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An alternative approach based on fuzzy PROMETHEE method for the supplier selection problem

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A B S T R A C T

In this paper, an alternative version of the fuzzy PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations) method is proposed. Differently from other studies, preference functions used in PROMETHEE method are handled in terms of fuzzy distances between alternatives with respect to each criterion. In order to indicate the applicability of this method, it is applied for a supplier selection problem in the literature. Ranking results are similar obtained by TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) and fuzzy ELECTRE (ELimination Et Choix Traduisant la REalité) methods. The implementation of the proposed method indicates that the amount of computations is decreased and decision makers can easily reach to desirable solution.

1. Introduction

Multi Criteria Decision Making (MCDM) is a decision support system for evaluating and ranking discrete set of alternatives by considering conflicting criteria (Bilsel et al., 2006; Li & Li, 2010). Sometimes, the number of decision makers can be more than one, which leads us to group decision making process. The role of the decision makers in this process is to provide qualitative and quantitative assessments of the performance of each alternative with respect to each criterion and the relative importance of criteria with respect to the overall objective of the problems (Kuo et al., 2007). But the main difficulty is to convert the human judgments including qualitative observations and preferences into quantitative input data (Goumas & Lygerou, 2000). So the complexity of MCDM problems arises from the decision environment in which the objectives and constraints are not precisely known and the problem cannot be exactly defined in crisp values (Bellman & Zadeh, 1970). To deal with such problems the usage of fuzzy set theory introduced by Zadeh (1965) may be a solution. A fuzzy set, an extension of a crisp set, allows partial membership between 0 and 1. In the fuzzy set theory, linguistic variables are tools that use fuzzy sets to express linguistic expression, mathematically.

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The PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations) method introduced by Brans et al. (1984), Brans and Vincke (1985), Brans et al. (1986), is an MCDM method to rank a discrete set of alternatives. The underlying philosophy of this method and the application steps are quite simple against the other methods (Goumas & Lygerou, 2000). There are a lot of application areas of PROMETHEE method including environment management, hydrology and water management, business and financial management, chemistry, logistics and transportation, manufacturing and assembly, energy management, production planning, social and other topics like medicine, agriculture, education, design, government and sports (Behzadian et al., 2010).

This paper presents an alternative approach for MCDM by integrating fuzzy set theory and PROMETHEE method to help decision makers for selection problems. This approach is applied to a problem from literature for the evaluation and ranking of alternative suppliers (Chen et al., 2006). The decision committee which includes decision makers analyzes the structure of the supplier selection problem and determines the weights of criteria. Decision makers use linguistic assessments represented by trapezoidal fuzzy numbers for describing the weights and preference functions of each criterion which are the main requirements of PROMETHEE method. Unlike other studies, the preference structure of PROMETHEE method is handled in terms of fuzzy distances. In other words, fuzzy Hamming distances are used while making pairwise comparisons between alternatives with respect to each criterion. Before presenting the details of the proposed approach, an information of PROMETHEE method is given in the second section. In the third section, fuzzy numbers and their fuzzy algebraic operations are introduced. In the same section, the concept of fuzzy PROMETHEE method is introduced briefly and the literature review and the formulation of fuzzy PROMETHEE method are given. In the fourth section, a numerical example is presented to demonstrate the details of the proposed method. Finally, conclusions and findings are interpreted to summarize the contribution of the proposed method.

2. PROMETHEE method

PROMETHEE is one of the outranking method, which is easy to understand and has a lot of applications in real life. PROMETHEE I and PROMETHEE II methods provide both partial and complete ranking of the alternatives (Ülengin et al., 2001).
The following steps are applied for the PROMETHEE method. It is assumed that there are m alternatives or actions ($A_1, A_2, \ldots, A_m$) and n decision criteria or attributes ($C_1, C_2, \ldots, C_n$) in the problem.

Step 1: A preference function P_j is associated with each criterion j. $P_j(g, f)$ is calculated for each pair of alternatives where g and f are two alternatives of a set of alternatives. The value of $P_j(g, f)$ varies from 0 to 1:

- $P_j(g, f) = 0$ means an indifference between g and f or no preference of g over f
- $P_j(g, f) \approx 0$ means weak preference of g over f
- $P_j(g, f) \approx 1$ means strong preference of g over f
- $P_j(g, f) = 1$ means strict preference of g over f

P_j is a function of the difference between the two evaluations so it can be written as:

$$P_j(g, f) = P_j(f(g) - f(f)) \quad (1)$$

The preference functions vary according to the problems. In the literature, there are six common functions (Brans et al., 1986).

Step 2: The preference index $\pi(g, f)$ which is weighted average of the preferences functions $P_j(g, f)$ for all criteria and it is calculated as (Giannopoulos & Founti 2010):

$$\pi(g, f) = \sum_{j=1}^{n} w_j P_j(g, f) / \sum_{j=1}^{n} w_j , \quad (2)$$

where w_j is the weight or relative importance of each criterion j (j=1,2,…,n). $\pi(g,f)$ represents the intensity of preference of the decision maker of alternative *g* over alternative *f*, by considering all the criteria, simultaneously. Its value is between 0 and 1:

- $\pi(g,f) \approx 0$ denotes a weak preference of *g* over *f* for all the criteria,
- $\pi(g,f) \approx 1$ denotes a strong preference of *g* over *f* for all the criteria (Brans et al., 1986)

Step 3: Flows for an alternative *g* are calculated. There are two types of flows as leaving and entering. The *leaving flow* at *g* indicates a preference of the alternative *g* over all other alternatives. It shows how 'good' the alternative *g* is. The leaving flow is calculated as follows:

$$\phi^+(g) = \sum_{\substack{f \neq g \\ f=1}}^{m} \pi(g,f) \tag{3}$$

The *entering flow* at *g* indicates a preference of all other alternatives, compared to *g*. It shows how 'weak' the alternative *g* is. The entering flow is calculated as follows:

$$\phi^-(g) = \sum_{\substack{f \neq g \\ f=1}}^{m} \pi(f,g) \tag{4}$$

A partial preorder between alternatives is obtained from the intersection of the two rankings induced by $\phi^+(g)$ and $\phi^-(g)$ with PROMETHEE I method. If the leaving flow is higher and the entering flow is lower, then the alternative is the better.

Step 4: Net flows are used for a complete ranking. The net flow of alternative *g* is calculated as follows:

$$\phi(g) = \phi^+(g) - \phi^-(g) \tag{5}$$

Finally net flow for each alternative is used to determine the final ranking of alternatives from the best to the worst. Higher net flow score means better performance of the alternative (Brans et al., 1986; Bilsel et al., 2006).

3. Fuzzy PROMETHEE method

The PROMETHEE method lacks the ability to process fuzzy data in the actual decision making environment. Fuzzy sets theory was introduced to deal with uncertainty of human judgment (Wang et al. 2008). Fuzzy set theory develops formulation and solution of problems, which are too complex or too ill-defined to be susceptible of analysis by conventional techniques (Kandel, 1986). Integration of fuzzy set theory and the PROMETHEE method was first proposed by Le Téno and Mareschal (1998). Geldermann et al. (2000) implemented fuzzy PROMETHEE method in iron and steel making industry. Goumas and Lygerou (2000) presented fuzzy PROMETHEE II method while evaluating the alternative energy exploitation projects. Bilsel et al. (2006) ranked web sites of Turkish hospitals using fuzzy PROMETHEE method. Wang et al. (2008) and Chen et al. (2011) applied a fuzzy PROMETHEE method for the information systems outsourcing suppliers' selection. Moreira et al. (2009) prioritized the failure modes of the diagnostic of electric power equipment by using PROMETHEE and fuzzy PROMETHEE methods. Aloini et al. (2009) applied a hybrid fuzzy PROMETHEE method for the logistic service provider selection. Zhou et al. (2009) presented pipe condition assessment problem by using PROMETHEE II method. Giannopoulos and Founti (2010) presented improved version of the PROMETHEE method which incorporates a reliable fuzzy ranking method. Li and Li (2010) applied a new PROMETHEE II method by considering generalized fuzzy numbers. Tuzkaya et al. (2010) presented alternative material handling equipment evaluation by fuzzy PROMETHEE method.

3.1 Basic concepts about fuzzy set theory

A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set $\tilde{A}$ of the real line R such that:

- It exists such that one $x_0 \in R$ with $\mu_{\tilde{A}}(x_0) = 1$ (x_0 is called mean value of $\tilde{A}$)

- $\mu_{\tilde{A}}(x)$ is piecewise continuous (Zimmermann 1992).

Trapezoidal fuzzy numbers are defined as a quadruplet (a_1, a_2, a_3, a_4) and the membership function of a fuzzy trapezoidal number is described as:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1, \\ (x-a_1)/(a_2-a_1), & a_1 \le x \le a_2, \\ 1 & a_2 \le x \le a_3, \\ (x-a_4)/(a_3-a_4), & a_3 \le x \le a_4, \\ 0, & x > a_4 \end{cases} \tag{6}$$

Given any two positive trapezoidal fuzzy numbers, $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ and a positive real number *r*, some main operations of fuzzy numbers $\tilde{A}$ and $\tilde{B}$ can be expressed as follows (Kaufmann & Gupta 1988):

$$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4) \tag{7}$$

$$\tilde{A} - \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1) \tag{8}$$

$$\tilde{A} \times r = (a_1 r, a_2 r, a_3 r, a_4 r) \tag{9}$$

$$\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4) \tag{10}$$

$$\tilde{A} \div \tilde{B} = (a_1 \div b_4, a_2 \div b_3, a_3 \div b_2, a_4 \div b_1) \tag{11}$$

In this paper, Hamming distance is used to determine the distance between two fuzzy numbers. For any fuzzy numbers $\tilde{A}$ and $\tilde{B}$, the Hamming distance $(\tilde{A}, \tilde{B})$ can be found as (Hatami-Marbini & Tavana 2011):

$$d(\tilde{A}, \tilde{B}) = \int_R \left| \mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x) \right| dx \tag{12}$$

In the literature, there are many defuzzification methods. In this paper Chen et al. (1997)'s defuzzification method is used. Let $\tilde{A}$ be a trapezoidal fuzzy number, $\tilde{A}$(a_1, a_2, a_3, a_4), then the defuzzified value $x_{\tilde{A}}$ of the fuzzy number $\tilde{A}$ is calculated as follows (Chen et al., 1997):

$$x_{\tilde{A}} = \frac{a_1 + a_2 + a_3 + a_4}{4} \tag{13}$$

3.2 An alternative approach for fuzzy PROMETHEE method

In this section an alternative approach is proposed for using PROMETHEE method under fuzzy environment. The following steps are required for the implementation of the method:

Step 1: First of all, a committee of decision makers is formed. In a decision committee that has *K* decision makers $DM_k\,(k = 1,2,\ldots K)$; fuzzy rating of each decision maker can be represented as trapezoidal fuzzy number $\tilde{R}_k\,(k = 1,2,\ldots K)$ with membership function $\mu_{\tilde{R}_k}(x)$.

Step 2: Then evaluation criteria are determined and feasible alternatives are generated. *m* alternatives (A_m) and *n* criteria (C_n) are supposed.

Step 3: Appropriate linguistic variables and their corresponding trapezoidal fuzzy numbers are chosen. They are used for evaluating the relative importance weights of criteria and rating alternatives under various criteria.

Step 4: $\tilde{R}_k = (a_k, b_k, c_k, d_k)$, $k = 1,2,\ldots,K$ is the fuzzy ratings of the *k*th decision makers in the form of trapezoidal fuzzy numbers. Then fuzzy rating is aggregated and denoted as $\tilde{R} = (a, b, c, d)$:

$$a = \min_k \{a_k\},\ \ b = \frac{1}{K}\sum_{k=1}^{K} b_k\ ,\quad c = \frac{1}{K}\sum_{k=1}^{K} c_k\ ,\ d = \max_k \{d_k\} \qquad k = 1,2,\ldots,K \tag{14}$$

$\tilde{x}_{ijk} = (a_{ijk}, b_{ijk}, c_{ijk}, d_{ijk})$ and $\tilde{w}_{jk} = (w^l_{jk}, w^p_{jk}, w^q_{jk}, w^u_{jk})$, $i = 1,2,\ldots m$, $j = 1,2,\ldots n$ are the fuzzy rating and importance weight of the *k*th decision maker, respectively. Then the fuzzy ratings $(\tilde{x}_{ij})$ of alternatives with respect to each criterion are aggregated as:

$$(\tilde{x}_{ij}) = (a_{ij}, b_{ij}, c_{ij}, d_{ij}) \tag{15}$$

$$a_{ij} = \min_k \{a_{ijk}\},\quad b_{ij} = \frac{1}{K}\sum_{k=1}^{K} b_{ijk}\ ,\ \ c_{ij} = \frac{1}{K}\sum_{k=1}^{K} c_{ijk}\ \ ,\ d_{ij} = \max_k \{d_{ijk}\} \tag{16}$$

Then the fuzzy weights $(\tilde{w}_{ij})$ of each criterion are aggregated as:

$$\tilde{w}_j = (w^l_j, w^p_j, w^q_j, w^u_j) \tag{17}$$

$$w^l_j = \min_k \{w^l_{jk}\},\ \ w^p_j = \frac{1}{K}\sum_{k=1}^{K} w^p_{jk}\ \ ,\ w^q_j = \frac{1}{K}\sum_{k=1}^{K} w^q_{jk}\ ,\ \ w^u_j = \max_k \{w^u_{jk}\} \tag{18}$$

Step 5: The fuzzy decision matrix is also constructed as;

$$\tilde{D} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix}, \qquad \tilde{W} = [\tilde{w}_1, \tilde{w}_2, \cdots \tilde{w}_n] \tag{19}$$

where $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ and $\tilde{w}_j = (w^l_j, w^p_j, w^q_j, w^u_j)$; $i = 1,2,\ldots m,\ \ j = 1,2,\ldots n$ can be approximated by positive trapezoidal fuzzy numbers.

Step 6: The fuzzy decision matrix is normalized with the linear normalization formula as follows:

$$\tilde{R} = [\tilde{r}_{ij}]_{mxn} \quad i = 1,2,\ldots,m;\ \ j = 1,2,\ldots n \tag{20}$$

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{d^*_j}, \frac{b_{ij}}{d^*_j}, \frac{c_{ij}}{d^*_j}, \frac{d_{ij}}{d^*_j}\right) \qquad d^*_j = \max_i d_{ij},\quad j \in \Omega_B$$
$$\tilde{r}_{ij} = \left(\frac{a^-_j}{d_{ij}}, \frac{a^-_j}{c_{ij}}, \frac{a^-_j}{b_{ij}}, \frac{a^-_j}{a_{ij}}\right) \qquad a^-_j = \min_i a_{ij}\ \ ,\ \ j \in \Omega_C \tag{21}$$

The normalized fuzzy decision matrix is denoted as $\tilde{R}$. Ω_B and Ω_C are the benefit and cost criteria index sets respectively.

Step 7: Normalized decision matrix is weighted by multiplying the importance weights of evaluation criteria and the values in the normalized fuzzy decision matrix. The weighted normalized fuzzy decision matrix $\tilde{V}$ is defined as:

$$\tilde{V} = \begin{bmatrix} \tilde{v}_{11} & \tilde{v}_{12} & \cdots & \tilde{v}_{1n} \\ \tilde{v}_{21} & \tilde{v}_{22} & \cdots & \tilde{v}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{v}_{m1} & \tilde{v}_{m2} & \cdots & \tilde{v}_{mn} \end{bmatrix} \tag{22}$$

where $\tilde{v}_{ij} = \tilde{r}_{ij}(.)\tilde{w}_j \quad i = 1,2,\ldots,m \quad j = 1,2,\ldots,n$ and here $\tilde{w}_j$ represents the importance weight of criterion j.

Step 8: In this paper, the Hamming distance method is used for comparing two alternatives *g* and *f* on each criterion. At first the maximum between two fuzzy numbers is computed. So their least upper bound is determined to find $\max\left(\tilde{v}_{gj}, \tilde{v}_{fj}\right)$. Then the Hamming distances $d\left(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{gj}\right)$ and $d\left(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{fj}\right)$ are calculated. $\tilde{v}_{gj} \geq \tilde{v}_{fj}$ if and only if $d\left(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{fj}\right) \geq d\left(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{gj}\right)$. Otherwise $\tilde{v}_{gj} < \tilde{v}_{fj}$ if and only if $d\left(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{fj}\right) < d\left(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{gj}\right)$ (Hatami-Marbini & Tavana 2011). Then preference function is constructed as:

$$P_j(g,f) = \begin{cases} d(\max(v_{gj}, v_{fj}), v_{gj}) & \tilde{v}_{gj} < \tilde{v}_{fj} \\ d(\max(v_{gj}, v_{fj}), v_{fj}) & \tilde{v}_{gj} \geq \tilde{v}_{fj} \end{cases} \tag{23}$$

If *g* is better than *f* then $P_j(g, f) > 0$; otherwise, $P_j(g, f) = 0$.

Step 9: Fuzzy preference index is calculated to determine the value of the outranking relation (j=1,2,…,n):

$$\tilde{\pi}(g,f) = \frac{\sum_{j=1}^{n}\left[\tilde{w}_j P_j(g,f)\right]}{\sum_{j=1}^{n}\tilde{w}_j} \tag{24}$$

Step 10: The leaving and entering flows are calculated for ranking of alternatives.

$$\tilde{\phi}^{+}(g) = \sum_{\substack{f \neq g \\ f=1}}^{m} \tilde{\pi}(g,f) \tag{25}$$

$$\tilde{\phi}^{-}(g) = \sum_{\substack{f \neq g \\ f=1}}^{m} \tilde{\pi}(f,g) \tag{26}$$

The results of leaving and entering flows are presented in the form of fuzzy numbers. Defuzzification is used for ranking of fuzzy flows that converts fuzzy numbers to appropriate crisp values (Giannopoulos & Founti 2010). In this paper Chen et al.'s method (Eq.13) is used for defuzzifying flows. After computing defuzzified leaving and entering flows for each alternative, the PROMETHEE I method partial ranking can be obtained as follows:

$$\begin{cases} gP^I f \quad \text{iff} & \begin{cases} \phi^{+}(g) > \phi^{+}(f) \text{ and } \phi^{-}(g) < \phi^{-}(f) \text{ or} \\ \phi^{+}(g) = \phi^{+}(f) \text{ and } \phi^{-}(g) < \phi^{-}(f) \text{ or} \\ \phi^{+}(g) > \phi^{+}(f) \text{ and } \phi^{-}(g) = \phi^{-}(f); \end{cases} \\ gI^I f \quad \text{iff} & \phi^{+}(g) = \phi^{+}(f) \text{ and } \phi^{-}(g) = \phi^{-}(f); \\ gR^I f \quad \text{iff} & \begin{cases} \phi^{+}(g) > \phi^{+}(f) \text{ and } \phi^{-}(g) > \phi^{-}(f) \text{ or} \\ \phi^{+}(g) < \phi^{+}(f) \text{ and } \phi^{-}(g) < \phi^{-}(f). \end{cases} \end{cases} \tag{27}$$

where P^I, I^I, R^I are preference, indifference and incomparability, respectively.

Step 11: Net flow is calculated by using this formula:

$$\phi(g) = \phi^{+}(g) - \phi^{-}(g) \tag{28}$$

$$\begin{cases} gP^{II} f & \text{iff} \quad \phi(g) > \phi(f) \\ gI^{II} f & \text{iff} \quad \phi(g) = \phi(f) \end{cases} \tag{29}$$

P^{II} and I^{II} are preference and indifference respectively (Brans et al., 1986; Brans & Mareschal 2005).

Step 12: Finally preference rank of each alternative is evaluated by constructing a value outranking graph.

4. Numerical example

In this paper, the numerical example used by Chen et al. (2006) is considered to demonstrate the proposed fuzzy PROMETHEE approach. The numerical example is associated with selection of a suitable material supplier to purchase the key components of new products for a high-technology manufacturing company. Firstly a committee of three decision makers, D_1, D_2 and D_3, is formed to select the most suitable supplier. Five supplier candidates (A_1, A_2, A_3, A_4, A_5) are determined. Then five benefit criteria are considered as profitability of supplier (C_1), relationship closeness (C_2), technological capability (C_3), conformance quality (C_4), conflict resolution (C_5). Three decision makers use the linguistic weighting variables shown in Table 1 to assess the importance of the criteria. And also they use the linguistic rating variables shown in Table 2 to evaluate the ratings of alternatives with respect to each criterion.

Table 1
The linguistic variables for the importance weights of the five criteria

Linguistic variable	Fuzzy number	Linguistic variable	Fuzzy number
Very Low (VL)	(0 , 0 , 0.1 , 0.2)	Medium high (MH)	(0.5 , 0.6 , 0.7 , 0.8)
Low (L)	(0.1 , 0.2 , 0.2 , 0.3)	High (H)	(0.7 , 0.8 , 0.8 , 0.9)
Medium low (ML)	(0.2 , 0.3 , 0.4 , 0.5)	Very high (VH)	(0.8 , 0.9 , 1 , 1)
Medium (M)	(0.4 , 0.5 , 0.5 , 0.6)		

Table 2
The linguistic variables for the performance ratings

Linguistic variable	Fuzzy number	Linguistic variable	Fuzzy number
Very poor (VP)	(0 , 0 , 1 , 2)	Medium good (MG)	(5 , 6 , 7 , 8)
Poor (P)	(1 , 2 , 2 , 3)	Good (G)	(7 , 8 , 8 , 9)
Medium poor (MP)	(2 , 3 , 4 , 5)	Very good (VG)	(8 , 9 , 10 , 10)
Fair (F)	(4 , 5 , 5 , 6)		

The importance weights of the criteria determined by these three decision makers are shown in Table 3. The ratings of the five alternatives by the decision makers under the various criteria are shown in Table 4.

Table 3
Importance weight of criteria from three decision makers

	D_1	D_2	D_3
C_1	H	H	H
C_2	VH	VH	VH
C_3	VH	VH	H
C_4	H	H	H
C_5	H	H	H

Table 4
Ratings of the five alternatives by decision makers under various criteria

Criteria	Suppliers	Decision makers		
		D₁	***D₂***	***D₃***
C₁	A_1	MG	MG	MG
	A_2	G	G	G
	A_3	VG	VG	G
	A_4	G	G	G
	A_5	MG	MG	MG
C₂	A_1	MG	MG	VG
	A_2	VG	VG	VG
	A_3	VG	G	G
	A_4	G	G	MG
	A_5	MG	G	G
C₃	A_1	G	G	G
	A_2	VG	VG	VG
	A_3	VG	VG	G
	A_4	MG	MG	G
	A_5	MG	MG	MG
C₄	A_1	G	G	G
	A_2	G	VG	VG
	A_3	VG	VG	VG
	A_4	G	G	G
	A_5	MG	MG	G
C₅	A_1	G	G	G
	A_2	VG	VG	VG
	A_3	G	VG	G
	A_4	G	G	VG
	A_5	MG	MG	MG

Then the linguistic evaluations shown in Tables 3 and Table 4 are converted into trapezoidal fuzzy numbers to construct the fuzzy decision matrix and determine the fuzzy weight of each criterion, as in Table 5. The normalized fuzzy decision matrix is constructed as in Table 6. Weighted normalized fuzzy decision matrix is constructed as in Table 7.

Table 5
The fuzzy decision matrix and the fuzzy weight of each criterion

	C_1	C_2	C_3	C_4	C_5
A_1	(5,6,7,8)	(5,7,8,10)	(7,8,8,9)	(7,8,8,9)	(7,8,8,9)
A_2	(7,8,8,9)	(8,9,10,10)	(8,9,10,10)	(7, 8.7 , 9.3 , 10)	(8,9,10,10)
A_3	(7, 8.7 , 9.3 , 10)	(7, 8.3 , 8.7 , 10)	(7, 8.7 , 9.3 , 10)	(8,9,10,10)	(7, 8.7 , 9.3 , 10)
A_4	(7,8,8,9)	(5 , 7.3 , 7.7 , 9)	(5, 6.7 , 7.3 , 9)	(7,8,8,9)	(7, 8.7 , 9.3 , 10)
A_5	(5,6,7,8)	(5 , 7.3 , 7.7 , 9)	(5,6,7,8)	(5, 6.7 , 7.3 , 9)	(5,6,7,8)
Weight	(0.7 , 0.8 , 0.8 , 0.9)	(0.8 , 0.9 , 1 , 1)	(0.7 , 0.87 , 0.93 , 1)	(0.7 , 0.8 , 0.8 , 0.9)	(0.7 , 0.8 , 0.8 , 0.9)

Table 6
The normalized fuzzy decision matrix

	C_1	C_2	C_3	C_4	C_5
A_1	(0.5 , 0.6 , 0.7 , 0.8)	(0.5 , 0.7 , 0.8 , 1)	(0.7 , 0.8 , 0.8 , 0.9)	(0.7 , 0.8 , 0.8 , 0.9)	(0.7 , 0.8 , 0.8 , 0.9)
A_2	(0.7 , 0.8 , 0.8 , 0.9)	(0.8 , 0.9 , 1 , 1)	(0.8 , 0.9 , 1 , 1)	(0.7 , 0.87 , 0.93 , 1)	(0.8 , 0.9 , 1 , 1)
A_3	(0.7 , 0.87 , 0.93 , 1)	(0.7, 0.83 , 0.87 , 1)	(0.7 , 0.87 , 0.93 , 1)	(0.8 , 0.9 , 1 , 1)	(0.7 , 0.87 , 0.93 , 1)
A_4	(0.7 , 0.8 , 0.8 , 0.9)	(0.5 , 0.73 , 0.77 , 0.9)	(0.5, 0.67 , 0.73 , 0.9)	(0.7 , 0.8 , 0.8 , 0.9)	(0.7 , 0.87 , 0.93 , 1)
A_5	(0.5 , 0.6 , 0.7 , 0.8)	(0.5 , 0.73 , 0.77 , 0.9)	(0.5 , 0.6 , 0.7 , 0.8)	(0.5, 0.67 , 0.73 , 0.9)	(0.5 , 0.6 , 0.7 , 0.8)

Table 7
The weighted normalized fuzzy decision matrix

	C_1	C_2	C_3	C_4	C_5
A_1	(0.35 , 0.48 , 0.56 , 0.72)	(0.4 , 0.63 , 0.8 , 1)	(0.49 , 0.7 , 0.74 , 0.9)	(0.49 , 0.64 , 0.64 , 0.81)	(0.49 , 0.64 , 0.64 , 0.81)
A_2	(0.49 , 0.64 , 0.64 , 0.81)	(0.64 , 0.81 , 1 , 1)	(0.56 , 0.78 , 0.93 , 1)	(0.49 , 0.7 , 0.74 , 0.9)	(0.56 , 0.72 , 0.8 , 0.9)
A_3	(0.49 , 0.7 , 0.74 , 0.9)	(0.56, 0.75 , 0.87 , 1)	(0.49 , 0.76 , 0.86 , 1)	(0.56 , 0.72 , 0.8 , 0.9)	(0.49 , 0.66 , 0.7 , 0.9)
A_4	(0.49 , 0.64 , 0.64 , 0.81)	(0.4 , 0.66 , 0.77 , 0.9)	(0.35, 0.58 , 0.68 , 0.9)	(0.49 , 0.64 , 0.64 , 0.81)	(0.49 , 0.66 , 0.7 , 0.9)
A_5	(0.35 , 0.48 , 0.56 , 0.72)	(0.4 , 0.66 , 0.77 , 0.9)	(0.35 , 0.52 , 0.65 , 0.8)	(0.35, 0.54 , 0.58 , 0.81)	(0.35 , 0.48 , 0.56 , 0.72)

The distances between two alternatives *g* and *f* with respect to each criterion are calculated using the Hamming distance method through Eq. (12) and shown in Table 8. For example, considering the first criterion if the first and the second alternative are compared, the distance of the first alternative to $\max(\widetilde{v}_{11}, \widetilde{v}_{21})$ is 0.065 while distance of the second alternative to $\max(\widetilde{v}_{11}, \widetilde{v}_{21})$ is 0. This means that the second alternative is preferred over the first alternative in terms of the first criterion.

Table 8
The distance between two alternatives *g* and *f* with respect to each criterion

	X_{11}	X_{21}	X_{31}	X_{41}	X_{51}
X_{11}	-	(0.065 , 0)	(0 , 0)	(0.065 , 0)	(0 , 0)
X_{21}	-	-	(0.065 , 0)	(0 , 0)	(0 , 0.065)
X_{31}	-	-	-	(0 , 0.065)	(0 , 0)
X_{41}	-	-	-	-	(0 , 0.065)
X_{51}	-	-	-	-	-
	X_{12}	X_{22}	X_{32}	X_{42}	X_{52}
X_{12}	-	(0.205 , 0)	(0.105 , 0)	(0.015 , 0.065)	(0.015 , 0.065)
X_{22}	-	-	(0 , 0.1)	(0 , 0.105)	(0 , 0.105)
X_{32}	-	-	-	(0 , 0.005)	(0 , 0.005)
X_{42}	-	-	-	-	(0 , 0)
X_{52}	-	-	-	-	-
	X_{13}	X_{23}	X_{33}	X_{43}	X_{53}
X_{13}	-	(0.07 , 0)	(0.08 , 0)	(0 , 0.1)	(0 , 0.065)
X_{23}	-	-	(0 , 0.1)	(0 , 0.03)	(0 , 0.005)
X_{33}	-	-	-	(0 , 0.02)	(0 , 0.015)
X_{43}	-	-	-	-	(0 , 0.039)
X_{53}	-	-	-	-	-
	X_{14}	X_{24}	X_{34}	X_{44}	X_{54}
X_{14}	-	(0.065 , 0)	(0.05 , 0)	(0 , 0)	(0 , 0.09)
X_{24}	-	-	(0.015 , 0)	(0 , 0.065)	(0 , 0.025)
X_{34}	-	-	-	(0 , 0.05)	(0 , 0.04)
X_{44}	-	-	-	-	(0 , 0.09)
X_{54}	-	-	-	-	-
	X_{15}	X_{25}	X_{35}	X_{45}	X_{55}
X_{15}	-	(0.05 , 0)	(0.065 , 0)	(0.065 , 0)	(0 , 0.065)
X_{25}	-	-	(0 , 0.015)	(0 , 0.015)	(0 , 0.015)
X_{35}	-	-	-	(0 , 0)	(0 , 0)
X_{45}	-	-	-	-	(0 , 0)
X_{55}	-	-	-	-	-

The distances shown in Table 8 are expressed as preference function for every pair of alternatives through Eq. (23) and shown in Table 9.

Table 9
The preference function in terms of distances

	C_1	C_2	C_3	C_4	C_5
P (1,2)	0	0	0	0	0
P (1,3)	0	0	0	0	0
P (1,4)	0	0,065	0,1	0	0
P (1,5)	0	0,065	0,065	0,09	0,065
P (2,1)	0,065	0,205	0,07	0,065	0,05
P (2,3)	0	0,1	0,1	0	0,015
P (2,4)	0	0,105	0,03	0,065	0,015
P (2,5)	0,065	0,105	0,005	0,025	0,015
P (3,1)	0	0,105	0,08	0,05	0,065
P (3,2)	0,065	0	0	0,015	0
P (3,4)	0,065	0,005	0,02	0,05	0
P (3,5)	0	0,005	0,015	0,04	0
P (4,1)	0,065	0,015	0	0	0,065
P (4,2)	0	0	0	0	0
P (4,3)	0	0	0	0	0
P (4,5)	0,065	0	0,039	0,09	0
P (5,1)	0	0,015	0	0	0
P (5,2)	0	0	0	0	0
P (5,3)	0	0	0	0	0
P (5,4)	0	0	0	0	0

The fuzzy preferences indexes shown in Table 10 are calculated by using the value of preference function and weights of criteria. Then fuzzy flows are determined for each alternative through Eq. (25) and Eq. (26) and shown in Table 11.

Table 10
The fuzzy preference index

	A_1	A_2	A_3	A_4	A_5
A_1	-	(0, 0, 0, 0)	(0, 0, 0, 0)	(0.026 , 0.034 , 0.038 , 0,046)	(0.044 , 0.055 , 0.060 , 0.075)
A_2	(0.072 , 0.090 , 0.099 , 0.121)	-	(0.034 , 0.044 , 0.049 , 0.059)	(0.034 , 0.043 , 0.047 , 0,058)	(0.034 , 0.042 , 0.046 , 0.057)
A_3	(0.0467 , 0.059 , 0.065 , 0.080)	(0.012 , 0.015 , 0.015 , 0.020)	-	(0.021 , 0.026, 0.028 , 0,036)	(0.009 , 0.011 , 0.012 , 0,016)
A_4	(0.025 , 0.031 , 0.033 , 0.042)	(0, 0, 0, 0)	(0, 0, 0, 0)	-	(0.029 , 0.036 , 0.038 , 0,050)
A_5	(0.006 , 0.007 , 0.008 , 0.010)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	-

Table 11
Fuzzy flows for each alternative

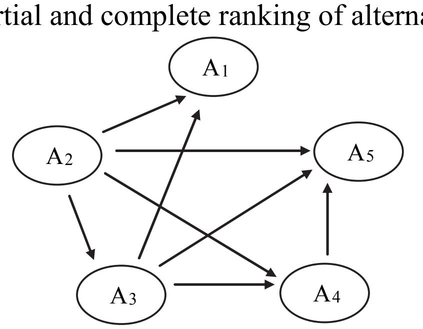
	Fuzzy leaving flows	Fuzzy entering flows
A_1	(0.070 , 0.089 , 0.098 , 0.121)	(0.150 , 0.188 , 0.206 , 0.253)
A_2	(0.175 , 0.218 , 0.242 , 0.295)	(0.012 , 0.015 , 0.015 , 0.020)
A_3	(0.089 , 0.112 , 0.120 , 0.151)	(0.034 , 0.044 , 0.049 , 0.059)
A_4	(0.054 , 0.068 , 0.072 , 0.092)	(0.081 , 0.103 , 0.113 , 0.139)
A_5	(0.006 , 0.007 , 0.008 , 0.010)	(0.116 , 0.145 , 0.157 , 0.197)

The fuzzy leaving and entering flows are defuzzified through Eq. (13). As a result of defuzzification, crisp leaving and entering flows are obtained and shown in Table 12. For complete ranking the net flows are calculated and shown in last column of Table 12.

Table 12
Crisp flows for each alternative

	Leaving flows	Entering flows	Net flows
A_1	0,094	0,199	-0,105
A_2	0,233	0,016	0,217
A_3	0,118	0,047	0,072
A_4	0,071	0,109	-0,038
A_5	0,008	0,154	-0,146

In fuzzy PROMETHEE I method, partial ranking of the alternatives gP^If and gR^If are $A_2P^IA_1$, $A_2P^IA_3$, $A_2P^IA_4$, $A_2P^IA_5$, $A_3P^IA_1$, $A_3P^IA_4$, $A_3P^IA_5$, $A_4P^IA_5$, $A_1R^IA_4$ and $A_1R^IA_5$. In fuzzy PROMETHEE II method complete ranking of $gP^{II}f$ are $A_2P^{II}A_3$, $A_3P^{II}A_4$, $A_4P^{II}A_1$ and $A_1P^{II}A_5$. Figure 1 and Figure 2 show the partial and complete ranking of alternatives respectively.



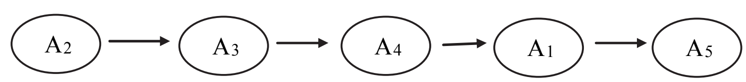


Fig. 1. Partial ranking of alternatives **Fig. 2.** Complete ranking of alternatives

When the results of our proposed methodology are compared with other results, it's seen that there is a similarity between our proposed methodology and the results of Chen et al. and Hatami-Marbini and Tavana according to Table 13 and Table 14.

Table 13
The results of proposed Fuzzy PROMETHEE method

Suppliers	Preferred alternatives	Incomparable alternatives	Ranking
A_1	-	A_4, A_5	4
A_2	A_1, A_3, A_4, A_5	-	1
A_3	A_1, A_4, A_5	-	2
A_4	A_5	A_1	3
A_5	-	A_1	5

Spearman's rank correlation coefficient is calculated to measure the correlation with other results. Spearman's rank correlation coefficients 1 and 0.8 are calculated between the results of our proposed approach and Chen et al.'s and Hatami-Marbini and Tavana's respectively. These coefficients support the similarity between the results.

Table 14
The results of Chen et al.'s TOPSIS method and Hatami-Marbini and Tavana's ELECTRE I method

Suppliers	The result of Chen et al.'s TOPSIS method		The result of Hatami-Marbini and Tavana's ELECTRE I method		
	CC_i	Ranking	Incomparable alternatives	Submissive alternatives	Ranking
A_1	0.5	4	A_4	A_5	3
A_2	0.64	1	A_3	A_1, A_4, A_5	1
A_3	0.62	2	A_2	A_1, A_4, A_5	1
A_4	0.51	3	A_1	A_5	3
A_5	0.4	5	-	-	5

5. Conclusion

In this paper an alternative approach based on fuzzy PROMETHEE method is proposed and it is used to solve the numerical example of Chen et al. (2006). Recently Hatami-Marbini and Tavana (2011) have solved the Chen et al. (2006)'s example with fuzzy ELECTRE method. Similar results have been found with these two studies by the proposed method. These results indicate that all these three methods are appropriate for the supplier selection problem. Also these three methods have some similarities and differences. Incomparability is taken into consideration in both fuzzy PROMETHEE and fuzzy ELECTRE methods but not in fuzzy TOPSIS method. Fuzzy PROMETHEE I method gives partial ranking and fuzzy PROMETHEE II method gives complete ranking of alternatives. But in fuzzy TOPSIS method only complete ranking of alternatives is found. Partial ranking can be found with fuzzy ELECTRE I method while complete ranking can be found by other variants of fuzzy ELECTRE method. When these three methods are compared with respect to the amount of computations fuzzy TOPSIS method requires less complex computations than fuzzy PROMETHEE and fuzzy ELECTRE methods. Both fuzzy PROMETHEE and fuzzy ELECTRE methods allow graphic representation and by this way decision makers can compare the alternatives easily. In all of these three methods distances between alternatives are considered for comparing alternatives under each criterion. In the literature there are fuzzy PROMETHEE methods proposed by different authors. Differently from these methods in this paper an alternative fuzzy PROMETHEE method is presented. The main advantage of proposed method is not requiring the use preference functions proposed by Brans et al. (1984). Preference functions are constructed in terms of distances between alternatives. By this way the amount of computations decreases and the method becomes convenient for decision makers. In future studies other fuzzy MCDM methods may be used to solve the supplier selection problem of the firms and the results of them may be compared. The other decision making problems may be considered. According to the problem different weights, criteria, linguistic variables and fuzzy numbers may be used. And also distance methods may be changed.

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