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Modeling of an inventory system with multi variate demand under volume flexibility and learning

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C H R O N I C L E

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A B S T R A C T

In this study, a volume flexible inventory system for deteriorating items with stock & time dependent demand has been developed over a finite planning horizon. Shortages are permitted with partial backorder. Uncertainties are inherent in real inventory problems due to complexities of market situation. This uncertainty can be handled by the concept of randomness. As a result, backorder rate is taken as random and follows a probability distribution. All the costs are influenced by the learning effect. The optimal number of production cycles that minimize the total cost is considered. Numerical illustrations together with sensitivity analysis are given to elucidate the model. Furthermore, the numerical results of the finite planning horizon model have been plotted graphically.

1. Introduction

Production is an organized activity of converting raw materials into useful products. This activity takes place in a wide range of manufacturing and service sectors. Production system requires the optimal utilization of natural resources like labor, money, machine, materials and time. Thus, it is essential that before starting the work of actual production, production planning is done in order to anticipate possible difficulties and to decide in advance as to how the production should be carried out in a best and economical way. In general, the Economic Production Quantity models are formulated with constant production. In real life, it may not be so. In the changing market scenario, flexibility is recognized as an important feature in manufacturing and volume flexibility is getting phenomenal importance amongst the researchers. Volume flexibility permits a manufacturing system to adjust production upwards or downwards within wide limits prior to the start of production of a lot.

The effect of learning from repetitive process cannot be ignored while developing the inventory model. Learning suggests that the performance of a person or an organization engaged in a repetitive task improves with time. This improvement is represented as a decrease in the cost of the product, but if the

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savings due to learning are significant, the effect on production time and hence inventory should also be significant. Factors contributing to this improved performance include more effective use of tools and machines, increased familiarity with operational tasks, the work environment and enhanced management efficiency.

There is almost unanimous agreement among practitioners and academicians that the learning curve is best described by a power as suggested by Wright (1936). It is worth noting that the learning curve in practice is an ‘S’-shaped curve (Jordon, 1958; Carlson, 1973). The theory in its most popular form states that as the total quantity of units produced becomes double, the cost per unit declines by some constant percentage (e.g., Yelle, 1979; Jaber, 2006). The form of the learning curve has been debate by Jaber (2006).

Zangwill (1966) discussed a production scheduling model with partial backlogging. They have taken the constant demand and backlogging rate. Hollier and Mak (1983) developed inventory replenishment policies for deteriorating items with negative exponentially demand and constant rate of deterioration. Wee (1999) considered an inventory model for deteriorating items with quantity discount, pricing and partial backordered. Wu (2001) formulated an order level inventory model for decaying items with time dependent demand and shortages were allowed with partial backlogging. Teng et al. (2002) developed an optimal replenishment policy for constant deteriorating items with time-varying demand and partial backlogging.

Aksen et al. (2003) considered the single item lot sizing inventory model with the effect of lost sales. Sana et al. (2004) considered a production inventory model for deteriorating items with trended demand. They allowed shortages with complete backlogging and production rate was taken as constant. Ouyang et al. (2005) presented an order level inventory model for deteriorating items with exponentially decreasing demand and partially backlogged. The backlogging rate was taken as time dependent in their model. Ouyang et al. (2006) developed an optimal ordering policy for deteriorating items with partial backlogging. Uthayakumar and Parvathi (2006) discussed a deterministic inventory model for deteriorating items with stock and time dependent demand and partially backlogged. Dye (2007) proposed joint pricing and ordering policy for deteriorating items with constant partial backlogging rate.

Singh and Singh (2008) developed an optimal ordering policy for decaying item with stock dependent demand. Singh et al. (2008) developed an inventory model for deteriorating items having stock dependent demand. They were allowed shortages with partial backlogging in their study. Arya et al. (2009) discussed an inventory system for perishable items with stock dependent demand and time dependent partial backlogging. In their model, constant holding cost has been taken. Singh et al. (2010) developed a volume flexible inventory model for defective Items with multi-variate demand and partial backlogging. Singh et al. (2012) studied an economic production lot-size (EPLS) model with rework and flexibility under allowable shortages. Singh et al. (2013) developed a supply chain inventory model for shortages with variable demand rate. Kumar et al. (2013) presented two-warehouse inventory model with K-release rule and learning effect. Singhal and Singh (2013) developed volume flexible multi items inventory system with imprecise environment.

In this model, volume flexible system for decaying items with stock and time dependent demand over a finite planning horizon has been developed. Our study includes the situation of shortages with partial backlogging, where backlogging rate depends upon stochastic environment. The combination of more than one parameter grants more genuineness to the formulation of the model and makes it more close to reality. We have discussed the learning effect on all cost. Numerical examples are presented to illustrate the theoretical results. The sensitivity of the optimal solutions with respect to system parameters is examined. Graphical analysis also has been discussed. The proposed model has a broad area of applicability.

2. Assumptions and Notations

The proposed inventory model is developed under the following assumptions and notations:

Assumptions

The following assumptions are as follows:

1. Demand rate is taken as both time and stock dependent.
2. The unit production cost is a function of production rate.
3. The rate of production is considered to be decision variable.
4. Shortages are allowed with partial backlogging.
5. Backlogging rate is random variable and follows a beta distribution of first kind.
6. Deterioration rate is taken as constant.
7. All the costs are taken with the effect of learning.
8. Time horizon is finite.
9. The finite time horizon is divided into a finite number of replenishment cycles, each of equal duration.

Notations

The following notations are used in our study:

$a+bt+cI(t)$	Time and stock dependent demand, a, b>0, 0<c<1
P	Production rate
θ	Deterioration rate, $0<\theta<1$
C_d	Deterioration cost per unit per unit time
H	Finite time horizon
$\eta_0(P)$	Unit production cost of an item and $\eta_0(P)=N+\frac{G}{P}+\alpha P,$ where N is material cost, α is tool or die cost and G is energy and labor cost
δ	The fraction of the demand during the stock-out period that will be backordered and a random variable, $0\le\delta\le1$. $M_\delta=\int_0^1\delta g(\delta)d\delta$:
$g(\delta)$	The probability density function (p.d.f.) of δ and follows Beta distribution of first kind $g(\delta)=\begin{cases}\frac{1}{B(\mu,\nu)}\delta^{\mu-1}(1-\delta)^{\nu-1}, \mu,\nu>0, 0<\delta<1\\ 0, otherwise\end{cases}$
n	Number of cycles in [0,H]

The learning effect is very much important; therefore in this model we studied the effect of learning. The earliest learning curve representation is a geometric progression that expresses the decreasing cost required to accomplish any repetitive operation. Several learning curve models were fitted to the collected data and the S-shaped logistic learning curve was found to fit well and it is of the form

$$R(n)=m+\frac{g}{n^\psi},$$

where m and $g>0$ are the model parameters, n is the cumulative number of shipments, and $R(n)$ is the percentage defective per shipment n.

From the Fig. 1: the first phase (incipient) is the phase during which the worker is getting acquainted with the set-up, the tooling, instruction, blueprints, the workplace arrangement and the conditions of the process. In this phase improvement is slow. The second phase (learning) is where most of the improvement, e.g., reduction in errors, changes in the distance moved takes place. The third and last phase (maturity) represents the learning of the curve.

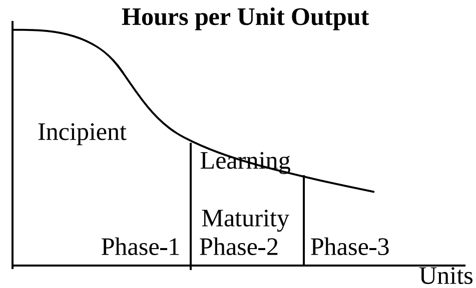

Fig. 1. The three phases of learning curve

The inventory carrying cost, backorder cost, lost sale cost, set up cost also follows the learning effect and function of these cost are $C_1 + \frac{C_{10}}{n^{\psi}}, C_s + \frac{C_{s_0}}{n^{\psi}}, C_{LS} + \frac{C_{LS_0}}{n^{\psi}}, C_{SP} + \frac{C_{SP0}}{n^{\psi}}$.

3. Model Formulation

In this model, volume flexible inventory system with the effect of learning has been developed. This inventory system considered four phases in each cycle. In ith cycle (i=1, 2, …., n), the initial inventory is zero and production starts at the very beginning of the cycle. As production continues, inventory begins to pile up continuously after meeting demand and deterioration. At time $t_i^{'}$, production stops. The accumulated inventory is just sufficient enough to account for demand and deterioration over the interval $[t_i^{'}, t_i]$. After time t_i, shortage starts with partial backlogging and reach to maximum shortage level at time S_i. Production restarts after S_i to fulfill the backlog demand and the cycle ends with zero inventory.

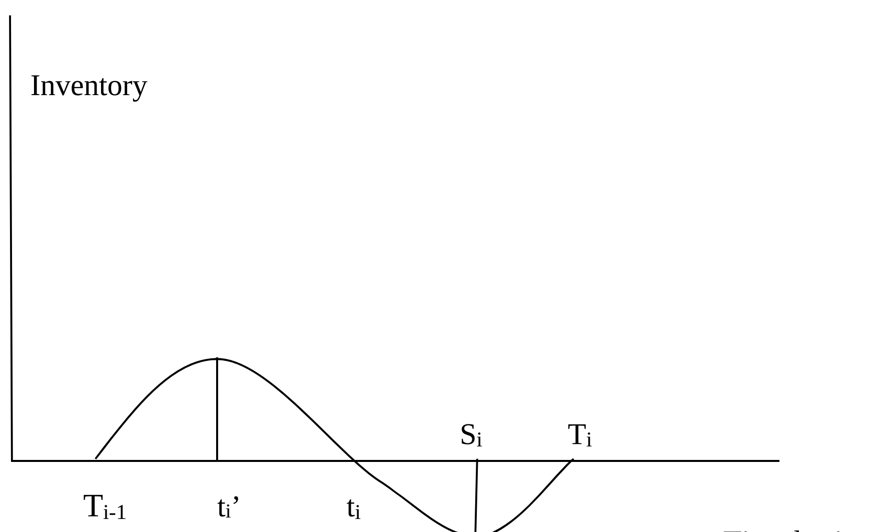

Fig. 2. Time versus inventory of ith cycle

The inventory level I(t) of the system at any time $t \in [T_{i-1}, T_i]$ is described by the following equations:

$$I^{'}(t)+\theta I(t)=R(n)P-(a+bt+cI(t)) \qquad T_{i-1}\le t\le t_i^{'} \tag{1}$$

$$I^{'}(t)+\theta I(t)=-(a+bt+cI(t)) \qquad t_i^{'}\le t\le t_i \tag{2}$$

$$I^{'}(t)=-\delta(a+bt) \qquad t_i\le t\le S_i \tag{3}$$

$$I^{'}(t)=R(n)P-(a+bt) \qquad S_i\le t\le T_i \tag{4}$$

With boundary conditions

$$I(T_{i-1})=0, I(t_i)=0, I(T_i)=0 \tag{5}$$

Solution of Eq. (1) is given by:

$$I(t)=\frac{(R(n)P-a)}{(\theta+c)}[1-e^{(\theta+c)(T_{i-1}-t)}]-\frac{b}{(\theta+c)^2}[\{(\theta+c)t-1\}-\{(\theta+c)T_{i-1}-1\}e^{(\theta+c)(T_{i-1}-t)}] \qquad T_{i-1}\le t\le t_i^{'} \tag{6}$$

Solution of Eq. (2) is given by:

$$I(t)e^{(\theta+c)}-I(t_i^{'})e^{(\theta+c)t_i^{'}}=\frac{a}{(\theta+c)}[e^{(\theta+c)t_i^{'}}-e^{(\theta+c)t}]-\frac{b}{(\theta+c)^2}[\{(\theta+c)t_i^{'}-1\}e^{(\theta+c)t_i^{'}}+\{(\theta+c)t-1\}e^{(\theta+c)t}] \tag{7}$$

From Eq. (6) substitute the value of $I(t_i^{'})$ in Eq. (7), this relation becomes:

$$I(t)=[\frac{R(n)P}{(\theta+c)}(e^{(\theta+c)t_i^{'}}-e^{(\theta+c)T_{i-1}})+\frac{a}{(\theta+c)}e^{(\theta+c)T_{i-1}}+\frac{b}{(\theta+c)^2}((\theta+c)T_{i-1}-1)e^{(\theta+c)T_{i-1}}]e^{(\theta+c)t}$$
$$-[\frac{a}{(\theta+c)}+\frac{b((\theta+c)t-1)}{(\theta+c)^2}] \qquad t_i^{'}\le t\le t_i \tag{8}$$

Using the conditions $I(t_i)=0$ in Eq. (8), one can have

$$K_i=\frac{n}{(\theta+c)rH}\ln[1+\frac{1}{R(n)P}\{a+\frac{bH}{n}(r+i-1-\frac{n}{(\theta+c)H})\}e^{\frac{(\theta+c)rH}{n}}-\frac{a}{R(n)P}$$
$$-\frac{bH}{nR(n)P}(i-1-\frac{n}{(\theta+c)H})] \tag{9}$$

Solution of Eq. (3) is given by:

$$I(t)=-\delta[(t-t_i)\{a+\frac{b}{2}(t+t_i)\}] \qquad t_i\le t\le S_i \tag{10}$$

Solution of Eq. (4) is given by:

$$I(t)-I(S_i)=(R(n)P-a)(t-S_i)-\frac{b}{2}(t^2-S_i^{\,2}) \qquad S_i\le t\le T_i \tag{11}$$

Substituting the value of $I(S_i)$ from Eq. (10), this relation becomes:

$$I(t)=(R(n)P-a)(t-S_i)-\frac{b}{2}(t^2-S_i^{\,2})-a\delta(S_i-t_i)-\frac{b\delta}{2}(S_i^{\,2}-t_i^{\,2}) \quad S_i\le t\le T_i \tag{12}$$

Using the conditions $I(T_i)=0$ in Eq. (12), one can have

$$d_i = 1 - \frac{a}{R(n)P} - \frac{bH(2i-1+r)}{2nR(n)P} \tag{13}$$

Using all the values of $t_i^{'}, t_i, S_i,\ T_{i-1}$ from Appendix, the following costs are as follows:

Holding cost occurs during the interval $[T_{i-1}, t_i]$ is given by:

$$\begin{aligned} C_{hi} &= \int_{T_{i-1}}^{t_i} (C_1 + \frac{C_{10}}{n^{\psi}}) I(t)dt \\ &= (C_1 + \frac{C_{10}}{n^{\psi}})[\frac{b(i-1)rH^2K_i}{n^2(\theta+c)} - \frac{brH^2K_i(rK_i+2i-2)}{2n^2(\theta+c)} + \frac{b(i-1)rH^2(1-K_i)}{n^2} - \frac{brH(1-K_i)}{n(\theta+c)} \\ &\quad - \frac{brH^2(1-K_i)(r+rK_i+2i-2)}{2n^2(\theta+c)} + \frac{brH(1-K_i)}{n(\theta+c)^2}] \end{aligned} \tag{14}$$

Deterioration cost occurs during the interval $[T_{i-1}, t_i]$ is given by:

$$\begin{aligned} C_{Di} &= \int_{T_{i-1}}^{t_i} \theta C_d I(t)dt \\ &= \theta C_d[\frac{b(i-1)rH^2K_i}{n^2(\theta+c)} - \frac{brH^2K_i(rK_i+2i-2)}{2n^2(\theta+c)} + \frac{b(i-1)rH^2(1-K_i)}{n^2} - \frac{brH(1-K_i)}{n(\theta+c)} \\ &\quad - \frac{brH^2(1-K_i)(r+rK_i+2i-2)}{2n^2(\theta+c)} + \frac{brH(1-K_i)}{n(\theta+c)^2}] \end{aligned} \tag{15}$$

Shortage cost occurs during the interval $[t_i, T_i]$ is given by:

$$\begin{aligned} C_{Si} &= \int_{t_i}^{T_i} (C_s + \frac{C_{s_0}}{n^{\psi}})[-I(t)]dt \\ &= (C_s + \frac{C_{s_0}}{n^{\psi}})[\delta[\frac{aH^2d_i^2(1-r)^2}{2n^2} + \frac{bH^3\{i-(1-d_i)(1-r)\}^3}{6n^3} - \frac{bH^3\{i-(1-d_i)(1-r)\}(i-1+r)^2}{2n^3} \\ &\quad + \frac{bH^3(i-1+r)^3}{3n^3}] + (P-a)[\frac{iH^2\{i-(1-d_i)(1-r)\}}{n^2} - \frac{H^2\{i-(1-d_i)(1-r)\}^2}{2n^2} - \frac{i^2H^2}{2n^2}] \\ &\quad + \frac{b}{2}[\frac{i^3H^3}{3n^3} + \frac{2H^3\{i-(1-d_i)(1-r)\}^3}{3n^3} - \frac{iH^3\{i-(1-d_i)(1-r)\}^2}{n^3}] \\ &\quad + a\delta[\frac{iH^2\{i-(1-d_i)(1-r)\}}{n^2} - \frac{H^2\{i-(1-d_i)(1-r)\}^2}{n^2} - \frac{iH^2(i-1+r)}{n^2} \\ &\quad + \frac{H^2\{i-(1-d_i)(1-r)\}(i-1+r)}{n^2}] + \frac{b\delta}{2}[\frac{iH^3\{i-(1-d_i)(1-r)\}}{n^3} - \frac{iH^3(i-1+r)^2}{n^3} \\ &\quad - \frac{H^3\{i-(1-d_i)(1-r)\}^3}{n^3} + \frac{H^3\{i-(1-d_i)(1-r)\}(i-1+r)^2}{n^3}]] \end{aligned} \tag{16}$$

Lost sale cost occurs during the interval $[t_i, S_i]$ is given by:

$$\begin{aligned} C_{LSi} &= \int_{t_i}^{S_i} (C_{LS} + \frac{C_{LS_0}}{n^{\psi}})[(1-\delta)(a+bt)]dt \\ &= (C_{LS} + \frac{C_{LS_0}}{n^{\psi}})(1-\delta)[\frac{aHd_i(1-r)}{n} + \frac{bH^2d_i(1-r)(2i-2+d_i+2r-rd_i)}{2n^2}] \end{aligned} \tag{17}$$

Production cost occurs during the interval $[T_{i-1}, t_i^{'}]$ & $[S_i, T_i]$ is given by:

$$\begin{aligned} P_{Ci} &= (N + \frac{G}{R(n)P} + \alpha R(n)P)[\int_{T_{i-1}}^{t_i^{'}} R(n)Pdt + \int_{S_i}^{T_i} R(n)Pdt] \\ &= (NR(n)P + G + \alpha R(n)P^2)\frac{H}{n}[\frac{r}{n} + (1-d_i)(1-r)] \end{aligned} \tag{18}$$

Set up cost occurs during the interval $[T_{i-1}, T_i]$ is given by:

$$A_{Si} = [\int_{T_{i-1}}^{T_i} (C_{SP} + \frac{C_{SP0}}{n^{\psi}}) dt] = (C_{SP} + \frac{C_{SP0}}{n^{\psi}}) \frac{H}{n} \quad (19)$$

The total cost is the sum of holding cost, deterioration cost, shortage cost, lost sale cost, production cost and set up cost. The average total cost during the time horizon (0, H) using the equations from Eq. (14) to Eq. (19) is given by:

$$Avc(r) = \frac{1}{H} \sum_{i=1}^{n} [C_{hi} + C_{Di} + C_{Si} + C_{LSi} + P_{Ci} + A_{Si}] \quad (20)$$

Since the backorder rate δ is a random variable with p.d.f. $g(\delta)$, the expected backorder rate is $M_{\delta} = \int_0^1 \delta g(\delta) d\delta$. Thus, the expected average total cost during the time horizon (0,H) is given by:

$$EAP(r) = E[Avc(r)] \quad (21)$$

This is the objective function which needs to be minimized. It is a function of service level ‘r’. For optimizing the expected average total cost

$$\frac{\partial \text{EAP(r)}}{\partial \text{r}} = 0 \quad (22)$$

The Eq. (22) is solved for different values of service level ‘r’ and the equation (21) is solved to find the values of total cost. These both equations are solved using the software for a fixed planning horizon H.

4. Numerical Illustrations

The numerical examples are given below to illustrate the above solution procedure. On the basis of previous studies, let us considered the following data in proper units:

C_S =5.5, C_{LS} =8.5, θ=0.05, a=250, b=6, c=0.07, H=10, n=1, G=3500, N=10, α =0.01, M_{δ} =0.75, d_1 =0.08, d_2 =0.02, K_1 =0.08, K_2 =0.01, C_1 =5.5, γ =0.05, P=300, ϕ =0.005, C_{10} =7.5, C_{S0} =5.5, C_{LS0} =6.7, m=16, g=4, C_{SP} =500, C_{SP0} =200

Table 1
Variation in Cycles

No. of Cycles ‘n’	Service level ‘r’	Total cost	Inventory cost	Production cost
1	0.995913	140404	-	1305730
2	0.895789	38980.7	15026.5	356888
3	0.795631	20306.8	14521.6	199768
4	0.695395	13818.1	11356.6	151045
5	0.59509	10837.8	8527.48	131362
6	0.494728	9231.89	6254.47	122223
7	0.394319	8271.52	4441.67	117647
8	0.293867	7653.47	2978.73	115283
9	0.193380	7233.39	1779.72	114079
10	0.0928603	6935.57	782.042	113515

The optimum values are: Service level ‘r’=0.995913, Total cost=140404 and Production cost=1305730. The graphical representation of the optimum values for n=1 has been shown by Fig. 3. The graphical representation of the service level ‘r’ and no. of cycles is shown by Fig. 4 and the graphical representation of the total cost and no. of cycles is shown by Fig. 5.

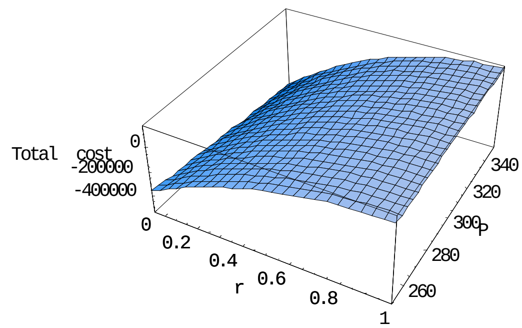

Fig. 3. Graphical representation of the system

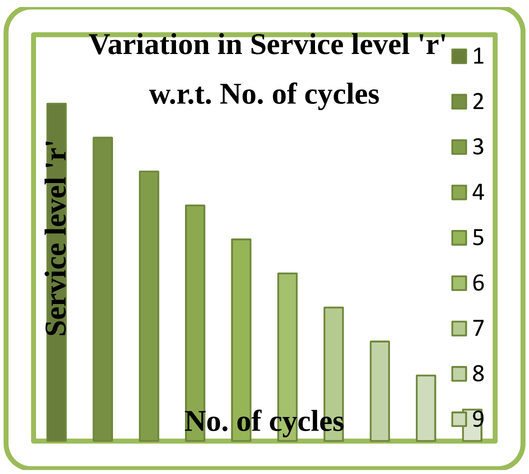

Fig. 4. Graphical representation of Service level 'r' w.r.t. No. of cycles

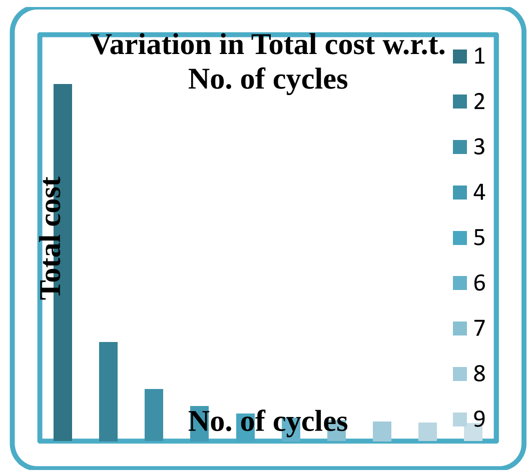

Fig. 5. Graphical representation of Total cost w.r.t. No. of cycles

5. Sensitivity Analysis

In this section, the effects of changes in the system parameters a, b, c, H, P and M_δ on the values of r, total cost and the production cost has been studied. The results are presented in Table 2.

Table 2

Effect of percentage change in system parameters of the inventory model

Parameter	% Change	-15%	-10%	-5%	5%	10%	15%
a	Service level 'r'	+0.0074	+0.0050	+0.0025	-0.0025	-0.0050	-0.0075
	Total cost	-1.3162	-0.8775	-0.4387	+0.4387	+0.8775	+1.3162
	Production cost	0.0	0.0	0.0	0.0	0.0	0.0
b	Service level 'r'	+0.1434	+0.0956	+0.0478	-0.0478	-0.0956	-0.1435
	Total cost	+0.4067	+0.2714	+0.1353	-0.1353	-0.2706	-0.4060
	Production cost	0.0	0.0	0.0	0.0	0.0	0.0
c	Service level 'r'	+0.0090	+0.0039	+0.0010	-0.0006	-0.0025	-0.0055
	Total cost	+0.5149	+0.3169	+0.1467	-0.1261	-0.2343	-0.3269
	Production cost	0.0	0.0	0.0	0.0	0.0	0.0
H	Service level 'r'	+0.2535	+0.1603	+0.0766	-0.0696	-0.1334	-0.1923
	Total cost	+0.1311	+0.0978	+0.0456	-0.0477	-0.0976	-0.1489
	Production cost	-14.9893	-9.9929	-4.9964	+4.9964	+9.9929	+14.981
P	Service level 'r'	-0.1758	-0.1105	-0.0522	+0.0471	+0.0899	+0.1288
	Total cost	-13.5423	-9.0411	-4.5269	+4.5390	+9.0909	+13.654
	Production cost	-11.4054	-9.7172	-4.8665	+4.8793	+9.7723	+14.679
M_δ	Service level 'r'	+0.0146	+0.0097	+0.0048	-0.0048	-0.0096	-0.0145
	Total cost	-1.2457	-0.8305	-0.4152	+0.4152	+0.8297	+1.2450
	Production cost	0.0	0.0	0.0	0.0	0.0	0.0

6. Observations

1 Service level 'r' is very slightly sensitive to change the parameters of demand ('a', 'b' and 'c').
2 The total cost is somewhat sensitive to change the demand parameters ('a', 'b' and 'c').
3 The total cost is decreases with the increases of the values of demand parameters ('b' and 'c'). The change in values of demand parameters (a, b and c) don't have any effect on the production cost.
4 The service level 'r' and total cost are faintly sensitive to change the parameter of planning horizon.
5 Production cost is highly sensitive to change the parameter of planning horizon.
6 The total cost and production cost are extremely sensitive and 'r' is slightly sensitive to change the parameter of production rate.
7 Service level 'r' and total cost are little sensitive to change the backlogging parameter. The values of backlogging rate don't give the effect on the production cost.

All the variations cited above have been shown graphically in Figs. (6-11).

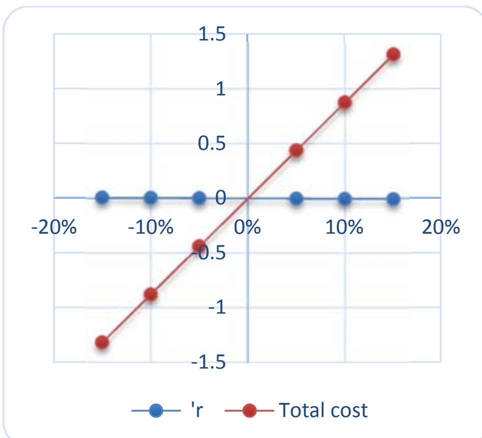

Fig. 6. Graphical representation of sensitivity of the 'r' and total cost w.r.t. 'a'

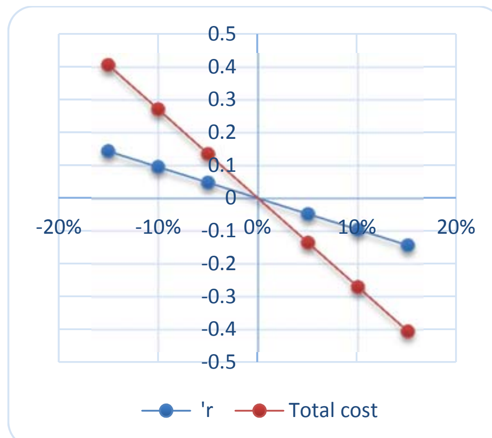

Fig. 7. Graphical representation of sensitivity of the 'r' and total cost w.r.t. 'b'

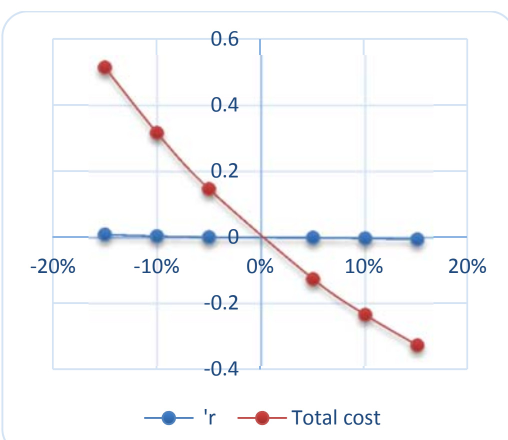

Fig. 8. Graphical representation of sensitivity of the 'r' and total cost w.r.t. 'c'

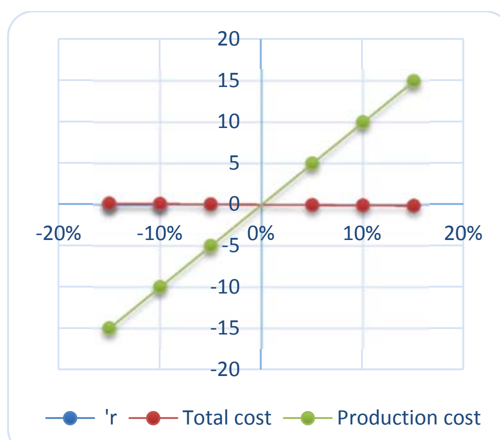

Fig. 9. Graphical representation of sensitivity of the 'r', total cost and production cost w.r.t. 'H'

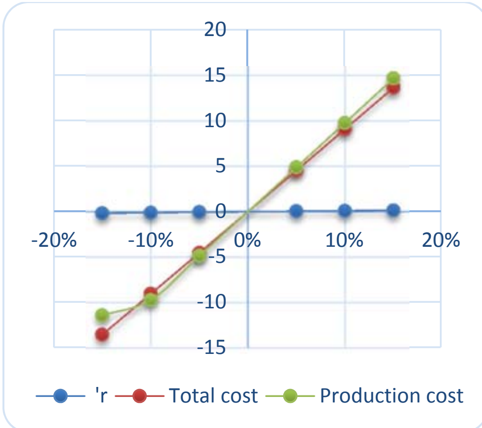

Fig. 10. Graphical representation of sensitivity of the ‘r’, total cost and production cost w.r.t. ‘P’

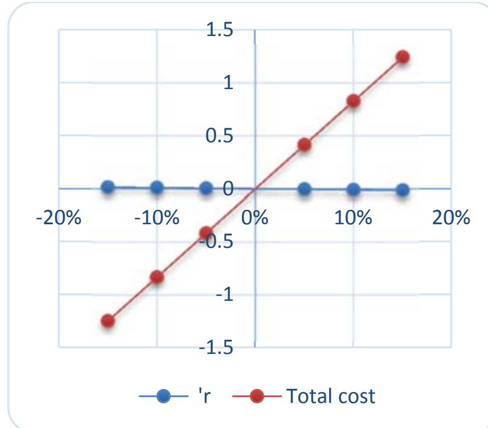

Fig. 11. Graphical representation of sensitivity of the ‘r’ and total cost w.r.t. ‘M_δ’

7. Conclusion

In this paper, an inventory model for deteriorating items with volume flexibility and stock & time dependent demand has been developed. Large quantities of goods displayed in market according to seasons lure the customer to buy more. In fact a customer’s demand is influenced by more than one parameter. It is very realistic to consider the practical demand rate, which depends upon both time and stock. It’s a well known fact that there are very much uncertainties in real life business scenarios with respect to lost sales. Therefore, it is worthwhile to consider the backorder rate stochastic in nature. The proposed model is very useful in the present market situation as almost every item having a demand rate varying according to time and stock available can be identified. This whole setup is very practical and can be applied to many commodities in today’s market. All these facts together make this study very unique and matter-of-fact.

Appendix

Here $t_i^{'}, t_i, S_i$ and T_i are connected by the following relations:

$$t_i^{'} = K_i t_i + (1-K_i)T_{i-1}, \quad t_i = rT_i + (1-r)T_{i-1}$$

$$S_i = d_i T_i + (1-d_i)t_i, \quad T_i = \frac{iH}{n}$$

and

$$t_i^{'} = \frac{H(rK_i + i - 1)}{n}, \quad t_i = \frac{H(r+i-1)}{n}$$

$$S_i = \frac{H}{n}\{i-(1-d_i)(1-r)\}, \quad T_{i-1} - t_i = -\frac{rH}{n}$$

$$S_i - t_i = \frac{Hd_i(1-r)}{n}, \quad S_i + t_i = \frac{H}{n}[2i - 2 + d_i + 2r - rd_i]$$

$0 < r < 1, 0 < K_i < 1, 0 < d_i < 1$ and i=1,2…n

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