

Uncertain Supply Chain Management

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Inventory model with Weibull time-dependent demand rate and completely backlogged permissible delay in payments

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CHRONICLE

Article history:
Received April 18, 2015
Received in revised format May 10, 2015
Accepted June 25 2015
Available online
June 26 2015

Keywords:
Inventory
EOQ
Shortage
Weibull time-dependent
Demand
Trade credit period

ABSTRACT

This paper considers the economic order quantity (EOQ) model over a finite horizon, in which demand rate follows a two-parameter Weibull time-dependent, shortages are allowed and completely backlogged. Mathematical formulations are derived under two different circumstances i.e. case I: The permissible delay is less than or equal time to finish positive inventory for settling the account, and case II: The permissible delay period is greater than time to finish positive inventory for settling the account. Based on the optimal solutions some important results are derived and numerical examples are provided to validate the proposed model. Finally, sensitivity analysis is provided to analyze the effect of changes with the variation in one parameter at a time on the optimal solution.

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1. Introduction

The classical inventory models consider the demand rate to be either constant or time dependent. The classical economic order quantity model was developed in 1915. Thereafter, many models have been developed in the inventory literature considering constant demand. However, in real market, the demand of many products is always, time dependent, price dependent or stock dependent. At present, it can be observed that suppliers often offer a certain fixed credit period to settle the account for stimulating retailer's demand. From the retailer's point of view, during the credit period before payment must be made, he/she can sell the commodities and accumulate the revenue and earn interest. Hence trade credit is an important payment behavior in real business transaction so that the decision of inventory strategies should involve the investment of working capital. In today's business transaction, supplier offers a certain fixed period to settle the account for stimulating his/ her demand. The retailer can accumulate the revenue and earn interest during this credit period. After this period the supplier charges interest on the unpaid balance. Thus a credit period reduces the cost of holding stock indirectly.

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However, in practice the supplier offers its customers a permissible delay in payments to attract new customers. To motivate faster payment, stimulate more sales or reduce credit expenses, the supplier also provides the customer a cash discount on the sales.

In inventory management, demand is the major factor. In the study of economic order quantity (EOQ) models, four types of demand are assumed i.e. (i) constant demand (ii) time dependent demand (iii) probabilistic demand and (iv) stock-dependent demand. In classical economic order quantity model the demand rate is assumed to be constant. But in real market situations the demand rate is not always constant but it varies with time. The stock-dependent demand has more relevant at present in the study of inventory models. Mandal and Phaujdar (1989) presented an inventory model for stock-dependent consumption rate. Tripathy and Mishra (2010) discussed an inventory model with time-dependent Weibull demand rate where shortages are allowed. Silver and Meal (1969) were the first to develop the EOQ model for the case of varying demand. Silver and Meal (1973) established an appropriate solution procedure for general case of a deterministic, time dependent demand pattern. Aggarwal et al. (2009) presented an inventory model by considering demand rate as exponentially increasing function of time. Lin and Julian (2012) investigated an inventory model with stock at the beginning and shortages allowed and then partially backlogged. There are many productions in the real world that demand is time-varying or time-dependent. Gupta and Vrat (1986) discussed an inventory model for stock-dependent demand rate. Ray and Chaudhuri (1997) investigated a finite time-horizon deterministic economic order quantity inventory model with shortages where the demand rate at any instant depends on the stock-level at that instant. In the past few decades many researchers have developed inventory models by considering time-dependent demand rate. Datta and Pal (1991) developed a finite time-horizon inventory model following the approach to Misra (1979) with linearly time-dependent demand rate allowing shortages and considering the effect of inflation and time value of money. Among the important papers published so far considering the time-dependent and stock-level dependent demand rate, the works of Baker and Urban (1988), Datta and Pal (1990), Urban (2012), Tripathi (2011), Tripathi et al. (2011), Tripathi and Kumar (2011) etc. are worth mentioning.

In most of the models mentioned above shortages does not have significant role to play in the inventory policy. However, some customers are willing to wait for back order and others would turn to buy from other sellers. In many situations shortages are allowed like crisis of raw material, labor strike, lock off, political disturbance, natural calamities, disaster etc. Recently Tripathi (2013) developed an inventory model with shortages and exponential demand rate under trade credits. Researchers such as Wee (1995), Park (1982), Hollier and Mak (1983), Uthayakumar and Geetha (2009) developed inventory models with partial backorders. Recently, Ouyang and Chang (2013) presented an economic production quantity (EPQ) model with imperfect production process and complete backlogging. Chen et al. (2014) established the retailer's optimal EOQ when the supplier offers conditionally permissible delay in payments linked to order quantity. Goyal and Giri (2003) developed production inventory model with shortages, partially backlogged. Wu (1998) developed economic ordering policy of deteriorating items under delay in payments and allowable shortages. Dye et al. (2007) developed a deterministic inventory model for deteriorating items with price-dependent demand rate with allowable shortages and the unsatisfied demand is partially backlogged at a negative exponential rate with the waiting time. Barron (2009) developed an EPQ type inventory model with planned backorders for determining the economic production quantity for a system that generates imperfect quantity product and all these defectives products are reworked in the same cycle. The inventory policies proposed by Jamal et al. (2004) and Sarker et al. (2008) did not consider planned backorders. Chiu (2003) presented an EPQ inventory model that consider the effects of the reworking of defective products on the lot size with backorders allowed, but his model only considered the linear backorder cost.

On the other hand in deriving the EOQ formula, it is tacitly considered that the supplier must pay for the items as soon as the items are received. In today's business transactions we see more and more that supplier would allow a certain fixed period (called credit period) for settling the amount owed to him

for the items supplied. Within the fixed period there is no charge given by the customer. After this period interest is charged by seller. Liao (2007) derived a production model for the lot size inventory system with finite production rate taking into consideration of the effect of decay and the condition of permissible delay in payments. Liao (2008) considered the impact of the trade credit policy on the classical EPQ model for an item subject to exponential decays. The extensive use of trade credit has been addressed by Goyal (1985) who developed an EOQ model under the condition of permissible delay in payments. Chung (1998) developed an alternative approach to the problem. Aggarwal and Jaggi (1995) extended Goyal's model to allow for deteriorating item. The valuable models related to delay in payments are Shah (1993), Shinn (1997), Hwang and Shinn (1997), Chiu et al. (1998), Khouja and Mehrez (1996), Sarker et al. (2000) and others.

This paper considers an EOQ model for non-deteriorating items with a Weibull time-dependent demand rate under trade credits with shortage. Mathematical models are also derived under two different conditions i.e. case 1: a credit period less than or equal to the time T_1 for settling the account and permissible credit period is greater than T_1 for settling the account. T_1 is the time in which inventory level becomes zero. Expressions for total cost are also derived for two aforesaid cases. Next, we provide the fundamental assumptions for the proposed model and notations in section 2; a mathematical model is established in section 3. In section 4 optimal solutions is given with some important deductions. In section 5, the numerical examples are provided to illustrate the proposed model and the sensitivity analysis of the optimal solution with respect to parameters is carried out in section 6. Finally, we draw the conclusions and future in section 7.

2. Notation and Assumptions

The following assumptions and notations are used to develop the proposed model:

s	per unit shortage cost
h	per unit holding cost excluding interest charges
p	per unit purchase cost
K	ordering cost of inventory per order
$I(t)$	inventory level at time ' t '
I_e	interest earned
I_r	interest charges invested in inventory $I_r \geq I_e$
m	permissible delay in settling the account $0 < m < T$
T	length of the replenishment cycle
T_1	time at which inventory level becomes zero, $0 \leq T_1 \leq T$
$Z(T_1, T)$	average total inventory cost per unit time when permissible delay period is m

$$Z(T_1, T) = \begin{cases} Z_1(T_1, T), & \text{for } T_1 \geq m \\ Z_2(T_1, T), & \text{for } T_1 < m \end{cases}$$

The following assumptions are used to develop the proposed model:

- (1). Shortages are allowed and completely backlogged
- (2). The inventory system involves only one item;
- (3). Replenishment occurs instantaneously on ordering i.e. lead time is zero.
- (4). The demand rate is time dependent and Weibull (two parameter) distribution function given by

$$D(t) = \alpha\beta t^{\beta-1}; \alpha, \beta > 0$$

- (5). No payment to the supplier is outstanding at the time of placing an order;
 (6). time horizon is infinite and is divided into subintervals of length T .

3. Mathematical Formulation

The inventory level $I(t)$ at time t generally decreases mainly to meet the demand only. Thus the variation of inventory with respect to time can be described by the following differential equation

$$\frac{dI(t)}{dt} = -\alpha\beta t^{\beta-1}; \quad 0 \leq t \leq T \quad (1)$$

The solution of Eq. (1) with $I(T_1) = 0$, is

$$I(t) = \alpha(T_1^\beta - t^\beta) \quad ; 0 \leq t \leq T \quad (2)$$

$$\text{In the interval } (0, T_1), \text{ holding cost } HC = h \int_0^{T_1} I(t) dt = \frac{h\alpha\beta}{\beta+1} T_1^\beta \quad (3)$$

The storage cost SC over the time interval (T_1, T) is

$$SC = s \int_{T_1}^T -I(t) dt = s\alpha\beta \left\{ \frac{T^{\beta+1}}{\beta(\beta+1)} - \frac{TT_1^\beta}{\beta} + \frac{T_1^{\beta+1}}{(\beta+1)} \right\} \quad (4)$$

Regarding interest payable and earned, the following two possible cases are based on the values of T_1 and m .

Case 1: $m \leq T_1$

Since the length of period with positive stock is larger than the credit period, the buyer can use the sales revenue to earn interest at an annual rate I_e in $(0, T_1)$, The interest earned E_1 is given by

$$E_1 = pI_e \int_0^{T_1} \alpha\beta t^{\beta-1} (T_1 - t) dt = \frac{pI_e \alpha}{(\beta+1)} T_1^{(\beta+1)} \quad (5)$$

Beyond the credit period, the unsold stock is assumed to be financed with an annual rate I_r and the interest payable I is given by

$$I = pI_r \int_m^{T_1} \alpha\beta t^{\beta-1} (T_1 - t) dt = pI_r \alpha\beta \left\{ \frac{T_1^{\beta+1}}{\beta(\beta+1)} - \frac{T_1 m^\beta}{\beta} + \frac{m^{\beta+1}}{(\beta+1)} \right\} \quad (6)$$

Thus, the total average cost per unit time is given by

$$Z_1(T_1, T) = \frac{K + HC + SC + I - E_1}{T} \quad (7)$$

$$Z_1(T_1, T) = \frac{1}{T} \left[K + \frac{\alpha}{\beta+1} \{ \beta(h+s) + p(I_r - I_e) \} T_1^{\beta+1} + \frac{s\alpha}{\beta+1} T^{\beta+1} - s\alpha T T_1^\beta - pI_r \alpha m^\beta T_1 + \frac{pI_r \alpha \beta}{\beta+1} m^{\beta+1} \right]$$

Case 2: $m > T_1$

In this case, the buyer pays no interest but earns interest at an annual rate I_e during the period $(0, m)$. Interest earned E_2 is given by

$$E_2 = pI_e \int_0^{T_1} \alpha\beta t^{\beta-1} (m - t) dt = pI_e \alpha\beta \left(\frac{m}{\beta} - \frac{T_1}{\beta+1} \right) T_1^\beta \quad (8)$$

The total average cost per unit time is given by $Z_2(T_1, T) = \frac{K + HC + SC - E_2}{T}$

$$Z_2(T_1, T) = \frac{1}{T} \left[K + \frac{h\alpha\beta}{\beta+1} T_1^{\beta+1} - pI_e\alpha\beta T_1^\beta \left(\frac{m}{\beta} - \frac{T_1}{\beta+1} \right) + \frac{s\alpha}{\beta+1} \{ T^{\beta+1} - (\beta+1)TT_1^\beta + \beta T_1^{\beta+1} \} \right] \tag{9}$$

4. Determination of Optimal Solution

Differentiating partially of Eq. (7) two times with respect to T_1 and T respectively yields,

$$\frac{\partial Z_1(T_1, T)}{\partial T_1} = \frac{1}{T} \left[\alpha \{ h\beta + s\beta + p(I_r - I_e) \} T_1^\beta - s\alpha\beta T T_1^{\beta-1} - pI_r\alpha m^\beta \right] \tag{10}$$

$$\frac{\partial^2 Z_1(T_1, T)}{\partial T_1^2} = \frac{T_1^{\beta-1}}{T} \left[\alpha\beta \{ h\beta + p(I_r - I_e) \} + s\alpha\beta \left\{ \beta + (1-\beta) \frac{T}{T_1} \right\} \right] > 0 \tag{11}$$

$$\frac{\partial Z_1(T_1, T)}{\partial T} = -\frac{1}{T^2} \left[K + \frac{\alpha}{\beta+1} \{ h\beta + s\beta + p(I_r - I_e) \} T_1^{\beta+1} - \frac{s\alpha\beta}{\beta+1} T^{\beta+1} - pI_r\alpha T_1 m^\beta + \frac{pI_r\alpha\beta}{\beta+1} m^{\beta+1} \right] \tag{12}$$

$$\frac{\partial^2 Z_1(T_1, T)}{\partial T^2} = \frac{2}{T^3} \left[K + \frac{\alpha}{\beta+1} \{ h\beta + s\beta + p(I_r - I_e) \} T_1^{\beta+1} + pI_r\alpha m^{\beta+1} \left(\frac{\beta}{\beta+1} - \frac{T_1}{m} \right) + \frac{s\alpha\beta(\beta-1)}{2(\beta+1)} T^{\beta+1} \right] > 0 \tag{13}$$

$\left(\frac{\partial^2 Z_1(T_1, T)}{\partial T_1^2} \right) \left(\frac{\partial^2 Z_1(T_1, T)}{\partial T^2} \right) - \left(\frac{\partial^2 Z_1(T_1, T)}{\partial T \partial T_1} \right)^2 > 0$, and $\frac{\partial^2 Z_1(T_1, T)}{\partial T_1^2} > 0$ and $\frac{\partial^2 Z_1(T_1, T)}{\partial T^2} > 0$, the optimal values of

$T_1 = T_1^*$ and $T = T^*$ are minimum. Solving $\frac{\partial Z_1(T_1, T)}{\partial T_1} = 0$ and $\frac{\partial Z_1(T_1, T)}{\partial T} = 0$ yields,

$$\alpha \{ h\beta + s\beta + p(I_r - I_e) \} T_1^\beta - s\alpha\beta T T_1^{\beta-1} - pI_r\alpha m^\beta = 0. \tag{14}$$

and

$$T^* = \left[\frac{(\beta+1)}{s\alpha\beta} K + \frac{T_1^{\beta+1}}{s\beta} \{ h\beta + s\beta + p(I_r - I_e) \} + \frac{pI_r(\beta+1)m^{\beta+1}}{s\beta} \left(\frac{\beta}{\beta+1} - \frac{T_1}{m} \right) \right]^{\frac{1}{\beta+1}} \tag{15}$$

For case 2 differentiating Eq. (9) with respect to T_1 and T two times respectively yields,

$$\frac{\partial Z_2(T, T_1)}{\partial T_1} = \frac{\alpha\beta T_1^{\beta-1}}{T} \{ (h + s + pI_e) T_1 - (sT + pI_e m) \} \tag{16}$$

$$\frac{\partial^2 Z_2(T, T_1)}{\partial T_1^2} = \frac{\alpha\beta T_1^{\beta-2}}{T} [(h + s + pI_e) \beta T_1 + (1-\beta)(sT + pI_e m)] \tag{17}$$

$$\frac{\partial Z_2(T, T_1)}{\partial T} = -\frac{1}{T^2} \left\{ K + \frac{\alpha\beta(h+s)}{\beta+1} T_1^{\beta+1} - \frac{s\alpha\beta}{\beta+1} T^{\beta+1} + \alpha pI_e \left(\frac{\beta}{\beta+1} - \frac{m}{T_1} \right) T_1^{\beta+1} \right\} \tag{18}$$

$$\frac{\partial^2 Z_2(T, T_1)}{\partial T^2} = \frac{2}{T^3} \left\{ K + \frac{\alpha\beta(h+s)}{\beta+1} T_1^{\beta+1} + \frac{s\alpha\beta(\beta-1)}{2(\beta+1)} T^{\beta+1} + \alpha pI_e \left(\frac{\beta}{\beta+1} - \frac{m}{T_1} \right) T_1^{\beta+1} \right\} > 0 \tag{19}$$

$\frac{\partial^2 Z_2(T_1, T)}{\partial T_1^2} > 0$, $\frac{\partial^2 Z_2(T_1, T)}{\partial T^2} > 0$, and $\left(\frac{\partial^2 Z_2(T_1, T)}{\partial T_1^2} \right) \left(\frac{\partial^2 Z_2(T_1, T)}{\partial T^2} \right) - \left(\frac{\partial^2 Z_2(T_1, T)}{\partial T \partial T_1} \right)^2 > 0$

The optimal values for $T_l=T_l^*$ and $T=T^*$ is minimum. The optimal values are obtained by solving

$$\frac{\partial Z_2(T, T_l)}{\partial T_l} = 0 \text{ and } \frac{\partial Z_2(T, T_l)}{\partial T} = 0, \text{ simultaneously, thus}$$

$$T_l^{**} = \frac{(sT + pI_e m)}{(h + s + pI_e)} \quad (20)$$

$$T^{**} = \left\{ \frac{(\beta+1)}{s\alpha\beta} K + \frac{(h+s)}{s} T_l^{\beta+1} + \frac{(\beta+1)pI_e}{s\beta} \left(\frac{\beta}{\beta+1} - \frac{m}{T_l} \right) T_l^{\beta+1} \right\}^{\frac{1}{\beta+1}} \quad (21)$$

The following properties are observed for optimal T_l , Z_l and Z_2 .

Deduction 1: Optimal T_l is an increasing function of T .

Proof: Differentiating equation (14) with respect to T we get,

$$\frac{dT_l^*}{dT} = \frac{sT_l^*}{\{\beta(h+s) + p(I_r - I_e)\} T_l^* + s(1-\beta)T} > 0 \quad (22)$$

as T_l^* and T are both positive and $I_r \geq I_e$. Also differentiating Eq. (20) with respect to T , we get,

$$\frac{dT_l^{**}}{dT} = \frac{s}{h + s + pI_e} > 0 \quad (23)$$

Hence optimal T_l is a non-decreasing function of T .

Deduction 2: Optimal T_l is an increasing function of m .

Proof: Differentiating Eq. (14) with respect to m , we get

$$\frac{dT_l^*}{dm} = \frac{pI_r m^{\beta-1}}{[\{\beta(h+s) + p(I_r - I_e)\} T_l^* + s(1-\beta)T] T_l^{*(\beta-2)}} > 0. \quad (24)$$

As T_l^* and m are both positive and $I_r \geq I_e$. Also differentiating Eq. (20) with respect to m , we get,

$$\frac{dT_l^{**}}{dm} = \frac{pI_e}{h + s + pI_e} > 0 \quad (25)$$

Hence optimal T_l is a non-decreasing function of m .

Deduction 3: $Z_l(T)$ is convex function of T .

Proof: Substituting T_l^* from Eq. (14) to Eq. (7) we get

$$Z_l(T) = \frac{1}{T} \left[K + \frac{\alpha}{\beta+1} \{\beta(h+s) + p(I_r - I_e)\} T_l^{*\beta+1} + \frac{s\alpha}{\beta+1} T^{\beta+1} - s\alpha T T_l^{*\beta} - pI_r \alpha m^\beta T_l^* + \frac{pI_r \alpha \beta}{\beta+1} m^{\beta+1} \right] \quad (26)$$

Differentiating Eq. (26) partially with respect to T , we get

$$\frac{\partial Z_1(T)}{\partial T} = -\frac{N}{T^2} + \frac{1}{T} \frac{dN}{dT}, \quad (27)$$

where N is the numerator of Eq. (26), for extreme values

$$\frac{\partial Z_1(T)}{\partial T} = 0 \text{ gives } N = T \frac{dN}{dT}, \quad (28)$$

also

$$\frac{\partial^2 Z_1(T)}{\partial T^2} = \frac{1}{T} \frac{d^2 N}{dT^2} = s\alpha\beta T^{\beta-2} > 0. \quad (29)$$

Hence $Z_1(T)$ is convex function of T .

Deduction 3: $Z_2(T)$ is convex function of T .

Proof : Substituting T_1^{**} from Eq. (20) to Eq. (9) we get

$$Z_2(T_1, T) = \frac{1}{T} \left[K + \frac{h\alpha\beta}{\beta+1} T_1^{**\beta+1} - pI_e\alpha\beta T_1^{**\beta} \left(\frac{m}{\beta} - \frac{T_1^{**}}{\beta+1} \right) + \frac{s\alpha}{\beta+1} \left\{ T^{\beta+1} - (\beta+1)TT_1^{**\beta} + \beta T_1^{**\beta+1} \right\} \right] \quad (30)$$

Differentiating Eq. (30) partially with respect to T , we get

$$\frac{\partial Z_2(T)}{\partial T} = -\frac{N^*}{T^2} + \frac{1}{T} \frac{dN^*}{dT}, \quad (31)$$

where ' N^* ' is the numerator of Eq. (30), for extreme values

$$\frac{\partial Z_2(T)}{\partial T} = 0, \text{ gives } N^* = T \frac{dN^*}{dT} \quad (32)$$

also

$$\frac{\partial^2 Z_2(T)}{\partial T^2} = \frac{1}{T} \frac{d^2 N^*}{dT^2} = s\alpha\beta T^{\beta-2} > 0 \quad (33)$$

Hence $Z_2(T)$ is convex function of T .

4. Examples

Example 1: Let $K = \$10.00$ per order, $h = \$20.00$ per unit, $p = \$100.00$ per unit, $s = \$50.00$ per unit, $\alpha = 0.2$, $\beta = 0.5$, $I_r = 0.2$, $I_e = 0.1$, find the optimal values of T_1 , T and $Z(T_1, T)$.

Solution1: Case 1: $m \leq T_1$

We take $m = 0.08333$ year, solving the Eqs. (14) and Eq. (15) simultaneously for T_1 , and T , we obtain $T_1^* = 1.12921$ year and $T^* = 1.78717$ years.

Putting the values in Eq. (7) we get optimal value of $Z_1(T_1^*, T^*) = \$7.19826$ / unit time

Optimality conditions holds as $\frac{\partial^2 Z_1(T_1, T)}{\partial T_1^2} \frac{\partial^2 Z_2(T_1, T)}{\partial T^2} - \left(\frac{\partial^2 Z_1(T_1, T)}{\partial T_1 \partial T} \right)^2 = 14.81 > 0$ and $\frac{\partial^2 Z_1(T_1, T)}{\partial T_1^2} = 4.4529 > 0$ on the given values.

Case 2: $T_1 < m$

We take $m = 1.25$ years, solving the Eqs. (20) and Eq. (21) simultaneously for T_1, T , we obtain $T_1^{**} = 1.19116$ year and $T^{**} = 1.65585$ years.

The optimal value of total cost is $Z_2(T_1^{**}, T^{**}) = \6.2433 per unit time.

Optimality conditions holds as $\frac{\partial^2 Z_2(T_1, T)}{\partial T_1^2} \frac{\partial^2 Z_2(T_1, T)}{\partial T^2} - \left(\frac{\partial^2 Z_2(T_1, T)}{\partial T_1 \partial T} \right)^2 = 16.58341 > 0$ and $\frac{\partial^2 Z_2(T_1, T)}{\partial T_1^2} = 4.4267 > 0$ on the given values. Also the variation of optimal solution for different values of m is given in Table 1(a) and Table 1(b) in section 6.

Example 2: Taking the values of the all parameters the same as the ones given in Example 1 except $\beta = 1.5$, the variation of optimal solution for different values of m is given in Table 2(a) and Table 2(b) in section 6.

6. Sensitivity Analysis

The variation of optimal solution for different values of m is shown in Tables 1(a), 1(b), 2(a), and 2(b), for $\beta = 0.5$ and $\beta = 1.5$ respectively. Also the variation of optimal solution for different values of 'h' and 's' is shown in Tables 3(a), and 3(b). Fig. 1, Fig. 2 and Fig. 3 show the behavior of total cost function with respect to T_1 and T . The graphs are drawn with the help of Mathematica7.0 software. Note: m, T_1 , and T are taken in years in Table 1 to Table 5 as follows.

Table 1(a)

Example 1, case-1, $\beta = 1.5$

m	T_1^*	T^*	$Z_1(T_1^*, T^*)$ (in \$)
0.08333	1.12921	1.78717	7.19826
0.16667	1.19200	1.78902	6.91605
0.25000	1.24378	1.79270	6.9975
0.33333	1.29026	1.79782	6.51877
0.41667	1.33363	1.80419	6.36102

Table 1(b)

Example 1, case-2, $\beta = 1.5$

m	T_1^*	T^*	$Z_1(T_1^*, T^*)$ (in \$)
1.25000	1.19116	1.65585	6.24331
1.33333	1.19362	1.64313	6.16600
1.41667	1.19602	1.63029	6.08813
1.50000	1.19833	1.61733	6.00974
1.58333	1.20057	1.60424	5.93080

Table 2(a)

Example 2, case-1, $\beta=1.5$

m	T_1^*	T^*	$Z_1(T_1^*, T^*)$ (in \$)
0.5000	1.82819	2.73349	11.4358
0.58333	1.85141	2.75151	11.3215
0.66667	1.87780	2.77337	11.2168
0.75000	1.90735	2.79918	11.1242
0.83330	1.93997	2.82899	11.0454

Table 2(b)

Example 2, case-2, $\beta = 1.5$

m	T_1^*	T^*	$Z_1(T_1^*, T^*)$ (in \$)
2.08333	2.05254	2.86740	9.43774
2.16667	2.07239	2.88249	9.31719
2.25000	2.09220	2.89752	9.19501
2.33333	2.11198	2.91249	9.07119
2.41667	2.13172	2.92741	8.94575

Table 3(a)

$m = 0.08333$ year, $s = \$50$, $\beta = 0.5$

m	T_1^*	T^*	$Z_1(T_1^*, T^*)$ (in \$)
20	1.129210	1.78717	7.19826
30	0.991201	1.75248	7.31956
40	0.882194	1.72392	7.47304
50	0.794039	1.69991	7.64073
60	0.721357	1.67939	7.81271

Table 3(b)

$m = 0.08333$ year, $h = \$20$, $\beta = 0.5$

m	T_1^*	T^*	$Z_1(T_1^*, T^*)$ (in \$)
50	1.129210	1.78717	7.19826
60	1.085220	1.60822	7.78962
70	1.042700	1.47009	8.33702
80	1.002580	1.35935	8.85215
90	0.965143	1.26805	9.34241

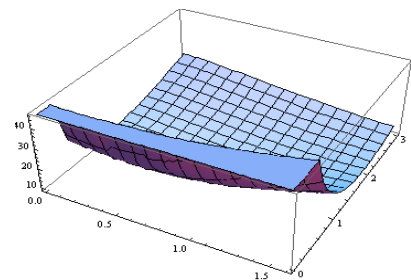
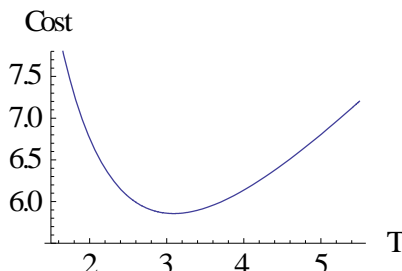
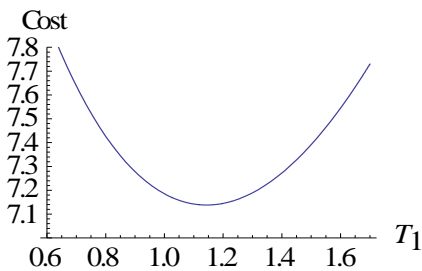


Fig.1. Total cost vs T_1 at fixed $T=1.81164$

Fig. 2. Total cost vs T at fixed $T_1=1.37498$

Fig. 3. Total cost vs T_1 and T

We draw the following conclusions from the above Tables and figures.

- (1) The numerical results shown in Table 1(a), for case-1, indicate that T_1^* is an increasing function of m as well as T . With an increase in permissible period ' m ' total cost $Z(T_1, T)$ decreases significantly.

- (2) The numerical results shown in Table 1(b), for case-1, indicate that the higher values of m allow the higher values of T_l simultaneously the lower values of T , Total cost $Z(T_l, T)$ decreases as ' m ' increases.
- (3) The numerical results shown in Table 2(a) and Table 2(b) for case-1 and case-2 indicate that T_l and T are both directly proportional to permissible period ' m '. Total cost $Z(T_l, T)$ is inversely proportional to ' m '.
- (4) In Table 3(a), it is seen that the higher values of holding cost ' h ' results the higher values in total cost $Z(T_l, T)$ and the lower values in T_l and T .
- (5) Fig.1 and Fig. 2 visualize that total cost function $Z(T_l, T)$ is convex with respect to T_l and T individually.
- (6) Graphics shown in Fig. 2 is data visualization of T_l , T verses total cost $Z(T_l, T)$ indicates that $Z(T_l, T)$ is convex in T_l and T simultaneously.

5. Conclusion and future Research

The present model has been based on a Weibull time dependent demand rate. While dealing with time varying demand pattern, the researcher usually takes the demand rate to be an exponential of the form $R(t) = ae^{bt}$, $a > 0$, $0 < b < 1$ or $b < 0$, which is rarely seen to occur for any product. Most of the authors considered constant demand or stock dependent demand. We developed inventory model for non-deteriorating and Weibull time dependent demand rate for finding optimal order policy when the supplier provides a trade credits. We have shown that Total cost function is convex with respect to T_l and T . The figures 1 and 2 show the validity of convexity of total cost function with respect to T_l and T . Numerical examples and sensitivity analysis are given to validate the model. From the sensitivity analysis it is clear that the variations are quite sensitive with respect to variation of parameters.

The proposed model can be extended in several ways. For instance; we could consider the demand rate as a function of selling price, product quantity and others. Finally, we could generalize the model to allow for shortages and quantity discount, freight charges and others.

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