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Economic production quantity model for defective items under deterioration

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A B S T R A C T

Traditional economic production quantity (EPQ) model assumes that the production products are perfect. However, this assumption does not hold for many real production systems due to several weaknesses. This paper considers production inventory model with defective items for deteriorating items. In this paper, production rate is considered to be greater than demand rate. Mathematical model is developed for finding optimal order quantity, cycle time and total profit. Moreover, a numerical example is provided to illustrate the proposed model. Next, sensitivity analysis is established to demonstrate the model developed. Finally, some conclusions and future research directions are proposed.

1. Introduction

Traditional economic production quantity (EPQ) model assumes that the production products are all perfect. However, in reality, the defective items may be produced due to imperfect production process or imperfect quality items. The economic production quantities are used to determine the optimal order quantity and the minimum total inventory cost. The aim of this paper is also to find the minimum total cost of the inventory system. Harries (1913) is believed to be the first who presented economic order quantity (EOQ) model and Taft (1918) established EPQ model. EOQ model with demand dependent unit cost was addressed by Cheng (1991). Rosenblatt and Lee (1986) studied the effects of an imperfect production process on the optimal production cycle time for the traditional economic manufacturing quantity (EMQ) model. The relationships between process quality control and lot sizing was introduced by Portens (1986). Jalan and Chaudhuri (1999) presented an order level inventory model for deteriorating items without shortages. Zhang and Gerchak (1990) established joint lot sizing and inspection policy in an EOQ model under random yield. Ben-Daya (2002) presented an EPQ model for preventive maintenance level under an imperfect process. Perumal and Arivarignan (2002) established a production inventory model with two rates of production and back orders. Lin et al. (2003) developed

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production inventory model for imperfect production process under inspection schedules. Samanta and Roy (2004) established a production inventory model of deteriorating items with shortages and assumed the production rate is changed to another at a time when the inventory level reaches a prefixed level. Sana (2010) established a production inventory model in an imperfect production system. Cardenas-Barron (2008) developed an EPQ type inventory model with planned backorders for deteriorating the EPQ for a single production. Chang and Ho (2010) established an inventory model for items with imperfect quality and shortage backordering. Ouyang and Chang (2013) studied the effects of the reworking imperfect quality items and trade credits on economic production quantity model with imperfect production processes and complete backlogging. Tripathi (2014) presented EPQ model by considering four different circumstances. Teng and Chang (2009) established EPQ model under two levels of trade credit policy. Taleizadeh et al. (2015) developed a vendor managed inventory (VMI) model for a two-level supply chain comprised of one vendor and several non-competing retailers in which both the raw materials and the finished products have different deterioration rates. Numerous studies in economic production quantity models with imperfect quality items have been discussed by Sana et al. (2007), Hsu and Hsu (2014), Rabbani et al. (2014), Wu and Sarker (2013), Lin and Lin (2007), Lee and Kim (2014).

Many products deteriorate due to evaporation spoilage etc. such as green vegetables, fruits, blood bank, volatile liquids. Ghare and Schrader (1963) established an EOQ model for exponentially decaying items. Covert and Philip (1973) extended model of Ghare and Schrader (1963) by considering for Weibull distribution deterioration. Hariga (1996) presented EOQ models for deteriorating items with log-concave time-varying demand. Manna and Chaudhuri (2006) established an order level inventory system for deteriorating items with demand rate as a ramp type function of time. Wu and Chen (2014) established optimal lot sizing policies for a retailer who sells a deteriorating item to credit risk customers by offering partial trade credit to reduce his/her risk. Ghiami and Williams (2015) established a production inventory model in which a manufacturer is delivering deteriorating product to retailer. Das et al. (2015) presented an integrated production EOQ model under interaction fuzzy credit period for deteriorating item with several markets. Yu et al. (2013) developed an inventory model in which there is only one buyer for Weibull distribution deteriorating items. Some relevant articles on deterioration were established by Saiedy and Moghadam (2011), Chen and Teng (2015), Taleizadeh (2014), Sana and Chaudhuri (2008), Singh and Prasher (2014), Li et al. (2010), Tripathi (2013), Tripathi, and Kumar (2014).

In this paper, a production–inventory model for deteriorating and defective item is developed and discussed. The production rate is considered to be greater than demand rate. The optimal strategies such as sales revenue, production cost, setup cost and holding cost are considered to optimize the total profit.

The rest of the paper is organized as follows: Section 2 provides assumptions and notation. Section 3 includes the mathematical formulation of the proposed model. Section 4 establishes determination of optimal solution. Section 5 provides the numerical example. Furthermore a sensitivity analysis with the variation of several key parameters is provided in section 6. Finally, a conclusion is presented and some future research directions are provided in the last section 7.

2. Notations and Assumption

The following notations are used throughout the manuscript:

P production rate in unit per unit time
λ demand rate in units per unit time
Q_1 on hand inventory level in $[0, T_1]$
Q production Quantity
p production cost per unit
x proportion of defective item from regular production ($0 \leq x \leq 0.1$)

D defective item during regular production ($D = Px$)
c setup cost per setup
h holding cost per unit/year
T cycle time
T_1 production time
T_2 consumption time
T cycle time
Θ deterioration rate
SC setup cost
$I(t)$ inventory level at any time 't'
PC production cost
HC holding cost
TC total cost
Q^* optimal production Quantity
TC^* optimal total cost
T_1^* optimal production time
T_2^* optimal consumption time
T^* optimal cycle time

Assumptions

In addition, the following assumptions are made throughout the manuscript:

1. The production rate P is always greater than the sum of demand rate and defective item D (= Px),
2. No shortages are allowed,
3. The model is considered for single product,
4. Two rates of production are considered,
5. The defective item are directly proportional to the square of the product of defective item from regular production i.e. $D = Px$, where $0 \leq x \leq 0.1$.

3. Mathematical Formulation

The production starts at $t = 0$ and finishes at $t = T_1$. During the period $[0, T_1]$ the demand rate is λ , and the defective rate is D (= Px). The process begins at $t = 0$ and the inventory accumulates at a rate of ($P - \lambda - D$). The consumption rate starts from $t = T_1$ and finishes at $t = T_2$. In the time interval $[T_1, T_2]$ product becomes obsolete. During the period $[T_1, T_2]$ the inventory level starts to decrease due to deterioration and demand. The inventory level is governed by the following differential equations:

$$\frac{dI(t)}{dt} = (P - \lambda - D),\ 0 \leq t \leq T_1, \tag{1}$$

and $$\frac{dI(t)}{dt} = -\theta I(t) - \lambda,\ T_1 \leq t \leq T_2. \tag{2}$$

The solution of Eq. (1) and EQ. (2) with the condition $I(0) = 0$, $I(T_1) = Q_1$ and $I(T_2) = 0$,is given by

$$I(t) = (P - \lambda - D)t,\ 0 \leq t \leq T_1 \tag{3}$$

$$I(t) = \frac{\lambda}{\theta}\left\{e^{\theta(T-t)} - 1\right\},\ T_1 \leq t \leq T_2, \tag{4}$$

where $T = T_1 + T_2$.

At $t = T_1$. Eq. (3) and Eq. (4) are the same i.e.

$$Q_1 = (P - \lambda - D)T_1 = \frac{\lambda}{\theta}\left(e^{\theta T_2} - 1\right). \quad (5)$$

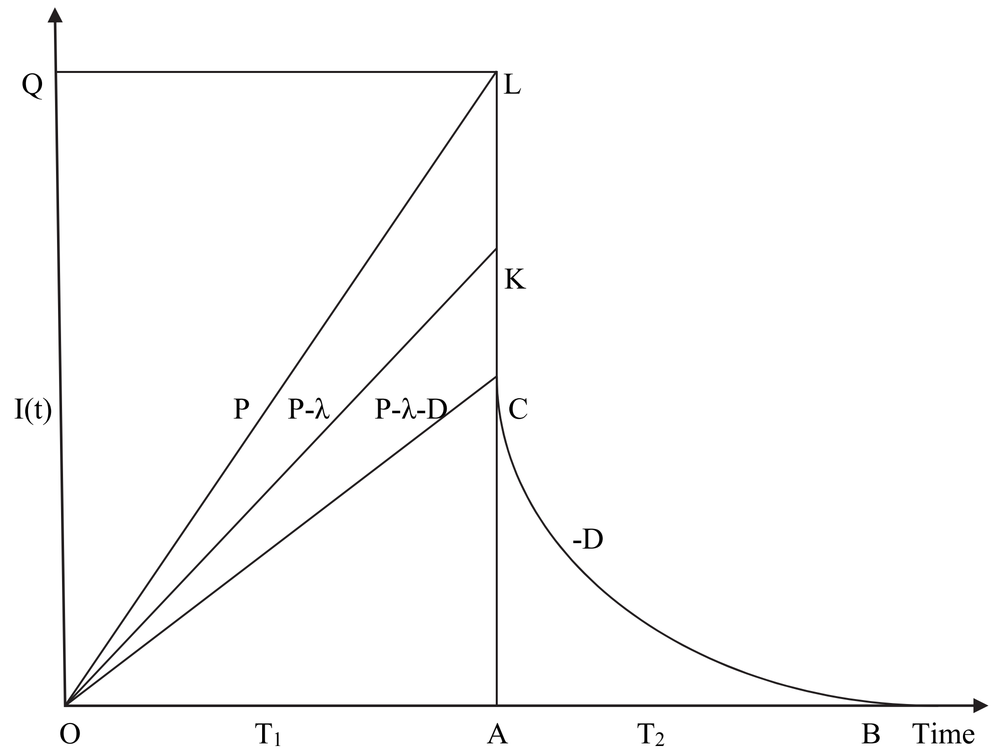

Fig. 1. I (t) vs time

The cycle time $T = T_1 + T_2$ needed to consume all units Q at demand rate i.e.

$$Q = \frac{\lambda}{\theta}\left(e^{\theta T} - 1\right), \quad (6)$$

$$\text{or, } T = \frac{1}{\theta}\log\left(1 + \frac{\theta Q}{\lambda}\right), \quad (7)$$

Fig.1 shows the change of the inventory system for one complete cycle. By assumption, the production rate of good item is always greater than or equal to the sum of demand rate and the rate of which defective items are produced (i.e. $P \geq \lambda + D$). Q_1 is the quantity of good item after consumption at the end of time $t = T_1$. During production, the cycle time is

$$T = T_1 + T_2 = \frac{1}{\theta}\log\left(1 + \frac{\theta Q}{\lambda}\right)(1 - x). \quad (8)$$

The total cost of the production system consists of three major costs; such as setup cost, production cost and holding cost. The total cost TC is given by

TC = Setup cost + Production Cost + Holding Cost

$$TC = SC + PC + HC. \quad (9)$$

All components of total profit are calculated as follows:

$$\text{(i). } SC = \frac{c}{T} = \frac{c\theta}{\log\left(1+\frac{\theta Q}{\lambda}\right)(1-x)}, \tag{10}$$

$$\text{(ii).} PC = \frac{pQ}{T} = \frac{p\theta Q}{\log\left(1+\frac{\theta Q}{\lambda}\right)(1-x)}, \tag{11}$$

$$\text{(iii).} HC = \frac{h}{T}\left[\int_0^{T_1}(P-\lambda-D)t.dt + \int_{T_1}^{T_2}\frac{\lambda}{\theta}\left(e^{\theta(T-t)}-1\right)dt\right] = \frac{\mathrm{h}}{\mathrm{T}}\left[\frac{(P-\lambda-D)T_1^2}{2} + \frac{\lambda}{\theta}\left\{\frac{e^{\theta T_2}-e^{\theta T_1}}{\theta} - (T_2-T_1)\right\}\right]. \tag{12}$$

Therefore total profit is given by

$$TC = \frac{\theta\left[c + pQ + h\left\{\frac{(P-\lambda-D)T_1^2}{2} + \frac{\lambda}{\theta}\left(\frac{e^{\theta T_2}-e^{\theta T_1}}{\theta} + T_1 - T_2\right)\right\}\right]}{\log\left(1+\frac{\theta Q}{\lambda}\right)(1-x)}. \tag{13}$$

Due to the presence of logarithmetic and exponential terms in the above Eq. (14), it is difficult to find closed form of the optimal solution. The second and the third approximations are used for logarithmetic and exponential terms i.e. $\log\left(1+\frac{\theta Q}{\lambda}\right) \approx \frac{\theta Q}{\lambda} - \frac{\theta^2 Q^2}{2\lambda^2}$ and $\mathrm{e}^{\theta T} \approx 1 + \theta T + \frac{\theta^2 T^2}{2}$, etc.

Total profit reduces to

$$TC \approx \frac{\lambda\left(c + pQ + hA\right)}{Q\left(1-\frac{\theta Q}{2\lambda}\right)(1-x)}, \tag{14}$$

$$\text{where } , A = \frac{1}{2}\left\{(P-2\lambda-D)T_1^2 + \lambda T_2^2 + \frac{\lambda\theta(T_2^3 - T_1^3)}{3}\right\}. \tag{15}$$

4. Determination of Optimal Solution

The necessary condition for total profit (TP) to be maximized is $\frac{\partial(TC)}{\partial Q} = 0; \frac{\partial(TC)}{\partial T_1} = 0,$ provided

$$\left(\frac{\partial^2(TC)}{\partial Q^2}\right)\left(\frac{\partial^2(TC)}{\partial T_1^2}\right) - \left(\frac{\partial^2(TC)}{\partial T_1 \partial Q}\right) > 0, \quad \frac{\partial^2(TC)}{\partial Q^2} > 0, \quad \frac{\partial^2(TC)}{\partial T_1^2} > 0.$$

Differentiating Eq. (15) partially respect to Q and T_1 (or T_2) yields

$$\frac{\partial(TC)}{\partial Q} = \frac{\theta pQ^2 - 2\left(c + hA\right)\left(\lambda - \theta Q\right)}{2Q^2\left(1-\frac{\theta Q}{2\lambda}\right)^2(1-x)}, \tag{16}$$

$$\frac{\partial(TC)}{\partial T_1}=\frac{h\lambda\left\{2(P-2\lambda-D)T_1+\frac{2T_2(P-\lambda-D)}{1+\theta T_2}+\frac{\theta T_2^{\,2}\left(P-\lambda-D\right)}{(1+\theta T_2)}-\lambda\theta T_1^{\,2}\right\}}{2Q\left(1-\frac{\theta Q}{2\lambda}\right)(1-x)}, \tag{17}$$

$$\frac{\partial^2(TC)}{\partial Q\partial T_1}=-\frac{\left(1-\frac{\theta Q}{\lambda}\right)A_1}{2Q^2\left(1-\frac{\theta Q}{2\lambda}\right)^2(1-x)}<0, \tag{18}$$

where $A_1=h\lambda\left\{2(P-2\lambda-D)T_1+2\lambda T_2+\frac{\theta T_2^{\,2}\left(P-\lambda-D\right)}{(1+\theta T_2)}-\lambda\theta T_1^2\right\}$.

$$\frac{\partial^2(TC)}{\partial Q^2}=\frac{\left(2\lambda-3\theta Q+\frac{3\theta^2Q^2}{2\lambda}\right)(c+hA)+\frac{p\theta^2Q^3}{2\lambda}}{Q^3\left(1-\frac{\theta Q}{2\lambda}\right)^3(1-x)}>0, \tag{19}$$

$$\frac{\partial^2(TC)}{\partial T_1^{\,2}}=\frac{h\lambda\left\{2(P-2\lambda-D)+\frac{(P-\lambda-D)^2\left\{2+\theta T_2(2+\theta T_2)\right\}}{\left(1+\theta T_2\right)^3}-2\lambda\theta T_1\right\}}{2Q\left(1-\frac{\theta Q}{2\lambda}\right)(1-x)}>0\cdot \tag{20}$$

From Eq. (18), Eq. (19) and Eq. (20), we get

$$\left(\frac{\partial^2(TC)}{\partial Q^2}\right)\left(\frac{\partial^2(TC)}{\partial T_1^{\,2}}\right)-\left(\frac{\partial^2(TC)}{\partial T_1\partial Q}\right)>0,\qquad \frac{\partial^2(TC)}{\partial Q^2}>0,\qquad \frac{\partial^2(TC)}{\partial T_1^{\,2}}>0\cdot$$

Putting $\frac{\partial(TC)}{\partial Q}=0,$ and $\frac{\partial(TC)}{\partial T_1}=0.$, we get

$$\theta pQ^2-\left[2c+h\lambda T_2^{\,2}\left\{\frac{(P-2\lambda-D)\lambda\left(1+\frac{\theta T_2}{2}\right)^2}{(P-\lambda-D)^2}+1+\frac{\theta T_2}{3}-\frac{\theta T_2\lambda^3\left(1+\frac{\theta T_2}{2}\right)^3}{3\left(P-\lambda-D\right)^3}\right\}\right](\lambda-\theta Q)=0, \tag{21}$$

and,

$$\frac{2(P-2\lambda-D)\lambda\left(1+\frac{\theta T_2}{2}\right)}{(P-\lambda-D)}+\frac{(2+\theta T_2)(P-\lambda-D)}{1+\theta T_2}-\frac{\theta\lambda^3T_2\left(1+\frac{\theta T_2}{2}\right)^2}{(P-\lambda-D)^2}=0 \tag{22}$$

It is clear from Eq. (19), Eq. (20) and Eq. (21), the total cost *TC* is a convex function of *Q* and T_1 (or T_2) or *T*. The optimal solution (minimum) is obtained by solving $\frac{\partial(TC)}{\partial Q}=0,$ and $\frac{\partial(TC)}{\partial T_1}=0,$ simultaneously.

4. Numerical Example 1. (With production of defective items from regular production x)

An example is derived to validate the effect of the model with the following data:

The inventory parametric values, P = 5000 units/year, λ = 2500 units/year, c = 20 per setup, x = 0.01, h = 10 per unit/year, p = 100 / unit, and Θ = 0.05 .Substituting these values in (21) , (22) and (15) solving for T_2 and Q , we get $Q = Q^*$ = 192.629 units $T_1 = T_1^*$ = 0.0383437 year $T_2 = T_2^*$ = 0.0375416 year , $T = T^*$ = 0.0758853 year and $TC = TC^*$ = \$ 4.82075x10^7.

5. Sensitivity Analysis

In this section we will discuss the variation of changes in the key parameters of the system on the optimal solution. In decision making the sensitivity analysis helps both vendor and buyers. Using the numerical example given in section 4, the sensitivity analysis of various parameters has been performed. Sensitivity analysis is performed by changing the key parameters and taking one parameter at a time keeping the remaining parameters at their original values. The results are summarized in the following Tables 1.

Table 1

Effect of change in x, Θ, h, p, λ, P, and c on the optimal replenishment policy

x	*Q**	T_2*	T_1*	*T**	*TC**
0.02	153.744	0.01761470	0.01835670	0.03597140	3.84861×107
0.03	146.084	0.01097420	0.01167700	0.02265210	3.65710×107
0.04	143.544	0.00766030	0.00832801	0.01598830	3.59360×107
0.05	142.461	0.00567782	0.00630958	0.01198740	3.56653×107
0.06	141.928	0.00436128	0.00495654	0.00931782	3.55320×107
θ	Q*	T2*	T1*	T*	TC*
0.06	175.781	0.0375268	0.0383358	0.0758626	4.39955×107
0.07	162.684	0.0375120	0.0383278	0.0758398	4.07213×107
0.08	152.125	0.0374971	0.0383197	0.0758168	3.80815×107
0.09	143.377	0.0374823	0.0383118	0.0757941	3.58946×107
0.10	135.975	0.0374675	0.0383038	0.0757713	3.40441×107
h	Q*	T2*	T1*	T*	TC*
15	213.725	0.0375416	0.0383437	0.0758853	5.34816×107
20	232.906	0.0375416	0.0383437	0.0758853	5.82769×107
25	250.614	0.0375416	0.0383437	0.0758853	6.27039×107
30	267.143	0.0375416	0.0383437	0.0758853	6.68362×107
35	282.700	0.0375416	0.0383437	0.0758853	7.07255×107
p	Q*	T2*	T1*	T*	TC*
110	183.681	0.0375416	0.0383437	0.0758853	5.05625×107
120	175.875	0.0375416	0.0383437	0.0758853	5.28127×107
130	168.987	0.0375416	0.0383437	0.0758853	5.49710×107
140	162.850	0.0375416	0.0383437	0.0758853	5.70478×107
150	157.337	0.0375416	0.0383437	0.0758853	5.90516×107
λ	Q*	T2*	T1*	T*	TC*
2600	145.386	0.005764050	0.006378170	0.012142220	3.78524×107
2700	146.976	0.002345190	0.002814390	0.005159580	3.97375×107
2800	149.505	0.001099720	0.001432230	0.003914110	4.19174×107
2900	152.121	0.000486061	0.000687607	0.001173668	4.41731×107
3000	154.720	0.000138082	0.000212435	0.000350517	4.64760×107
P	Q*	T2*	T1*	T*	TC*
4500	141.281	0.00144387	0.00184645	0.00329032	3.53703×107
4600	141.382	0.00222655	0.00271017	0.00493672	3.53955×107
4700	141.653	0.00347378	0.00403400	0.00750778	3.54633×107
4800	142.491	0.00574165	0.00637486	0.01211651	3.56728×107
4900	146.165	0.01106330	0.01176770	0.02283100	3.65913×107
c	Q*	T2*	T1*	T*	TC*
22	197.723	0.0375416	0.0383437	0.0758853	4.94860×107
24	202.688	0.0375416	0.0383437	0.0758853	5.07322×107
26	207.533	0.0375416	0.0383437	0.0758853	5.19485×107
28	212.263	0.0375416	0.0383437	0.0758853	5.31360×107
30	216.898	0.0375416	0.0383437	0.0758853	5.42970×107

Based on the computational results shown in Table 1, the following inferences can be made from managerial point of view:

(i). It can be seen that the decrease of *T*, Q** and *TC** with an increase in the proportion of defective item from regular production *x*. It indicates that if the proportion of defective item from regular production increases, cycle time, order quantity and total cost decreases.

(ii). It can be observed that *Q** and *T** will decrease whereas *TC** increases with the increase of deterioration rate *Θ*. It represents that change of deterioration rate will cause negative change in optimal order quantity and cycle time but positive change in total cost.

(iii). It can be seen that *T** remains constant when there is an increase in *Q** and *TC** with the increase of holding cost *h*. It indicates that if the holding cost changes there is a positive change in Q* and TC* while T* remain constant.

(iv). When production cost, p, increases, the optimal cycle time remains constant while optimal order quantity *Q** decreases and total cost *TC** increases. That is, any change in *p* will lead to negative change on *Q** and positive change on *TC**.

(v). The increase on demand rate, *λ,* results a decrease in optimal cycle time *T**, increase in optimal order quantity *Q** and total cost *TC**. That is, any change in *λ* will lead to negative change in *T** and positive change in *Q** and *TC**.

(vi). It can be seen that as the optimal cycle time *T** increases, optimal order quantity *Q** approximately remains constant and the optimal total cost increases with the increase of production rate P. That is, any change in *P* will lead to positive change in *T** and total cost *TC** and *Q** remain constant.

(vii). When setup cost *c* increases, optimal cycle time *T** remains constant while increases the order quantity *Q** and the total cost *TC**. That is, any change in *c* will lead to positive change in *Q** and *TC**.

Special Case I: If production of defective items from regular production x = 0, then D = 0. The total cost in the situation is as follows,

$$TC \approx \frac{\lambda\left[c + pQ + \frac{h}{2}\left\{(P-2\lambda)T_1^2 + \lambda T_2^2 + \frac{\lambda\theta(T_2^3 - T_1^3)}{3}\right\}\right]}{Q\left(1 - \frac{\theta Q}{2\lambda}\right)} \quad (23)$$

Differentiating Eq. (23) partially with respect to *Q* and T_2 yields

$$24\theta pQ^2\left(P-\lambda\right)^3 - \left[48c\left(P-\lambda\right)^3 + h\lambda T_2^2\left\{6(P-2\lambda)\lambda\left(2+\theta T_2\right)^2(P-\lambda) + 8\left(3+\theta T_2\right)(P-\lambda)^3 - \theta T_2\lambda^3\left(2+\theta T_2\right)^3\right\}\right](\lambda-\theta Q) = 0 \quad (24)$$

$$\&, 4\lambda(P-\lambda)(P-2\lambda)\left(2+\theta T_2\right)\left(1+\theta T_2\right) + 4(2+\theta T_2)(P-\lambda)^3 - \theta\lambda^3 T_2\left(2+\theta T_2\right)^2\left(1+\theta T_2\right) = 0 \quad (25)$$

Example 2 on special case I. (Without production of defective items from regular production x)

Taking the same numerical date as mentioned in Example 1 and solving Eq. (24) and Eq. (25) simultaneously for Q and T_2, we get Q = Q* = 27658.1 units, $T_1 = T_1^* = 22.2677$ years, $T_2 = T_2^* = 15.9264$ years and total cost TC = TC* = 6.91479×10^9.

Note: If we compare the results obtained from numerical examples 1 and 2, we see that large differences (jump) in the order quantity Q, production time T_1 , consumption time T_2, and total cost TC.

6. Conclusion and Future Research

In real life the use of commodities decreases over time for deteriorating items. For more profit it is suggested that the cycle time should have smaller length. In this paper, EPQ model has been developed considering for defective items under deterioration. We have given mathematical formulation of the problem to find the optimal order quantity, cycle time and total profit. We have also shown that the total profit was a concave function of order quantity and cycle time. Finally, the sensitivity of the solution to changes in the values of different key parameters has been discussed. We have also compared the results with and without production of defective items from regular production x. From managerial point of view the following inferences can be made:

- The change of the proportion of defective item from regular production will cause negative change in optimal cycle time, order quantity and total cost.
- The change on deterioration rate will cause negative change in optimal order quantity and cycle time but positive change in total cost.
- If the holding cost changes there is a positive change in optimal order quantity and total cost.
- The change in p will lead to negative change on order quantity and positive change in total cost.
- The change in λ will lead to negative change in cycle time and positive change in order quantity and total cost.
- The change in c will lead to positive change in order quantity and total cost.

The model proposed in this study can be extended in several ways. We may extend the constant demand rate to a time dependent demand. Also, we could consider the constant deterioration to a two-parameter Weibull distribution. We may also generalize the model to allow for shortages, inflation rates, demand rate as a function of quality and others

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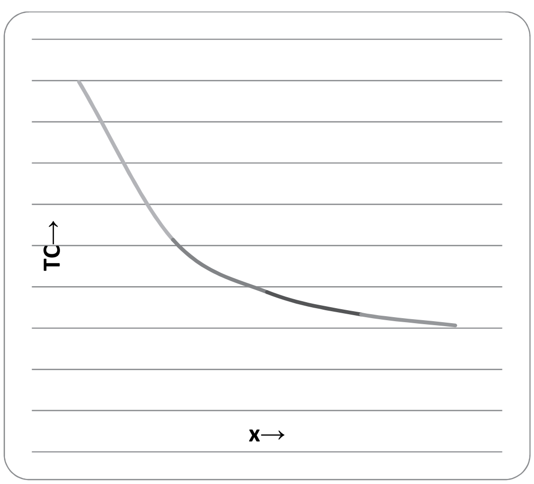
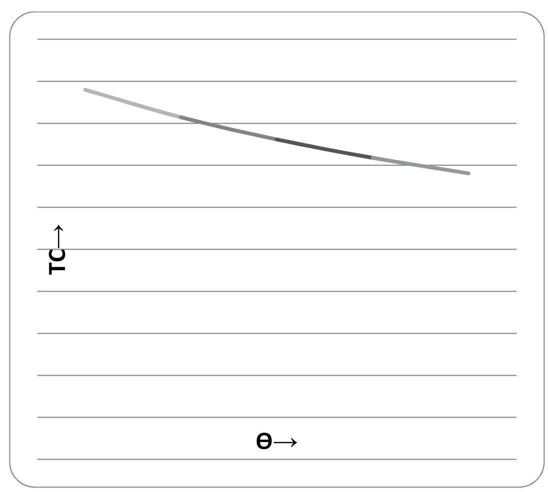
Appendix





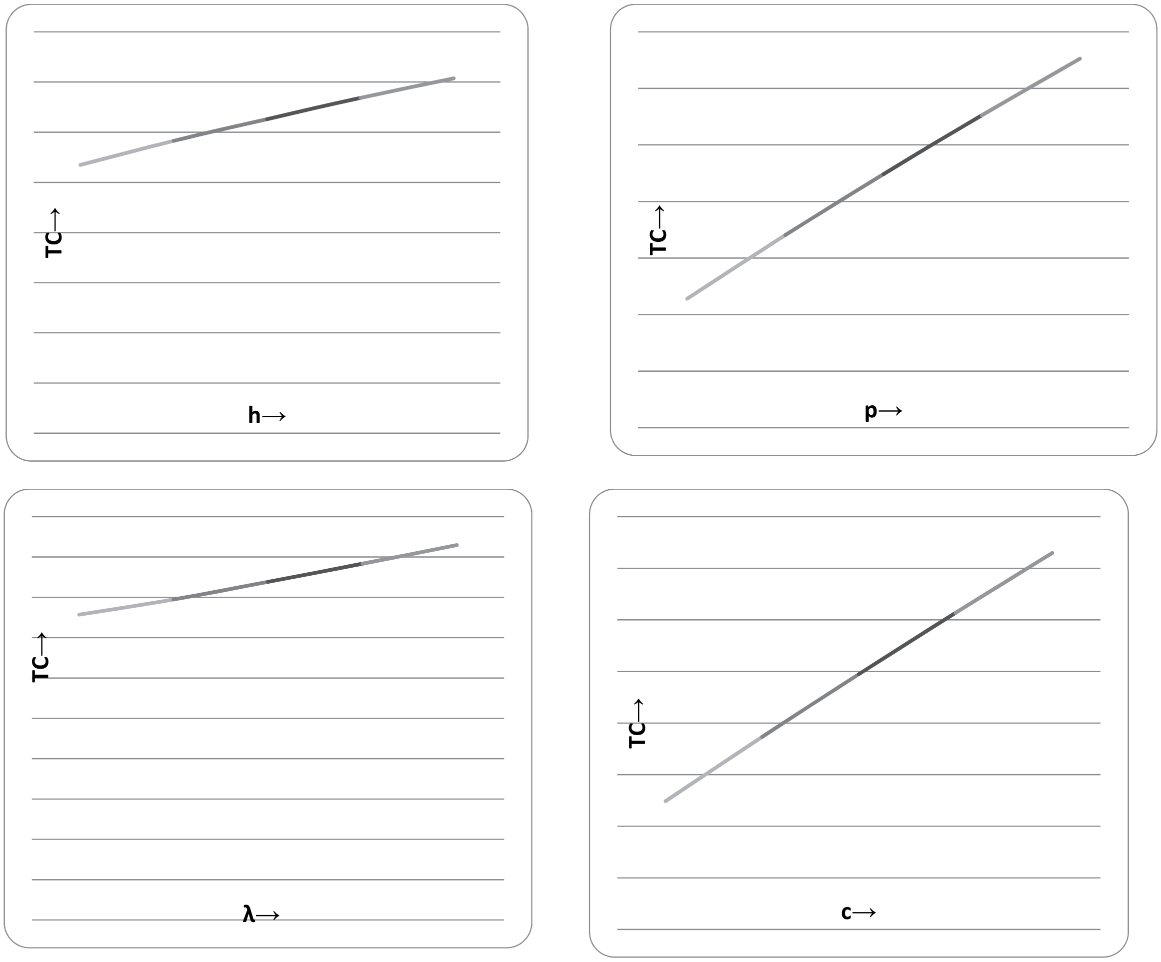


Fig. 2. *x, Θ, ,h, p, λ , c* and *TC*