int_{t_{i1}}^{t_i} I_{ri}(t)e^{-rt}dt$$

$$=\frac{\lambda_0 S_r}{pq\alpha}\left[\frac{(e^{(\alpha+r)\frac{t_i}{n_d n_r}}-e^{(\alpha-r)\frac{t_{i1}}{n_d n_r}})}{(\alpha-r)}+\frac{e^{\alpha\frac{t_{i1}}{n_d n_r}}}{r}(e^{-r\frac{t_i}{n_d n_r}}-e^{-r\frac{t_{i1}}{n_d n_r}})\right]e^{(\alpha-r)\frac{t_{i-1}}{n_d n_r}} \quad (12)$$

Present worth of total cost

The present worth of the total variable cost of the system during the entire time horizon H is given by

$$TC_r=\sum_{i=1}^{n}(A_{ri}+P_{ri}+H_{ri}+S_{ri}) \quad (13)$$

3.2 Distributor's model

Assumptions for distributor is also continuous compounding of inflation, the ordering cost, unit cost of the item, out-of-pocket inventory carrying cost and shortage cost at any time t are

$$A(t)=A_d e^{\alpha t},\ U(t)=U_d e^{\alpha t},\ C(t)=C_d e^{\alpha t},\ S(t)=S_d e^{\alpha t} \quad (14)$$

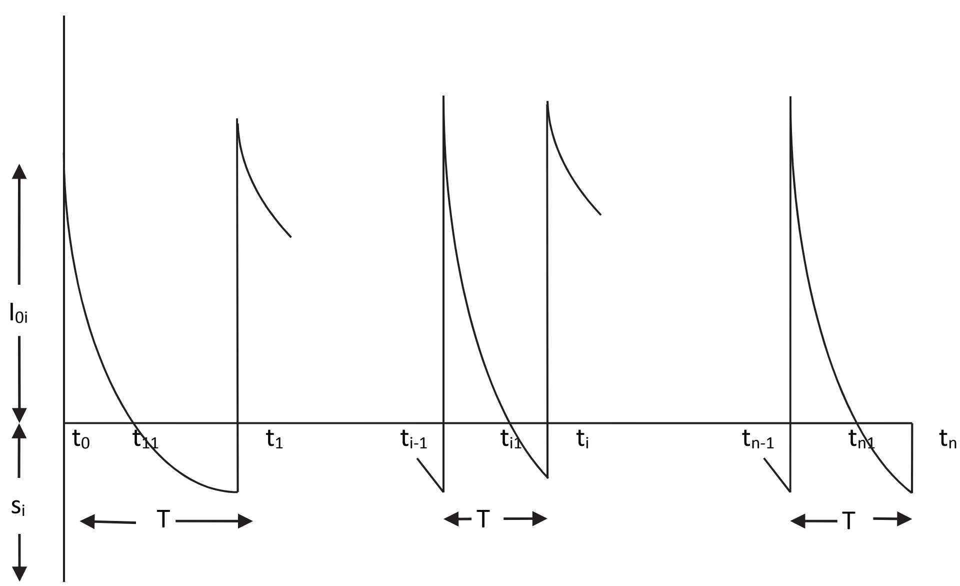

Fig. 2. Graphical representation of inventory model of the distributor

The differential equations describing the instantaneous states of $I_i(t)$ over (t_{i-1}, t_i) are

$$I'_{di}(t)+\theta I_{di}(t)=-\frac{\lambda_0}{q}e^{\alpha t} \qquad ,\frac{t_{i-1}}{n_d}\le t\le\frac{t_{i1}}{n_d}, \qquad i = 1, 2, \ldots, n \tag{15}$$

$$I'_{di}(t)=-\frac{\lambda_0}{q}e^{\alpha t} \qquad ,\frac{t_{i1}}{n_d}\le t\le\frac{t_i}{n_d}, \qquad i = 1, 2, \ldots, n \tag{16}$$

The solution of the above differential equations along with the boundary conditions $I_{di}(t_{i-1}) = I_{oi}$ and $I_{di}(t_{i1}) = 0$ are

$$I_{di}(t)=\begin{cases} I_{0i}e^{\theta(\frac{t_{i-1}}{n_d}-t)}-\dfrac{\lambda_0 e^{-\theta t}}{q(\alpha+\theta)}(e^{(\theta+\alpha)t}-e^{(\theta+\alpha)\frac{t_{i-1}}{n_d}}) & ,\dfrac{t_{i-1}}{n_d}\le t\le\dfrac{t_{i1}}{n_d} \\ -\dfrac{\lambda_0}{q\alpha}(e^{\alpha t}-e^{\alpha\frac{t_{i1}}{n_d}}) & ,\dfrac{t_{i1}}{n_d}\le t\le\dfrac{t_i}{n_d} \end{cases} \tag{17a, 17b}$$

Since $I_{di}(t_{i1}) = 0$ and $I_{di}(t_i) = -s_{di}$, Eqs. (17a-17b) give

$$I_{0i}=\frac{\lambda_0 e^{-\theta\frac{t_{i-1}}{n_d}}}{q(\alpha+\theta)}(e^{(\theta+\alpha)\frac{t_{i1}}{n_d}}-e^{(\theta+\alpha)\frac{t_{i-1}}{n_d}}) \tag{18}$$

and

$$s_{di}=\frac{\lambda_0}{q\alpha}(e^{\alpha\frac{t_i}{n_d}}-e^{\alpha\frac{t_{i1}}{n_d}}) \tag{19}$$

Substituting I_{0i} from Eq. (18), Eq. (17a) becomes

$$I_{di}(t)=\frac{\lambda_0 e^{-\theta t}}{q(\alpha+\theta)}(e^{(\theta+\alpha)\frac{t_{i1}}{n_d}}-e^{(\theta+\alpha)t}), \qquad \frac{t_{i-1}}{n_d}\le t\le\frac{t_{i1}}{n_d} \tag{20}$$

Further, batch size q_i for the ith cycle is $q_{di} = I_{0i} + s_{di}$. From Eqs. (4-5), we get

$$q_{di}=\frac{\lambda_0 e^{-\theta\frac{t_{i-1}}{n_d}}}{q(\alpha+\theta)}(e^{(\theta+\alpha)\frac{t_{i1}}{n_d}}-e^{(\theta+\alpha)\frac{t_{i-1}}{n_d}})+\frac{\lambda_0}{q\alpha}(e^{\alpha\frac{t_i}{n_d}}-e^{\alpha\frac{t_{i1}}{n_d}}) \tag{21}$$

Present worth of ordering cost for the ith cycle, A_i, is

$$A_{di}=A_d e^{(\alpha-r)\frac{t_{i-1}}{n_d}}, \qquad i=1,2,\ldots\ldots,n \tag{22}$$

Present worth of purchase cost for the ith cycle, P_i, is

$$P_{di}=q_{di}U_d e^{(\alpha-r)\frac{t_{i-1}}{n_d}}, \qquad i=1,2,\ldots\ldots,n \tag{23}$$

Present worth of holding cost for the ith cycle, H_i, is

$$H_{di}=C(t_{i-1})e^{-r\frac{t_{i-1}}{n_d}}\int_{t_{i-1}}^{t_{i1}}I_i(t)e^{-rt}dt$$

$$=\frac{\lambda_0 C_d}{q(\alpha+\theta)}\left[\frac{(e^{-(r+\theta)\frac{t_{i-1}}{n_d}}e^{(\alpha+\theta)\frac{t_{i1}}{n_d}}-e^{(\alpha-r)\frac{t_{i1}}{n_d}})}{(r+\theta)}-\frac{(e^{(\alpha-r)\frac{t_{i1}}{n_d}}-e^{(\alpha-r)\frac{t_{i-1}}{n_d}})}{(\alpha-r)}\right]e^{(\alpha-r)\frac{t_{i-1}}{n_d}} \tag{24}$$

Present worth of shortage cost for the ith cycle, S_i, is

$$S_i=S(t_{i-1})e^{-r\frac{t_{i-1}}{n_d}}\int_{t_{i1}}^{t_i}I_i(t)e^{-rt}dt=\frac{\lambda_0 S_d}{q\alpha}\left[\frac{(e^{(\alpha+r)\frac{t_i}{n_d}}-e^{(\alpha-r)\frac{t_{i1}}{n_d}})}{(\alpha-r)}+\frac{e^{\alpha\frac{t_{i1}}{n_d}}}{r}(e^{-r\frac{t_i}{n_d}}-e^{-r\frac{t_{i1}}{n_d}})\right]e^{(\alpha-r)\frac{t_{i-1}}{n_d}} \tag{25}$$

The present worth of the total variable cost of the system during the entire time horizon H is given by

$$TC_d = \sum_{i=1}^{n}(A_{di} + P_{di} + H_{di} + S_{di})\,. \tag{26}$$

3.3 Producer's model

The whole planning horizon has been divided equally into *n* cycles. The cycle starts at time $t = 0$, when the production starts. We assume that there are n_d numbers of deliveries made by the producer in any cycle, all at equal intervals of time. Since, we have considered an inflation induced demand; hence, we assume that every lot sent off by the producer is of equal amount. During time interval [0, T_1], the inventory reduces due to both demand and deterioration continuously. At time $t = T_1$, the production is stopped, and now the inventory diminishes due to deterioration only, except at the times when a lot is sent off, and then there is a sudden reduction in the inventory level. Hence, the producer's cycle looks like Fig. 3. Assumptions for producer is also continuous compounding of inflation, the setup cost, unit cost of the item, out-of-pocket inventory carrying cost at any time *t* are

$$A(t) = A_p e^{\alpha t},\ U(t) = U_p e^{\alpha t},\ C(t) = C_p e^{\alpha t},\ S(t) = S_p e^{\alpha t} \tag{27}$$

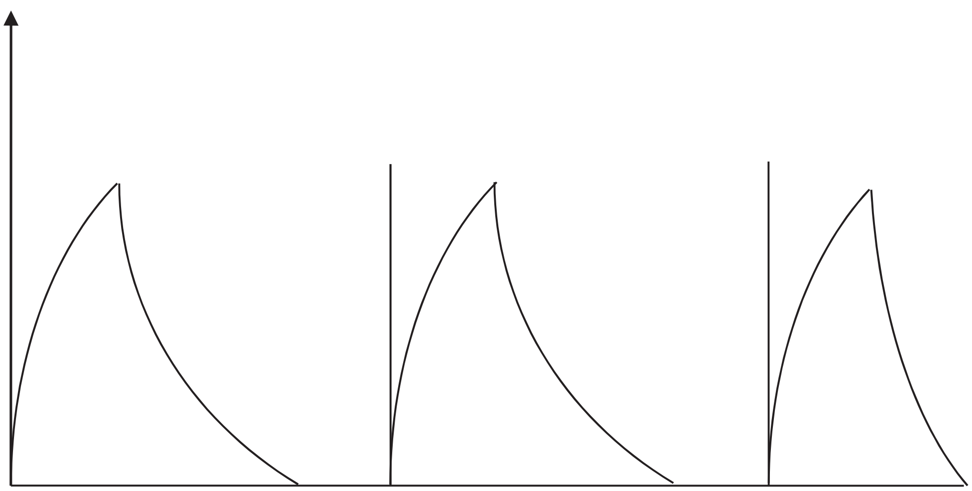

Fig. 3. Graphical representation of the cycles of the producer

The differential equations describing the instantaneous states of $I_i(t)$ over (T_{i-1}, T_i) are

$$I'_{p1i}(t) + \theta I_{p1i}(t) = P - \lambda_0 e^{\alpha t} \qquad ,T_{i-1} \le t \le T_{i1}, \qquad i = 1, 2, \ldots, n \tag{28}$$

$$I'_{p2i}(t) + \theta I_{p2i}(t) = -\lambda_0 e^{\alpha t} \qquad ,T_{i1} \le t \le T_i, \qquad i = 1, 2, \ldots, n \tag{29}$$

The solution of the above differential equations along with the boundary conditions $I_{p1i}(0) = 0$ and $I_{p2i}(T_i) = 0$ are

$$I_{pi}(t) = \begin{cases} \dfrac{(K-1)\lambda_0}{(\alpha+\theta)}(e^{\alpha t} - e^{-\theta t}) & ,T_{i-1} \le t \le T_{i1} \\ \dfrac{\lambda_0 e^{-\theta t}}{\alpha+\theta}(e^{(\alpha+\theta)T_{i2}} - e^{(\alpha+\theta)t}) & ,T_{i1} \le t \le T_i \end{cases} \tag{30a, 30b}$$

From $I_{p1i}(T_{i1}) = q_p = I_{pi}(0)$, one can derive the following equation:

$$\frac{(K-1)\lambda_0}{(\alpha+\theta)}(e^{\alpha T_{i1}} - e^{-\theta T_{i1}}) = \frac{\lambda_0}{\alpha+\theta}(e^{(\alpha+\theta)T_{i2}} - 1) \tag{31}$$

By Taylor's series expansion and assuming small value of θ, one has

$$T_{i1} = \frac{T_i}{K-1} \tag{32}$$

We now compute the different costs associated with the producer.

Present worth of ordering cost for the ith cycle, A_i, is

$$A_{pi} = A_p e^{(\alpha-r)T_{i-1}}, \qquad i = 1,2,.....,n \tag{33}$$

Present worth of purchase cost for the ith cycle, P_i, is

$$P_{pi} = KT_{i1}U_p e^{(\alpha-r)T_{i-1}} \int_{T_{i-1}}^{T_{i1}} \lambda_0 e^{-\alpha t} dt, \qquad i = 1,2,.....,n \tag{34}$$

Present worth of holding cost for the ith cycle, H_i, is

$$H_{pi} = C(T_{i-1})e^{-rT_{i-1}} \left[\int_{T_{i-1}}^{T_{i1}} I_{p1i}(t)e^{-rt}dt + \int_{T_{i1}}^{T_i} I_{p2i}(t)e^{-rt}dt \right]$$

$$= \frac{C_p\lambda_0}{(\alpha+\theta)} \left[K\left(\frac{e^{(\alpha-r)\frac{T_i}{K-1}}}{\alpha-r} - \frac{e^{-(r+\theta)\frac{T_i}{K-1}}}{r+\theta} \right) + \left(\frac{e^{(\alpha+\theta)T_{i2}}e^{-(r+\theta)\frac{T_i}{K-1}}}{r+\theta} - \frac{1}{r+\theta} - \frac{e^{(\alpha-r)T_i}}{\alpha-r} - \frac{e^{-(r+\theta)\frac{T_i}{K-1}}}{r+\theta} \right) \right. \tag{35}$$

$$\left. K\left(-\frac{e^{(\alpha-r)T_{i-1}}}{\alpha-r} - \frac{e^{-(r+\theta)T_{i-1}}}{r+\theta} \right) + \left(\frac{e^{(\alpha-r)T_{i-1}}}{\alpha-r} - \frac{e^{-(r+\theta)T_{i-1}}}{r+\theta} \right) \right] e^{(\alpha-r)T_{i-1}}$$

Present worth of total cost

The present worth of the total variable cost of the system during the entire time horizon H is given by

$$TC_p = \sum_{i=1}^{n} (A_{pi} + P_{pi} + H_{pi}) \tag{36}$$

4. Numerical Illustrations

The following numerical data has been used to find the optimal solution of the three players, the producer, the distributor and the retailer. First we find the separate optimal solutions for the three individuals and then a combined optimal solution has been arrived at. $r = 0.12$, $\theta = 0.01$, $p = 6$, $q = 4$, $\lambda_0 = 1000$, $\alpha = 0.05$. Producer data: $U_p = 10$ $C_p = 1$ $A_p = 100$ $K = 2$, Distributor data: $U_d = 20$ $C_d = 1$ $A_d = 50$, Retailer data: $U_r = 25$ $C_r = 1$ $A_r = 50$.

We find the solution for different values of 'n', 'n_d' and 'n_r' and the total cost of the supplier, distributor and the retailer have been found for every combination of these three variables. From the solution obtained, we find the optimal solution for each one of them, separately. In the end we obtain the optimal solution for the whole supply chain using a commercial software package and Table 1 summarizes the results for various values of n.

Table 1
The results of optimal solutions

n	T	n_r	n_d	K	TC_r	TC_d	TC_p	TC
	1	3	5	0.00955	119.675	1060.96	5163.45	6344.08
		1	1	0.00984	1144.0	5326.13	9899.21	16369.34
1		2	4	0.00960	181.018	1317.14	4100.84	5598.998
		3	4	0.00953	137.165	1747.16	6545.77	8430.095
		6	2	0.00951	137.165	2618.71	7278.51	10034.385
n	T	n_r	n_d	K	TC_r	TC_d	TC_p	TC
	0.5	3	5	0.00955	134.687	600.943	1975.67	2711.3
		1	1	0.00984	625.14	2627.09	5010.40	8262.63
2		2	4	0.00960	165.076	726.529	1956.89	2848.495
		3	4	0.00953	143.366	726.529	1978.92	2848.815
		6	2	0.00951	143.366	726.71	2010.56	2880.636
n	T	n_r	n_d	K	TC_r	TC_d	TC_p	TC
	0.33	3	5	0.00955	172.845	480.021	1610.23	2263.085
		1	1	0.00984	493.725	1805.15	3100.78	5399.655
3		2	4	0.00960	192.842	562.605	1789.47	2544.917
		3	4	0.00953	178.557	562.605	1809.66	2550.822
		6	2	0.00951	178.557	975.995	2189.18	3343.732
n	T	n_r	n_d	K	TC_r	TC_d	TC_p	TC
	0.25	3	5	0.00955	217.275	449.815	1565.60	2232.69
		1	1	0.00984	459.431	1450.62	2845.19	4755.241
4		2	4	0.00960	232.393	512.293	1742.23	2486.916
		3	4	0.00953	221.594	512.293	1799.45	2533.337
		6	2	0.00951	221.594	824.828	2045.89	3092.312
n	T	n_r	n_d	K	TC_r	TC_d	TC_p	TC
	0.2	3	5	0.00955	263.796	449.744	1589.71	2303.25
		1	1	0.00984	457.02	1249.19	3545.95	2852.16
5		2	4	0.00960	275.868	499.687	1688.22	2463.775
		3	4	0.00953	267.245	499.687	1675.89	2442.822
		6	2	0.00951	267.245	749.449	1989.34	3006.034

5. Observations

The objective of our study is to derive the optimal number of delivery for the producer, distributor and the retailer. Using the solution procedure, the computational result for the optimal solution is given in Table 1.

1. Optimal solution for the retailer is $n=1$, $n_d=5$, $n_r=3$. For the distributor it is $n=5$, $n_d=3$, $n_r=5$ while for the producer, it is $n=4$, $n_d=5$, $n_r=3$. The overall optimal solution which ultimately minimizes the cost across the whole supply chain is $n=4$, $n_d=5$, $n_r=3$.

2. The total cost for the system has been computed for one producer, four distributors and twenty four retailers.

Hence, we observe that in spite of the fact that individual optimal solutions are attained different, but the final optimal for the complete supply chain is very different from the sub optimal solutions.

6. Conclusion

In this chapter we first formulated an inventory model with the assumptions that demand is induced by inflation and shortages over a finite planning horizon. Presence of inflation in cost and its impact on demand suggest larger cycle length. As inflation goes up the value of money decreases, which erodes the future worth of savings and suggests more current spending. Usually, these spending are on peripherals and luxury items that give rise to demand of these items. A numerical example is used to demonstrate the feasibility and properties of the proposed integration model in this chapter.

References

Bhunia, A. K., & Maiti, M. (1994). A two warehouse inventory model for a linear trend in demand. *Opsearch*, *31*, 318-318.

Buzacott, J. A. (1975). Economic order quantities with inflation. *Operational Research Quarterly*, *26*(3), 553-558.

Chang, C. T. (2004). An EOQ model with deteriorating items under inflation when supplier credits linked to order quantity. *International Journal of Production economics*, *88*(3), 307-316.

Chandra, M. J., & Bahner, M. L. (1985). The effects of inflation and the time value of money on some inventory systems. *International Journal of Production Research*, *23*(4), 723-730.

Cohen, M. A., & Lee, H. L. (1988). Strategic analysis of integrated production-distribution systems: Models and methods. *Operations Research*,*36*(2), 216-228.

Goyal, S. K. (1977). An integrated inventory model for a single supplier-single customer problem. *The International Journal of Production Research*,*15*(1), 107-111.

Goyal, S. K., & Nebebe, F. (2000). Determination of economic production–shipment policy for a single-vendor–single-buyer system. *European Journal of Operational Research*, *121*(1), 175-178.

Kar, S., Bhunia, A. K., & Maiti, M. (2001). Deterministic inventory model with two levels of storage, a linear trend in demand and a fixed time horizon.*Computers & Operations Research*, *28*(13), 1315-1331.

Lee, H. T., & Wu, J. C. (2006). A study on inventory replenishment policies in a two-echelon supply chain system. *Computers & Industrial Engineering*,*51*(2), 257-263.

Lo, S. T., Wee, H. M., & Huang, W. C. (2007). An integrated production-inventory model with imperfect production processes and Weibull distribution deterioration under inflation. *International Journal of Production Economics*,*106*(1), 248-260.

Misra, R. B. (1979). A note on optimal inventory management under inflation.*Naval Research Logistics Quarterly*, *26*(1), 161-165.

Moon, I., & Lee, S. (2000). The effects of inflation and time-value of money on an economic order quantity model with a random product life cycle.*European Journal of Operational Research*, *125*(3), 588-601.

Murdeshwar, T. M., & Sathe, Y. S. (1985). Some aspects of lot size model with two levels of storage. *Opsearch*, *22*(4), 255-262.

Pyke, D. F., & Cohen, M. A. (1993). Performance characteristics of stochastic integrated production-distribution systems. *European Journal of Operational Research*, *68*(1), 23-48.

Sarma, K. V. S. (1987). A deterministic order level inventory model for deteriorating items with two storage facilities. *European Journal of Operational Research*, *29*(1), 70-73.

Sarma, K. V. S. (1983). A deterministic inventory model with two levels of storage and an optimum release rule. *Opsearch*, *20*(3), 175-180.

El Saadany, A. M., & Jaber, M. Y. (2008). Coordinating a two-level supply chain with production interruptions to restore process quality. *Computers & Industrial Engineering*, *54*(1), 95-109.

Yang, P. C., & Wee, H. M. (2003). An integrated multi-lot-size production inventory model for deteriorating item. *Computers & Operations Research*,*30*(5), 671-682.

Yang, H. L. (2006). Two-warehouse partial backlogging inventory models for deteriorating items under inflation. *International Journal of Production Economics*, *103*(1), 362-370.

Zhou, Y. W., & Yang, S. L. (2005). A two-warehouse inventory model for items with stock-level-dependent demand rate. *International Journal of Production Economics*, *95*(2), 215-228.