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Two-warehouse optimized inventory model for time dependent decaying items with ramp type demand rate under inflation

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A B S T R A C T

This paper deals with developing an inventory model for two warehouses. In today's business era, there are various types of conditions such as discounts, bulk storage and seasonal products forcing the buyer to purchase the order more than owned warehouse capacity. To store the excess unit of purchase order, buyer arrange additional storage space called as rented warehouse. It is known that the demand of the seasonal products (as woolen garments) increases at the beginning of the season up to a certain time and then stabilizes to a constant rate for the remaining time of the season. The ramp type demand rate forces the buyer to store a higher quantity of the product at the beginning of the season. Most of the physical goods undergo decay or deterioration over time so we study deteriorating seasonal products in this paper. This two warehouse inventory model is developed with inflation and shortages. The model starts with rent warehouse, in first rent warehouse's inventory level is depleted due to demand and deterioration. At this time own warehouse is depleted due to deterioration only. But after that the inventory level of owned warehouse is depleted due to both demand and deterioration. The shortages are considered in owned warehouse, which is partially backlogged. Numerical solution of the model is obtained to verify the optimal solution. Comprehensive sensitivity analysis has been carried out for showing the effect of variations in the parameters. The model is solved analytically by minimizing the total cost.

1. Introduction

It is often seen that the capacity of warehouses may be limited. But in the super markets when an attractive price discount is offered to customers, the demand of the products increases. The customer intends to buy more goods, which creates greater demand for the goods. This condition motivates the buyers to increase their order quantity in an attempt to earn more profit and to increase the revenue. To store the excess unit of purchase order, buyer arranges the storage space (rent warehouse). It is assumed that the holding cost in rent warehouse (RW) is greater than owned warehouse (OW) due to additional rent charge. Sharma (1983) introduced a two warehouse inventory model by assuming the cost of transporting unit from RW to OW as constant. Goswami and Chaudhuri (1992) proposed an economic order quantity model for items with two levels of storages for a linear trend in demand. In the first

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phase of the paper, a deterministic model without shortage was developed and in the second phase deterministic model with shortage was considered. In both cases the authors assumed that the stock of RW were transported in 'n' shipments after an optimum time interval between successive shipments taking a linear increasing trend in demand with time.

A two warehouse inventory model for a linear trend in demand was proposed by Bhunia and Maiti (1994) for single item with infinite rate of replenishment and linear increasing demand where shortage was completely backlogged. Bhunia and Maiti (1998) developed a deterministic inventory model with two warehouses for deteriorating items taking linearly increasing demand with time, shortages were acceptable and surplus demand was backlogged as well. Stock was transferred from RW to OW and the deterioration rate was different in both the warehouses.

Kar et al. (2001) developed a deterministic inventory model for a single item having two separate storage facilities (owned and rented warehouses) due to limited capacity of the existing storage (owned warehouse) with linearly time-dependent demand (increasing) over a fixed limited time. The model was formulated by assuming that the rate of replenishment is infinite. Shortages are permissible and totally backlogged. Zhou (2003) developed a deterministic replenishment model with warehouse possessing limited storage capacity. In this model, the replenishment rate is unlimited. The demand rate is time dependent and increases at a decreasing rate. The stocks of rented warehouse are transported to owned warehouse in continuous release pattern. In this model shortages are allowed in owned warehouse and permits part of the backlogged shortages to turn into lost sales which are assumed to be a function of the currently backlogged amount. As a special case of the model, the parallel models with fully backlogged shortages and without shortages are also presented.

An inventory model with two warehouses and stock-dependent demand rate was proposed by Zhou and Yang (2005). Shortages were not allowed in the model and the transportation cost for transferring items from RW to OW was taken to be dependent on the transported amount. Skouri and Konstantaras (2013) developed two warehouse inventory models for deteriorating products with ramp type demand rate.

The condition becomes more complex when the inventory deteriorates in nature. Deterioration of goods is one of the important factors in any inventory and production systems. Many researchers have worked for inventory with deteriorating items in recent years because most of the physical goods undergo decay or deterioration over time. Ghare and Schrader (1963) suggested a model for an exponentially decaying inventory. Inventory models with a time dependent rate of deterioration were considered by Covert and Philip (1973), Chung and Ting (1993), Hariga and Benkherouf (1994), Wee (1995), Giri and Chaudhuri (1997), Giri et al. (2003). They have done significant works in the field of structural properties of an inventory system with deterioration and trended demand. Singh et al. (2008) introduced an ordering policy for perishable items having stock dependent demand with partial backlogging and inflation. Chaudhary and Vikas (2013a) proposed an inventory model for deteriorating items with Weibull deterioration with time dependent demand and shortages. In general holding cost is assumed to be known and constant. Chaudhary and Vikas (2013b) suggested Retailer's profit maximization model for Weibull deteriorating items with permissible delay on payments and shortages. Optimal inventory model for time dependent decaying items with stock dependent demand rate and shortages was introduced by Chaudhary and Vikas (2013c,d). Chaudhary and Vikas (2015) proposed an optimal policy for Weibull deteriorating items with power demand pattern and permissible delay on payments. Chaudhary and Vikas (2016) developed supply chain model with multi distributer and multi retailer with deterioration.

In this paper, we develop an optimal inventory model in which deterioration rate follows Weibull distribution with two parameters. Shortages are considered as partially backlogged. Demand rate is price dependent. We solve the model to optimize the total profit which is maximum. Model is illustrated with numerical examples and comprehensive sensitivity analysis.

2. Assumptions and Notations

The proposed inventory model is developed under the following assumptions and notations:

2.1 Assumptions

- The system operates for a prescribed period of T units of time and the replenishment rate is infinite.
- Lead time is zero.
- Shortages are partially backlogged and backlogged rate is δ which is constant.
- The inflation is also consider, the inflation rate is assume r and it is defined as follows, $f(t) = e^{-rt} \quad r > 0$
- The deterioration rate is time dependent given by the deterioration rate $\theta_1(t)$ in RW as $\theta_1(t) = \alpha t$ where α is deterioration rate parameter with $\alpha > 0$. Deterioration rate $\theta_2(t)$ in OW is given by $\theta_2(t) = \beta t$ where β is deterioration rate parameter with $\beta > 0$.

- Demand rate $D(t)$ is ramp type function as follows,

$$D(t) = \begin{cases} f(t) = a + bt, & t < \mu \\ f(\mu) = a + b\mu, & t \geq \mu \end{cases}$$

where $f(t)$ is a linear function of time, and $f(\mu)$ is a linear function of μ.

- The ordering cost A_0 is constant.
- The cycle length is assumed $0 < t < T$.

2.2 Notations

The following notations are made for development of mathematical model:

$I_O(t)$ is the inventory level in OW at time t $(0 \leq t < T)$.
$I_R(t)$ is the inventory level in RW at time t $(0 \leq t < T)$.
t_1 is the time at which the inventory level reaches zero in OW.
x_1 is the time at which the inventory level reaches zero in RW.
C_1 is the inventory holding cost per unit item per unit time in RW.
C_2 is the inventory holding cost per unit item per unit time in OW. ($C_1 > C_2$).
C_3 is the shortage cost per unit item per unit time.
C_4 is the deterioration cost per unit item per unit time.
C_5 is the per unit item opportunity cost due to the lost sales.
A_0 is the ordering cost.
W is the capacity of owned warehouse.
μ is the point where increasing demand becomes steady.

3. A two-warehouse with ramp type demand rate under inflation

The length of the cycle is T. During the interval $[0, x_1]$ the inventory level in RW depleted due to demand and deterioration and it vanishes at $t = x_1$. In OW, the inventory level W decreases during $[0,x_1]$ due to deterioration only, But during $[x_1,t_1]$ the inventory level is depleted due to both demand and deterioration. At time t_1 both warehouses are empty and after that the shortages occurring in the period (t_1, T) which is partially backlogged. Backlogging rate is δ.

The differential equation can be expressed for inventory level at time t at RW and OW when the instantaneous state over (0, T) are given by

$$I_R'(t) + \alpha t.I_R(t) = -D(t) \quad 0 \le t \le x_1 \quad \text{with } I_R(x_1) = 0 \tag{1}$$
$$I_o'(t) + \beta t.I_O(t) = 0 \quad 0 \le t \le x_1 \quad \text{with } I_O(0) = W \tag{2}$$
$$I_o'(t) + \beta t.I_O(t) = -D(t) \quad x_1 \le t \le t_1 \quad \text{with } I_O(t_1) = 0 \tag{3}$$
$$I_o'(t) = -\delta D(t) \quad t_1 \le t \le T \quad \text{with } I_O(t_1) = 0 \tag{4}$$

The solution of these equations are defined by the relation between x_1, t_1, and μ with respect to demand rate function so that the following three cases may arise

3.1 Case I: $x_1 \le t_1 \le \mu$

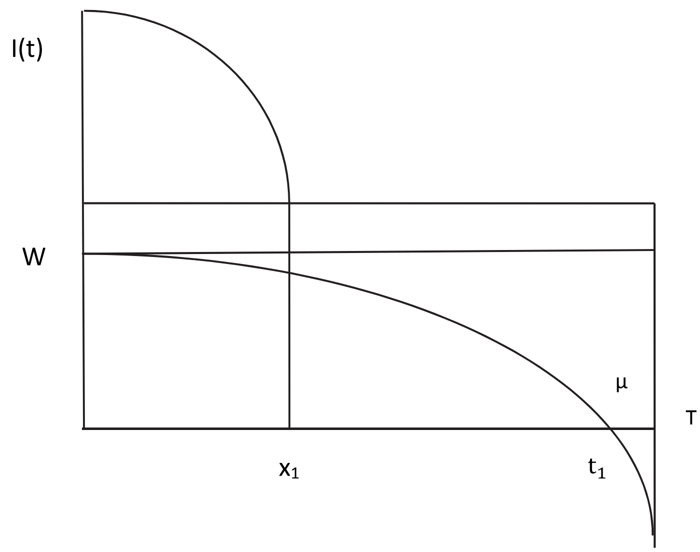

Fig. 1. Inventory system for the case $x_1 \le t_1 \le \mu$

In this case the above equations are defined as follow,

$$I_R'(t) + \alpha t.I_R(t) = -(a+bt) \quad 0 \le t \le x_1 \text{ with } I_R(x_1) = 0 \tag{5}$$
$$I_o'(t) + \beta t.I_O(t) = 0 \quad 0 \le t \le x_1 \text{with} \quad I_O(0) = W \quad I_o'(t) + \beta t.I_O(t) = -(a+bt) \quad x_1 \le t \le t_1 \text{ with } I_O(t_1) = 0 \tag{6}$$
$$I_o'(t) = -\delta(a+bt) \quad t_1 \le t \le \text{ with } I_O(t_1) = 0 \tag{7}$$
$$I_o'(t) = -\delta(a+b\mu) \quad \mu \le t \le T \quad \text{with } I_O(\mu) = 0 \tag{8}$$

The solution of the above equations can be derived as follows,

$$I_R(t) = \left[1-\alpha\frac{t^2}{2}\right]\left[\alpha(x_1-t)+\frac{b}{2}(x_1^2-t^2)+\frac{\alpha a}{6}(x_1^3-t^3)+\frac{\alpha b}{8}({x_1}^4-t^4)\right] \quad 0 \le t \le x_1 \tag{9}$$
$$I_{O1}(t) = W - \beta t \quad 0 \le t \le x_1 \tag{10}$$
$$I_{O2}(t) = \left[1-\beta\frac{t^2}{2}\right]\left[\alpha(t_1-t)+\frac{b}{2}(t_1^2-t^2)+\frac{\beta a}{6}(t_1^3-t^3)\right] \quad x_1 \le t \le t_1 \tag{11}$$
$$I_{O3}(t) = \left[\delta a(t_1-t)+\frac{\delta b}{2}(t_1^2-t^2)\right] \quad t_1 \le t \le \mu \tag{12}$$

$$I_{O4}(t) = [\delta a(\mu - t) + \delta b(\mu^2 - \mu t)] \qquad \mu \le t \le T \qquad (13)$$

3.1.1 Holding Cost for the warehouses during the time period 0 to t_1 under the inflation rate r

The holding cost for rent warehouse (H_R') during the time interval 0 to x_1 is as follows,

$$H_R' = \int_0^{x_1} e^{-rt}.I(t)dt.$$

The total holding cost during the time period 0 to x_1 is given as follows,

$$H_R' = c_1 \int_0^{x_1} e^{-rt}.I(t)dt.$$

Now total holding cost for rent warehouse is given by

$$H_R' = c_1 \left[\frac{a}{2}x_1^2 - \left(\frac{1}{6}ar - \frac{1}{3}b\right)x_1^3 - \left(\frac{1}{24}a\alpha - \frac{1}{8}br + \frac{1}{8}\alpha a\right)x_1^4 + \left(\frac{1}{40}a\alpha r - \frac{1}{30}b + \frac{1}{45}\alpha ar + \frac{1}{10}b\alpha\right)x_1^5 + \left(\frac{1}{48}a\alpha b - \frac{1}{72}\alpha^2 a - \frac{1}{24}\alpha br\right)x_1^6 + \left(\frac{1}{112}ar - \frac{1}{84}\alpha^2 b\right)x_1^7 + \left(\frac{1}{128}\alpha^2 rb\right)x_1^8\right]$$

The holding cost for own warehouse (H_{O1}') during the time interval 0 to x_1 is as follows,

$$H_{O1}' = \int_0^{x_1} e^{-rt}.I_{O1}(t)dt.$$

The total holding cost during the time period 0 to x_1 is stated as follows,

$$H_{O1}' = c_2 \int_0^{x_1} e^{-rt}.I_{O1}(t)dt.$$

Now total holding cost will be during the time period 0 to x_1 is stated as follows,

$$H_{O1}' = c_2 \left[wx_1 - \frac{1}{2}(wr + \beta)x_1^2 + \frac{1}{3}\beta rbx_1^8\right].$$

The holding cost for own warehouse (H_{O2}') during the time interval x_1 to t_1 are given as follows,

$$H_{O2}' = \int_{x1}^{t_1} e^{-rt}.I_{O2}(t)dt.$$

The total holding cost during the time period x_1 to t_1 is also given by

$$H_{O2}' = c_2 \int_{x1}^{t_1} e^{-rt}.I_{O2}(t)dt.$$

$$H_{O2}' = c_2 \left[\frac{a}{2}(t_1^2 - x_1^2) - \left(\frac{1}{6}ar - \frac{1}{3}b\right)(t_1^3 - x_1^3) - \left(\frac{1}{24}a\beta - \frac{1}{8}br + \frac{1}{8}\beta a\right)(t_1^4 - x_1^4) + \left(\frac{1}{40}a\beta r - \frac{1}{30}b + \frac{1}{45}\beta ar + \frac{1}{10}b\beta\right)(t_1^5 - x_1^5) + \left(\frac{1}{48}a\beta b - \frac{1}{72}\beta^2 a - \frac{1}{24}\beta br\right)(t_1^6 - x_1^6) + \left(\frac{1}{112}ar - \frac{1}{84}\beta^2 b\right)(t_1^7 - x_1^7) + \left(\frac{1}{128}\beta^2 rb\right)(t_1^8 - x_1^8)\right].$$

Now total holding cost for warehouses

$$H_C' = H_R' + H_{O1}' + H_{O2}'. \qquad (14)$$

3.1.2 Deterioration cost for the warehouses during the time period 0 to t_1 under the inflation rate r

The deterioration cost for rent warehouse (D_R) during the time interval 0 to x_1 is as follows,

$$D_R' = \int_0^{x_1} e^{-rt}.\theta_1(t).I_R(t)dt.$$

The total deterioration cost during the time period 0 to x_1 is stated as follows,

$$D_R' = c_3\int_0^{x_1} e^{-rt}.\alpha t.I_R(t)dt.$$

Now total deterioration cost for rent warehouse can be computed as

$$D_R' = c_3\left[\frac{a}{6}x_1^3 + (b+2ar)x_1^4 + (3a\alpha - 2br)\frac{x_1^5}{30} - \left(\frac{1}{36}ar(1-3\alpha) + \quad b\left(\frac{r}{16} - \frac{\alpha}{24}\right)\right)x_1^6 - \alpha\left(\frac{1}{105}br + \frac{1}{112}\alpha a\right)x_1^7 + \alpha^2\left(\frac{1}{160}ar - \frac{1}{128}b\right)x_1^8 + \quad \left(\frac{1}{180}\alpha^2 rb\right)x_1^9\right].$$

The deterioration cost for own warehouse (D_{O1}') during the time interval 0 to x_1 is as follows,

$$D_{O1}' = c_3\int_0^{x_1} e^{-rt}.\theta_1(t).I_{O1}(t)dt.$$

The total deterioration cost during the time period 0 to x_1 is as follows,

$$D_{O1}' = c_3\int_0^{x_1} e^{-rt}.\beta t.I_{O1}(t)dt.$$

Now total deterioration cost during the time period 0 to x_1 is as follows,

$$D_{O1}' = c_3\left[w\beta\frac{x_1^2}{2} - \frac{1}{3}(w\beta r + \beta^2)x_1^3 + \frac{1}{4}\beta^2 r x_1^4\right].$$

The deterioration cost for own warehouse (D_{O2}') during the time interval x_1 to t_1 is as follows,

$$D_{O2}' = \int_{x1}^{t_1} e^{-rt}.\beta t.I_{O2}(t)dt.$$

The total deterioration cost during the time period x_1 to t_1 is stated as follows,

$$D_{O2}' = c_2\int_{x1}^{t_1} e^{-rt}.\beta t.I_{O2}(t)dt.$$

Now total holding cost during the time period x_1 to t_1 is as follows,

$$D_{O2}' = c_3\left[\frac{a}{6}(t_1^3 - x_1^3) + (b+2ar)(t_1^4 - x_1^4) + (3a\alpha - 2br)\frac{(t_1^5 - x_1^5)}{30} - \left(\frac{1}{36}ar(1-3\alpha) + b\left(\frac{r}{16} - \frac{\alpha}{24}\right)\right)(t_1^6 - x_1^6) - \alpha\left(\frac{1}{105}br + \frac{1}{112}\alpha a\right)(t_1^7 - x_1^7) + \alpha^2\left(\frac{1}{160}ar - \frac{1}{128}b\right)(t_1^8 - x_1^8) + \left(\frac{1}{180}\alpha^2 rb\right)(t_1^9 - x_1^9)\right].$$

Now total deterioration cost for warehouses is given by

$$D_C' = D_R' + D_{O1}' + D_{O2}' \tag{15}$$

3.1.3 Shortage cost for the own warehouses (Sh_C') during the time period t_1 to T under the inflation rate r

The shortages are occur for own warehouse only so the shortages cost will be for own warehouse.

The shortage cost for own warehouse (Sh_{O1}') during the time interval t_1 to μ is given by,

$$Sh_{O1}' = -\int_{t_1}^{\mu} e^{-rt}.I_{\ O3}(t)dt.$$

The total shortage cost during the time period t_1 to μ.

$$Sh_{O1}' = -c_4 \int_{0}^{x_1} e^{-rt}.I_{O3}\ (t)dt.$$

Now total shortage cost

$$Sh_{O1}' = c_4\left[(\mu^2 - t_1^2)\left(\frac{\delta a}{2} + \frac{\delta rat}{2} + \frac{\delta brt_1^2}{2}\right) - (\mu - t_1)\left(\delta a t_1 + \frac{\delta b t_1^2}{2}\right) + (\mu^3 - t_1^3)\left(\frac{\delta b}{6} - \frac{\delta ra}{3}\right) - (\mu^4 - t_1^4)\left(\frac{\delta br}{8}\right)\right].$$

The shortage cost for own warehouse (Sh_{O2}') during the time interval μ to T is also as follows,

$$Sh_{O2}' = -\int_{\mu}^{T} e^{-rt}.I_{O4}\ (t)dt.$$

The total shortage cost during the time period μ to T is given by,

$$Sh_{O2}' = -c_4 \int_{\mu}^{T} e^{-rt}.I_{O4}\ (t)dt.$$

Now total shortage cost during the time period 0 to x_1 is as follows,

$$Sh_{O2}' = c_4\left[(a + b\mu)\left(\delta r\left(\frac{\mu t^2}{2} - \frac{T^3}{3} - \frac{\mu^3}{6}\right) - \delta\left(\frac{T^2}{2} + \frac{\mu^2}{2}\right) - \delta \mu t\right)\right].$$

Now total shortage cost for own warehouse during the time period t_1 to T is define as

$$Sh_C' = Sh_{O1}' + Sh_{O2}'. \tag{16}$$

3.1.4 Lost Sale Cost during the time period t_1 to T

The lost sale cost (L_{S1}') during the time interval t_1 to μ.

$$L_{S1}' = -\int_{t_1}^{\mu} (1 - \delta)e^{-rt}(a + bt)dt.$$

The total lost sale cost (Ls_1) during the time period t_1 to μ is as follows,

$$L_{S1}' = -c_5 \int_{t_1}^{\mu} (1 - \delta)e^{-rt}(a + bt)dt.$$

$$L_{S1}' = c_5\left[(\delta - 1)\left(a(\mu - t_1) + \frac{(b - ar)}{2}(\mu^2 - t_1^2) + \frac{rb}{3}(t_1^3 - \mu^3)\right)\right].$$

The lost sale cost (L_{S2}') during the time interval μ to T is given by

$$L_{S2}' = -\int_{\mu}^{T} (1 - \delta)e^{-rt}(a + b\mu)dt.$$

The total lost sale cost (L_{S2}') during the time period μ to T is as follows,

$$L_{S2}' = -c_5 \int_{\mu}^{T} (1-\delta) e^{-rt} (a + b\mu) dt.$$

Now total lost sale cost (L_{S2}') is as follows,

$$L'_{S2} = c_5(\delta - 1)(a + b\mu)\left[a(\mu - t_1) + \frac{(b - ar)}{2}(\mu^2 - t_1^2) + \frac{rb}{3}(t_1^3 - \mu^3)\right].$$

Lost sale cost (L_{SC}') during the time period t_1 to T is define as

$$L_{SC}' = L_{S1}' + L_{S2}' . \tag{17}$$

3.1.5 Total Cost

Expected total cost can be define as follow,

$T_{C1}(\mathrm{T}, \mathrm{t}_1) = [Ordering\ cost + Total\ holding\ cost + Total\ deterioration\ cost + total\ shortage\ cost + Total\ lost\ sale\ cost]$

$$T_{C1}(\mathrm{T}, \mathrm{t}_1) = [A_0 + H_C' + D_C' + Sh_C' + L_{SC}']. \tag{18}$$

3.2 Case II: $x_1 \le \mu \le t_1$

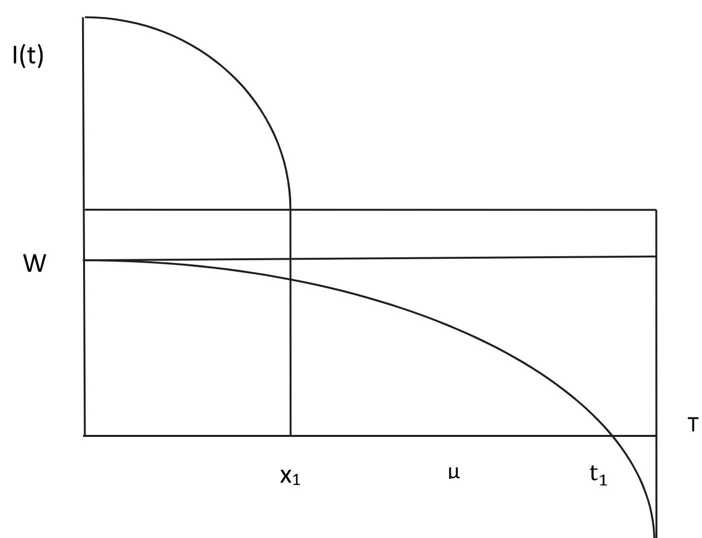

Fig. 2. Inventory system for the case $x_1 \le \mu \le t_1$

So that the differential equation can be expressed for Inventory level at time t at RW and OW when the instantaneous state over *(0,T)* are given by

$$\mathrm{I'}_R(t) + \alpha t.\mathrm{I}_R(t) = -(a + bt) \quad 0 \le t \le x_1 \text{ with } \mathrm{I}_R(x_1) = 0 \tag{19}$$
$$\mathrm{I'}_{O1}(t) + \beta t.\mathrm{I}_O(t) = 0 \quad 0 \le t \le x_1 \text{ with } \mathrm{I}_O(0) = W \tag{20}$$
$$\mathrm{I'}_{O2}(t) + \beta t.\mathrm{I}_O(t) = -(a + bt) \quad x_1 \le t \le \mu \text{ with } \mathrm{I}_O(\mu) = 0 \tag{21}$$
$$\mathrm{I'}_{O3}(t) + \beta t.\mathrm{I}_O(t) = -(a + b\mu) \quad \mu \le t \le t_1 \text{ with } \mathrm{I}_O(t_1) = 0 \tag{22}$$
$$\mathrm{I'}_{O4}(t = -\delta(a + b\mu) \quad t_1 \le t \le T \text{ with } \mathrm{I}_O(t_1) = 0 \tag{23}$$

The solution of the above equations can be derived as follows,

$$I_R(t) = \left[1-\alpha\frac{t^2}{2}\right]\left[\alpha(x_1-t)+\frac{b}{2}(x_1^2-t^2)+\frac{\alpha a}{6}(x_1^3-t^3)+\frac{\alpha b}{8}(x_1{}^4-t^4)\right] 0 \le t \le x_1 \quad (24)$$

$$I_{O1}(t) = W-\beta t \qquad 0 \le t \le x_1 \quad (25)$$

$$I_{O2}(t) = \left[1-\beta\frac{t^2}{2}\right]\left[\alpha(\mu-t)+\frac{b}{2}(\mu^2-t^2)+\frac{\beta a}{6}(\mu^3-t^3)+\frac{\beta b}{8}(\mu^4-t^4)\right] x_1 \le t \le \mu \quad (26)$$

$$I_{O3}(t) = \left[1-\beta\frac{t^2}{2}\right]\left[(a+b\mu)\left((t_1-t)+\frac{\beta}{6}(t_1^3-t^3)\right)\right] \mu \le t \le t_1 \quad (27)$$

$$I_{O4}(t) = \delta[a(t_1-t)+b\mu(t_1-t)] \qquad t_1 \le t \le T \quad (28)$$

3.2.1 Holding Cost for the warehouses during the time period 0 to t_1 under the inflation rate r

The holding cost for rent warehouse (H_R'') during the time interval 0 to x_1 is as follows,

$$H_R'' = \int_0^{x_1} e^{-rt}.I_R(t)dt.$$

The total holding cost during the time period 0 to x_1 is as follows,

$$H_R'' = c_1\int_0^{x_1} e^{-rt}.I_R(t)dt.$$

Now total holding cost is

$$H_R'' = c_1\left[\frac{a}{2}x_1^2-\left(\frac{1}{6}ar-\frac{1}{3}b\right)x_1^3-\left(\frac{1}{24}a\alpha-\frac{1}{8}br+\frac{1}{8}\alpha a\right)x_1^4+\left(\frac{1}{40}a\alpha r-\frac{1}{30}b+\frac{1}{45}\alpha ar+\frac{1}{10}b\alpha\right)x_1^5+\left(\frac{1}{48}a\alpha b-\frac{1}{72}\alpha^2 a-\frac{1}{24}\alpha br\right)x_1^6+\left(\frac{1}{112}ar-\frac{1}{84}\alpha^2 b\right)x_1^7+\left(\frac{1}{128}\alpha^2 rb\right)x_1^8\right].$$

The holding cost for own warehouse (H_{O1}'') during the time interval 0 to x_1 is as follows,

$$H_{O1}'' = \int_0^{x_1} e^{-rt}.I_{O1}(t)dt.$$

The total holding cost during the time period 0 to x_1 is as follows,

$$H_{O1}'' = c_2\int_0^{x_1} e^{-rt}.I_{O1}(t)dt.$$

Now total holding cost during the time period 0 to x_1 is calculated as follows,

$$H_{O1}'' = c_2\left[wx_1-\frac{1}{2}(wr+\beta)x_1^2+\frac{1}{3}\beta rbx_1^8\right].$$

The holding cost for own warehouse (H_{O2}'') during the time interval x_1 to μ is also given as follows,

$$H_{O2}'' = \int_{x1}^{\mu} e^{-rt}.I_{O2}(t)dt.$$

The total holding cost during the time period x_1 to μ is as follows,

$$H_{O2}'' = c_2\int_{x1}^{\mu} e^{-rt}.I_{O2}(t)dt.$$

Now total holding cost during the time period x_1 to μ is as follows,

$$H_{O2}'' = c_2\left[\frac{a}{2}(\mu^2-x_1^2)-\left(\frac{1}{6}ar-\frac{1}{3}b\right)(\mu^3-x_1^3)-\left(\frac{1}{24}a\beta-\frac{1}{8}br+\frac{1}{8}\beta a\right)(\mu^4-x_1^4)+\left(\frac{1}{40}a\beta r-\frac{1}{30}b+\frac{1}{45}\beta ar+\frac{1}{10}b\beta\right)(\mu^5-x_1^5)+\left(\frac{1}{48}a\beta b-\frac{1}{72}\beta^2 a-\frac{1}{24}\beta br\right)(\mu^6-x_1^6)+\left(\frac{1}{112}ar-\frac{1}{84}\beta^2 b\right)(\mu^7-x_1^7)+\left(\frac{1}{128}\beta^2 rb\right)(\mu^8-x_1^8)\right].$$

The holding cost for own warehouse (H_{O3}'') during the time interval μ to t_1 is as follows,

$$H_{O3}'' = \int_{\mu}^{t_1} e^{-rt}.I_{O3}\,(t)dt.$$

The total holding cost during the time period μ to t_1 is as follows,

$$H_{O3}'' = c_2 \int_{\mu}^{t_1} e^{-rt}.I_{O3}\,(t)dt.$$

Now total holding cost during the time period x_1 to μ is as follows,

$$H_{O3}'' = c_2 \int_{\mu}^{t_1} e^{-rt}.\left[1 - \beta\frac{t^2}{2}\right]\left[(a + b\mu)\left((t_1 - t) + \frac{\beta}{6}(t_1^3 - t^3)\right)\right]dt.$$

Now total holding cost for warehouse is given by

$$H_C'' = H_R'' + H_{O1}'' + H_{O2}'' + H_{O3}''. \tag{29}$$

3.2.2 Deterioration cost for the warehouses during the time period 0 to t_1 under the inflation rate r

The deterioration cost for rent warehouse (D_R'') during the time interval 0 to x_1 is

$$D_R'' = \int_0^{x_1} e^{-rt}.\theta_1(t).I_R(t)dt.$$

The total deterioration cost during the time period 0 to x_1 is as follows,

$$D_R'' = c_3 \int_0^{x_1} e^{-rt}.\alpha t.I_R\,(t)dt.$$

Now total deterioration cost will become

$$D_R'' = c_3\left[\frac{a}{6}x_1^3 + (b + 2ar)x_1^4 + (3a\alpha - 2br)\frac{x_1^5}{30} - \left(\frac{1}{36}ar(1 - 3\alpha) + b\left(\frac{r}{16} - \frac{\alpha}{24}\right)\right)x_1^6 - \alpha\left(\frac{1}{105}br + \frac{1}{112}\alpha a\right)x_1^7 + \alpha^2\left(\frac{1}{160}ar - \frac{1}{128}b\right)x_1^8 + \left(\frac{1}{180}\alpha^2 rb\right)x_1^9\right].$$

The deterioration cost for own warehouse (D_{O1}'') during the time interval 0 to x_1 is as follows,

$$D_{O1}'' = c_3 \int_0^{x_1} e^{-rt}.\theta_2(t).I_{O1}\,(t)dt.$$

The total deterioration cost during the time period 0 to x_1 is as follows,

$$D_{O1}'' = c_3 \int_0^{x_1} e^{-rt}.\beta t.I_{O1}\,(t)dt.$$

Now total deterioration cost during the time period 0 to x_1 is as follows,

$$D_{O1}'' = c_3\left[w\beta\frac{x_1^2}{2} - \frac{1}{3}(w\beta r + \beta^2)x_1^3 + \frac{1}{4}\beta^2 r x_1^4\right].$$

The deterioration cost for own warehouse (D_{O2}'') during the time interval x_1 to μ is as follows,

$$D_{O2}'' = \int_{x1}^{\mu} e^{-rt}.\beta t.I_{O2}\,(t)dt.$$

The total deterioration cost during the time period x_1 to t_1 is given by

$$D_{O2}{''} = c_3 \int_{x1}^{\mu} e^{-rt}.\beta t. I_{O2}\,(t)dt.$$

Now total deterioration cost during the time period x_1 to μ is as follows,

$$D_{O2}{''} = c_3 \int_{x1}^{\mu} e^{-rt}.\beta t.\left[1-\beta\frac{t^2}{2}\right]\left[\alpha(\mu-t)+\frac{b}{2}(\mu^2-\,t^2)+\frac{\beta a}{6}(\mu^3-t^3)+\frac{\beta b}{8(\mu^4-t^4)}\right]dt.$$

The deterioration cost for own warehouse ($D_{O3}{''}$) during the time interval μ to t_1 is as follows,

$$D_{O3}{''} = \int_{\mu}^{t_1} e^{-rt}.\beta t. I_{O3}\,(t)dt.$$

The total deterioration cost during the time period μ to t_1 is as follows,

$$D_{O3}'' = c_3 \int_{\mu}^{t_1} e^{-rt}.\beta t. I_{O3}\,(t)dt.$$

Now total deterioration cost during the time period x_1 to t_1 is as follows,

$$D_{O3}{''} = c_3 \int_{\mu}^{t_1} e^{-rt}.\beta t.\left[1-\beta\frac{t^2}{2}\right]\left[(a+b\mu)\left((t_1-\,t)+\frac{\beta}{6}(t_1^3-t^3)\right)\right]dt.$$

Now total deterioration cost for warehouses is summarized as,

$$D_C'' = D_R'' + D_{O1}{''} + D_{O2}{''} + D_{O3}{''}. \tag{30}$$

3.2.3 Shortage Cost for the Own Warehouses ($Sh_C{''}$) during the time Period t_1 To T under the Inflation Rate r

The shortage cost for own warehouse ($Sh_C{''}$) during the time interval t_1 to T is as follows,

$$Sh_C'' = -\int_{t_1}^{T} e^{-rt}.I_{\,O4}(t)dt.$$

The total shortage cost during the time period t_1 to μ is as follows,

$$Sh_C'' = -c_4\int_{t_1}^{T} e^{-rt}.I_{\,O4}(t)dt.$$

Now total shortage cost is as follows,

$$Sh_C'' = c_4\left[(a+bt_1)\left(\delta r\left(\frac{t_1t^2}{2}-\frac{T^3}{3}-\frac{t_1^3}{6}\right)+\delta\left(\frac{T^2}{2}-\frac{t_1^2}{2}-\delta t_1 T\right)\right)\right]. \tag{31}$$

3.2.4 Lost Sale Cost during the time period t_1 to T

The lost sale cost ($L_{SC}{''}$) during the time interval t_1 to T is given by

$$L_{SC}{''} = -\int_{t_1}^{T}(1-\delta)e^{-rt}(a+b\mu)dt.$$

The total lost sale ($L_{SC}{''}$) during the time period t_1 to μ is as follows,

$$L_{SC}{''} = -c_5\int_{t_1}^{T}(1-\delta)e^{-rt}(a+b\mu)dt.$$

Now total lost sale ($L_{SC}{''}$) is calculated as,

$$L''_{SC} = c_5\left[(\delta - 1)\left((a + b\mu)\left(T - t_1 - \frac{r}{2}(T^2 - t_1^2)\right)\right)\right]. \quad (32)$$

3.2.5 Total Cost

Total cost can be define as follow

$T_{C2}(\mathrm{T}, \mathrm{t}_1) = \ [Ordering\ cost + Total\ holding\ cost + Total\ deterioration\ cost + Total\ shortage\ cost\ + Total\ lost\ sale\ cost]$

$$T_{C2}(\mathrm{T}, \mathrm{t}_1) = [A_0 + H''_C + D''_C + Sh''_C + L''_{SC}]. \quad (33)$$

6.3.3 Case III: $\mu \leq x_1 \leq t_1$

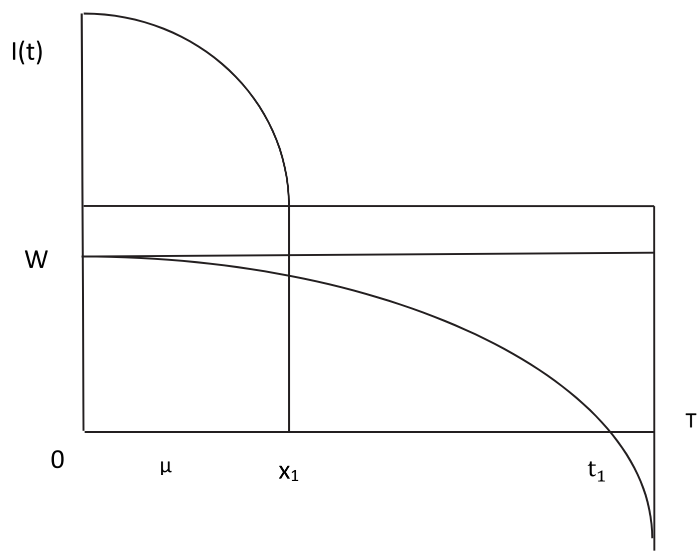

Fig. 3. Inventory system for the case $\mu \leq x_1 \leq t_1$

In this case the above equations are defined as follow,

$$I'_{R1}(t) + \alpha t.I_{R1}(t) = -(a + bt) \quad 0 \leq t \leq \mu \text{ with } I_R(\mu) = 0 \quad (34)$$
$$I'_{R2}(t) + \alpha t.I_{R2}(t) = -(a + b\mu) \quad \mu \leq t \leq x_1 \text{ with } I_R(x_1) = 0 \quad (35)$$
$$I'_{O1}(t) + \beta t.I_{O1}(t) = 0 \quad 0 \leq t \leq \mu \text{ with } I_O(0) = W \quad (36)$$
$$I'_{O2}(t) + \beta t.I_{O2}(t) = 0 \quad \mu \leq t \leq x_1 \text{ with } I_O(x_1) = W \quad (37)$$
$$I'_{O3}(t) + \beta t.I_O(t) = -(a + b\mu) \quad x_1 \leq t \leq t_1 \text{ with } I_O(t_1) = 0 \quad (38)$$
$$I'_{O4}(t) = -\delta(a + b\mu) \quad t_1 \leq t \leq T \text{ with } I_O(t_1) = 0 \quad (39)$$

The solution of the above equations can be derived as below,

$$I_{R1}(t) = \left[1 - \alpha\frac{t^2}{2}\right]\left[a(\mu - t) + \frac{b}{2}(\mu^2 - t^2) + \frac{\alpha a}{6}(\mu^3 - t^3) + \frac{\alpha b}{8}(\mu^4 - t^4)\right] \quad 0 \leq t \leq \mu \quad (40)$$
$$I_{R2}(t) = \left[1 - \alpha\frac{t^2}{2}\right]\left[(a + b\mu)\left(x_1 - t + \frac{\alpha}{6}(x_1^3 - t^3)\right)\right] \quad \mu \leq t \leq x_1 \quad (41)$$
$$I_{O1}(t) = W - \beta t \quad 0 \leq t \leq \mu \quad (42)$$
$$I_{O2}(t) = W - \beta t \quad \mu \leq t \leq x_1 \quad (43)$$
$$I_{O3}(t) = \left[1 - \beta\frac{t^2}{2}\right]\left[(a + b\mu)\left((t_1 - t) + \frac{\beta}{6}(t_1^3 - t^3)\right)\right] \quad x_1 \leq t \leq t_1 \quad (44)$$
$$I_{O4}(t) = \delta[a(t_1 - t) + b\mu(t_1 - t)] \quad t_1 \leq t \leq T \quad (45)$$

3.3.1 Holding Cost for the warehouses during the time period 0 to t_1 under the inflation rate r

The holding cost for rent warehouse (H_{R1}''') during the time interval 0 to μ is as follows,

$$H_{R1}''' = \int_0^{\mu} e^{-rt}.I_{R1}(t)dt.$$

The total holding cost during the time period 0 to x_1 is as follows,

$$H_{R1}''' = c_1 \int_0^{\mu} e^{-rt}.I_{R1}(t)dt.$$

Now total holding cost is calculated as follows,

$$H_{R1}''' = c_1 \int_0^{\mu} e^{-rt}.\left[1-\alpha\frac{t^2}{2}\right]\left[a(\mu-t)+\frac{b}{2}(\mu^2-t^2)+\frac{\alpha a}{6}(\mu^3-t^3)+\frac{\alpha b}{8}(\mu^4-t^4)\right]dt.$$

The holding cost for rent warehouse (H_{R2}''') during the time interval μ to x_1 is as follows,

$$H_{R2}''' = \int_{\mu}^{x_1} e^{-rt}.I_{R2}(t)dt.$$

The total holding cost during the time period 0 to x_1 is as follows,

$$H_{R2}''' = c_1 \int_0^{\mu} e^{-rt}.I_{R2}(t)dt.$$

Now total holding cost is as follows,

$$H_{R2}''' = c_1 \int_0^{\mu} e^{-rt}.\left[1-\alpha\frac{t^2}{2}\right]\left[(a+b\mu)\left(x_1-t+\frac{\alpha}{6}(x_1^3-t^3)\right)\right]dt.$$

The holding cost for own warehouse (H_{O1}''') during the time interval 0 to μ is as follows,

$$H_{O1}''' = \int_0^{\mu} e^{-rt}.I_{O1}(t)dt.$$

The total holding cost during the time period 0 to x_1 is as follows,

$$H_{O1}''' = c_2 \int_0^{\mu} e^{-rt}.I_{O1}(t)dt = c_2\left[w\mu-\frac{1}{2}\beta\mu^2+\frac{1}{2}wr\mu^2+\frac{1}{3}\beta r\mu^3\right].$$

The holding cost for own warehouse (H_{O2}''') during the time interval μ to x_1 is as follows,

$$H_{O2}''' = \int_{\mu}^{x_1} e^{-rt}.I_{O2}(t)dt.$$

The total holding cost during the time period x_1 to μ is as follows,

$$H_{O2}''' = c_2 \int_{\mu}^{x_1} e^{-rt}.I_{O2}(t)dt.$$

Now total holding cost during the time period x_1 to μ is as follows,

$$H_{O2}''' = c_2\left[w(x_1-\mu)-(x_1^2-\mu^2)\left(\frac{\beta}{2}+wr\right)-\frac{\beta r}{3}(x_1^3-\mu^3)\right].$$

The holding cost for own warehouse (H_{O3}''') during the time interval μ to t_1 is as follows,

$$H_{O3}''' = \int_{x_1}^{t_1} e^{-rt}.I_{O3}(t)dt.$$

The total holding cost during the time period μ to t_1 is given by

$$H_{O3}^{'''} = c_2 \int_{\mu}^{t_1} e^{-rt}. I_{O3}\,(t)dt.$$

Now total holding cost during the time period x_1 to μ is as follows,

$$H_{O3}^{'''} = c_2 \int_{\mu}^{t_1} e^{-rt}. \left[1 - \beta \frac{t^2}{2}\right]\left[(a + b\mu)\left((t_1 - t) + \frac{\beta}{6}(t_1^3 - t^3)\right)\right] dt.$$

Now total holding cost for own warehouse during the time period 0 to t_1 is defined as

$$H_O = H_{R1}''' + H_{R2}''' + H_{O1}''' + H_{O2}''' + H_{O3}''' \,. \tag{46}$$

3.3.2 Deterioration cost for the warehouses during the time period 0 to t_1 under the inflation rate r

The deterioration cost for rent warehouse (D_{R1}''') during the time interval 0 to x_1 is as follows,

$$D_{R1}^{'''} = \int_{0}^{\mu} e^{-rt}. \theta_1(t). I_{R1}(t)dt.$$

The total deterioration cost during the time period 0 to x_1 is as follows,

$$D_{R1}^{'''} = c_3 \int_{0}^{\mu} e^{-rt}. \alpha t. I_{R1}\,(t)dt.$$

Now total deterioration cost is as follows,

$$D_{R1}^{'''} = c_3 \int_{0}^{\mu} e^{-rt}. \alpha t. \left[1 - \alpha \frac{t^2}{2}\right]\left[a(\mu - t) + \frac{b}{2}(\mu^2 - t^2) + \frac{\alpha a}{6}(\mu^3 - t^3) + \frac{\alpha b}{8(\mu^4 - t^4)}\right] dt.$$

The deterioration cost for rent warehouse (D_{R2}''') during the time interval 0 to x_1 is as follows,

$$D_{R2}^{'''} = \int_{\mu}^{x_1} e^{-rt}. \theta_1(t). I_{R2}(t)dt.$$

The total deterioration cost during the time period 0 to x_1 is as follows,

$$D_{R2}^{'''} = c_3 \int_{\mu}^{x_1} e^{-rt}. \theta_1(t). I_{R2}(t)dt.$$

Now total deterioration cost is as follows,

$$D_{R2}''' = c_3 \int_{\mu}^{x_1} e^{-rt}. \alpha t. \left[1 - \alpha \frac{t^2}{2}\right]\left[(a + b\mu)\left(x_1 - t + \frac{\alpha}{6}(x_1^3 - t^3)\right)\right] dt.$$

The deterioration cost for own warehouse (D_{O1}''') during the time interval 0 to μ is as follows,

$$D_{O1}^{'''} = c_3 \int_{0}^{\mu} e^{-rt}. \theta_2(t). I_{O1}\,(t)dt.$$

The total deterioration cost during the time period 0 to x_1 is as follows,

$$D_{O1}''' = c_3 \int_{0}^{\mu} e^{-rt}. \beta t. I_{O1}\,(t)dt.$$

Now total deterioration cost during the time period 0 to x_1 is as follows,

$$D_{O1}^{'''} = c_3 \beta \left[w \frac{\mu^2}{2} - \frac{1}{3}\beta\mu^3 - \frac{1}{3} r w \mu^3 + \frac{1}{4}\beta r \mu^4\right].$$

The deterioration cost for own warehouse (D_{O2}) during the time interval μ to x_1 is as follows,

$$D_{O2}^{'''} = \int_{\mu}^{x_1} e^{-rt}.\beta t.I_{O2}\,(t)dt.$$

The total deterioration cost during the time period μ to x_1 is as follows,

$$D_{O2}''' = c_3 \int_{\mu}^{x_1} e^{-rt}.\beta t.I_{O2}\,(t)dt.$$

Now total holding cost during the time period μ to x_1 is as follows,

$$D_{O2}^{'''} = c_3\beta\left[\frac{w}{2}(x_1^2-\mu^2)-(x_1^3-\mu^3)\left(\frac{\beta}{3}+\frac{rw}{3}\right)+\beta r(x_1^4-\mu^4)\right].$$

The deterioration cost for own warehouse (D_{O3}) during the time interval μ to t_1 is as follows,

$$D_{O3}''' = \int_{x_1}^{t_1} e^{-rt}.\beta t.I_{O3}\,(t)dt.$$

The total deterioration cost during the time period μ to t_1 is as follows,

$$D_{O3}^{'''} = c_3 \int_{x_1}^{t_1} e^{-rt}.\beta t.I_{O3}\,(t)dt.$$

Now total deterioration cost will be during the time period x_1 to t_1 is as follows,

$$D_{O3}^{'''} = c_3 \int_{\mu}^{t_1} e^{-rt}.\beta t.\left[1-\beta\frac{t^2}{2}\right]\left[(a+b\mu)\left((t_1-\ t)+\frac{\beta}{6}(t_1^3-t^3)\right)\right]dt.$$

Now total deterioration cost for own warehouse will be during the time period 0 to t_1 is define as

$$D_C''' = D_{R1}''' + D_{R2}''' + D_{O1}''' +\ D_{O2}''' + D_{O3}'''. \quad (47)$$

3.3.3 Shortage cost for the own warehouses (Sh_C''')) during the time period t_1 to T under the inflation rate r

The shortage cost for own warehouse (Sh_C''') during the time interval t_1 to T is as follows,

$$Sh_C''' = -\int_{t_1}^{\mathrm{T}} e^{-rt}.I_{\ O4}(t)dt.$$

The total shortage cost during the time period t_1 to μ is as follows,

$$Sh_C^{'''} = -c_4 \int_{t_1}^{\mathrm{T}} e^{-rt}.I_{\ O4}(t)dt.$$

Now total shortage cost is:

$$Sh_C''' = c_4\left[(a+bt_1)\left(\delta r\left(\frac{t_1t^2}{2}-\frac{T^3}{3}-\frac{t_1^3}{6}\right)+\delta\left(\frac{T^2}{2}-\frac{t_1^2}{2}-\delta t_1 T\right)\right)\right]. \quad (48)$$

3.3.4 Lost Sale Cost during the time period t_1 to T

The lost sale cost (L_{SC}''') during the time interval t_1 to T is given by

$$L_{SC}^{'''} = -\int_{t_1}^{T}(1-\delta)e^{-rt}(a+b\mu)dt.$$

The total lost sale (Ls) during the time period t_1 to μ is also given by

$$L_{SC}^{'''} = -c_5 \int_{t_1}^{T} (1-\delta)e^{-rt}(a+b\mu)dt.$$

Now total lost sale (Ls_1) is stated as

$$L_{SC}^{'''} = c_5\left[(\delta-1)\left((a+b\mu)\left(T-t_1-\frac{r}{2}(T^2-t_1^2)\right)\right)\right]. \tag{49}$$

3.3.5 Total cost

$$T_{C3}(\mathrm{T},\mathrm{t}_1) = [Ordering\ cost + Total\ holding\ cost + Total\ deterioration\ cost + total\ shortage\ cost\ + Total\ lost\ sale\ cost]$$

$$T_{C3}(\mathrm{T},\mathrm{t}_1) = \left[A_0 + H_C^{'''} + D_C^{'''} + Sh_C^{'''} + L_{SC}^{'''}\right]. \tag{50}$$

3.4 Total Inventory Cost

From Eq. (18), Eq. (33) and Eq. (50) the total Inventory cost per unit item per unit time is as follows,

$$T_C(\mathrm{T},\mathrm{t}_1) = T_{C1}(\mathrm{T},\mathrm{t}_1) + T_{C2}(\mathrm{T},\mathrm{t}_1) + T_{C3}(\mathrm{T},\mathrm{t}_1). \tag{51}$$

3.4.1 Mathematical formulation of the model

Our main objective to minimize the Total cost function $T_C(\mathrm{T},\mathrm{t}_1)$ the necessary condition for minimize the total inventory cost are

$$\frac{\partial T_C(T,t_1)}{\partial T}, 0\ \ and\ \ \frac{\partial T_C(T,t_1)}{\partial t_1} = 0 \tag{52}$$

Using the software mathematica-5.8, we can calculate the optimal value of T* and t_1* by equation (53). And the optimal value $T_C^*(\mathrm{T},\mathrm{t}_1)$ of the total Inventory cost is determined by equation (52). The optimal value of T*and t_1*, satisfy the sufficient conditions for minimizing the total inventory cost function $T_C^*(\mathrm{T},\mathrm{t}_1)$ are $\frac{\partial^2 T_C(T,t_1)}{\partial T^2} < 0$ $\frac{\partial^2 T_C(T,t_1)}{\partial t_1^2} < 0$ and $\frac{\partial^2 T_C(T,t_1)}{\partial T^2}.\frac{\partial^2 T_C(T,t_1)}{\partial t_1^2} - \frac{\partial^2 T_C(T,t_1)}{\partial T \partial t} > 0$. In addition, at T = T* optimal value $t_1 = t_1$

4. Numerical Illustration

Example 1: Let us consider A = 600, a = 175, b=2.4, r = 0.5, c_1 = 1.7,

c_2 = 1.4, $\alpha = 0.2$ $\beta = 0.1$, $c_3 = 0.2$, $c_4 = 0.3$ μ = 0.99, δ = 0. 2

Based on above input data and using the software mathematica-5.8, we calculate the optimal value of $T_C(\mathrm{T},\mathrm{t}_1)$, T* and t_1* simultaneously by Eq. (51) and Eq. (52)

$T_C(\mathrm{T},\mathrm{t}_1)$= 4420.61, T* = 4.37312, t_1* = 2.878721

Example 2: Let us consider A = 250, a = 125, b=1.4, r = 0.5, c_1 = 1.2, c_2 = 0.91, $\alpha = 0.1$ $\beta = 0.091$, $c_3 = 0.1$, $c_4 = 0.2$, μ = 0.99 ,δ = 0. 2

$T_C(\mathrm{T},\mathrm{t}_1)$= 1815.70, T* = 2.03141, t_1* = 1.237592

5. Sensitivity analysis and observations

We have studied the effects of changes of the parameters on the optimal values of $T_C(\mathrm{T},\mathrm{t}_1)$, T* and t_1* derived by the proposed method. The sensitivity analysis is performed in view of the numerical example. We have executed sensitivity analysis by changing the parameters a , b , α , r, and β as

+20%, +50%, -20% and -50%. All remaining parameters have original values with respect to these changes. The corresponding changes in $T_C(\mathrm{T}, \mathrm{t}_1)$, T* are t_1* are shown in Table 1 as follows,

Table 1
Sensitivity Analysis of Optimal Solution { $T_C(\mathrm{T}, \mathrm{t}_1)$} w.r.t various Parameters

Parameters	% change	T*	t_1*	$T_C(\mathrm{T}, \mathrm{t}_1)$
a	-50	5.60571	3.67621	4557.39
	-20	4.99079	3.2528	4473.95
	20	3.94993	2.0668	3439.06
	50	2.98009	2.79768	2253.59
α	-50	4.27685	2.82883	4657.77
	-20	4.3299	2.82109	4599.39
	20	4.45075	2.93656	4081.87
	50	4.51244	3.0099	3928.53
β	-50	4.10878	2.62883	3828.73
	-20	4.27408	2.61109	3991.93
	20	4.39004	2.89656	4499.77
	50	4.49729	3.0199	4557.99
b	-50	5.97687	2.67387	9905.6
	-20	4.87172	2.97911	7819.88
	20	3.99856	2.69506	3023.15
	50	4.19306	2.80501	802.353
r	-50	5.30571	3.67621	762.353
	-20	4.39079	3.2528	2923.15
	20	4.04993	2.0668	8019.88
	50	3.98009	2.79768	9305.6

We study above table brings out the following:

We have observed that as parameters a and b increase the optimal values of T* and t_1* decrease and the average total cost $T_C(\mathrm{T}, \mathrm{t}_1)$ of an inventory system also decreases, but as parameters a and b decrease, optimal values of T*, t_1* and $T_C(\mathrm{T}, \mathrm{t}_1)$ increase. It is interesting to observe that as deterioration parameter α increases, optimal values of T* and t_1* decrease and the average total cost $T_C(\mathrm{T}, \mathrm{t}_1)$ of an inventory system increases. If deterioration parameter α decreases, optimal values of T* and t_1* increase while the average total cost $T_C(\mathrm{T}, \mathrm{t}_1)$ of an inventory system decreases. Second as deterioration parameter β increases, optimal values of T* and t_1* slightly decrease while the average total cost $T_C(\mathrm{T}, \mathrm{t}_1)$ of an inventory system increases. If deterioration parameter β decreases, optimal values of T* and t_1* increase while the average total cost $T_C(\mathrm{T}, \mathrm{t}_1)$ of an inventory system decreases.

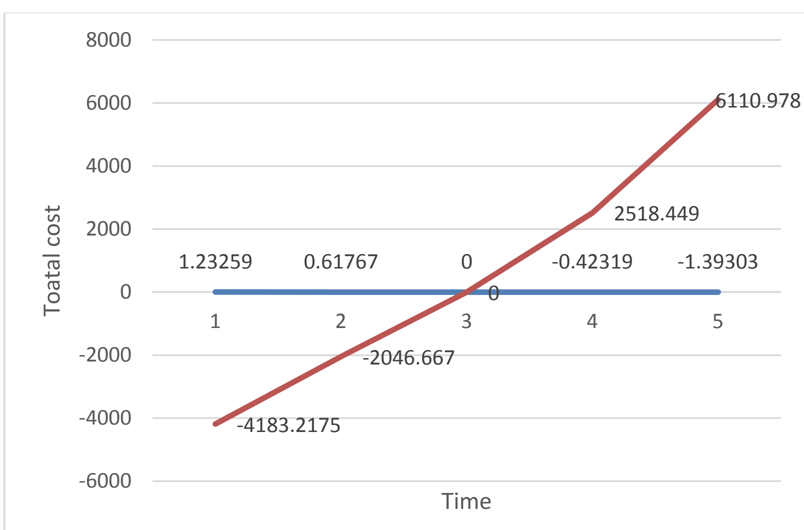

Fig. 4. Graphical representation of sensitivity of the Time and Total cost versus a

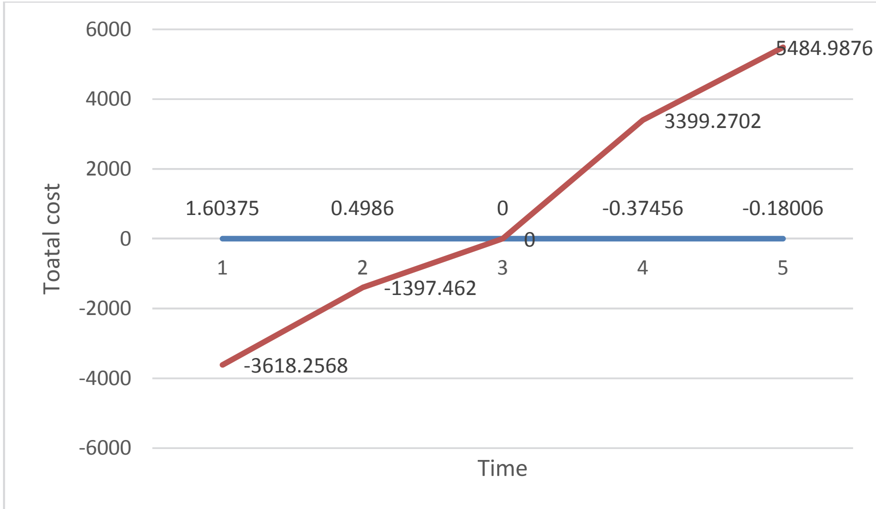

Fig. 5. Graphical representation of sensitivity of the Time and Total cost versus b

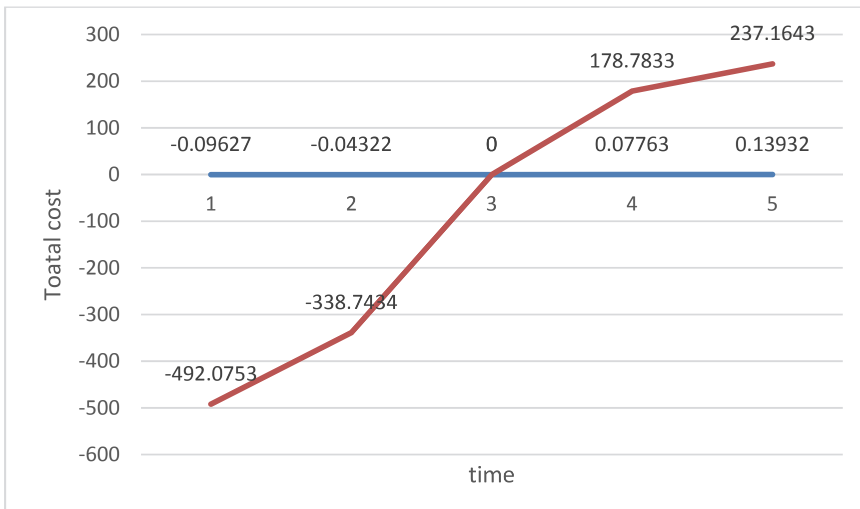

Fig. 6. Graphical representation of sensitivity of the Time and Total versus α

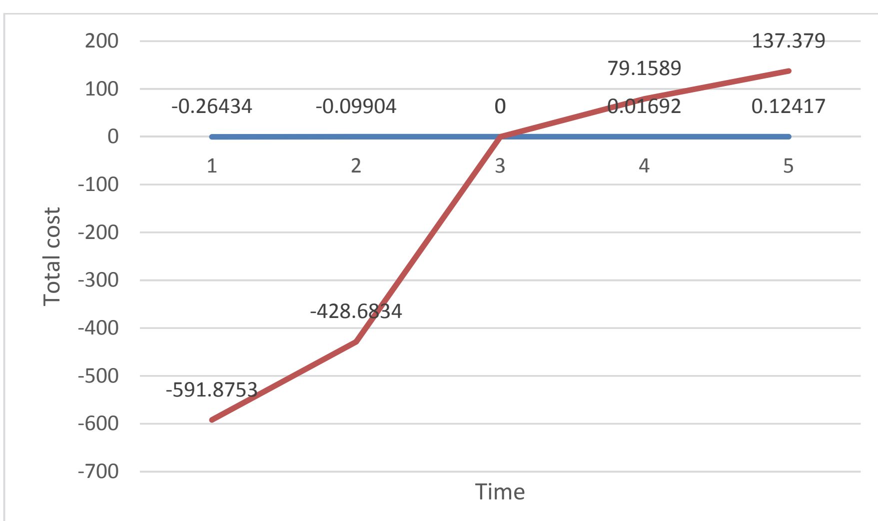

Fig. 7. Graphical representation of sensitivity of the Time and Total cost versus β

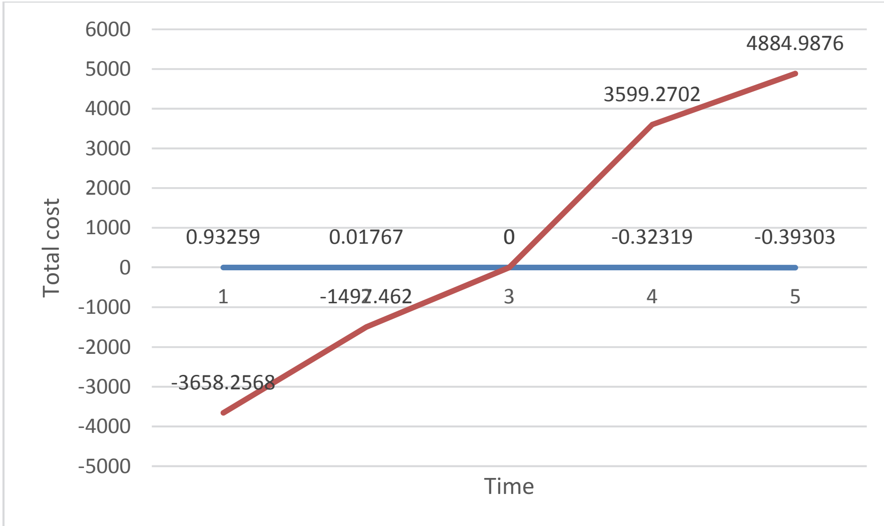

Fig. 8. Graphical representation of sensitivity of the Time and Total cost versus r

7 Conclusions

In this paper, we have developed a partially backlogging inventory model for two warehouse problem. In our study we have considered two warehouse problems under the inflation with deterioration, one with limited storage space and one with rented warehouse with unlimited storage space. This helps in reducing inventory costs as well as in obtaining the best prices due to large volume of the purchases. The rate of deterioration is time dependent. The ramp type demand rate is assumed in the present model. The shortages are allowed and shortages are partially backlogged. The deterioration cost, inventory holding cost and shortage cost are considered in this model. The numerical examples are given to illustrate the model developed. Comprehensive sensitivity analysis with graph has been carried out for showing the effect of variation in the parameter. The model has been solved analytically by minimizing the total cost under inflation. Convexity shows that the model is developed for minimum inventory cost.

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