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Supply chain management under the effect of trade credit for deteriorating items with ramp-type demand and partial backordering under inflationary environment

Aditya Shastri[a*], S.R. Singh[b] and Shalley Gupta[a]

[a]*Banasthali Univeristy, Rajasthan, India*
[b]*D.N (P.G) College, Meerut-U.P, India*

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ABSTRACT

In this paper, a supply chain inventory model is developed in inflationary environment by incorporating some realistic features such as ramp type demand, deterioration, partial backlogging, inflation, and trade credit. Here, rate of deterioration is linear and partial backlogging rate is variable and dependent on the waiting time for the next replenishment. Depending on the trade credit period, three different situations arise. For each model the optimal replenishment policy is determined. Numerical examples are provided to illustrate the proposed inventory model and sensitivity analyses of optimal solutions are given for each case of trapezoidal demand function.

1. Introduction

In today's competitive business transactions, it is common for the supplier to offer a certain credit period to the retailer for stimulating his/her demand. During this credit period the retailer can accumulate the revenue and earn interest on that revenue. However, beyond this period the supplier charges interest on the unpaid balance. Hence, a permissible delay indirectly reduces the cost of holding stock. On the other hand, trade credit offered by the supplier encourages the retailer to buy more. Thus, it is also a powerful promotional tool that attracts new customers, who consider it as an alternative incentive policy to quantity discounts. Hence, trade credit can play a major role in inventory control for both the supplier as well as the retailer. Owing this fact, during the last few years, many articles dealing with various inventory models under a variety of trade credit have appeared in the literature.

In most of the above papers, two types of time varying demand rate have been considered: (i) linear positive/negative trend in demand rate and (ii) exponentially increasing/decreasing demand rate. However, demand cannot increase continuously over time. For example, demand rate for fashionable

* Corresponding author
E-mail address: meshalley@gmail.com (S. Gupta)

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products, increases with time up to a certain moment (only if customers are satisfied with quality and price) and then stabilizes to a constant rate. The term ‘‘ramp type’’ is used to represent such demand pattern. Therefore, a ramp type demand function has two different time segments. In its first segment, the demand is any increasing function of time. But demand remains constant in its second time segment. Hill (1995) proposed an inventory model with increasing demand (general power of time) followed by a constant demand. Mandal and Pal (1998) considered an inventory model for exponentially decaying items by allowing shortages. Wu et al. (1999) related the backlogging rate to the waiting time up to the next replenishment (partial backlogging). Wu and Ouyang (2000) studied the Mandal and Pal’s (1998) inventory model under two different replenishment policies: (a) those starting with no shortages and (b) those starting with shortages. Wu (2001) investigated an inventory model with ramp type demand rate, Weibull distributed deterioration rate and partial backlogging. Giri et al. (2003) extended the ramp type demand inventory model with a more generalized Weibull deterioration distribution. Deng et al. (2007) revisit the inventory model considered by Mandal and Pal (1998) and Wu and Ouyang (2000). They study it by considering the two cases given above and comment on these questionable results. Panda et al. (2007) built an inventory model for deteriorating items with generalized exponential ramp type demand rate and complete backlogging. Skouri et al. (2009) extend the work of Deng et al. (2007) by introducing a general ramp type demand rate and Weibull deterioration rate. Skouri et al. (2011) developed an order level inventory model for deteriorating items deteriorated with constant rate with ramp type demand rate under the conditions of permissible delay in payment.

In the most of the above referred papers, complete backlogging of unsatisfied demand is assumed. In practice, there are customers who are willing to wait and receive their orders at the end of shortage period, while others are not. Inventory models, which consider a mixture of backorders and lost sales for non-deteriorating products, were proposed by Montgomery et al. (1973), Park (1982), and Rosenberg (1979). These authors assumed that only a fixed fraction of demand during the stockout time is backlogged and the rest is lost. In the last few years, considerable attention has been paid to inventory models with partial backlogging. The backlogging rate can be modeled taking into account the customers’ behavior. The first paper in which customers’ impatience functions are proposed seems to be that by Abad (1996). Chang and Dye (1999) developed a finite horizon inventory model using Abad’s reciprocal backlogging rate. Skouri and Papachristos (2002) studied a multi-period inventory model using the negative exponential backlogging rate proposed by Abad. Teng et al. (2002) extended the Chang and Dye’s (1999) and Skouri and Papachristos’ (2002) models, assuming as backlogging rate any decreasing function of the waiting time up to the next replenishment. Research on models with partial backlogging continues with Wang (2002), San Jose et al. (2006), Agrawal and Banerjee (2011), and Goyal et al. (2015).

In all of the models mentioned above, the inflation and time value of money were disregarded. It has happened mostly because of the belief that the inflation and the time value of money would not influence the inventory policy to any significant degree. However, most of the countries have suffered from large-scale inflation and sharp decline in the purchasing power of money last several years. As a result, while determining the optimal inventory policy, the effects of inflation and time value of money cannot be ignored. The pioneer research in this direction was Buzacott (1975), who developed an EOQ model with inflation subject to different types of pricing policies. In the same year, Misra (1975) also developed an EOQ model incorporating inflationary effects. Vrat and Padmanabhan (1990) developed an inventory model under a constant inflation rate for initial stock-dependent consumption rate. Datta and Pal (1991) developed a model with linear time-dependent demand rate and shortages to investigate the effects of inflation and time value of money on ordering policy over a finite time horizon. Hariga (1995) extended Datta and Pal (1991) model by relaxing the assumption of equal inventory carrying time during each replenishment cycle and modified their mathematical formulation. Hariga and Ben-Daya (1996) then extended Hariga (1995) by removing the restriction of equal replenishment cycle and provided two solution procedures with and without shortages. Later, Ray and Chaudhuri (1997), Chen (1998), Sarker et al. (2000), Chung and Lin (2001), and Wee and Law (2001), Hou (2006), Yadav et al. (2015) all have investigated the effects of inflation, time value of money and deterioration on inventory models.

In this paper, an inventory model under inflationary environment with ramp type demand rate, linear deterioration rate, partial backlogging rate and conditions of trade credit in payment. Its study requires exploring the feasible ordering relations between the times parameters appeared, which leads to three models. For each model the optimal replenishment policy is determined. Numerical examples are provided to illustrate the proposed inventory model and sensitivity analyses of optimal solutions are given for each case of trapezoidal demand assumption.

2. Notations and Assumptions

The following notations and assumptions are used throughout in this paper for developing the model.

Notations

T	Cycle time
t_1	Time at which the inventory level falls to zero
S	Maximum inventory level at each cycle time
$C_P(t)$	Unit purchase cost per item, i.e., $C_P(t)=C_Pe^{kt}$, where C_P is the purchasing cost at time zero
$C_H(t)$	Holding cost per item, i.e., $C_H(t)=C_He^{kt}$, where C_H is the holding cost at time zero
$C_S(t)$	Shortage cost, i.e., $C_S(t)=C_Se^{kt}$, where C_S is the shortage cost at time zero
$C_D(t)$	Deterioration cost per item, i.e., $C_D(t)=C_De^{kt}$, where C_D is the deterioration cost at time zero
$C_L(t)$	Unit opportunity cost due to lost sales, i.e., $C_L(t)=C_Le^{kt}$, where C_L is unit opportunity cost due to lost sales at time zero
$C_s(t)$	Unit selling price per item, i.e., $C_s(t)=C_se^{kt}$, where C_s is the selling price at time zero
I_e	Interest rate earned per unit per unit of time
I_p	Interest rate charged per unit per unit of time
M	Credit period offered by retailer
μ	The parameter of the ramp type demand function (break point)

Assumptions

1. The ordering quantity brings the inventory level up to the order level S.

2. Replenishment rate is infinite.

3. Shortages are allowed. Unsatisfied demand is backlogged, and the fraction of shortages backordered is $1/(1 + \delta x)$, where x is the waiting time up to the next replenishment and δ is a positive constant.

4. The retailer can use the sale revenue to earn the interest with annual rate I_e during the period $[0,t_1]$. Beyond the fixed credit period, the product still in stock is assumed to be financed with an annual rate I_c.

5. The on hand inventory deteriorates at the rate (a+bt) per unit of time where a,b>0. The deteriorated items are withdrawn immediately from the warehouse and there is no provision for repair or replacement.

6. Demand rate D(t) is a ramp type function of time given by:

$$D(t) = \begin{cases} a_1 + b_1 t & t < \mu \\ a_1 + b_1 \mu & t \geq \mu \end{cases}$$

7. The time horizon of the inventory system is infinite.

8. The unit price is subject to the same inflation rate as other inventory related costs, thereby implying that the ordering size can be determined by minimizing the total cost over a planning period.

9. The inflation rate is constant.

10. Lead time is zero.

3. Retailer's Inventory Model

Here, the deterministic inventory model for deteriorating items for retailer where shortages occur at the end of the cycle is being discussed. At time t=0, a lot size of S units enters into the system. During the time interval [0, t_1], the inventory S in retailer warehouse decreases due to demand and deterioration and it vanishes at t=t_1. At time t=t_1, the inventory in retailer's warehouse reaches to zero and thereafter the shortages occur during the time interval [t_1, T]. The shortage quantity is supplied to customers at the beginning of the next cycle. The objective of the inventory system is to determine the timing t_1 in order to keep the total relevant cost per unit of time as low as possible.

The inventory level, I(t), satisfies the following differential equations:

$$\frac{dI(t)}{dt} = -D(t) - (a + bt)I(t), \quad 0 \le t \le t_1 \tag{1}$$

With boundary condition I(t_1)=0 and

$$\frac{dI(t)}{dt} = -\frac{D(t)}{1+\delta(T-t)}, \quad t_1 \le t \le T \tag{2}$$

With boundary condition I(t_1)=0

There are two possible relations between parameters t_1 and μ:

(i) $t_1 \le \mu$
(ii) $t_1 > \mu$

Now, we discuss two cases one by one.

(i) Case-1: $t_1 \le \mu$

In this, Eq. (1) and Eq. (2) reduces as follows:

$$\frac{dI(t)}{dt} = -(a_1 + b_1 t) - (a + bt)I(t), \quad 0 \le t \le t_1 \tag{3}$$

with boundary condition I(t_1)=0 and

$$\frac{dI(t)}{dt} = -\frac{a_1 + b_1 t}{1+\delta(T-t)}, \quad t_1 \le t \le \mu \tag{4}$$

with boundary condition I(t_1)=0

$$\frac{dI(t)}{dt} = -\frac{a_1 + b_1 \mu}{1+\delta(T-t)}, \quad \mu \le t \le T \tag{5}$$

with boundary condition I(μ_-)= I(μ_+)

Solutions of Eqs. (3-5) are as follows:

$$I(t)=a_1\left((t_1-t)+\frac{a}{2}(t_1^2-t^2)+\frac{b}{6}(t_1^3-t^3)\right)e^{-\left(at+\frac{bt^2}{2}\right)}+b_1\left(\frac{1}{2}(t_1^2-t^2)+\frac{a}{3}(t_1^3-t^3)+\frac{b}{8}(t_1^4-t^4)\right)e^{-\left(at+\frac{bt^2}{2}\right)},0\le t\le t_1 \quad (6)$$

$$I(t)=\frac{b_1(t-t_1)}{\delta}+\frac{(b_1+(a_1+b_1)\delta)log\left(\frac{1-\delta(T-t)}{1-\delta(T-t_1)}\right)}{\delta^2},t_1\le t\le\mu \quad (7)$$

$$I(t)=\frac{b_1(\mu-t_1)}{\delta}+\frac{(b_1+(a_1+b_1)\delta)log\left(\frac{1-\delta(T-\mu)}{1-\delta(T-t_1)}\right)}{\delta^2}+\frac{(a_1+b_1\mu)log\left(\frac{1+\delta(T-t)}{1+\delta(T-\mu)}\right)}{\delta}, \quad \mu\le t\le T \quad (8)$$

Total amount of deteriorated items during $[0,t_1]$ is

$$D_1=I(0)-\int_0^{t_1}(a+bt)dt \quad (9)$$

$$D_1=a_1\left(t_1+\frac{at_1^2}{2}+\frac{bt_1^3}{6}\right)+b_1\left(\frac{t_1^2}{2}+\frac{at_1^3}{3}+\frac{bt_1^4}{8}\right)-\left(at_1+\frac{bt_1^2}{2}\right)$$

Total inventory during the time interval $[0,t_1]$ is

$$I_{I1}=\int_0^{t_1}I(t)dt$$

$$I_{I1}=\int_0^{t_1}\left(a_1\left((t_1-t)+\frac{a}{2}(t_1^2-t^2)+\frac{b}{6}(t_1^3-t^3)\right)e^{-\left(at+\frac{bt^2}{2}\right)}+b_1\left(\frac{1}{2}(t_1^2-t^2)+\frac{a}{3}(t_1^3-t^3)+\frac{b}{8}(t_1^4-t^4)\right)e^{-\left(at+\frac{bt^2}{2}\right)}\right)dt \quad (10)$$

$$I_{I1}=a_1\left(\frac{t_1^2}{2}+\frac{at_1^3}{6}+\frac{5bt_1^4}{48}\right)+b_1\left(\frac{t_1^3}{3}+\frac{at_1^4}{8}\right)$$

Total amount of backorders due to shortages in the interval $[t_1,T]$ is

$$I_{B1}=\int_{t_1}^{T}-I(t)dt=\int_{t_1}^{\mu}-I(t)dt+\int_{\mu}^{T}-I(t)dt$$

$$I_{B1}=\int_{t_1}^{\mu}-\left(\frac{b_1(t-t_1)}{\delta}+\frac{(b_1+(a_1+b_1)\delta)log\left(\frac{1-\delta(T-t)}{1-\delta(T-t_1)}\right)}{\delta^2}\right)dt+\int_{\mu}^{T}-\left(\frac{b_1(\mu-t_1)}{\delta}+\frac{(b_1+(a_1+b_1)\delta)log\left(\frac{1-\delta(T-\mu)}{1-\delta(T-t_1)}\right)}{\delta^2}+\frac{(a_1+b_1\mu)log\left(\frac{1+\delta(T-t)}{1+\delta(T-\mu)}\right)}{\delta}\right)dt \quad (11)$$

$$I_{B1}=\left(-\frac{b_1(\mu-t_1)^2}{2\delta}-\frac{(b_1+(a_1+b_1)\delta)}{\delta^2}\left[\left(\mu+\frac{1-\delta T}{\delta}\right)log\left(\frac{1-\delta(T-\mu)}{1-\delta(T-t_1)}\right)+(\mu-t_1)\right]\right)-\left(\frac{b_1(\mu-t_1)}{\delta}+\frac{(b_1+(a_1+b_1)\delta)log\left(\frac{1-\delta(T-\mu)}{1-\delta(T-t_1)}\right)}{\delta^2}\right)(T-\mu)-\frac{(a_1+b_1\mu)}{\delta}\left(\left(T-\frac{1-\delta T}{\delta}\right)log\left(1+\delta(T-\mu)\right)+(T-\mu)\right)$$

The amount of lost sales during [t_1,T] is

$$L_1 = \int_{t_1}^{\mu}\left(1-\frac{1}{1+\delta(T-t)}\right)(a_1+b_1t)dt + \int_{\mu}^{T}\left(1-\frac{1}{1+\delta(T-t)}\right)(a_1+b_1\mu)dt$$

$$L_1 = \left(a_1+\frac{b_1}{\delta}\right)(\mu-t_1)+\frac{b_1}{2}(\mu^2-t^2)+\left(a_1\delta+b_1(1+\delta T)\right)log\frac{1+\delta(T-\mu)}{1+\delta(T+t_1)}+(a_1+b_1\mu)\left[(T-\mu)-\frac{1}{\delta}log\left(1+\delta(T-\mu)\right)\right] \quad (12)$$

Case-2: $t_1>\mu$

In this case, Eq. (1) and Eq. (2) reduces as follows:

$$\frac{dI(t)}{dt} = -(a_1+b_1t)-(a+bt)I(t), \quad 0\le t\le\mu \quad (13)$$

with boundary condition I(μ^-)= I(μ^+) and

$$\frac{dI(t)}{dt} = -(a_1+b_1\mu)-(a+bt)I(t), \quad \mu\le t\le t_1 \quad (14)$$

with boundary condition I(t_1)= 0

$$\frac{dI(t)}{dt} = -\frac{a_1+b_1\mu}{1+\delta(T-t)}, \quad t_1\le t\le T \quad (15)$$

with boundary condition I(t_1)=0

Solutions of Eqs. (13-14) are as follows:

$$I(t) = a_1\left((\mu-t)+\frac{a}{2}(\mu^2-t^2)+\frac{b}{6}(\mu^3-t^3)\right)e^{-\left(at+\frac{bt^2}{2}\right)}$$
$$+b_1\left(\frac{1}{2}(\mu^2-t^2)+\frac{a}{3}(\mu^3-t^3)+\frac{b}{8}(\mu^4-t^4)\right)e^{-\left(at+\frac{bt^2}{2}\right)} \quad (16)$$
$$+(a_1+b_1\mu)\left(\frac{a}{2}(t_1^2-\mu^2)+\frac{b}{6}(t_1^3-\mu^3)\right)e^{-\left(at+\frac{bt^2}{2}\right)}, 0\le t\le\mu$$

$$I(t) = (a_1+b_1\mu)\left(\frac{a}{2}(t_1^2-t^2)+\frac{b}{6}(t_1^3-t^3)\right)e^{-\left(at+\frac{bt^2}{2}\right)}, \mu\le t\le t_1 \quad (17)$$

$$I(t) = \frac{(a_1+b_1\mu)log\left(\frac{1+\delta(T-t)}{1+\delta(T-t_1)}\right)}{\delta}, \quad t_1\le t\le T \quad (18)$$

Total amount of deteriorated items during [0,t_1] is

$$D_2 = I(0)-\int_0^{\mu}(a+bt)dt-\int_{\mu}^{t_1}(a+b\mu)dt$$

$$D_2 = a_1\left(\mu + \frac{a\mu^2}{2} + \frac{b\mu^3}{6}\right) + b_1\left(\frac{\mu^2}{2} + \frac{a\mu^3}{3} + \frac{b\mu^4}{8}\right) + (a_1 + b_1\mu)\left(\frac{a}{2}(t_1^2 - \mu^2) + \frac{b}{6}(t_1^3 - \mu^3)\right) - \left(a\mu + \frac{b\mu^2}{2}\right) - (a + b\mu)(t_1 - \mu) \quad (19)$$

Total inventory during the time interval [0,t_1] is

$$I_{I2} = \int_0^{t_1} I(t)dt = \int_0^{\mu} I(t)dt + \int_{\mu}^{t_1} I(t)dt$$

$$I_{I2} = \int_0^{\mu}\left(a_1\left((\mu - t) + \frac{a}{2}(\mu^2 - t^2) + \frac{b}{6}(\mu^3 - t^3)\right)e^{-\left(at + \frac{bt^2}{2}\right)} + +b_1\left(\frac{1}{2}(\mu^2 - t^2) + \frac{a}{3}(\mu^3 - t^3) + \frac{b}{8}(\mu^4 - t^4)\right)e^{-\left(at + \frac{bt^2}{2}\right)} + (a_1 + b_1\mu)\left(\frac{a}{2}(t_1^2 - \mu^2) + \frac{b}{6}(t_1^3 - \mu^3)\right)e^{-\left(at + \frac{bt^2}{2}\right)}\right)dt + \int_{\mu}^{t_1}\left((a_1 + b_1\mu)\left(\frac{a}{2}(t_1^2 - t^2) + \frac{b}{6}(t_1^3 - t^3)\right)e^{-\left(at + \frac{bt^2}{2}\right)}\right)dt \quad (20)$$

$$I_{I2} = a_1\left(\frac{\mu^2}{2} + \frac{a\mu^3}{3} + \frac{b\mu^4}{12}\right) + b_1\left(\frac{\mu^3}{3} + \frac{a\mu^4}{8} + \frac{b\mu^5}{15}\right) + (a_1 + b_1\mu)\left(\frac{a}{2}(t_1^2 - \mu^2) + \frac{b}{6}(t_1^3 - \mu^3)\right)\left(\mu - \left(\frac{a\mu^2}{2} + \frac{b\mu^3}{6}\right)\right)$$

Total amount of backorders due to shortages in the interval [t_1,T] is

$$I_{B2} = \int_{t_1}^{T} -I(t)dt$$

$$I_{B2} = \int_{t_1}^{T} -\left(\frac{(a_1 + b_1\mu)log\left(\frac{1+\delta(T-t)}{1+\delta(T-t_1)}\right)}{\delta}\right)dt \quad (21)$$

$$I_{B2} = -\left(\frac{(a_1 + b_1\mu)}{\delta}\left[(T - t_1) + \left(\frac{1 - 2\delta T}{\delta}\right)log\big(1 + \delta(T - t_1)\big)\right]\right)$$

The amount of lost sales during [t_1,T] is

$$L_2 = \int_{t_1}^{T}\left(1 - \frac{1}{1 + \delta(T - t)}\right)(a_1 + b_1\mu)dt \quad (22)$$

$$L_2 = (a_1 + b_1\mu)\left[(T - t_1) - \frac{1}{\delta}log\big(1 + \delta(T - t_1)\big)\right]$$

It is assumed that T>μ otherwise, demand rate is increasing but not ramp-type through the entire planning horizon T. For the given value of M, following three possibilities arises:

i. M≤μ
ii. μ≤M≤T
iii. T≤M

In different three cases different cost are different. So, we discuss all the three models one by one.

3.1 Derivation of Retailer's cost function for model-1 (M≤μ)

From the possible values of t_1, we obtain the three possible sub-cases, $t_1 \le M \le \mu < T$, $M \le t_1 \le \mu < T$ and $M \le \mu \le t_1 \le T$. For details, see Figs. 1-3.

3.1.a The case $t_1 \le M \le \mu < T$

The interest earned during the credit period where the inventory level is positive is

$$I_{E1,1} = \sum_{n=0}^{\infty} C_s(nT) \left[I_e \int_0^{t_1} \left(\int_0^{t} (a_1 + b_1 x) dx \right) dt + I_e (M - t_1) \int_0^{t_1} (a_1 + b_1 x) dx \right] \quad (23)$$

$$I_{E1,1} = C_s I_e \left[M - \frac{a_1 t_1^2}{2} - \frac{b_1 t_1^3}{3} \right] \frac{1}{1 - e^{-kT}}$$

Since $t_1 \le M$, so there is no interest paid in that duration.

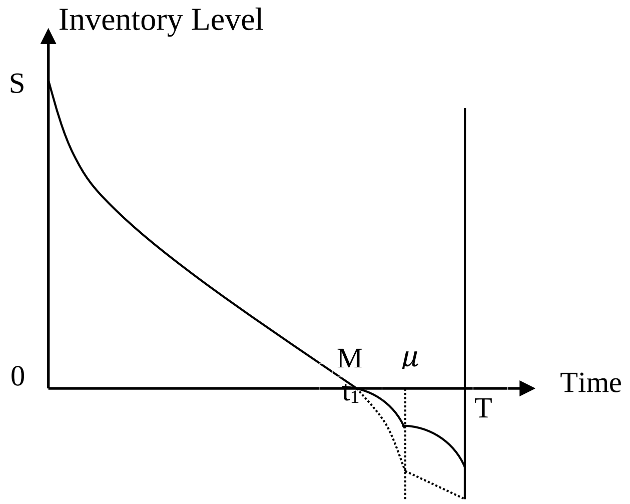

Fig. 1. Inventory level for the model M≤ μ (the case $t_1 \le M \le \mu < T$)

$$Holding\ Cost = \sum_{n=0}^{\infty} C_H(nT) \left(a_1 \left(\frac{t_1^2}{2} + \frac{a t_1^3}{6} + \frac{5 b t_1^4}{48} \right) + b_1 \left(\frac{t_1^3}{3} + \frac{a t_1^4}{8} \right) \right)$$

$$= C_H \left(a_1 \left(\frac{t_1^2}{2} + \frac{a t_1^3}{6} + \frac{5 b t_1^4}{48} \right) + b_1 \left(\frac{t_1^3}{3} + \frac{a t_1^4}{8} \right) \right) \frac{1}{1 - e^{-kT}}$$

$$Backordering\ Cost = \sum_{n=0}^{\infty} C_B(nT) \left(\left(-\frac{b_1(\mu - t_1)^2}{2\delta} - \frac{(b_1 + (a_1 + b_1)\delta)}{\delta^2} \left[\left(\mu + \frac{1 - \delta T}{\delta} \right) log \left(\frac{1 - \delta(T - \mu)}{1 - \delta(T - t_1)} \right) + (\mu - t_1) \right] \right) - \left(\frac{b_1(\mu - t_1)}{\delta} + \frac{(b_1 + (a_1 + b_1)\delta) log \left(\frac{1 - \delta(T - \mu)}{1 - \delta(T - t_1)} \right)}{\delta^2} \right) (T - \mu) - \frac{(a_1 + b_1 \mu)}{\delta} \left(\left(T - \frac{1 - \delta T}{\delta} \right) log(1 + \delta(T - \mu)) + (T - \mu) \right) \right)$$

$$= C_B \left(\left(-\frac{b_1(\mu - t_1)^2}{2\delta} - \frac{(b_1 + (a_1 + b_1)\delta)}{\delta^2} \left[\left(\mu + \frac{1 - \delta T}{\delta} \right) log \left(\frac{1 - \delta(T - \mu)}{1 - \delta(T - t_1)} \right) + (\mu - t_1) \right] \right) - \left(\frac{b_1(\mu - t_1)}{\delta} + \frac{(b_1 + (a_1 + b_1)\delta) log \left(\frac{1 - \delta(T - \mu)}{1 - \delta(T - t_1)} \right)}{\delta^2} \right) (T - \mu) - \frac{(a_1 + b_1 \mu)}{\delta} \left(\left(T - \frac{1 - \delta T}{\delta} \right) log(1 + \delta(T - \mu)) + (T - \mu) \right) \right) \frac{1}{1 - e^{-kT}}$$

$$Lost\ Sale\ Cost = \sum_{n=0}^{\infty} C_L(nT)\left(\left(a_1 + \frac{b_1}{\delta}\right)(\mu - t_1) + \frac{b_1}{2}(\mu^2 - t^2) + \left(a_1\delta + b_1(1+\delta T)\right) log\frac{1+\delta(T-\mu)}{1+\delta(T+t_1)} + (a_1 + b_1\mu)\left[(T-\mu) - \frac{1}{\delta}log\left(1+\delta(T-\mu)\right)\right]\right)$$

$$= C_L\left(\left(a_1 + \frac{b_1}{\delta}\right)(\mu - t_1) + \frac{b_1}{2}(\mu^2 - t^2) + \left(a_1\delta + b_1(1+\delta T)\right) log\frac{1+\delta(T-\mu)}{1+\delta(T+t_1)} + (a_1 + b_1\mu)\left[(T-\mu) - \frac{1}{\delta}log\left(1+\delta(T-\mu)\right)\right]\right)\frac{1}{1-e^{-kT}}$$

$$Deterioration\ Cost = \sum_{n=0}^{\infty} C_D(nT)\left(a_1\left(t_1 + \frac{at_1^2}{2} + \frac{bt_1^3}{6}\right) + b_1\left(\frac{t_1^2}{2} + \frac{at_1^3}{3} + \frac{bt_1^4}{8}\right) - \left(at_1 + \frac{bt_1^2}{2}\right)\right)$$

$$= \left(a_1\left(t_1 + \frac{at_1^2}{2} + \frac{bt_1^3}{6}\right) + b_1\left(\frac{t_1^2}{2} + \frac{at_1^3}{3} + \frac{bt_1^4}{8}\right) - \left(at_1 + \frac{bt_1^2}{2}\right)\right)\frac{1}{1-e^{-kT}}$$

$$TC_{1,1} = Holding\ Cost + Backorering\ Cost + Deterioration\ Cost + Lost\ Sale\ Cost - Interest\ Earned \quad (24)$$

3.1.b The case $M < t_1 \leq \mu < T$

The interest earned during the credit period where the inventory level is positive is

$$I_{E1,2} = \sum_{n=0}^{\infty} C_S(nT)\, I_e \int_0^{t_1}\left(\int_0^{t}(a_1 + b_1x)dx\right)dt \quad (25)$$

$$I_{E1,2} = C_S I_e\left(\frac{a_1t_1^2}{2} + \frac{b_1t_1^3}{6}\right)\frac{1}{1-e^{-kT}}$$

The interest charged $P_{P1,2}$, for the inventory not being sold after the due date M, is given by:

$$I_{P1,2} = \sum_{n=0}^{\infty} C_P(nT)\, I_P \int_M^{t_1}\left(a_1\left((t_1 - t) + \frac{a}{2}(t_1^2 - t^2) + \frac{b}{6}(t_1^3 - t^3)\right)e^{-\left(at+\frac{bt^2}{2}\right)} + +b_1\left(\frac{1}{2}(t_1^2 - t^2) + \frac{a}{3}(t_1^3 - t^3) + \frac{b}{8}(t_1^4 - t^4)\right)e^{-\left(at+\frac{bt^2}{2}\right)}\right)dt \quad (26)$$

$$I_{P1,2} = C_P I_P\left(a_1\left[\frac{(t_1-M)^2}{2} + \frac{at_1^3}{6} - \frac{aM}{2}\left(t_1^2 - \frac{M^2}{3}\right) + \frac{bt_1^4}{8} - \frac{bM}{6}\left(t_1^3 - \frac{M^3}{4}\right) + aM^2\left(\frac{t_1}{2} - \frac{M}{3}\right)\right] + b_1\left[\frac{t_1^3}{3} - \frac{M}{2}\left(t_1^2 - \frac{M^2}{3}\right) + \frac{at_1^4}{4} - \frac{aM}{3}\left(t_1^3 - \frac{M^3}{4}\right) + \frac{bt_1^5}{10} - \frac{bM}{8}\left(t_1^4 - \frac{M^4}{5}\right)\right]\right)\frac{1}{1-e^{-kT}}$$

Since $t_1 \leq M$, so there is no interest paid in that duration.

$$Holding\ Cost = \sum_{n=0}^{\infty} C_H(nT)\left(a_1\left(\frac{t_1^2}{2} + \frac{at_1^3}{6} + \frac{5bt_1^4}{48}\right) + b_1\left(\frac{t_1^3}{3} + \frac{at_1^4}{8}\right)\right)$$

$$= C_H\left(a_1\left(\frac{t_1^2}{2} + \frac{at_1^3}{6} + \frac{5bt_1^4}{48}\right) + b_1\left(\frac{t_1^3}{3} + \frac{at_1^4}{8}\right)\right)\frac{1}{1-e^{-kT}}$$

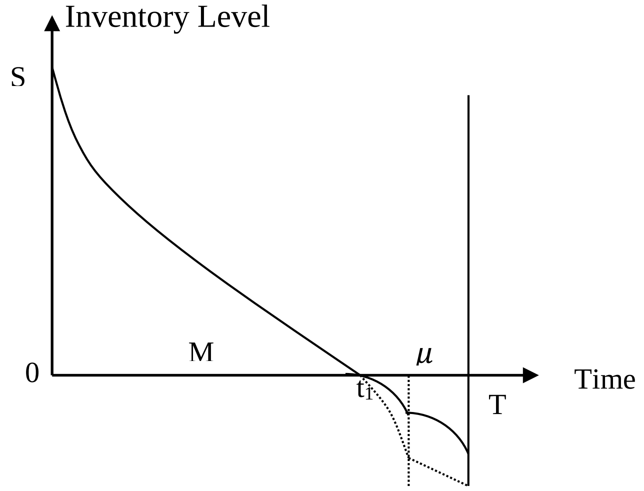

Fig. 2. Inventory level for the model M≤ μ (the case M<t1≤ μ<T)

$$Backordering\ Cost = \sum_{n=0}^{\infty} C_B(nT)\left(\left(-\frac{b_1(\mu-t_1)^2}{2\delta} - \frac{(b_1+(a_1+b_1)\delta)}{\delta^2}\left[\left(\mu+\frac{1-\delta T}{\delta}\right)log\left(\frac{1-\delta(T-\mu)}{1-\delta(T-t_1)}\right)+(\mu-t_1)\right]\right) - \left(\frac{b_1(\mu-t_1)}{\delta}+\frac{(b_1+(a_1+b_1)\delta)log\left(\frac{1-\delta(T-\mu)}{1-\delta(T-t_1)}\right)}{\delta^2}\right)(T-\mu) - \frac{(a_1+b_1\mu)}{\delta}\left(\left(T-\frac{1-\delta T}{\delta}\right)log\big(1+\delta(T-\mu)\big)+(T-\mu)\right)\right)$$

$$= C_B\left(\left(-\frac{b_1(\mu-t_1)^2}{2\delta} - \frac{(b_1+(a_1+b_1)\delta)}{\delta^2}\left[\left(\mu+\frac{1-\delta T}{\delta}\right)log\left(\frac{1-\delta(T-\mu)}{1-\delta(T-t_1)}\right)+(\mu-t_1)\right]\right) - \left(\frac{b_1(\mu-t_1)}{\delta}+\frac{(b_1+(a_1+b_1)\delta)log\left(\frac{1-\delta(T-\mu)}{1-\delta(T-t_1)}\right)}{\delta^2}\right)(T-\mu) - \frac{(a_1+b_1\mu)}{\delta}\left(\left(T-\frac{1-\delta T}{\delta}\right)log\big(1+\delta(T-\mu)\big)+(T-\mu)\right)\right)\frac{1}{1-e^{-kT}}$$

$$Lost\ Sale\ Cost = \sum_{n=0}^{\infty} C_L(nT)\left(\left(a_1+\frac{b_1}{\delta}\right)(\mu-t_1)+\frac{b_1}{2}(\mu^2-t^2)+\big(a_1\delta+b_1(1+\delta T)\big) log\frac{1+\delta(T-\mu)}{1+\delta(T+t_1)}+(a_1+b_1\mu)\left[(T-\mu)-\frac{1}{\delta}log\big(1+\delta(T-\mu)\big)\right]\right)$$

$$= C_L\left(\left(a_1+\frac{b_1}{\delta}\right)(\mu-t_1)+\frac{b_1}{2}(\mu^2-t^2)+\big(a_1\delta+b_1(1+\delta T)\big) log\frac{1+\delta(T-\mu)}{1+\delta(T+t_1)}+(a_1+b_1\mu)\left[(T-\mu)-\frac{1}{\delta}log\big(1+\delta(T-\mu)\big)\right]\right)\frac{1}{1-e^{-kT}}$$

$$Deterioration\ Cost = \sum_{n=0}^{\infty} C_D(nT)\left(a_1\left(t_1+\frac{at_1^2}{2}+\frac{bt_1^3}{6}\right)+b_1\left(\frac{t_1^2}{2}+\frac{at_1^3}{3}+\frac{bt_1^4}{8}\right)-\left(at_1+\frac{bt_1^2}{2}\right)\right)$$

$$= C_D\left(a_1\left(t_1+\frac{at_1^2}{2}+\frac{bt_1^3}{6}\right)+b_1\left(\frac{t_1^2}{2}+\frac{at_1^3}{3}+\frac{bt_1^4}{8}\right)-\left(at_1+\frac{bt_1^2}{2}\right)\right)\frac{1}{1-e^{-kT}}$$

$$TC_{1,2} = Holding\ Cost + Backorering\ Cost + Deterioration\ Cost + Lost\ Sale\ Cost + Interest\ paid - Interest\ Earned \quad (27)$$

3.1.c The case M ≤μ≤t1 ≤T

The interest earned during the credit period where the inventory level is positive is

$$I_{E1,3} = \sum_{n=0}^{\infty} C_S(nT)\, I_e \left[\int_0^{\mu}\left(\int_0^t (a_1+b_1x)dx\right)dt + \int_{\mu}^{t_1}\left(\int_0^{\mu}(a_1+b_1x)dx\right)dt + \int_{\mu}^{t_1}\left(\int_{\mu}^{t}(a_1+b_1\mu)dx\right)dt\right]$$

$$= C_S\left[\frac{a_1t_1^2}{2}+\frac{b_1t_1^3}{6}+\frac{(a_1+b_1\mu)(t_1-\mu)^2}{2}\right]\frac{1}{1-e^{-kT}} \quad (28)$$

The interest charged $P_{P1,2}$, for the inventory not being sold after the due date M, is given by:

$$I_{P1,3} = \sum_{n=0}^{\infty} C_P(nT)\, I_P \left[\int_M^{\mu}\left[a_1(\mu-t)+\frac{a}{2}(\mu^2-t^2)+\frac{b}{6}(\mu^3-t^3)\, e^{-\left(at+\frac{bt^2}{2}\right)}\right.\right.$$
$$(a_1+b_1\mu)\left(\frac{a}{2}(t_1^2-t^2)+\frac{b}{6}(t_1^3-t^3)\right)e^{-\left(at+\frac{bt^2}{2}\right)}$$
$$\left.+b_1\left(\frac{1}{2}(\mu^2-t^2)+\frac{a}{3}(\mu^3-t^3)+\frac{b}{8}(\mu^4-t^4)\right)e^{-\left(at+\frac{bt^2}{2}\right)}\right]dt$$
$$\left.+\int_{\mu}^{t_1}\left[(a_1+b_1\mu)\left(\frac{a}{2}(t_1^2-\mu^2)+\frac{b}{6}(t_1^3-\mu^3)\right)e^{-\left(at+\frac{bt^2}{2}\right)}\right]dt\right]$$

$$= C_PI_P\left(-\frac{1}{36}(a_1+b_1\mu)(-3at_1^2-bt_1^3+3a\mu^2+b\mu^3)\left(6t_1-3at_1^2-bt_1^3+\mu(-6+3a\mu+b\mu^2)\right)\right. \quad (29)$$
$$+\frac{1}{5040}\left(70a_1\left(36(M-\mu)^2+2b^2(t_1^3-M^3)(M^3-\mu^3)\right.\right.$$
$$\left.-3b(M-\mu)(4t_1^3+M^3-3M^2\mu-3M\mu^2+\mu^3)\right)$$
$$+42a^2\left(30a_1(t_1^2-M^2)(M^2-\mu^2)\right.$$
$$\left.+b_1\left(-8M^5-15M^4\mu-30t_1^2\mu^3+3\mu^5+10M^2(3t_1^2\mu+2\mu^3)\right)\right)$$
$$+b_1\left(-126bM^5-45b^2M^7+210bM^4\mu-70b^2M^6\mu\right.$$
$$-210M\mu(4bt_1^3+12\mu+3b\mu^3)+35M^3\left(24+12b\mu^2+b^2(4M^3\mu+3\mu^4)\right)$$
$$\left.+2\mu^2\left(840\mu+21b(20t_1^3+3\mu^3)+b^2(-70\mu^2t_1^3+5\mu^5)\right)\right)$$
$$+35a(M$$
$$-\mu)\left(12a_1\left(bt_1^3(M+\mu)+t_1^2\left(-6+b(M^2+M\mu+\mu^2)\right)\right.\right.$$
$$\left.-M\left(-6\mu+bM(2M^2+2M\mu+\mu^2)\right)\right)$$
$$+b_1\left(-7bM^5-19bM^4\mu+M^2\mu(18+12bt_1^2-11b\mu^2)-M^3(6+19b\mu^2)\right.$$
$$+2\mu\left(6b\mu t_1^3+3\mu^2-b\mu^4+6t_1^2(-6+b\mu^2)\right)$$
$$\left.\left.\left.\left.+2M\mu\left(27\mu+b(6t_1^3+6t_1^2\mu-\mu^3)\right)\right)\right)\right)\right)\frac{1}{1-e^{-kT}}$$

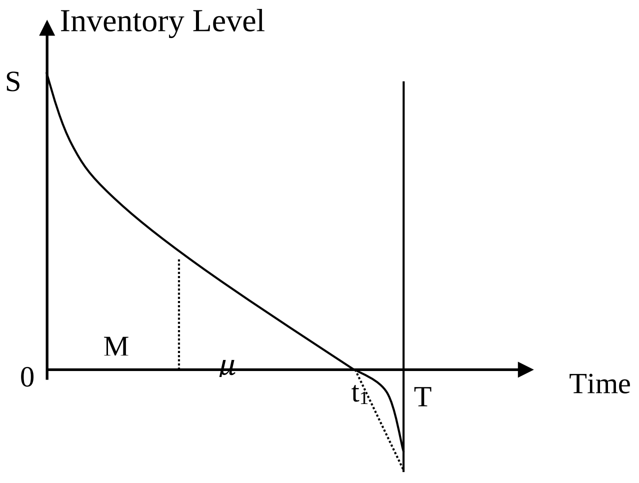

Fig. 3. Inventory level for the model M≤ μ (the case M< μ ≤t_1<T)

$$Holding\ Cost = \sum_{n=0}^{\infty} C_H(nT)\left[\int_0^{\mu}\left[a_1(\mu - t) + \frac{a}{2}(\mu^2 - t^2) + \frac{b}{6}(\mu^3 - t^3)\, e^{-\left(at+\frac{bt^2}{2}\right)}\right.\right.$$

$$(a_1 + b_1\mu)\left(\frac{a}{2}(t_1^2 - t^2) + \frac{b}{6}(t_1^3 - t^3)\right)e^{-\left(at+\frac{bt^2}{2}\right)}$$

$$\left.\left.+b_1\left(\frac{1}{2}(\mu^2 - t^2) + \frac{a}{3}(\mu^3 - t^3) + \frac{b}{8}(\mu^4 - t^4)\right)e^{-\left(at+\frac{bt^2}{2}\right)}\right]dt + \int_{\mu}^{t_1}\left[(a_1 + b_1\mu)\left(\frac{a}{2}(t_1^2 - \mu^2) + \frac{b}{6}(t_1^3 - \mu^3)\right)e^{-\left(at+\frac{bt^2}{2}\right)}\right]dt\right]$$

$$= C_H\left(\tfrac{1}{2}aa_1t_1^2\mu + \tfrac{1}{6}ba_1t_1^3\mu + \tfrac{1}{2}a_1\mu^2 - \tfrac{1}{4}a^2a_1t_1^2\mu^2 - \tfrac{1}{12}aa_1bt_1^3\mu^2 + \tfrac{1}{6}bb_1t_1^3\mu^2 + \tfrac{1}{3}b_1\mu^3 - \tfrac{1}{12}aa_1bt_1^2\mu^3 - \tfrac{1}{4}a^2b_1t_1^2\mu^3 - \tfrac{1}{36}a_1b^2t_1^3\mu^3 - \tfrac{1}{12}abb_1t_1^3\mu^3 + \tfrac{1}{24}a_1b\mu^4 - \tfrac{1}{24}ab_1\mu^4 - \tfrac{1}{12}abb_1t_1^2\mu^4 - \tfrac{1}{36}b^2b_1t_1^3\mu^4 + \tfrac{1}{40}a^2b_1\mu^5 + \tfrac{1}{40}bb_1\mu^5 + \tfrac{1}{72}abb_1\mu^6 + \tfrac{1}{504}b^2b_1\mu^7 - \tfrac{1}{36}(a_1 + b_1\mu)(-3at_1^2 - bt_1^3 + 3a\mu^2 + b\mu^3)\left(6t_1 - 3at_1^2 - bt_1^3 + \mu(-6 + 3a\mu + b\mu^2)\right)\right)\frac{1}{1-e^{-kT}}$$

$$Backordering\ Cost = \textstyle\sum_{n=0}^{\infty} C_B(nT)(a_1 + b_1\mu)\int_{t_1}^{T}\frac{(T-x)}{1+\delta(T-x)}dx$$

$$= \frac{C_B(a_1+b_1\mu)}{\delta}\left[(T - t_1) - \frac{1}{\delta}log\big(1 + \delta(T - t_1)\big)\right]\frac{1}{1-e^{-kT}}$$

$$Lost\ Sale\ Cost = \textstyle\sum_{n=0}^{\infty} C_L(nT)(a_1 + b_1\mu)\int_{t_1}^{T}\left(1 - \frac{1}{1+\delta(T-x)}\right)dx$$

$$= C_L(a_1 + b_1\mu)\left[(T - t_1) - \frac{1}{\delta}log\big(1 + \delta(T - t_1)\big)\right]\frac{1}{1 - e^{-kT}}$$

$$Deterioration\ Cost = \textstyle\sum_{n=0}^{\infty} C_D(nT)\left[a_1\left(\mu + \frac{a\mu^2}{2} + \frac{b\mu^3}{6}\right) + b_1\left(\frac{\mu^2}{2} + \frac{a\mu^3}{3} + \frac{b\mu^4}{8}\right) + (a_1 + b_1\mu)\left(\frac{a}{2}(t_1^2 - \mu^2) + \frac{b}{6}(t_1^3 - \mu^3)\right) - \left(a\mu + \frac{b\mu^2}{2}\right) - (a + b\mu)(t_1 - \mu)\right]$$

$$= C_D\left[a_1\left(\mu + \frac{a\mu^2}{2} + \frac{b\mu^3}{6}\right) + b_1\left(\frac{\mu^2}{2} + \frac{a\mu^3}{3} + \frac{b\mu^4}{8}\right) + (a_1 + b_1\mu)\left(\frac{a}{2}(t_1^2 - \mu^2) + \frac{b}{6}(t_1^3 - \mu^3)\right) - \left(a\mu + \frac{b\mu^2}{2}\right) - (a + b\mu)(t_1 - \mu)\right]\frac{1}{1 - e^{-kT}}$$

$$TC_{1,3} = Holding\ Cost + Backorering\ Cost + Deterioration\ Cost + Lost\ Sale\ Cost + Interest\ paid - Interest\ Earned \quad (30)$$

3.1.d Algorithm to Determine Optimal Replenishment Policy

The total cost function in different cases can be combined as follows:

$$TC_1 = \begin{cases} TC_{1,1}(t_1) & t_1 \leq M, \\ TC_{1,2}(t_1) & M < t_1 \leq \mu, \\ TC_{1,3}(t_1) & \mu \leq t_1 \end{cases}$$

and the problem is

$\min TC_1(t_1)$

We follow the following step obtain the optimal replenishment policy:

Step-1: Find the global say $\min TC_{1,1}(t_1)$, say $t_{1,1}^*$ as follows:

Step-1a: Compute $t_{1,1}$ on solving the equation $\frac{dTC_1(t_1)}{dt_1}$=0 if $t_{1,1}$<M then set $t_{1,1}^* = t_{1,1}$ and compute $TC_{1,1}(t_{1,1}^*)$, else go to step 1b.

Step-1b: Find the $\min\{TC_{1,1}(0), TC_{1,1}(M)\}$ and accordingly set $t_{1,1}^*$.

Step-2: Find the global say $\min TC_{1,2}(t_1)$, say $t_{1,2}^*$ as follows:

Step-2a: Compute $t_{1,2}$ on solving the equation $\frac{dTC_{1,2}(t_1)}{dt_1}$=0 if M< $t_{1,2}$ <μ then set $t_{1,2}^* = t_{1,2}$ and compute $TC_{1,2}(t_{1,2}^*)$, else go to step 2b.

Step-2b: Find the $\min\{TC_{1,2}(M), TC_{1,2}(\mu)\}$ and accordingly set $t_{1,2}^*$.

Step-3: Find the global say $\min TC_{1,3}(t_1)$, say $t_{1,3}^*$ as follows:

Step-3a: Compute $t_{1,3}$ on solving the equation $\frac{dTC_{1,3}(t_1)}{dt_1}$=0 if M< $t_{1,3}$ <μ then set $t_{1,3}^* = t_{1,3}$ and compute $TC_{1,3}(t_{1,3}^*)$, else go to step 3b.

Step-3b: Find the $\min\{TC_{1,3}(M), TC_{1,3}(\mu)\}$ and accordingly set $t_{1,3}^*$.

Step-4: Find the $\min\{TC_{1,1}(t_{1,1}^*), TC_{1,2}(t_{1,2}^*), TC_{1,3}(t_{1,3}^*)\}$ and accordingly set t_1 .

3.2 Derivation of Retailer's cost function for model-2 (μ≤M≤T)

In this case, the offered credit period *M* is longer than the demand stabilizing point μ and shorter than the planning horizon *T*. To obtain the total cost function the three cases arises. We discuss all the three cases one by one.

3.2.a The case $t_1 \leq \mu < M < T$

The interest earned during the period of positive inventory level is:

$$I_{E2,1} = \sum_{n=0}^{\infty} C_S(nT)\, I_e \left[\int_0^{t_1}\left(\int_0^{t}(a_1 + b_1 x)dx\right)dt + (M - t_1)\int_0^{t_1}(a_1 + b_1 x)dx\right]$$

$$= C_S I_e \left[\frac{a_1 t_1^2}{2} + \frac{b_1 t_1^3}{6} + (M - t_1)\left(a_1 t + \frac{b_1 t_1^2}{2}\right)\right]\frac{1}{1 - e^{-kT}}$$

Since $t_1 \leq M$, so there is no interest paid in that duration.

$$Holding\ Cost = \sum_{n=0}^{\infty} C_H(nT) \quad \left(a_1\left(\frac{t_1^2}{2} + \frac{at_1^3}{6} + \frac{5bt_1^4}{48}\right) + b_1\left(\frac{t_1^3}{3} + \frac{at_1^4}{8}\right)\right)$$

$$= C_H\left(a_1\left(\frac{t_1^2}{2} + \frac{at_1^3}{6} + \frac{5bt_1^4}{48}\right) + b_1\left(\frac{t_1^3}{3} + \frac{at_1^4}{8}\right)\right)\frac{1}{1-e^{-kT}}$$

$$Backordering\ Cost = \sum_{n=0}^{\infty} C_B(nT)\left(\left(-\frac{b_1(\mu-t_1)^2}{2\delta} - \frac{(b_1+(a_1+b_1)\delta)}{\delta^2}\left[\left(\mu + \frac{1-\delta T}{\delta}\right)log\left(\frac{1-\delta(T-\mu)}{1-\delta(T-t_1)}\right) + (\mu - t_1)\right]\right) - \left(\frac{b_1(\mu-t_1)}{\delta} + \frac{(b_1+(a_1+b_1)\delta)log\left(\frac{1-\delta(T-\mu)}{1-\delta(T-t_1)}\right)}{\delta^2}\right)(T-\mu) - \frac{(a_1+b_1\mu)}{\delta}\left(\left(T - \frac{1-\delta T}{\delta}\right)log\big(1+\delta(T-\mu)\big) + (T-\mu)\right)\right)$$

$$= C_B\left(\left(-\frac{b_1(\mu-t_1)^2}{2\delta} - \frac{(b_1+(a_1+b_1)\delta)}{\delta^2}\left[\left(\mu + \frac{1-\delta T}{\delta}\right)log\left(\frac{1-\delta(T-\mu)}{1-\delta(T-t_1)}\right) + (\mu - t_1)\right]\right) - \left(\frac{b_1(\mu-t_1)}{\delta} + \frac{(b_1+(a_1+b_1)\delta)log\left(\frac{1-\delta(T-\mu)}{1-\delta(T-t_1)}\right)}{\delta^2}\right)(T-\mu) - \frac{(a_1+b_1\mu)}{\delta}\left(\left(T - \frac{1-\delta T}{\delta}\right)log\big(1+\delta(T-\mu)\big) + (T-\mu)\right)\right)\frac{1}{1-e^{-kT}}$$

$$Lost\ Sale\ Cost = \sum_{n=0}^{\infty} C_L(nT)\left(\left(a_1 + \frac{b_1}{\delta}\right)(\mu - t_1) + \frac{b_1}{2}(\mu^2 - t^2) + \big(a_1\delta + b_1(1+\delta T)\big) log\frac{1+\delta(T-\mu)}{1+\delta(T+t_1)} + (a_1 + b_1\mu)\left[(T-\mu) - \frac{1}{\delta}log\big(1+\delta(T-\mu)\big)\right]\right)$$

$$= C_L\left(\left(a_1 + \frac{b_1}{\delta}\right)(\mu - t_1) + \frac{b_1}{2}(\mu^2 - t^2) + \big(a_1\delta + b_1(1+\delta T)\big) log\frac{1+\delta(T-\mu)}{1+\delta(T+t_1)} + (a_1 + b_1\mu)\left[(T-\mu) - \frac{1}{\delta}log\big(1+\delta(T-\mu)\big)\right]\right)\frac{1}{1-e^{-kT}}$$

$$Deterioration\ Cost = \sum_{n=0}^{\infty} C_D(nT)\left(a_1\left(t_1 + \frac{at_1^2}{2} + \frac{bt_1^3}{6}\right) + b_1\left(\frac{t_1^2}{2} + \frac{at_1^3}{3} + \frac{bt_1^4}{8}\right) - \left(at_1 + \frac{bt_1^2}{2}\right)\right)$$

$$= \left(a_1\left(t_1 + \frac{at_1^2}{2} + \frac{bt_1^3}{6}\right) + b_1\left(\frac{t_1^2}{2} + \frac{at_1^3}{3} + \frac{bt_1^4}{8}\right) - \left(at_1 + \frac{bt_1^2}{2}\right)\right)\frac{1}{1-e^{-kT}}$$

$$TC_{2,1} = Holding\ Cost + Backorering\ Cost + Deterioration\ Cost + Lost\ Sale\ Cost - Interest\ Earned \tag{31}$$

3.2.b The case $\mu < t_1 \leq M < T$

The interest earned during the period of positive inventory level is:

$I_{E2,2} = \sum_{n=0}^{\infty} C_S(nT)\, I_e \left[\int_0^{\mu}\left(\int_0^t (a_1+b_1x)dx\right)dt + \int_{\mu}^{M}\left(\int_0^{\mu}(a_1+b_1x)dx\right)dt + \int_{\mu}^{t_1}\left(\int_{\mu}^{t}(a_1+b_1\mu)dx\right)dt + \int_{t_1}^{M}\left(\int_{\mu}^{t_1}(a_1+b_1\mu)dx\right)dt\right]$

$$= C_S I_e\left[\frac{a_1\mu^2}{2}+\frac{b_1\mu^3}{3}+\left(a_1\mu+\frac{b_1\mu^2}{2}\right)(M-\mu)+\frac{(a_1+b_1\mu)(t_1-\mu)^2}{2}+(a_1+b_1\mu)(t_1-\mu)(M-t_1)\right]\frac{1}{1-e^{-kT}}$$

Since $t_1 \leq M$, so there is no interest paid in that case.

$Holding\ Cost = \sum_{n=0}^{\infty} C_H(nT)\left[\int_0^{\mu}\left[a_1(\mu-t)+\frac{a}{2}(\mu^2-t^2)+\frac{b}{6}(\mu^3-t^3)\, e^{-\left(at+\frac{bt^2}{2}\right)}\right.\right.$

$(a_1+b_1\mu)\left(\frac{a}{2}(t_1^2-t^2)+\frac{b}{6}(t_1^3-t^3)\right)e^{-\left(at+\frac{bt^2}{2}\right)}$

$\left.\left.+b_1\left(\frac{1}{2}(\mu^2-t^2)+\frac{a}{3}(\mu^3-t^3)+\frac{b}{8}(\mu^4-t^4)\right)e^{-\left(at+\frac{bt^2}{2}\right)}\right]dt + \int_{\mu}^{t_1}\left[(a_1+b_1\mu)\left(\frac{a}{2}(t_1^2-\mu^2)+\frac{b}{6}(t_1^3-\mu^3)\right)e^{-\left(at+\frac{bt^2}{2}\right)}\right]dt\right]$

$$\begin{aligned} = C_H\Bigg(&\frac{1}{2}aa_1t_1^2\mu+\frac{1}{6}ba_1t_1^3\mu+\frac{1}{2}a_1\mu^2-\frac{1}{4}a^2a_1t_1^2\mu^2-\frac{1}{12}aa_1bt_1^3\mu^2+\frac{1}{6}bb_1t_1^3\mu^2+\frac{1}{3}b_1\mu^3\\ &-\frac{1}{12}aa_1bt_1^2\mu^3-\frac{1}{4}a^2b_1t_1^2\mu^3-\frac{1}{36}a_1b^2t_1^3\mu^3-\frac{1}{12}abb_1t_1^3\mu^3+\frac{1}{24}a_1b\mu^4\\ &-\frac{1}{24}ab_1\mu^4-\frac{1}{12}abb_1t_1^2\mu^4-\frac{1}{36}b^2b_1t_1^3\mu^4+\frac{1}{40}a^2b_1\mu^5+\frac{1}{40}bb_1\mu^5+\frac{1}{72}abb_1\mu^6\\ &+\frac{1}{504}b^2b_1\mu^7\\ &-\frac{1}{36}(a_1+b_1\mu)(-3at_1^2-bt_1^3+3a\mu^2+b\mu^3)\big(6t_1-3at_1^2-bt_1^3\\ &+\mu(-6+3a\mu+b\mu^2)\big)\Bigg)\frac{1}{1-e^{-kT}}\end{aligned}$$

$$Backordering\ Cost = \sum_{n=0}^{\infty} C_B(nT)(a_1+b_1\mu)\int_{t_1}^{T}\frac{(T-x)}{1+\delta(T-x)}dx$$

$= \frac{C_B(a_1+b_1\mu)}{\delta}\left[(T-t_1)-\frac{1}{\delta}log\big(1+\delta(T-t_1)\big)\right]\frac{1}{1-e^{-kT}}$

$$Lost\ Sale\ Cost = \sum_{n=0}^{\infty} C_L(nT)(a_1+b_1\mu)\int_{t_1}^{T}\left(1-\frac{1}{1+\delta(T-x)}\right)dx$$

$$= C_L(a_1+b_1\mu)\left[(T-t_1)-\frac{1}{\delta}log\big(1+\delta(T-t_1)\big)\right]\frac{1}{1-e^{-kT}}$$

$Deterioration\ Cost = \sum_{n=0}^{\infty} C_D(nT)\left[a_1\left(\mu+\frac{a\mu^2}{2}+\frac{b\mu^3}{6}\right)+b_1\left(\frac{\mu^2}{2}+\frac{a\mu^3}{3}+\frac{b\mu^4}{8}\right)+(a_1+b_1\mu)\left(\frac{a}{2}(t_1^2-\mu^2)+\frac{b}{6}(t_1^3-\mu^3)\right)-\left(a\mu+\frac{b\mu^2}{2}\right)-(a+b\mu)(t_1-\mu)\right]$

$= C_D\left[a_1\left(\mu+\frac{a\mu^2}{2}+\frac{b\mu^3}{6}\right)+b_1\left(\frac{\mu^2}{2}+\frac{a\mu^3}{3}+\frac{b\mu^4}{8}\right)+(a_1+b_1\mu)\left(\frac{a}{2}(t_1^2-\mu^2)+\frac{b}{6}(t_1^3-\mu^3)\right)-\left(a\mu+\frac{b\mu^2}{2}\right)-(a+b\mu)(t_1-\mu)\right]\frac{1}{1-e^{-kT}}$

$$TC_{2,2} = Holding\ Cost + Backorering\ Cost + Deterioration\ Cost + Lost\ Sale\ Cost + Interest\ paid - Interest\ Earned \qquad (32)$$

3.2.c The case $\mu < M < t_1 \leq T$

The interest earned during the period of positive inventory level is:

$$I_{E2,3} = \sum_{n=0}^{\infty} C_S(nT)\, I_e \left[\int_0^{\mu} \left(\int_0^{t}(a_1 + b_1 x)dx\right) dt + \int_{\mu}^{t_1}\left(\int_0^{\mu}(a_1 + b_1 x)dx\right)dt + \int_{\mu}^{t_1}\left(\int_{\mu}^{t}(a_1 + b_1\mu)dx\right)dt\right]$$

$$= C_S I_e \left[\frac{a_1\mu^2}{2} + \frac{b_1\mu^3}{3} + \left(a_1\mu + \frac{b_1\mu^2}{2}\right)(t_1 - \mu) + \frac{(a_1 + b_1\mu)(t_1 - \mu)^2}{2}\right]\frac{1}{1 - e^{-kT}}$$

The interest payable for the inventory not being sold after the due date M is

$$I_{P2,3} = \sum_{n=0}^{\infty} C_P(nT)\, I_P \int_M^{t_1} \left[(a_1 + b_1\mu)\left(\frac{a}{2}(t_1^2 - t^2) + \frac{b}{6}(t_1^3 - t^3)\right) e^{-\left(at + \frac{bt^2}{2}\right)}\right] dt$$

$$= C_P I_P \left[\frac{at_1^3}{3} - \frac{aM}{2}(t_1^2 - M^2) + \frac{bt_1^4}{8} - \frac{bM}{2}(t_1^3 - M^3)\right]\frac{1}{1-e^{-kT}}$$

$$Holding\ Cost = \sum_{n=0}^{\infty} C_H(nT)\left[\int_0^{\mu}\left[a_1(\mu - t) + \frac{a}{2}(\mu^2 - t^2) + \frac{b}{6}(\mu^3 - t^3)\, e^{-\left(at + \frac{bt^2}{2}\right)}\right.\right.$$

$$(a_1 + b_1\mu)\left(\frac{a}{2}(t_1^2 - t^2) + \frac{b}{6}(t_1^3 - t^3)\right) e^{-\left(at + \frac{bt^2}{2}\right)}$$

$$\left.\left. + b_1\left(\frac{1}{2}(\mu^2 - t^2) + \frac{a}{3}(\mu^3 - t^3) + \frac{b}{8}(\mu^4 - t^4)\right) e^{-\left(at + \frac{bt^2}{2}\right)}\right] dt + \int_{\mu}^{t_1}\left[(a_1 + b_1\mu)\left(\frac{a}{2}(t_1^2 - \mu^2) + \frac{b}{6}(t_1^3 - \mu^3)\right) e^{-\left(at + \frac{bt^2}{2}\right)}\right] dt\right]$$

$$= C_H\left(\frac{1}{2}aa_1t_1^2\mu + \frac{1}{6}ba_1t_1^3\mu + \frac{1}{2}a_1\mu^2 - \frac{1}{4}a^2a_1t_1^2\mu^2 - \frac{1}{12}aa_1bt_1^3\mu^2 + \frac{1}{6}bb_1t_1^3\mu^2 + \frac{1}{3}b_1\mu^3 - \frac{1}{12}aa_1bt_1^2\mu^3 - \frac{1}{4}a^2b_1t_1^2\mu^3 - \frac{1}{36}a_1b^2t_1^3\mu^3 - \frac{1}{12}abb_1t_1^3\mu^3 + \frac{1}{24}a_1b\mu^4 - \frac{1}{24}ab_1\mu^4 - \frac{1}{12}abb_1t_1^2\mu^4 - \frac{1}{36}b^2b_1t_1^3\mu^4 + \frac{1}{40}a^2b_1\mu^5 + \frac{1}{40}bb_1\mu^5 + \frac{1}{72}abb_1\mu^6 + \frac{1}{504}b^2b_1\mu^7 - \frac{1}{36}(a_1 + b_1\mu)(-3at_1^2 - bt_1^3 + 3a\mu^2 + b\mu^3)\left(6t_1 - 3at_1^2 - bt_1^3 + \mu(-6 + 3a\mu + b\mu^2)\right)\right)\frac{1}{1-e^{-kT}}$$

$$Backordering\ Cost = \sum_{n=0}^{\infty} C_B(nT)(a_1 + b_1\mu)\int_{t_1}^{T}\frac{(T-x)}{1+\delta(T-x)}dx$$

$$= \frac{C_B(a_1+b_1\mu)}{\delta}\left[(T - t_1) - \frac{1}{\delta}log\left(1 + \delta(T - t_1)\right)\right]\frac{1}{1-e^{-kT}}$$

$$Lost\ Sale\ Cost = \sum_{n=0}^{\infty} C_L(nT)(a_1 + b_1\mu)\int_{t_1}^{T}\left(1 - \frac{1}{1+\delta(T-x)}\right)dx$$

$$= C_L(a_1 + b_1\mu)\left[(T - t_1) - \frac{1}{\delta}log\left(1 + \delta(T - t_1)\right)\right]\frac{1}{1-e^{-kT}}$$

$$Deterioration\ Cost = \sum_{n=0}^{\infty} C_D(nT)\left[a_1\left(\mu + \frac{a\mu^2}{2} + \frac{b\mu^3}{6}\right) + b_1\left(\frac{\mu^2}{2} + \frac{a\mu^3}{3} + \frac{b\mu^4}{8}\right) + (a_1 + b_1\mu)\left(\frac{a}{2}(t_1^2 - \mu^2) + \frac{b}{6}(t_1^3 - \mu^3)\right) - \left(a\mu + \frac{b\mu^2}{2}\right) - (a + b\mu)(t_1 - \mu)\right]$$

$= C_D \left[a_1\left(\mu + \frac{a\mu^2}{2} + \frac{b\mu^3}{6}\right) + b_1\left(\frac{\mu^2}{2} + \frac{a\mu^3}{3} + \frac{b\mu^4}{8}\right) + (a_1 + b_1\mu)\left(\frac{a}{2}(t_1^2 - \mu^2) + \frac{b}{6}(t_1^3 - \mu^3)\right) - \left(a\mu + \frac{b\mu^2}{2}\right) - (a + b\mu)(t_1 - \mu)\right]\frac{1}{1-e^{-kT}}$

$$TC_{2,3} = Holding\ Cost + Backorering\ Cost + Deterioration\ Cost + Lost\ Sale\ Cost + Interest\ paid - Interest\ Earned \quad (33)$$

3.2.d Algorithm to Determine Optimal Replenishment Policy

The total cost function in different cases can be combined as follows:

$$TC_2 = \begin{cases} TC_{2,1}(t_1) & t_1 \le \mu, \\ TC_{2,2}(t_1) & \mu < t_1 \le M, \\ TC_{2,3}(t_1) & M \le t_1 \end{cases}$$

and the problem is

$\min TC_2(t_1)$

We follow the following step obtain the optimal replenishment policy:

Step-1: Find the global say $\min TC_{2,1}(t_1)$, say $t_{1,1}^*$ as follows:

Step-1a: Compute $t_{2,1}$ on solving the equation $\frac{dTC_{2,1}(t_1)}{dt_1}$=0 if $t_{1,1}<\mu$ then set $t_{1,1}^* = t_{1,1}$ and compute $TC_{2,1}(t_{1,1}^*)$, else go to step 1b.

Step-1b: Find the $\min\{TC_{2,1}(0), TC_{2,1}(\mu)\}$ and accordingly set $t_{1,1}^*$.

Step-2: Find the global say $\min TC_{2,2}(t_1)$, say $t_{1,2}^*$ as follows:

Step-2a: Compute $t_{1,2}$ on solving the equation $\frac{dTC_{2,2}(t_1)}{dt_1}$=0 if $\mu< t_{1,2} = t_{1,1}<M$ then set $t_{1,2}^* = t_{1,1}$ and compute $TC_{2,2}(t_{1,2}^*)$, else go to step 2b.

Step-2b: Find the $\min\{TC_{2,2}(M), TC_{2,2}(\mu)\}$ and accordingly set $t_{1,2}^*$.

Step-3: Find the global say $\min TC_{2,3}(t_1)$, say $t_{1,3}^*$ as follows:

Step-3a: Compute $t_{1,3}$ on solving the equation $\frac{dTC_{2,3}(t_1)}{dt_1}$=0 if $M< t_{1,3} <T$ then set $t_{1,3}^* = t_{1,3}$ and compute $TC_{2,3}(t_{1,3}^*)$, else go to step 3b.

Step-3b: Find the $\min\{TC_{2,3}(M), TC_{2,3}(T)\}$ and accordingly set $t_{1,3}^*$.

Step-4: Find the $\min\{TC_{2,1}(t_{1,1}^*), TC_{2,2}(t_{1,2}^*), TC_{2,3}(t_{1,3}^*)\}$ and accordingly set t_1 .

3.3 Derivation of Retailer's cost function for model-3 (T<M)

In this case the credit period M is longer than the planning horizon T and the following sub cases can arises.

3.3.a The case $t_1 \le \mu$

The interest earned during the period with positive inventory is:

$$I_{E3,1} = \sum_{n=0}^{\infty} C_S(nT)\, I_e \left[\int_0^{t_1}\left(\int_0^{t}(a_1 + b_1 x)dx\right)dt + \int_{t_1}^{M}\left(\int_0^{t_1}(a_1 + b_1 x)dx\right)dt + \int_T^{M}\left(\int_{t_1}^{\mu}\frac{(a_1+b_1 x)}{1+\delta(T-x)}dx\right)dt + \int_T^{M}\left(\int_{\mu}^{T}\frac{(a_1+b_1\mu)}{1+\delta(T-x)}dx\right)dt\right]$$

$$= C_S I_e\left[\frac{a_1 t_1^2}{2} + \frac{b_1 t_1^6}{6} + \left(a_1 t_1 + \frac{b_1 t_1^2}{2}\right)(M - t_1) - \frac{b_1(\mu - t_1)}{\delta} - \left(\frac{a_1}{\delta} + \frac{b_1(1+\delta T)}{\delta^2}\right) log\frac{1+\delta(T-\mu)}{1+\delta(T-t_1)}\right]\frac{1}{1-e^{-kT}}$$

$$Holding\ Cost = \sum_{n=0}^{\infty} C_H(nT)\quad \left(a_1\left(\frac{t_1^2}{2} + \frac{at_1^3}{6} + \frac{5bt_1^4}{48}\right) + b_1\left(\frac{t_1^3}{3} + \frac{at_1^4}{8}\right)\right)$$

$$= C_H\left(a_1\left(\frac{t_1^2}{2} + \frac{at_1^3}{6} + \frac{5bt_1^4}{48}\right) + b_1\left(\frac{t_1^3}{3} + \frac{at_1^4}{8}\right)\right)\frac{1}{1-e^{-kT}}$$

$$Backordering\ Cost = \sum_{n=0}^{\infty} C_B(nT)\left(\left(-\frac{b_1(\mu-t_1)^2}{2\delta} - \frac{(b_1+(a_1+b_1)\delta)}{\delta^2}\left[\left(\mu + \frac{1-\delta T}{\delta}\right) log\left(\frac{1-\delta(T-\mu)}{1-\delta(T-t_1)}\right) + (\mu - t_1)\right]\right) - \left(\frac{b_1(\mu-t_1)}{\delta} + \frac{(b_1+(a_1+b_1)\delta) log\left(\frac{1-\delta(T-\mu)}{1-\delta(T-t_1)}\right)}{\delta^2}\right)(T-\mu) - \frac{(a_1+b_1\mu)}{\delta}\left(\left(T - \frac{1-\delta T}{\delta}\right) log\left(1+\delta(T-\mu)\right) + (T-\mu)\right)\right)$$

$$= C_B\left(\left(-\frac{b_1(\mu-t_1)^2}{2\delta} - \frac{(b_1+(a_1+b_1)\delta)}{\delta^2}\left[\left(\mu + \frac{1-\delta T}{\delta}\right) log\left(\frac{1-\delta(T-\mu)}{1-\delta(T-t_1)}\right) + (\mu - t_1)\right]\right) - \left(\frac{b_1(\mu-t_1)}{\delta} + \frac{(b_1+(a_1+b_1)\delta) log\left(\frac{1-\delta(T-\mu)}{1-\delta(T-t_1)}\right)}{\delta^2}\right)(T-\mu) - \frac{(a_1+b_1\mu)}{\delta}\left(\left(T - \frac{1-\delta T}{\delta}\right) log\left(1+\delta(T-\mu)\right) + (T-\mu)\right)\right)\frac{1}{1-e^{-kT}}$$

$$Lost\ Sale\ Cost = \sum_{n=0}^{\infty} C_L(nT)\left(\left(a_1 + \frac{b_1}{\delta}\right)(\mu - t_1) + \frac{b_1}{2}(\mu^2 - t^2) + \left(a_1\delta + b_1(1+\delta T)\right) log\frac{1+\delta(T-\mu)}{1+\delta(T+t_1)} + (a_1 + b_1\mu)\left[(T-\mu) - \frac{1}{\delta} log\left(1+\delta(T-\mu)\right)\right]\right)$$

$$= C_L\left(\left(a_1 + \frac{b_1}{\delta}\right)(\mu - t_1) + \frac{b_1}{2}(\mu^2 - t^2) + \left(a_1\delta + b_1(1+\delta T)\right) log\frac{1+\delta(T-\mu)}{1+\delta(T+t_1)} + (a_1 + b_1\mu)\left[(T-\mu) - \frac{1}{\delta} log\left(1+\delta(T-\mu)\right)\right]\right)\frac{1}{1-e^{-kT}}$$

$$Deterioration\ Cost = \sum_{n=0}^{\infty} C_D(nT)\left(a_1\left(t_1 + \frac{at_1^2}{2} + \frac{bt_1^3}{6}\right) + b_1\left(\frac{t_1^2}{2} + \frac{at_1^3}{3} + \frac{bt_1^4}{8}\right) - \left(at_1 + \frac{bt_1^2}{2}\right)\right)$$

$$= \left(a_1\left(t_1 + \frac{at_1^2}{2} + \frac{bt_1^3}{6}\right) + b_1\left(\frac{t_1^2}{2} + \frac{at_1^3}{3} + \frac{bt_1^4}{8}\right) - \left(at_1 + \frac{bt_1^2}{2}\right)\right)\frac{1}{1-e^{-kT}}$$

$$TC_{3,1} = Holding\ Cost + Backorering\ Cost + Deterioration\ Cost + Lost\ Sale\ Cost - Interest\ Earned \qquad (34)$$

3.3.b The case μ<t1

The interest earned during the period with positive inventory is:

$$I_{E3,2} = \sum_{n=0}^{\infty} C_S(nT)\, I_e \left[\int_0^{\mu}\left(\int_0^{t}(a_1+b_1x)dx\right)dt + \int_{\mu}^{t_1}\left(\int_0^{\mu}(a_1+b_1x)dx\right)dt + \int_{\mu}^{t_1}\left(\int_{\mu}^{t}(a_1+b_1\mu)dx\right)dt + \int_{t_1}^{M}\left(\int_0^{\mu}(a_1+b_1x)dx\right)dt + \int_{t_1}^{M}\left(\int_{\mu}^{t_1}(a_1+b_1\mu)dx\right)dt + \int_{T}^{M}\left(\int_{t_1}^{T}\frac{(a_1+b_1\mu)}{1+\delta(T-x)}dx\right)dt\right]$$

$$= C_S I_e\left[\frac{a_1\mu^2}{2}+\frac{b_1\mu^3}{6}+\left(a_1\mu+\frac{b_1\mu^2}{2}\right)(t_1-\mu)+\frac{(a_1+b_1\mu)(t_1-\mu)^2}{2}+\left(a_1\mu+\frac{b_1\mu^2}{2}\right)(M-t_1)+(a_1+b_1\mu)(t_1-\mu)(M-t_1)+(a_1+b_1\mu)(M-T)log\big(1+\delta(T-t_1)\big)\right]\frac{1}{1-e^{-kT}}$$

In this case, no interest is paid.

$$Holding\ Cost = \sum_{n=0}^{\infty} C_H(nT)\left[\int_0^{\mu}\left[a_1(\mu-t)+\frac{a}{2}(\mu^2-t^2)+\frac{b}{6}(\mu^3-t^3)\,e^{-\left(at+\frac{bt^2}{2}\right)}\right.\right.$$

$$(a_1+b_1\mu)\left(\frac{a}{2}(t_1^2-t^2)+\frac{b}{6}(t_1^3-t^3)\right)e^{-\left(at+\frac{bt^2}{2}\right)}$$

$$\left.+b_1\left(\frac{1}{2}(\mu^2-t^2)+\frac{a}{3}(\mu^3-t^3)+\frac{b}{8}(\mu^4-t^4)\right)e^{-\left(at+\frac{bt^2}{2}\right)}\right]dt+\int_{\mu}^{t_1}\left[(a_1+b_1\mu)\left(\frac{a}{2}(t_1^2-\mu^2)+\frac{b}{6}(t_1^3-\mu^3)\right)e^{-\left(at+\frac{bt^2}{2}\right)}\right]dt\Bigg]$$

$$= C_H\left(\frac{1}{2}aa_1t_1^2\mu+\frac{1}{6}ba_1t_1^3\mu+\frac{1}{2}a_1\mu^2-\frac{1}{4}a^2a_1t_1^2\mu^2-\frac{1}{12}aa_1bt_1^3\mu^2+\frac{1}{6}bb_1t_1^3\mu^2+\frac{1}{3}b_1\mu^3-\frac{1}{12}aa_1bt_1^2\mu^3-\frac{1}{4}a^2b_1t_1^2\mu^3-\frac{1}{36}a_1b^2t_1^3\mu^3-\frac{1}{12}abb_1t_1^3\mu^3+\frac{1}{24}a_1b\mu^4-\frac{1}{24}ab_1\mu^4-\frac{1}{12}abb_1t_1^2\mu^4-\frac{1}{36}b^2b_1t_1^3\mu^4+\frac{1}{40}a^2b_1\mu^5+\frac{1}{40}bb_1\mu^5+\frac{1}{72}abb_1\mu^6+\frac{1}{504}b^2b_1\mu^7-\frac{1}{36}(a_1+b_1\mu)(-3at_1^2-bt_1^3+3a\mu^2+b\mu^3)\big(6t_1-3at_1^2-bt_1^3+\mu(-6+3a\mu+b\mu^2)\big)\right)\frac{1}{1-e^{-kT}}$$

$$Backordering\ Cost = \sum_{n=0}^{\infty} C_B(nT)(a_1+b_1\mu)\int_{t_1}^{T}\frac{(T-x)}{1+\delta(T-x)}dx$$

$$= \frac{C_B(a_1+b_1\mu)}{\delta}\left[(T-t_1)-\frac{1}{\delta}log\big(1+\delta(T-t_1)\big)\right]\frac{1}{1-e^{-kT}}$$

$$Lost\ Sale\ Cost = \sum_{n=0}^{\infty} C_L(nT)(a_1+b_1\mu)\int_{t_1}^{T}\left(1-\frac{1}{1+\delta(T-x)}\right)dx$$

$$= C_L(a_1+b_1\mu)\left[(T-t_1)-\frac{1}{\delta}log\big(1+\delta(T-t_1)\big)\right]\frac{1}{1-e^{-kT}}$$

$$Deterioration\ Cost = \sum_{n=0}^{\infty} C_D(nT)\left[a_1\left(\mu+\frac{a\mu^2}{2}+\frac{b\mu^3}{6}\right)+b_1\left(\frac{\mu^2}{2}+\frac{a\mu^3}{3}+\frac{b\mu^4}{8}\right)+(a_1+b_1\mu)\left(\frac{a}{2}(t_1^2-\mu^2)+\frac{b}{6}(t_1^3-\mu^3)\right)-\left(a\mu+\frac{b\mu^2}{2}\right)-(a+b\mu)(t_1-\mu)\right]$$

$$= C_D\left[a_1\left(\mu+\frac{a\mu^2}{2}+\frac{b\mu^3}{6}\right)+b_1\left(\frac{\mu^2}{2}+\frac{a\mu^3}{3}+\frac{b\mu^4}{8}\right)+(a_1+b_1\mu)\left(\frac{a}{2}(t_1^2-\mu^2)+\frac{b}{6}(t_1^3-\mu^3)\right)-\left(a\mu+\frac{b\mu^2}{2}\right)-(a+b\mu)(t_1-\mu)\right]\frac{1}{1-e^{-kT}}$$

$$TC_{3,2} = Holding\ Cost + Backorering\ Cost + Deterioration\ Cost + Lost\ Sale\ Cost + Interest\ paid - Interest\ Earned \quad (35)$$

3.3.c Algorithm to Determine Optimal Replenishment Policy

The total cost function in different cases can be combined as follows:

$$TC_3 = \begin{cases} TC_{3,1}(t_1) & t_1 \leq \mu, \\ TC_{3,2}(t_1) & \mu < t_1 \end{cases}$$

and the problem is

$\min TC_3(t_1)$

We follow the following step obtain the optimal replenishment policy:

Step-1: Compute $t_1^* = t_{1,1} = t_{1,2}$ on solving the equation $\frac{dTC_{3,1}(t_1)}{dt_1} = 0$

Step-2: Compare t_1^* and μ:

Step-2a: $t_1^* \leq$ μ then optimal cost can be obtained from the case 3.3.a

Step-2b: $t_1^* >$ μ then optimal cost can be obtained from the case 3.3.b

4. Numerical Analysis

In this section, we provide numerical example to illustrate the model which we develop in previous sections and also carry out sensitive analysis with respect to different parameters to illustrate how they affect our optimal solutions. Following data has been used to illustrate the model.

C_H=\$3per unit per month, C_S=\$15 per unit per month, C_D=\$5per unit, C_L=\$20 per unit, C_s=\$12per unit, C_P=\$10per unit, I_e=0.09, I_P=0.1, μ=3month, a=0.002, b=0.0001, T=5month, a_1=3, b_1=5, δ=0.5, k=0.05, M=1month

By using algorithm which we discussed in section 3.3.c and MATHEMATICA software, we obtained the optimal values as

$$t_1^* = 3.95, Q^* = 3978569, TC_1^* = \$39577889$$

Now, we perform sensitivity analysis with the same parameters as we taken in example-1.

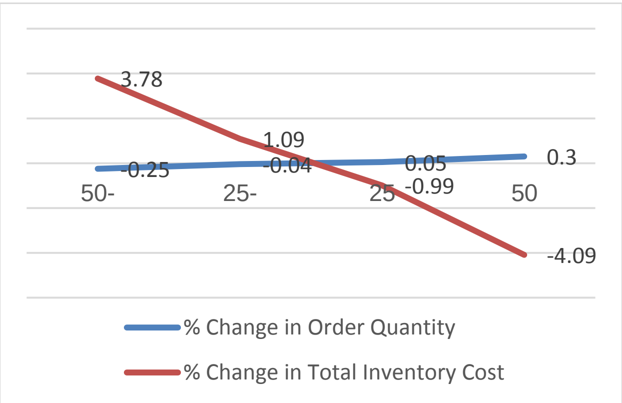

Fig. 4. Effect of I_e on the Optimal Order Quantity and on Total Inventory Cost

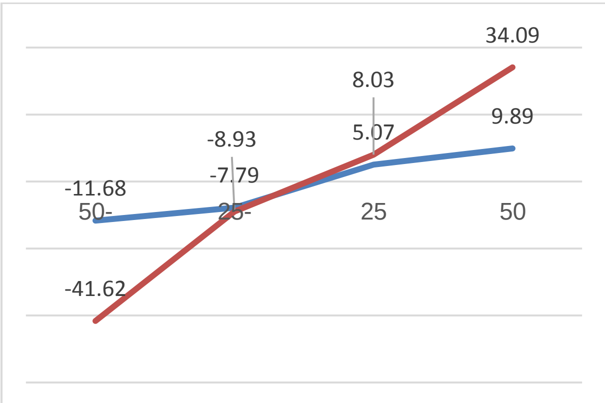

Fig. 5. Effect of 'k' on the Optimal Order Quantity and on Total Inventory Cost

Fig. 4 shows that as rate of interest earned increases, order quantity decreases whereas total inventory cost decreases. These two quantities are not so much affected by rate of interest earned. Fig. 5 indicates that a higher value of inflation rate causes higher value of order quantity and total inventory cost. This indicates that while taking the decision related to inventory control, effect of inflation cannot be ignored.

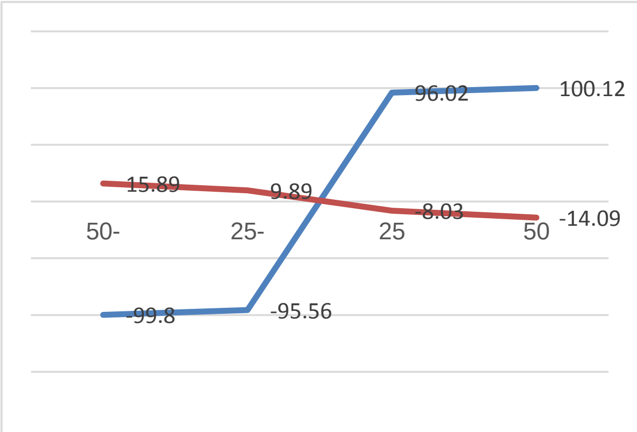

Fig. 6. Effect of 'M' on the Optimal Order Quantity and on Total Inventory Cost

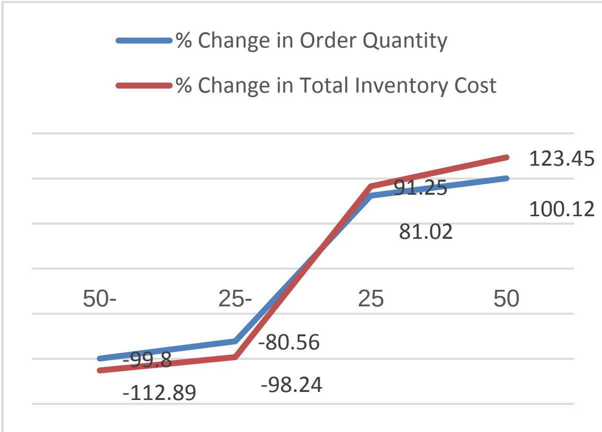

Fig. 7. Effect of 'μ' on the Optimal Order Quantity and on Total Inventory Cost

Fig. 6 shows that as the value of trade credit increases retailer's order quantity increases where inventory cost decreases total.

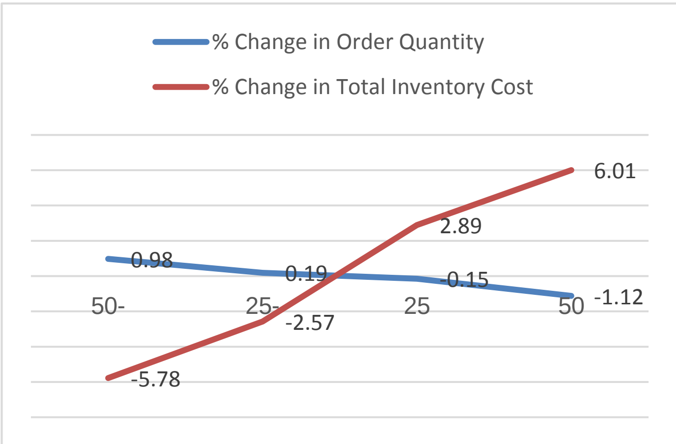

Fig. 8. Effect of 'I_P' on the Optimal Order Quantity and on Total Inventory Cost

Fig. 8 shows the effect of interest paid rate on the optimal order quantity and on the total inventory cost. Order quantity is inversely related to I_P where total inventory cost is directly related. Thus as the I_P increases total inventory cost of the system is also increases.

5. Conclusions

In present paper, a model has been presented that incorporates some realistic features that are likely to be associated with some types of inventory. These features include ramp type demand, deterioration, partial backlogging, inflation and trade credit. It is observed from market that the demand rate is increases during the growth stage and then the market grows into a stable stage such that the demand becomes a constant until the end of the inventory cycle. Such type of demand pattern is generally seen in the case of any fashionable or seasonal goods coming to market. We think that such type of demand rate is quite realistic. Deterioration of many items during storage period is a real fact. Customer may turn back during the condition of stock out state in inventory management system, to seek out better representation of such situations, we have allowed shortages in this study and partial backlogging approach has taken in to fulfill the total backordered demand at the receiving of next replenishment. The partial backlogging rate is the decreasing function of waiting time for the next replenishment.

Next, from a financial point of view, inventory represents a capital investment and must compete with other assets because of a firm's limited capital funds. Hence, the effect of inflation on the inventory system cannot be ignored. Finally, in real life situations, the supplier frequently offers a trade credit to the customers especially when the economy turns sour. We have studied optimal policies for three cases that are occurred by the combination of shortage point in model and trapezoidal interval. Since this type of demand is time-consuming so to give better demonstration of cost under consideration, we have also considered the effect of inflation during entire order cycle.

Present study can be used for the determination of the optimal replenishment policy under a specific trade credit and for supplier's selection on the basis of trade credit they are providing. Since this study deals with decaying item with trapezoidal demand and inflation, inventory managers may use this study in wide extend of decision making in inventory controlling. There are several hopeful areas for further research. In this study, there is a trapezoidal demand rate with linear pattern in every interval is considered. In the future version of the present study, this model can be extended by considering for exponentially increasing demand in every interval of trapezoidal demand function.

References

Abad, P.L. (1996). Optimal pricing and lot sizing under conditions of perishability and partial backordering. *Management Science*, *42*, 1093–1104.

Agrawal, S., & Banerjee, S. (2011). Two-warehouse inventory model with ramp type demand and partially backlogged shortages. *International Journal of Systems Science, 42*(7), 1115-1126.

Alfares, H.K. (2007). Inventory model with stock-level dependent demand rate and variable holding cost. *International Journal of Production Economics*, *108*(1-2), 259–265.

Buzacott, J.A. (1975). Economic order quantities with inflation. *Operational Research Quarterly,* 26, 553–558.

Chang, H.J., & Dye, C.Y. (1999). An EOQ model for deteriorating items with time varying demand and partial backlogging. *Journal of the Operational Research Society, 50,* 1176–1182.

Chen, J.M. (1998). An inventory model for deteriorating items with time-proportional demand and shortages under inflation and time discounting. *International Journal of Production Economics, 55*, 21–30.

Chung, K.J., & Lin, C.N. (2001). Optimal inventory replenishment models for deteriorating items taking account of time discounting. *Computers & Operations Rese*arch, *28*, 67–83.

Covert, R.P., & Philip, G.C. (1973). An EOQ model for items with Weibull distribution deterioration. *AIIE Transaction, 5*, 323–326.

Datta, T.K., & Pal, A.K. (1988). Order level inventory system with power demand pattern for items with variable rate of deterioration. *Indian Journal of Pure and Applied Mathematics, 19*(11), 1043–1053.

Datta, T.K., & Pal, A.K. (1991). Effects on inflation and time value of money on an inventory model with linear time dependent demand rate and shortages. *European Journal of Operational Research, 52*, 326–333.

Dave, U., & Patel, L.K. (1981). (T,Si) policy inventory model for deteriorating items with time proportional demand. *Journal of the Operational Research Society, 32*, 137–142.

Deng, P.S., Lin, R.H.-J., & Chu, P. (2007). A note on the inventory models for deteriorating items with ramp type demand rate. *European Journal of Operational Research, 178,* 112–120.

Donaldson, W.A. (1977). Inventory replenishment policy for a linear trend in demand: An analytic solution. *Operational Research Quarterly, 28,* 663–670.

Ferguson, M., Hayaraman, V., & Souza, G.C. (2007). Note: An application of the EOQ model with nonlinear holding cost to inventory management of perishables. *European Journal of Operational Research*, *180*(1), 485–490.

Ghare, P.M., & Schrader, G.F. (1963). A model for exponentially decaying inventories. *Journal of Industrial Engineering, 14,* 238–243.

Giri, B.C., Jalan, A.K., & Chaudhuri, K.S. (2003). Economic order quantity model with Weibull deterioration distribution, shortage and ramp-type demand. *International Journal of Systems Science, 34*(4), 237–243.

Goswami, A., & Chaudhuri, K.S. (1991). An EOQ model for deteriorating items with shortages and a linear trend in demand. *Journal of the Operation Research Society, 42*, 1105–1110.

Goyal, S.K. (1987). Economic ordering policy for deteriorating items over an infinite time horizon. *European Journal of Operational Research, 28*, 298–301.

Goyal, S.K., & Giri, B.C. (2001). Recent trends in modeling of deteriorating inventory. *European Journal of Operational Research, 134*, 1–16.

Hariga, M. (1996). Optimal EOQ models for deteriorating items with time-varying demand. *Journal of the Operational Research Society, 47,* 1228–1246.

Hariga, M.A. (1995). Effects of inflation and time value of money on an inventory model with time-varying demand rate and shortages. *European Journal of Operational Research, 81*, 512–520.

Hariga, M.A., & Ben-Daya, M. (1996). Optimal time varying lot-sizing models under inflationary conditions. *European Journal of Operational Research, 89*, 313–325.

Hill, R.M. (1995). Inventory model for increasing demand followed by level demand. *Journal of the Operational Research Society*, *46*, 1250–1259.

Hou, K.L. (2006). An inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting, *European Journal of Operational Research, 168*, 463–474.

Mandal, B., & Pal, A.K. (1998). Order level inventory system with ramp type demand rate for deteriorating items. *Journal of Interdisciplinary Mathematics, 1*, 49–66.

Mishra, V.K., & Singh, L.S. (2011). Deteriorating inventory model for time dependent demand and holding cost with partial backlogging. *International Journal of Management Science and Engineering Management, 6*(4), 267-271.

Misra, R.B. (1975). Optimum production lot size model for a system with deteriorating inventory. *International Journal of Production Research, 13*, 495–505.

Montgomery, D.C., Bazaraa, M.S., & Keswani, A.K. (1973). Inventory models with a mixture of backorders and lost sales. *Naval Research Logistics Quarterly, 20*, 255–263.

Naddor, E. (1966). Inventory Systems. *John Wiley and Sons, Inc*.

Panda, S., Saha, S., & Basu, M. (2007). An EOQ model with generalized ramp-type demand and Weibull distribution deterioration. *Asia-Pacific Journal of Operational Research, 24*(1), 93-109.

Park, K.S. (1982). Inventory model with partial backorders. *International Journal of Systems Sciences, 13*, 1313–1317.

Raafat, F., (1991). Survey of literature on continuously deteriorating inventory model. *Journal of the Operational Research Society*, 42, 27–37.

Ray, J., & Chaudhuri, K.S. (1997). An EOQ model with stock-dependent demand, shortage, inflation and time discounting. *International Journal of Production Economics, 53,* 171–180.

Resh, M., Friedman, M., & Barbosa, L.C. (1976). On a general solution of the deterministic lot size problem with time-proportional demand. *Operations Research, 24*, 718–725.

Rosenberg, D. (1979). A new analysis of a lot size model with partial backlogging. *Naval Research Logistics Quarterly, 26,* 346–353.

Roy, A. (2008). An inventory model for deteriorating items with price dependent demand and time varying holding cost. *Advanced Modeling and Optimization, 10,* 25–37.

Sachan, R.S., (1984). On (T,Si) inventory policy model for deteriorating items with time proportional demand. *Journal of the Operational Research Society, 35*(11), 1013–1019.

San Jose, L.A., Sicilia, J., & Garcia-Laguna, J. (2005). An inventory system with partial backlogging modeled according to a linear function. *Asia-Pacific Journal of Operational Research*, *22*, 189–209.

San Jose, L.A., Sicilia, J., & Garcia-Laguna, J. (2006). Analysis of an inventory system with exponential partial backordering. *International Journal of Production Economics, 100,* 76–86.

Sarker, B.R., Jamal, A.M.M., & Wang, S. (2000). Supply chain models for perishable products under inflation and permissible delay in payment. *Computers & Operations Research, 27,* 59–75.

Skouri, K., & Papachristos, S. (2002). A continuous review inventory model, with deteriorating items, time-varying demand, linear replenishment cost, partially time-varying backlogging. *Applied Mathematical Modelling*, *26*(5), 603–617.

Skouri, K., & Papachristos, S. (2003). Four inventory models for deteriorating items with time varying demand and partial backlogging: A cost comparison. *Optimal Control Application and Methods, 24*, 315–330.

Skouri, K., Konstantaras, I., Papachristos, S., & Ganas, I. (2009). Inventory models with ramp-type demand rate, partial backlogging and Weibull deterioration rate. *European Journal of Operational Research*, *192*(1), 79-92.

Skouri, K., Konstantaras, I., Papachristos, S., & Teng, J.T. (2011). Supply chain models for deteriorating products with ramp type demand rate under permissible delay in payment. *Expert Systems with Application*, *38,* 14861-14869.

Tadikamalla, P.R. (1978). An EOQ inventory model for items with Gamma distribution. *AIIE Transaction, 5*, 100–103.

Teng, J.T., Chang, H.J., Dye, & C.Y., Hung, C.H. (2002). An optimal replenishment policy for deteriorating items with time-varying demand and partial backlogging. *Operations Research Letters, 30*, 387–393.

Teng, J.-T., Chern, M.-S., & Yang, H.-L. (1997). An optimal recursive method for various inventory replenishment models with increasing demand and shortages. *Naval Research Logistics, 44,* 791–806.

Vrat, P., & Padmanabhan, G. (1990). An inventory model under inflation for stock dependent consumption rate items. *Engineering Costs and Production Economics, 19,* 379–383.

Wang, S.P. (2002). An inventory replenishment policy for deteriorating items with shortages and partial backlogging. *Computers and Operations Research, 29*, 2043–2051.

Wee, H.M., & Law, S.T., (2001). Replenishment and pricing policy for deteriorating items taking into account the time value of money. *International Journal of Production Economics, 71*, 213–220.

Weiss, H.J. (1982). Economic order quantity models with nonlinear holding costs. *European Journal of Operational Research*, *9*(1), 56–60.

Wu, J.-W., Lin, C., Tan, B., & Lee, W.-C. (1999). An EOQ inventory model with ramp type demand rate for items with Weibull deterioration. *Information and Management Science, 10*(3), 41–51.

Wu, K.-S. (2001). An EOQ inventory model for items with Weibull distribution deterioration, ramp type demand rate and partial backlogging. *Production Planning and Control, 12*(8), 787–793.

Wu, K.-S., & Ouyang, L.-Y. (2000). A replenishment policy for deteriorating items with ramp type demand rate. *Proceedings of the National Science Council, Republic of China, Part A: Physical Science and Engineering*, *24*(4), 279–286.

Yang, H.L., Teng, J.T., & Chern, M.S. (2001). Deterministic inventory lot-size models under inflation with shortages and deterioration for fluctuating demand. *Naval Research Logistics*, *48*, 144–158.