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Replenishment-shipment decision for a multiproduct producer-client coordinated FPR model with postponement, rework and subcontracting

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ABSTRACT

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This research constructs a mathematical scheme to explore replenishment-shipment decisions for a multiproduct producer-client coordinated finite production rate (FPR) model with the postponement, rework, and subcontracting plan. The considered multiple goods have a common component, and a batch FPR fabrication with postponement is planned to meet the annual multiproduct requirements. The first fabricating phase makes only the standard components needed for a batch and subcontracts a proportion of them (with additional cost) to expedite the process. In contrast, the second fabricating phase produces the finished multiple merchandise in sequence. The in-house rework processes with extra expense help retain the desirable quality. Each merchandise's finished batch is transported to the clients in equal-sized numerous shipments. This study derives the optimal batch cycle length and transporting frequency by minimizing the overall fabricating-shipment expenses (including clients' holding costs). This work offers a numerical example demonstrating various crucial system features influenced by the factors of subcontracting, postponement, rework, and transportation policies to facilitate managerial decision making in industries.

1. Introduction

Satisfying client expectations of variety and quality merchandise in a shorter order-response time becomes today's intra-supply-chain managers' everyday mission. With this incentive, the present work studies the replenishing-shipment decision for a multiproduct producer-client coordinated FPR model with the postponement, rework and subcontracting. The postponement strategy may generate potential benefits/savings in fabricating process/time and expense. Boone et al. (2007) examined the influence of implementing postponement on the operations of supply chains to identify potential gaps/directions for future study. By surveying/reviewing postponement-related works from 1999 to 2006, the study identified previous, current, and future challenges/ opportunities derived from implementing postponement strategies and suggested potential future study directions. Gualandris and Kalchschmidt (2015) examined the collective interaction among supply interruptions (unavailable/failed supplies), supply-chain risk probability, and postponement/modularity implementation. The study aims to learn the best timing and the correct extent to implement the postponement to minimize the supply-chain disruption risk. The researcher built a conceptual scheme and verified it by surveying 54 Italian production firms. As a result, they revealed that the uncertain technology used and difficult supply market increase higher risk and negatively influence the supply-chain interruptions (unavailable/failed supplies). Additionally, they showed evidence that postponement/modularity can ease the

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impact of uncertain technology on supply-chain disruption. Jafari et al. (2022) explored the postponement's impact and the extent of uncertain demand and logistics integration's influence on logistics flexibility and related retailers' performance. Using a survey of 261 Sweden retailers, this study finds that logistics flexibility mediates the relationship between postponement and performance. The research showed the relationships between (1) postponement and logistics-flexibility and (2) logistics-flexibility and retailers' performance are deepened when uncertainty is above medium level. The postponement may not be beneficial in increasing logistics flexibility and subsequent performance when demand uncertainty is significantly high or low. Furthermore, the retailers cannot gain the expected performance from postponement's flexibility when they select logistics integration. Additional works (Galizia et al., 2020; Bolaños and Barbalho, 2021; Ohmori and Yoshimoto, 2021; Ramón-Lumbierres et al., 2021; Kiani et al., 2022; Lin et al., 2022) studied various postponement strategies' influence on supply chains and multiproduct fabricating operations and management.

A multiproduct postponement fabricating system seeks to make all standard (common) components in the first phase. Subcontracting a proportion of them will expedite the process. Choi (2007) explored whether to outsource or not in the global markets with uncertainty. The manufacturing activity can be divided into a few separate processes to be subcontracted to different countries with different outsourcing prices. The researchers explored the outsourcing factors and approaches influencing the competitive/cost-minimized integrated production activity. Chukwunweike et al. (2015) studied the outsourcing production strategy as the key continued existence of small- and medium-sized firms in Nigeria. They focused on crucial business functions, especially gaining professional resources and firming the direct integration of economic factors with outsourcing partners to ensure the survival of Nigeria's small- and medium-sized firms. Blom and Niemann (2022) were concerned that supply-chain disruptions may cause different long-term influences, primarily the firm's reputation relating to the company's image and values for the stakeholders. The researchers looked into managing reputation risk in recovering the supply-chain disruption existing within the external logistics suppliers and customers in the South African firms. The study used a qualitative scheme to examine the interview data collected from 5 different logistics triads to find that reputational risk positively impacts the recovery process of supply-chain disruptions. The researcher further suggested using a control center and involving the primary account manager to enhance the firm's reputation management. Lastly, the study provided awareness and valuable insight into managing reputation risk while recovering the supply-chain disruption. Additional works (Harap, 2010; Kavčič et al., 2015; Chiu et al., 2021; Thongrawd et al., 2020; Cornelius et al., 2021; Akhtar, 2022; Chiu et al., 2022a; de Carvalho et al., 2022; Lahiri et al., 2022; Suharmono et al., 2022) studied various subcontracting strategies' influence on fabricating systems and supply chains' operations and management.

Manufacturing managers often use the rework process to help retain the desirable in-house items' quality and transport the finished batch of merchandise to meet clients' requirements in equal-sized numerous shipments. Garcĺa and de las Morenas (2012) stated that a growing trend of product identification technology can be applied to an enhanced manufacturing-distribution integration in supply chains. The optimization modeling comprising structural and operational features can help this integration for factory configuration. The researchers focused on potential problems and techniques in implementing optimization modeling in manufacturing, storage, and shipping planning. Roy et al. (2014) considered a stochastic demand economic manufacturing lot-size system with exponentially distributed equipment stability times. The random shifting from an in-control status to an out-of-control may cause defective goods, and a rework process repairs the faulty items to ensure the product quality. Also, the study allows a partial/complete backlogging of the permitted shortages due to the stochastic requirements. Then, the researchers employed an optimization process using calculus to maximize the system's expected annual profits comprising selling price, fabricating, stock holding, lost sale, rework, and backlogging expenses, to derive the optimal fabricating rate and batch size. The study justifies its model with a few numerical and graphical demonstrations. Yassine (2020) studied a manufacturing system with various random quality (with known probability distribution) raw materials/components used and their delivery emissions tax. The study developed a math model comprising a set of components' quality-percentage-related random variables to derive the closed-form optimal (approximating) solution that minimizes the expected overall production-inventory expenses. Specifically, the study explored uniformly-distributed quality-related variables and illustrated them with a numerical example. The research provided a simulation algorithm to locate the optimal batch size for non-uniform distribution quality-related variables. Taheri-Tolgari and Mirzazadeh (2021) examined a multiproduct fabrication quantity model featuring single source, scraps, backlogging, screening errors, and repair failure. The screening task of the proposed fabricating system identifies goods made as perfect and imperfect. A rework process repairs some repairable faulty goods; however, the screening and the rework processes are not error-free. The studied system allows backlogging unsatisfied demand, and the objective of their study is to decide the optimal/cost-minimized cycle time and backlogging amount simultaneously. Lastly, the study offers a numerical demonstration and sensitivity analyses for the research results. Additional works (Nachiappan et al., 2008; Singh and Singh, 2013; Chiu et al., 2021; Di Nardo et al., 2021; Uzen et al., 2021; Aljumah et al., 2022; Chiu et al., 2022b; Comert et al., 2022; Lin et al., 2022b; Riaño et al., 2022; Seçkin and Seçkin, 2022) studied various random faulty goods produced and quality assurance actions, and shipping policies influence fabricating systems and supply chains' operations and management. Few earlier works examined the collective/individual impact of scraps, subcontracting strategy, rework, and postponement on the multiproduct replenishing-transportation decisions; we intend to fill the gap.

2. Materials and Methods

This study explores the replenishment-shipment decision for a multiproduct producer-client coordinated finite production rate (FPR) model with the postponement, subcontracting, and rework. Appendix-A gives definition of our model's notation.

2.1. Assumption and modeling

Fig. 1 exhibits the inventory status in our proposed two-stage multiproduct producer-client coordinated FPR model with the postponement, rework and subcontracting compared with the same problem without subcontracting. The assumptions of the proposed study include the following: (1) It considers an L different multiproduct producer-client coordinated FPR system with the constant demand rate λ_i (for $i = 1, 2, \ldots, L$); (2) The standard component exists in multiple merchandise, and this common component has a constant completion rate γ as compared to its finished merchandise; suppose $\gamma = 0.5$, then the manufacturing rates $P_{1,i}$ and $P_{1,0}$ become twice as much as their regular rate as in a single-stage production system; (3) Assume a two-stage manufacturing plan to incorporate the postponement strategy, and both manufacturing processes face random faulty fractions x_0 and x_i, the reworking of these nonconforming items ensure their required quality; (4) Shortages are not permitted, so $P_{1,0} - d_{1,0} > 0$ and $P_{1,i} - d_{1,i} - \lambda_i > 0$ must hold; (5) A subcontracting fraction π_0 of the standard components helps to cut short its manufacturing uptime; the schedule receipt time for external components is predetermined at the starting of components' depleting time, and the following different setup cost $K_{\pi0}$ and unit cost $C_{\pi0}$ are associated with this subcontracting activity.

$$C_{\pi0} = \left(1 + \beta_{2,0}\right) C_0 \tag{1}$$

$$K_{\pi0} = \left(1 + \beta_{1,0}\right) K_0 \tag{2}$$

Fig. 1 indicates that when the standard component uptime completes, its inventory accumulates to $H_{1,0}$, and as the rework time achieves, it rises to $H_{2,0}$. Meantime, upon receipt of subcontracting components, its inventory peaks at $H_{3,0}$. Fig. 2 shows the standard components' inventory status during the uptime of each finished merchandise i. One can observe $t_{3,0}$ in Fig. 1 along with Fig. 2 and find the following relevant formulas relating to the depletion time $t_{3,0}$:

$$H_1 = H_{3,0} - Q_1 \tag{3}$$

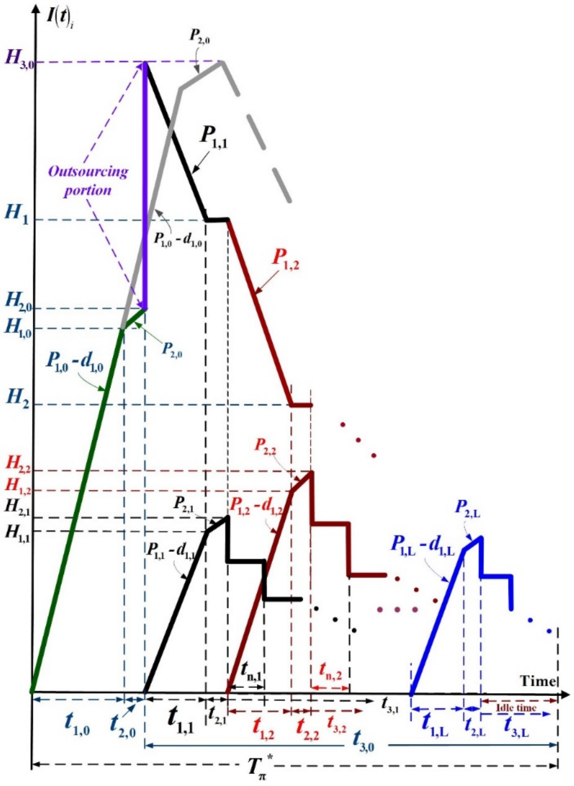

Fig. 1. Inventory status in our proposed two-stage multiproduct producer-client coordinated FPR model with the postponement, rework and subcontracting compared with the same problem without subcontracting (in grey)

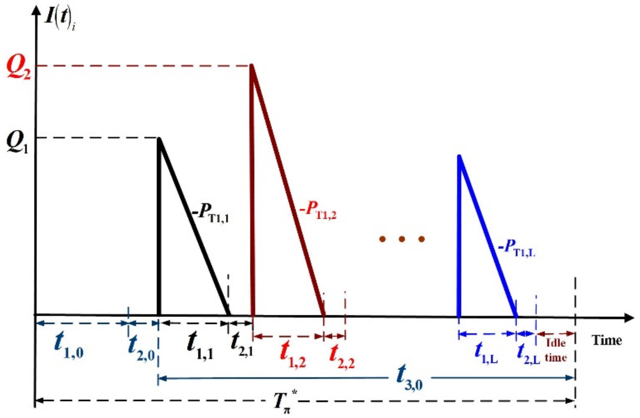

Fig. 2. Inventory status of the standard components during $t_{3,0}$

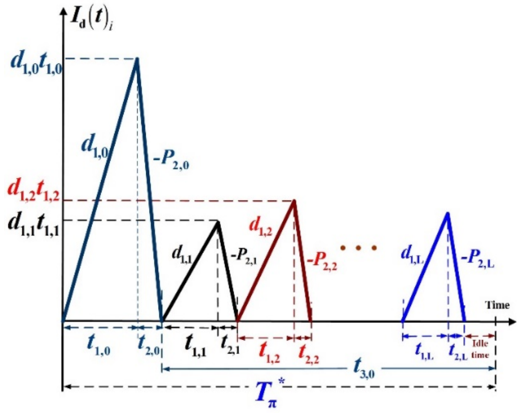

Fig. 3. Inventory status of the random faulty components and merchandise

$$H_i = H_{(i-1)} - Q_i \text{ , } for\ i = 2,\ 3,\ ...,L \tag{4}$$

$$H_L = H_{(L-1)} - Q_L = 0 \tag{5}$$

Fig. 3 depicts our studied model's inventory status of the random faulty components and merchandise. It exposes that when uptimes achieve, the maximal amounts of defective parts and merchandise reach ($d_{1,0}\,t_{1,0}$) and ($d_{1,i}\,t_{1,i}$). They deplete to zero when the rework times end.

2.2. Model formulation

According to our model's assumptions and by observing the demonstrations from Figs. (1-3), we first show the following straightforward relationships of specific system parameters in stage 2 of our model for making each finished merchandise i ($i = 1, 2, \ldots, L$):

$$Q_i = \lambda_i T_\pi \tag{6}$$

$$H_{1,i} = t_{1,i}\left(P_{1,i} - d_{1,i}\right) \tag{7}$$

$$H_{2,i} = H_{1,i} + t_{2,i}P_{2,i} \tag{8}$$

$$T_\pi = t_{1,i} + t_{2,i} + t_{3,i} = \frac{Q_i}{\lambda_i} \tag{9}$$

$$t_{1,i} = \frac{H_{1,i}}{P_{1,i} - d_{1,i}} = \frac{Q_i}{P_{1,i}} \tag{10}$$

$$t_{2,i} = \frac{x_i Q_i}{P_{2,i}} = \frac{H_{2,i} - H_{1,i}}{P_{2,i}} \tag{11}$$

$$t_{3,i} = T_\pi - \left(t_{1,i} + t_{2,i}\right) \tag{12}$$

Fig. 4 displays the finished merchandise i's inventory status in distribution time $t_{3,i}$.The equal-amount merchandise is distributed to the client in $t_{3,i}$. The total inventories are expressed in formulas (13).

$$\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right)\left(t_{3,i}\right)H_{2,i} = \left(\frac{1}{n^2}\right)\left(t_{3,i}\right)H_{2,i}\left[\frac{n(n-1)}{2}\right] = \left(\frac{n-1}{2n}\right)\left(t_{3,i}\right)H_{2,i} \tag{13}$$

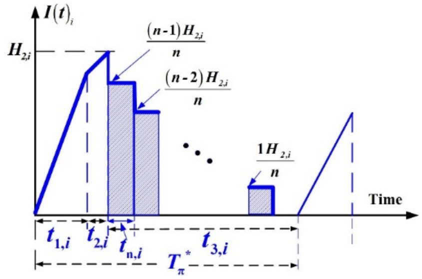

Fig. 4. The finished merchandise i's inventory status in distribution time $t_{3,i}$

Fig. 5 exhibits the finished merchandise *i*'s inventory status on the client side, and the total inventories for each finished merchandize *i* are shown in Eq. (14).

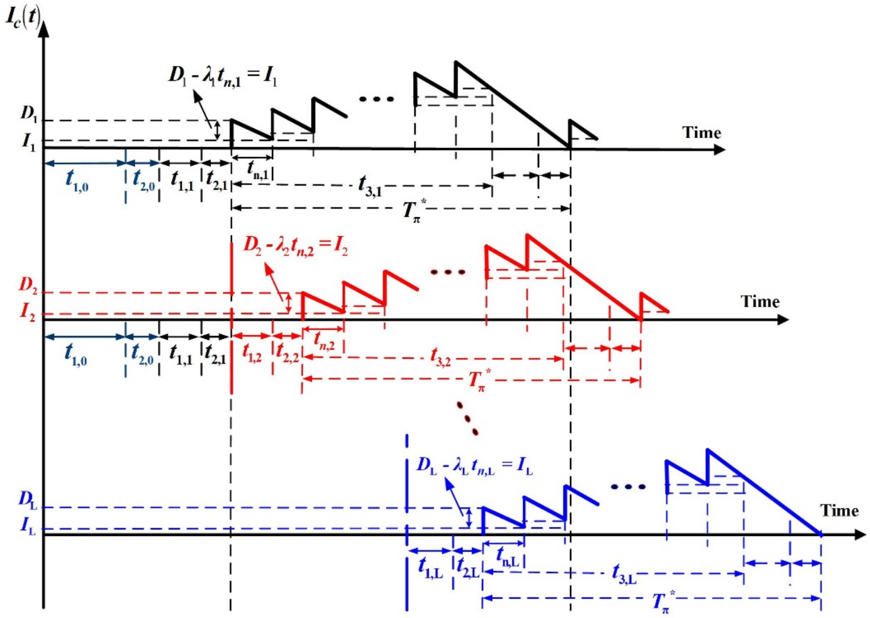

Fig. 5. The finished merchandise *i*'s inventory status on the client side

$$\left[\frac{n(D_i - I_i)t_{n,i}}{2} + \frac{nI_i\left(t_{1,i} + t_{2,i}\right)}{2} + \frac{n(n+1)}{2} I_i t_{n,i}\right] \tag{14}$$

where

$$I_i = D_i - \lambda_i\left(t_{n,i}\right) \tag{15}$$

$$t_{n,i} = \frac{t_{3,i}}{n} \tag{16}$$

$$D_i = \frac{H_{2,i}}{n} \tag{17}$$

Meantime, by observing the illustrations from Figs. (1) to (3), and based on our assumptions, we can obtain the following straightforward system parameters' relationships:

$$H_{3,0} = \sum_{i=1}^{L} Q_i = \sum_{i=1}^{L} \lambda_i T_\pi \tag{18}$$

$$\lambda_0 = \frac{\sum_{i=1}^{L} Q_i}{T_\pi} \tag{19}$$

$$\pi_0\left(\sum_{i=1}^{L} Q_i\right) = H_{3,0} - H_{2,0} \tag{20}$$

$$Q_0 = H_{2,0} \tag{21}$$

$$T_\pi = t_{1,0} + t_{2,0} + t_{3,0} \tag{22}$$

$$t_{1,0} = \frac{Q_0}{P_{1,0}} = \frac{H_{1,0}}{P_{1,0} - d_{1,0}} \tag{23}$$

$$H_{1,0} = \left(P_{1,0} - d_{1,0}\right)t_{1,0} \tag{24}$$

$$t_{2,0} = \frac{x_0 Q_0}{P_{2,0}} = \frac{H_{2,0} - H_{1,0}}{P_{2,0}} \tag{25}$$

$$H_{2,0} = P_{2,0} t_{2,0} + H_{1,0} \tag{26}$$

$$H_{2,0} = \left(1 - \pi_0\right)\left(\sum_{i=1}^{L} Q_i\right) \tag{27}$$

$$t_{3,0} = T_{\pi} - \left(t_{1,0} + t_{2,0}\right) \tag{28}$$

2.3. The system's operating expense & optimal policies

The total operating expenses, $TC(T_{\pi}, n)$ include: (a) subcontracting and in-house manufacturing setup and variable expenses, in-house rework, and inventory holding expenses; (b) the sum of finished merchandise i's manufacturing setup, variable, inventory holding, rework, and distributing expenses; and (c) the client's inventory holding expense. Hence, $TC(T_{\pi}, n)$ becomes:

$$\begin{aligned} TC\left(T_{\pi}, n\right) = {} & C_{\pi 0}\left(\sum_{i=1}^{L} Q_i\right)\pi_0 + K_{\pi 0} + C_0 Q_0 + K_0 + C_{R,0} Q_0 x_0 + h_{2,0}\left(\frac{d_{1,0} t_{1,0}}{2}\right)\left(t_{2,0}\right) \\ & + h_{1,0}\left[\frac{H_{1,0} t_{1,0}}{2} + \sum_{i=1}^{L}\left[\frac{Q_i}{2}\left(t_{1,i}\right) + H_i\left(t_{1,i} + t_{2,i}\right)\right] + \frac{d_{1,0} t_{1,0}\left(t_{1,0}\right)}{2} + \frac{\left(H_{2,0} + H_{1,0}\right)\left(t_{2,0}\right)}{2}\right] \\ & + \sum_{i=1}^{L}\left\{\begin{array}{l} Q_i C_i + K_i + C_{R,i} Q_i x_i + h_{2,i}\left(\dfrac{d_{1,i} t_{1,i}}{2}\right)\left(t_{2,i}\right) + C_{D,i} Q_i + n K_{D,i} \\ + h_{1,i}\left[\dfrac{H_{1,i} t_{1,i}}{2} + \dfrac{H_{2,i} + H_{1,i}}{2}\left(t_{2,i}\right) + \dfrac{d_{1,i} t_{1,i}}{2}\left(t_{1,i}\right) + \left(\dfrac{n-1}{2n}\right) H_{2,i}\left(t_{3,i}\right)\right] \\ + h_{3,i}\left[\dfrac{n\left(D_i - I_i\right) t_{n,i}}{2} + \dfrac{n\left(n+1\right)}{2} I_i t_{n,i} + \dfrac{n I_i\left(t_{1,i} + t_{2,i}\right)}{2}\right] \end{array}\right\} \end{aligned} \tag{29}$$

By employing $E[x_i]$ (for $i = 0, 1, 2, \ldots, L$) to cope with the random faulty rates and substituting Eqs. (1) to (28) in Eq. (29), plus extra efforts in derivation, the expected annualized system expense, $E[TCU(T_{\pi}, n)]$ becomes (for details, refer to Appendix B):

$$\begin{aligned} E\left[TCU\left(T_{\pi}, n\right)\right] = {} & \frac{\left(1+\beta_{1,0}\right) K_0}{T_{\pi}} + \left(1 + \beta_{2,0}\right) C_{\pi 0} \pi_0 \lambda_0 + C_0\left(1 - \pi_0\right)\lambda_0 + \frac{K_0}{T_{\pi}} + h_{2,0}\frac{\left(1 - \pi_0\right)^2 \lambda_0^2 T_{\pi} E\left[x_0\right]^2}{2 P_{2,0}} \\ & + C_{R,0} E\left[x_0\right]\left(1 - \pi_0\right)\lambda_0 + h_{1,0}\left[\frac{\left(1 - \pi_0\right)^2 \lambda_0^{\,2} T_{\pi}}{2} E_{0P} + \sum_{i=1}^{L}\left[\frac{\lambda_i^2 T_{\pi}}{2 P_{1,i}}\right] + \sum_{i=1}^{L}\left[\left(\sum_{i=1}^{L}\lambda_i T_{\pi} - \sum_{j=1}^{i}\lambda_j T_{\pi}\right)\lambda_i E_{2i}\right]\right] \\ & + \sum_{i=1}^{L}\left\{\begin{array}{l} C_i \lambda_i + \dfrac{K_i}{T_{\pi}} + C_{R,i} E\left[x_i\right]\lambda_i + \dfrac{n K_{D,i}}{T_{\pi}} + C_{D,i}\lambda_i + h_{2,i}\left(\dfrac{\lambda_i^2 T_{\pi} E\left[x_i\right]^2}{2 P_{2,i}}\right) + \dfrac{h_{3,i}\left(\lambda_i^2 T_{\pi}\right)}{2} E_{2i} \\ + h_{1,i}\left(\dfrac{\lambda_i^2 T_{\pi}}{2}\right) E_{3i} + \dfrac{\lambda_i^{\,2}\left(h_{3,i} - h_{1,i}\right) T_{\pi}}{2n}\left[\dfrac{1}{\lambda_i} - E_{2i}\right] \end{array}\right\} \end{aligned} \tag{30}$$

2.4. Optimization process

By employing the Hessian Matrix equations to $E[TCU(T_Z, n)]$ (Rardin, 1998):

$$\left[T_{\pi} \quad n\right] \cdot \begin{pmatrix} \dfrac{\partial^2 E\left[TCU\left(T_{\pi}, n\right)\right]}{\partial T_{\pi}^{\,2}} & \dfrac{\partial^2 E\left[TCU\left(T_{\pi}, n\right)\right]}{\partial T_{\pi}\partial n} \\ \dfrac{\partial^2 E\left[TCU\left(T_{\pi}, n\right)\right]}{\partial T_{\pi}\partial n} & \dfrac{\partial^2 E\left[TCU\left(T_{\pi}, n\right)\right]}{\partial n^2} \end{pmatrix} \cdot \begin{bmatrix} T_{\pi} \\ n \end{bmatrix} = \left[\sum_{i=1}^{L}\left\{\frac{2K_i}{T_{\pi}}\right\} + \frac{2\left(1+\beta_{1,0}\right) K_0}{T_{\pi}} + \frac{2K_0}{T_{\pi}}\right] > 0 \tag{31}$$

Since K_i, T_{π}, $(1 + \beta_{1,0})$, and K_0 are positive, Eq. (31) yields a positive value. Hence, $E[TCU(T_{\pi}, n)]$ is strictly convex for all n and T_{π} values > 0, and it has the minimum. By applying the 1st and 2nd derivatives of $E[TCU(T_{\pi}, n)]$, one obtains Eqs. (32) and (33).

$$\begin{aligned}\frac{\partial E\left[TCU\left(T_\pi,n\right)\right]}{\partial T_\pi}&=-\frac{K_0\left(1+\beta_{1,0}\right)}{{T_\pi}^2}-\frac{K_0}{{T_\pi}^2}+h_{2,0}\frac{\left(1-\pi_0\right)^2\lambda_0^2\left[E_{10}\right]^2}{2P_{2,0}}\\&+h_{1,0}\left[\frac{\left(1-\pi_0\right)^2{\lambda_0}^2}{2}E_{0P}+\sum_{i=1}^{L}\left[\frac{\lambda_i^2}{2P_{1,i}}\right]+\sum_{i=1}^{L}\left[\left(\sum_{i=1}^{L}\lambda_i-\sum_{j=1}^{i}\lambda_j\right)\lambda_iE_{2i}\right]\right]\\&+\sum_{i=1}^{L}\left\{-\frac{K_i}{{T_\pi}^2}-\frac{nK_{D,i}}{{T_\pi}^2}+h_{2,i}\left(\frac{\lambda_i^2E\left[x_i\right]^2}{2P_{2,i}}\right)+\frac{h_{3,i}\left(\lambda_i^2\right)}{2}E_{2i}+h_{1,i}\left(\frac{\lambda_i^2}{2}\right)E_{3i}+\frac{{\lambda_i}^2\left(h_{3,i}-h_{1,i}\right)}{2n}\left[\frac{1}{\lambda_i}-E_{2i}\right]\right\}\end{aligned}\tag{32}$$

$$\frac{\partial E\left[TCU\left(T_\pi,n\right)\right]}{\partial n}=\sum_{i=1}^{L}\left\{\frac{K_{D,i}}{T_\pi}-\frac{{\lambda_i}^2\left(h_{3,i}-h_{1,i}\right)T_\pi}{2n^2}\left[\frac{1}{\lambda_i}-E_{2i}\right]\right\}\tag{33}$$

By letting Eqs. (32) and (33) equal to zero and solve them simultaneously, and one derives the following T_π* and n*:

$${T_\pi}^*=\sqrt{\frac{2\left\{\sum_{i=1}^{L}\left[K_i+nK_{D,i}\right]+K_0\left(2+\beta_{1,0}\right)\right\}}{\begin{aligned}&\sum_{i=1}^{L}\left\{h_{2,i}E\left[x_i\right]^2\frac{\lambda_i^2}{P_{2,i}}+h_{1,i}\lambda_i^2E_{3i}+\left(\frac{\lambda_i^2}{n}\right)\left(h_{3,i}-h_{1,i}\right)\left[\frac{1}{\lambda_i}-E_{2i}\right]+h_{3,i}\lambda_i^2E_{2i}\right\}+h_{2,0}E\left[x_0\right]^2\frac{\left(1-\pi_0\right)^2\lambda_0^2}{P_{2,0}}\\&+h_{1,0}\left[\left(1-\pi_0\right)^2{\lambda_0}^2E_{0P}+\sum_{i=1}^{L}\left(\frac{\lambda_i^2}{P_{1,i}}\right)+2\sum_{i=1}^{L}\left(\lambda_i\right)\sum_{i=1}^{L}\lambda_iE_{2i}-2\sum_{i=1}^{L}\left[\left(\sum_{j=1}^{i}\left(\lambda_j\right)\right)\left(\lambda_iE_{2i}\right)\right]\right]\end{aligned}}}\tag{34}$$

and

$$n^*=\sqrt{\frac{\left[\sum_{i=1}^{L}K_i+K_0\left(2+\beta_{1,0}\right)\right]\cdot\sum_{i=1}^{L}\left\{{\lambda_i}^2\left(h_{3,i}-h_{1,i}\right)\left(\frac{1}{\lambda_i}-E_{2i}\right)\right\}}{\sum_{i=1}^{L}\left\{2K_{Di}\right\}\cdot\sum_{i=1}^{L}\left\{\begin{aligned}&\sum_{i=1}^{L}\left\{h_{2,i}E\left[x_i\right]^2\frac{\lambda_i^2}{P_{2,i}}+h_{1,i}\left[\lambda_i^2E_{3i}\right]+h_{3,i}\left(\lambda_i^2\right)E_{2i}\right\}+h_{2,0}E\left[x_0\right]^2\left(\frac{\left(1-\pi_0\right)^2\lambda_0^2}{P_{2,0}}\right)\\&+h_{1,0}\left[\left(1-\pi_0\right)^2{\lambda_0}^2E_{0P}+\sum_{i=1}^{L}\left(\frac{\lambda_i^2}{P_{1,i}}\right)+2\sum_{i=1}^{L}\left(\lambda_i\right)\sum_{i=1}^{L}\lambda_iE_{2i}-2\sum_{i=1}^{L}\left[\left(\sum_{j=1}^{i}\left(\lambda_j\right)\right)\left(\lambda_iE_{2i}\right)\right]\right]\end{aligned}\right\}}}\tag{35}$$

2.5. Prerequisite and setup times circumstances

When planning a multiproduct single-equipment fabrication featuring rework, subcontracting, and postponement, the prerequisite situation, as exhibited in Eq. (36), must hold to guarantee the single equipment's manufacturing capacity in our model (Nahmias, 2009).

$$\left[\sum_{i=1}^{L}\left(t_{1,i}+t_{2,i}\right)+\left(t_{1,0}+t_{2,0}\right)\right]<{T_\pi}^*\ \text{or}\ \left[\sum_{i=1}^{L}Q_i\left(\frac{1}{P_{1,i}}+\frac{E\left[x_i\right]}{P_{2,i}}\right)+Q_0\left(\frac{1}{P_{1,0}}+\frac{E\left[x_0\right]}{P_{2,0}}\right)\right]<{T_\pi}^*\tag{36}$$

or

$$\left\{\sum_{i=1}^{L}\lambda_i\left[\frac{1}{P_{1,i}}+\frac{E\left[x_i\right]}{P_{2,i}}\right]+\lambda_0\left(1-\pi_0\right)\left(\frac{1}{P_{1,0}}+\frac{E\left[x_0\right]}{P_{2,0}}\right)\right\}<1\tag{37}$$

Furthermore, if the total setup times S_i (for $i = 0, 1, 2, \ldots, L$) in our model is greater than the idle time (as illustrated in Fig. 1), then one should first calculate the $T_{\min}$ (Nahmias (2009)). Finally, choose the maximum of $T_{\min}$ and T_π* as the finalized solution for the rotation cycle length to guarantee the single equipment's manufacturing capacity in our model.

$$T_{\min}=\frac{\sum_{i=0}^{L}\left(S_i\right)}{1-\left\{\lambda_0\left(1-\pi_0\right)\left(\frac{1}{P_{1,0}}+\frac{E\left[x_0\right]}{P_{2,0}}\right)+\sum_{i=1}^{L}\lambda_i\left[\frac{1}{P_{1,i}}+\frac{E\left[x_i\right]}{P_{2,i}}\right]\right\}}\tag{38}$$

3. Demonstrative example

The following demonstrative example shows how one can apply our research results to solve the proposed problem's replenishment-shipment decision and expose essential system information. Tables 1 and 2 exhibits the assumed values in this demonstrative example for both stages' relevant system variables. In contrast, Table C-1 in Appendix C shows its corresponding variable values in the single-stage production scheme.

Table 1
Assumed variables values in stage one of this demonstrative example

$C_{R,0}$	λ_0	$h_{2,0}$	K_0	i_0	π_0	$\beta_{2,0}$	x_0
\$25	17406	\$8	\$8500	0.2	0.4	0.4	0.025
$P_{2,0}$	C_0	$\beta_{1,0}$	γ	δ	$h_{1,0}$	$P_{1,0}$	
96000	\$40	-0.7	0.5	0.5	\$8	120000	

Table 2
Assumed variables values in stage two of this demonstrative example

Item i	$h_{1,i}$	$h_{3,i}$	K_i	$C_{D,i}$	$P_{2,i}$	$K_{D,i}$	$C_{R,i}$	λ_i	i_i	$P_{1,i}$	$h_{2,i}$	x_i	C_i
1	\$8	\$70	\$8500	\$0.1	89806	\$1800	\$25	3000	0.2	112258	\$8	0.025	\$40
2	\$10	\$75	\$9000	\$0.2	92852	\$1900	\$30	3200	0.2	116066	\$10	0.075	\$50
3	\$12	\$80	\$9500	\$0.3	96000	\$2000	\$35	3400	0.2	120000	\$12	0.125	\$60
4	\$14	\$85	\$10000	\$0.4	99254	\$2100	\$40	3600	0.2	124068	\$14	0.175	\$70
5	\$16	\$90	\$10500	\$0.5	102621	\$2200	\$45	3800	0.2	128276	\$16	0.225	\$80

We first demonstrate the optimal operating batch policy for delivery frequency and replenishing cycle. By computing Eq. (35) and Eq. (34), one decides the optimal $n^* = 4$ and $T_\pi^* = 0.5451$. It follows that by calculating equation (30), one finds the optimal operating expense $E[TCU(T_\pi^*, n^*)]$ = \$2,242,716. Figures 6 and 7 depict $E[TCU(T_\pi, n)]$'s behavior and convexity relating to n and T_π, respectively. One notices that $E[TCU(T_\pi, n)]$ significantly upsurges as T_π and n depart from their optimal point.

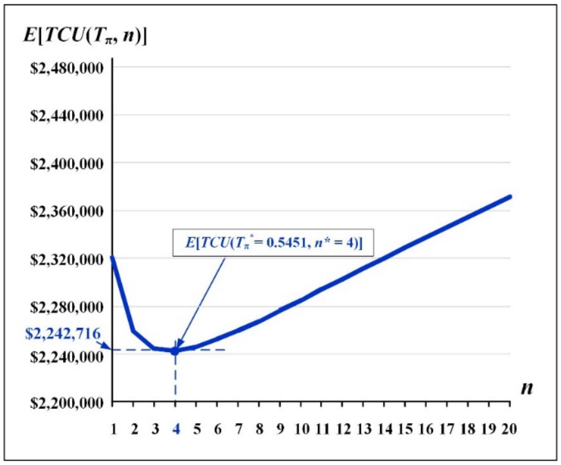

Fig. 6. $E[TCU(T_\pi^*, n^*)]$'s convexity vis-à-vis n

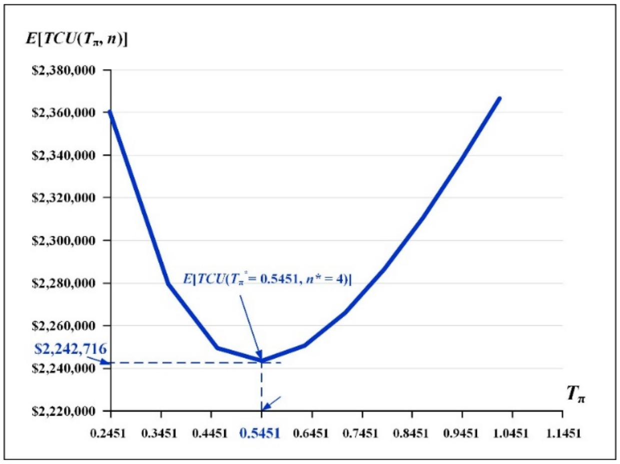

Fig. 7. $E[TCU(T_\pi^*, n^*)]$'s convexity concerning T_π

The present study allows one to explicitly evaluate all relevant system expenditure in $E[TCU(T_\pi^*, n^*)]$. Figure 8 uncovers all contributors of $E[TCU(T_\pi^*, n^*)]$. Among them, three major contributors (added up to 88.71%) comprise:

(i) 51.55% contribute from the finished merchandise's variable and setup costs;
(ii) 19.28% donate by the standard components' variable expense; and
(iii) 17.88% come from subcontracting 40% of the needed standard parts.

Other expenses comprise: client holding cost 4.73%, finished merchandise distribution cost 3.65%; total rework expenses 2.18% (i.e., 0.15% in stage one and 2.03% in stage two); and stage one's setup cost 0.72 %.

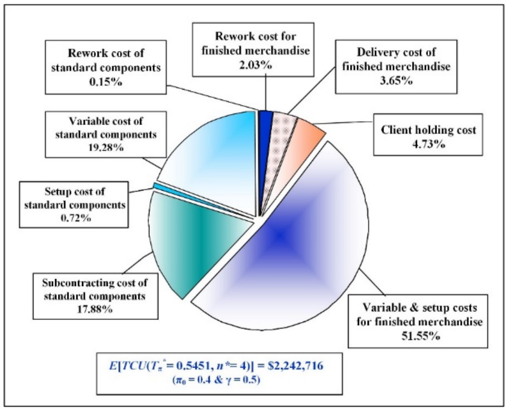

Fig. 8. $E[TCU(T_\pi^*, n^*)]$'s detailed expense contributors

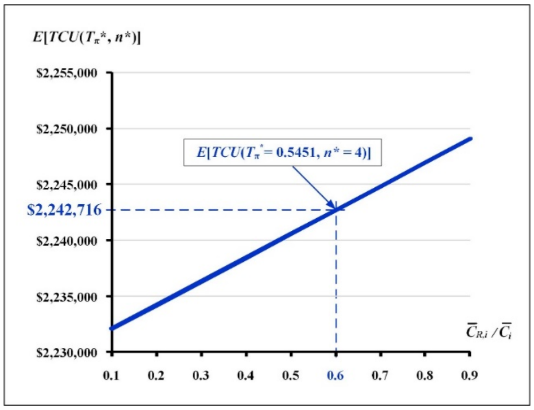

Fig. 9. $E[TCU(T_\pi^*, n^*)]$'s performance concerning the ratio of mean-$C_{R,i}$ over mean-C_i

Our study can explore the influence of extra quality-assurance expense (in terms of the ratio of mean-$C_{R,i}$ over mean-C_i) on $E[TCU(T_\pi^*, n^*)]$. Fig. 9 reveals as the rework expense ratio rises, $E[TCU(T_\pi^*, n^*)]$ surges accordingly. Fig. 10 depicts the explorative result of γ (i.e., the standard part's completing percentage) on the optimal operating expense $E[TCU(T_\pi^*, n^*)]$. It uncovers that as γ rises, $E[TCU(T_\pi^*, n^*)]$ knowingly surges. For this example's assumption $\gamma = 0.5$, one finds that t_0^* (i.e., the sum of the optimal $t^*_{1,0}$ and $t^*_{2,0}$) surges to 0.0471, and we also confirm that the optimal $E[TCU(T_\pi^*, n^*)] = \$2,242,176$.

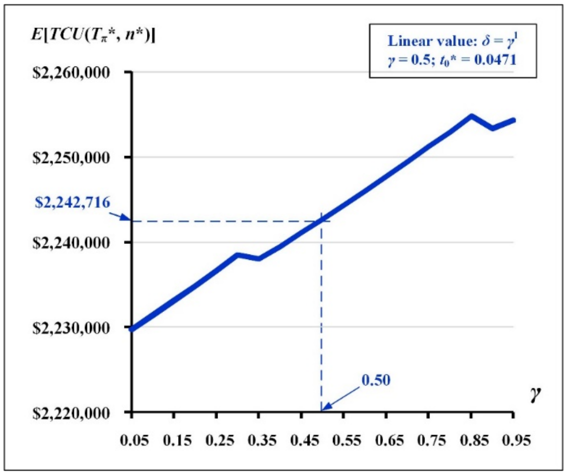

Fig. 10. $E[TCU(T_\pi^*, n^*)]$'s performance vis-à-vis γ

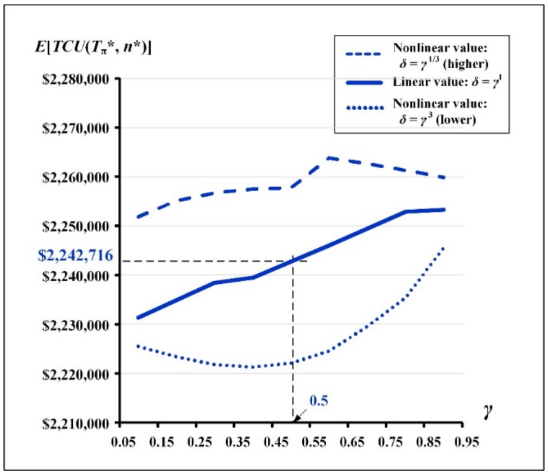

Fig. 11. $E[TCU(T_\pi^*, n^*)]$'s performance vis-à-vis diverse relationship δ in terms of γrates

Our demonstrative example takes a linear relationship δ between the standard part's value and its relevant completing rate γ. For $\gamma = 0.5$, we assume the common component's value is 1/2 of its finished goods. But, for certain types of merchandise, this may not be the case. To cope with this matter, we demonstrate that the present study can explore other relationships, for example, $\delta = \gamma^1$ (the linear case) and $\delta = \gamma^{1/3}$ (a nonlinear case). Fig. 11 illustrates $E[TCU(T_\pi^*, n^*)]$'s performance vis-à-vis diverse relationship δ in terms of γrates. Our study can investigate the collective influence of the subcontracting proportion π_0 and the standard part's completing rate γ on the sum of the optimal $t^*_{1,0}$ and $t^*_{2,0}$ (i.e., t_0^*; see Fig. 12).

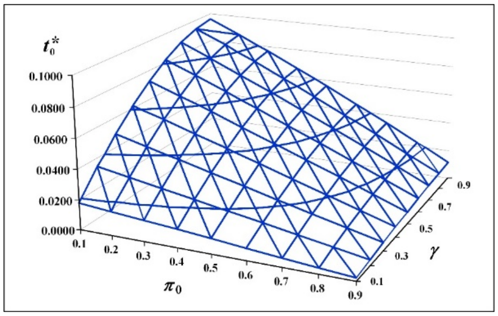

Fig. 12. The behavior of optimal t_0^* concerning π_0 and γ

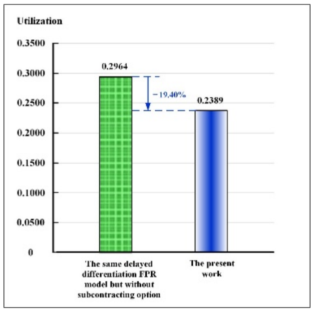

Fig. 13. Reduction of machine utilization concerning our example's assumption $\pi_0 = 0.4$

It reveals that as π_0 increases, the producer makes fewer standard parts, so the optimal uptime plus rework times t_0* considerably decreases. In contrast, as γ increases, the producer will need to spend more time making each standard part, so t_0* upsurges significantly. The example's assumption $\pi_0 = 0.4$ cuts producer's uptime in making the standard parts, consequently, it reduces the machine utilization by 19.40% (i.e., declines from 0.2964 to 0.2389) as exhibited in Fig. 13. Table D-1 exhibits the investigative outcomes of different essential manufacturing-time relevant variables affected by π_0 (see Appendix D). Fig. 14 demonstrates the analytical result for the price paid to reduce 19.40% utilization. It uncovers that bringing utilization down 19.40% requires an extra expense 5.06% in $E[TCU(T_\pi*, n*)]$ (i.e., from \$2,134,736 to \$2,2427,16). Table D-2 discloses the explorative outcomes of different essential manufacturing-expenditure relevant variables affected by π_0 (refer to Appendix D).

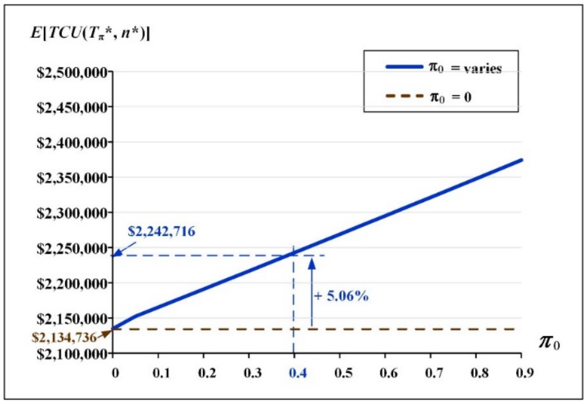

Fig. 14. Analytical result for the price paid in $E[TCU(T_\pi*, n*)]$'to reduce 19.40% utilization

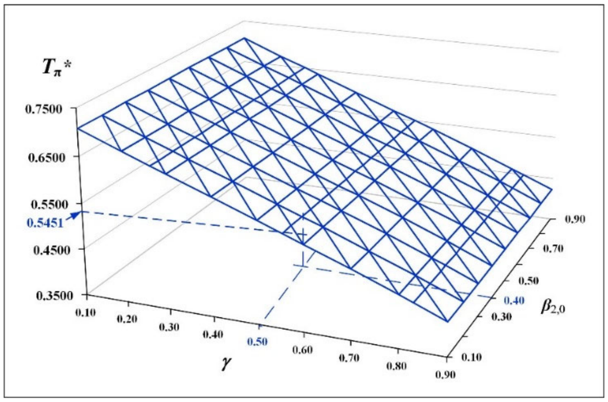

Fig. 15. The optimal cycle length T_π*'s performance concerning γ and $\beta_{2,0}$

We further show our study can explore in-depth particular parameters' effect on specific crucial system operating policies. For example, Figure 15 illustrates the optimal cycle length T_π*'s performance concerning γ and $\beta_{2,0}$ (the subcontracting added unit cost of standard parts). It exposes that as $\beta_{2,0}$ increases, T_π* decreases irrelevantly; and as γ rises, T_π* upsurges radically.

4. Conclusions

Simultaneously meeting clients' expectations of variety and quality goods in a short lead time has become the current intra-supply-chain manager's ordinary task. With such a motivation, this study examines the replenishment-shipment decision for a multiproduct producer-client coordinated FPR model with postponement, rework, and subcontracting. We build a two-phase mathematical model to interpret the proposed problem explicitly. The formulation and optimization method help us obtain annual expected annual system expenses and the best-operating policies concerning the rotation fabricating cycle time and distribution frequency (as exhibited in Section 2). To demonstrate our research outcomes, we present a numerical illustration to examine the performance of crucial system parameters influenced by rework, subcontracting, postponement, and finished goods transportation (see the details in Tables D-1 and D-2). Figures 6 through 15 (in Section 3) depict how this study facilitates industries in their managerial decision makings concerning the following aspects:

(1) $E[TCU(T_\pi*, n*)]$'s convexity vis-à-vis n and T_π* (refer to Fig. 6 and Fig. 7);
(2) Detailed expense contributors of $E[TCU(T_\pi*, n*)]$ (Figure 8);
(3) $E[TCU(T_\pi*, n*)]$'s performance concerning the rework relating factor $C_{R,i}$, standard part's completion rate γ, and diverse relationship δ in terms of γrates (see Fig. 9, Fig. 10, and Fig. 11);
(4) The behavior of the optimal uptime plus rework time t_0* concerning π_0 and γ (Fig. 12);
(5) Utilization reduction and $E[TCU(T_\pi*, n*)]$ surge concerning subcontracting factor π_0 (refer to Fig. 13 and Fig. 14);
(6) T_π*'s performance concerning γ and subcontracting cost-factor $\beta_{2,0}$.

Examining stochastic finished merchandise requirements in the same setting as the studied problem is worth investigating for future research.

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Appendix - A

Notations' definitions in stage 2 when making each finished merchandise i ($i = 1, 2, \ldots, L$):

L = the number of finished merchandise,
T_π = system's rotation cycle length,
n = distribution frequency,
λ_i = requirement per year,
Q_i = batch size,
S_i = setup time,
$t_{1,i}$ = manufacturing uptime,
$P_{1,i}$ = annual manufacturing rate,
C_i = unit cost,
$h_{1,i}$ = unit holding cost,
K_i = setup cost,
x_i = random faulty fraction,
$d_{1,i}$ = manufacturing rate of faulty merchandise ($d_{1,i} = x_i P_{1,i}$),,
$H_{1,i}$ = inventory status when manufacturing uptime completes,
$C_{R,i}$ = unit rework cost,
$h_{2,i}$ = unit holding cost of reworked merchandise,
$t_{2,i}$ = rework time,
$P_{2,i}$ = annual reworking rate,
t_i^* = total optimal uptimes and rework times,
$I_d(t)_i$ = faulty inventory status at time t,
$H_{2,i}$ = inventory status when the rework completes,
$t_{3,i}$ = finished merchandise's distribution time,
$h_{3,i}$ = client unit holding cost,
$t_{n,i}$ = fixed time-interval between each distribution,
$K_{D,i}$ = setup cost of distribution,
$C_{D,i}$ = unit distribution cost,
D_i = fixed-amount of each distribution,
I_i = number of merchandise left when $t_{n,i}$ completes,
$I(t)_i$ = inventory status at time t,
$I_c(t)_i$ = client's inventory status at time t.

Notations' definitions in stage 1 when producing standard components:

λ_0 = requirement per year,
Q_0 = in-house batch size,
C_0 = in-house manufacturing unit cost,
K_0 = in-house manufacturing setup cost,
S_0 = setup time,
$h_{1,0}$ = unit holding cost,
π_0 = standard components' subcontracting fraction,
$C_{\pi 0}$ = unit subcontracting cost,
$\beta_{1,0}$ = linking parameter between $K_{\pi 0}$ and K_0,
$K_{\pi 0}$ = subcontracting setup cost,
$\beta_{2,0}$ = linking parameter between $C_{\pi 0}$ and C_0,

i_0 = linking ratio of $h_{1,i}$ and C_i (for $i = 0, 1, 2, \ldots, L$; for example, $h_{1,i} = i_0C_i$),
$P_{1,0}$ = in-house manufacturing rate,
$t_{1,0}$ = uptime,
x_0 = random faulty fraction,
$C_{R,0}$ = unit rework cost,
$d_{1,0}$ = manufacturing rate of faulty components ($d_{1,0} = P_{1,0}x_0$),
$P_{2,0}$ = annual reworking rate,
$h_{3,i}$ = client's unit holding cost,
$h_{2,0}$ = reworked items' unit holding cost,
γ = standard part's completion rate (as compared with a finished merchandise),
$H_{1,0}$ = stock level when fabricating process ends,
$t_{2,0}$ = repairable faulty standard parts' rework time,
t_0^* = the sum of the optimal $t^*_{1,0}$ and $t^*_{2,0}$,
$H_{2,0}$ = stock level when reworking process ends,
$H_{3,0}$ = inventory status when subcontracting components are received,
$t_{3,0}$ = standard components' depletion time,
$E[T_\pi]$ = expected rotation cycle length,
$TC(T_\pi, n)$ = total system expense per cycle,
$E[TC(T_\pi, n)]$ = expected total system expense per cycle,
$E[TCU(T_\pi, n)]$ = expected annualized system expense.

Appendix B:

Derivation of Eq. (30) has the following details:

By employing $E[x_i]$ (for $i = 0, 1, 2, \ldots, L$) to cope with the random faulty rates and substituting Eqs. (1) to (28) in Eq. (29), plus extra efforts in deriving $E[TCU(T_\pi, n)] = E[TC(T_\pi, n)] / E[T_\pi]$, one can obtain $E[TCU(T_\pi, n)]$ as exhibited in Eq. (B-1).

$$\begin{aligned} E\left[TCU\left(T_\pi,n\right)\right] &= \frac{\left(1+\beta_{1,0}\right)K_0}{T_\pi} + \left(1+\beta_{2,0}\right)C_{\pi 0}\pi_0\lambda_0 + C_0\left(1-\pi_0\right)\lambda_0 + \frac{K_0}{T_\pi} + C_{R,0}E\left[x_0\right]\left(1-\pi_0\right)\lambda_0 \\ &+ h_{2,0}\frac{\left(1-\pi_0\right)^2\lambda_0^2 T_\pi E\left[x_0\right]^2}{2P_{2,0}} + h_{1,0}\left[\begin{array}{l} \dfrac{\left(1-\pi_0\right)^2\lambda_0^{\,2}T_\pi}{2}\left[\dfrac{1}{P_{1,0}} + \dfrac{E\left[x_0\right]\left(2-E\left[x_0\right]\right)}{P_{2,0}}\right] + \displaystyle\sum_{i=1}^{L}\left[\frac{\lambda_i^2 T_\pi}{2P_{1,i}}\right] \\ + \displaystyle\sum_{i=1}^{L}\left[\left(\sum_{i=1}^{L}\lambda_i T_\pi - \sum_{j=1}^{i}\lambda_j T_\pi\right)\cdot\left(\frac{\lambda_i}{P_{1,i}} + \frac{\lambda_i E\left[x_i\right]}{P_{2,i}}\right)\right] \end{array}\right] \\ &+ \sum_{i=1}^{L}\left\{\begin{array}{l} C_i\lambda_i + \dfrac{K_i}{T_\pi} + C_{R,i}E\left[x_i\right]\lambda_i + \dfrac{nK_{D,i}}{T_\pi} + C_{D,i}\lambda_i + h_{2,i}\left(\dfrac{\lambda_i^2 T_\pi E\left[x_i\right]^2}{2P_{2,i}}\right) + \dfrac{h_{3,i}\left(\lambda_i^2 T_\pi\right)}{2}\left(\dfrac{1}{P_{1,i}} + \dfrac{E\left[x_i\right]}{P_{2,i}}\right) \\ + h_{1,i}\left(\dfrac{\lambda_i^2 T_\pi}{2}\right)\left(\dfrac{1}{\lambda_i} + \dfrac{E\left[x_i\right]\left(1-E\left[x_i\right]\right)}{P_{2,i}}\right) + \dfrac{\lambda_i^{\,2}\left(h_{3,i}-h_{1,i}\right)T_\pi}{2n}\left[\dfrac{1}{\lambda_i} - \dfrac{1}{P_{1,i}} - \dfrac{E\left[x_i\right]}{P_{2,i}}\right] \end{array}\right\} \end{aligned} \tag{B-1}$$

Let E_{0P}, E_{2i}, and E_{3i} represent the following:

$$E_{0P} = \left[\frac{1}{P_{1,0}} + \frac{E\left[x_0\right]\left[2-E\left[x_0\right]\right]}{P_{2,0}}\right] \tag{B-2}$$

$$E_{2i} = \left[\frac{1}{P_{1,i}} + \frac{E\left[x_i\right]}{P_{2,i}}\right];\ E_{3i} = \left[\frac{1}{\lambda_i} + \frac{E\left[x_i\right]\left(1-E\left[x_i\right]\right)}{P_{2,i}}\right]\ \textit{for}\ \ i = 1,\ldots,L. \tag{B-3}$$

Substitute Eqs. (B-2) and (B-3) in Eq. (B-1), $E[TCU(T_\pi, n)]$ becomes as shown in Eq. (30).

$$\begin{aligned} E\left[TCU\left(T_\pi,n\right)\right] &= \frac{\left(1+\beta_{1,0}\right)K_0}{T_\pi} + \left(1+\beta_{2,0}\right)C_{\pi 0}\pi_0\lambda_0 + C_0\left(1-\pi_0\right)\lambda_0 + \frac{K_0}{T_\pi} + h_{2,0}\frac{\left(1-\pi_0\right)^2\lambda_0^2 T_\pi E\left[x_0\right]^2}{2P_{2,0}} \\ &+ C_{R,0}E\left[x_0\right]\left(1-\pi_0\right)\lambda_0 + h_{1,0}\left[\frac{\left(1-\pi_0\right)^2\lambda_0^{\,2}T_\pi}{2}E_{0P} + \sum_{i=1}^{L}\left[\frac{\lambda_i^2 T_\pi}{2P_{1,i}}\right] + \sum_{i=1}^{L}\left[\left(\sum_{i=1}^{L}\lambda_i T_\pi - \sum_{j=1}^{i}\lambda_j T_\pi\right)\lambda_i E_{2i}\right]\right] \\ &+ \sum_{i=1}^{L}\left\{\begin{array}{l} C_i\lambda_i + \dfrac{K_i}{T_\pi} + C_{R,i}E\left[x_i\right]\lambda_i + \dfrac{nK_{D,i}}{T_\pi} + C_{D,i}\lambda_i + h_{2,i}\left(\dfrac{\lambda_i^2 T_\pi E\left[x_i\right]^2}{2P_{2,i}}\right) + \dfrac{h_{3,i}\left(\lambda_i^2 T_\pi\right)}{2}E_{2i} \\ + h_{1,i}\left(\dfrac{\lambda_i^2 T_\pi}{2}\right)E_{3i} + \dfrac{\lambda_i^{\,2}\left(h_{3,i}-h_{1,i}\right)T_\pi}{2n}\left[\dfrac{1}{\lambda_i} - E_{2i}\right] \end{array}\right\} \end{aligned} \tag{30}$$

Appendix C

Table C-1

Assumed variables values of this example in a corresponding single-stage system

Product i	$C_{R\,i}$	$h_{3\,i}$	$K_{D\,i}$	i_i	K_i	$h_{2\,i}$	$P_{2\,i}$	C_i	$P_{1\,i}$	x_i	$C_{D\,i}$	λ_i	$h_{1\,i}$
1	\$50	\$70	\$1800	0.2	\$17000	\$16	46400	\$80	58000	0.05	\$0.1	3000	\$16
2	\$55	\$75	\$1900	0.2	\$17500	\$18	47200	\$90	59000	0.10	\$0.2	3200	\$18
3	\$60	\$80	\$2000	0.2	\$18000	\$20	48000	\$100	60000	0.15	\$0.3	3400	\$20
4	\$65	\$85	\$2100	0.2	\$18500	\$22	48800	\$110	61000	0.20	\$0.4	3600	\$22
5	\$70	\$90	\$2200	0.2	\$19000	\$24	49600	\$120	62000	0.25	\$0.5	3800	\$24

Appendix D

Table D-1

Different essential manufacturing-time relevant variables affected by π_0

π_0	Utilization (A)	(A) % drop	T_π^*	t_0^* (B)	(B)% drop	System's Total rework time (C)	System's total uptime (D)	(C) % drop	(D) % drop
0.00	0.2964	-	0.5329	0.0767	-	0.00726	0.15077	-	-
0.05	0.2892	-2.43%	0.5407	0.0739	-3.65%	0.00723	0.14915	-0.41%	-1.07%
0.10	0.2820	-4.85%	0.5414	0.0701	-8.60%	0.00718	0.14552	-1.10%	-3.48%
0.15	0.2748	-7.28%	0.5421	0.0663	-13.56%	0.00713	0.14186	-1.79%	-5.91%
0.20	0.2676	-9.71%	0.5428	0.0625	-18.51%	0.00707	0.13819	-2.62%	-8.34%
0.25	0.2604	-12.14%	0.5434	0.0586	-23.60%	0.00702	0.13450	-3.31%	-10.79%
0.30	0.2532	-14.56%	0.5440	0.0547	-28.68%	0.00697	0.13080	-3.99%	-13.25%
0.35	0.2461	-16.99%	0.5445	0.0509	-33.64%	0.00692	0.12707	-4.68%	-15.72%
0.40	**0.2389**	**-19.40%**	**0.5451**	**0.0470**	**-38.72%**	**0.00686**	**0.12333**	**-5.51%**	**-18.20%**
0.45	0.2317	-21.84%	0.5455	0.0432	-43.68%	0.00681	0.11957	-6.20%	-20.69%
0.50	0.2245	-24.27%	0.5460	0.0393	-48.76%	0.00675	0.11580	-7.02%	-23.19%
0.55	0.2173	-26.70%	0.5463	0.0353	-53.98%	0.00670	0.11201	-7.71%	-25.71%
0.60	0.2101	-29.12%	0.5467	0.0315	-58.93%	0.00664	0.10821	-8.54%	-28.23%
0.65	0.2029	-31.55%	0.5470	0.0275	-64.15%	0.00658	0.10440	-9.37%	-30.76%
0.70	0.1957	-33.98%	0.5473	0.0237	-69.10%	0.00653	0.10057	-10.06%	-33.30%
0.75	0.1885	-36.41%	0.5475	0.0197	-74.32%	0.00647	0.09673	-10.88%	-35.84%
0.80	0.1813	-38.83%	0.5477	0.0157	-79.53%	0.00641	0.09289	-11.71%	-38.39%
0.85	0.1741	-41.26%	0.5478	0.0118	-84.62%	0.00635	0.08903	-12.53%	-40.95%
0.90	0.1669	-43.69%	0.5479	0.0079	-89.70%	0.00629	0.08517	-13.36%	-43.51%
0.95	0.1597	-46.11%	0.5480	0.0040	-94.78%	0.00623	0.08130	-14.19%	-46.08%
1.00	0.1525	-48.54%	0.5238	0.0000	-100.0%	0.00590	0.07400	-18.73%	-50.92%

Table D-2

Different essential manufacturing-expenditure relevant variables affected by π_0

π_0	n^*	$E[TCU(T_\pi^*, n^*)]$ (A)	(A) % drop	Finished products delivery expenditure	Client inventory holding expense	Stage 1's total manufacturing expenditure	Stage one's Subcontract-ing expenses (B)	(B) / (A) %	Stage one's other manufacturing-related cost	Stage 1's in-house rework-expense	Stage 2's rework-expense
0.00	4	\$2.134.736	-	\$80.365	\$99.716	\$712.018	\$0	0.00%	\$706.705	\$5.314	\$43.833
0.05	4	\$2,152,301	0.82%	\$79,281	\$101,176	\$729,472	\$52,316	2.43%	\$672,108	\$5,048	\$43,833
0.10	4	\$2,165,138	1.42%	\$79,180	\$101,315	\$742,296	\$99,910	4.61%	\$637,604	\$4,782	\$43,833
0.15	4	\$2,178,001	2.03%	\$79,084	\$101,446	\$755,145	\$147,504	6.77%	\$603,125	\$4,516	\$43,833
0.20	4	\$2,190,891	2.63%	\$78,994	\$101,570	\$768,021	\$195,098	8.90%	\$568,673	\$4,251	\$43,833
0.25	4	\$2,203,807	3.24%	\$78,909	\$101,687	\$780,924	\$242,693	11.01%	\$534,247	\$3,985	\$43,833
0.30	4	\$2,216,750	3.84%	\$78,830	\$101,797	\$793,854	\$290,288	13.10%	\$499,847	\$3,719	\$43,833
0.35	4	\$2,229,719	4.45%	\$78,756	\$101,900	\$806,811	\$337,883	15.15%	\$465,475	\$3,454	\$43,834
0.40	**4**	**\$2,242,716**	**5.06%**	**\$78,687**	**\$101,995**	**\$819,796**	**\$385,478**	**17.19%**	**\$431,129**	**\$3,188**	**\$43,834**
0.45	4	\$2,255,738	5.67%	\$78,624	\$102,083	\$832,807	\$433,074	19.20%	\$396,811	\$2,922	\$43,834
0.50	4	\$2,268,788	6.28%	\$78,566	\$102,163	\$845,847	\$480,671	21.19%	\$362,519	\$2,657	\$43,834
0.55	4	\$2,281,865	6.89%	\$78,514	\$102,236	\$858,914	\$528,267	23.15%	\$328,256	\$2,391	\$43,834
0.60	4	\$2,294,968	7.51%	\$78,467	\$102,302	\$872,009	\$575,864	25.09%	\$294,019	\$2,125	\$43,834
0.65	4	\$2,308,099	8.12%	\$78,426	\$102,360	\$885,132	\$623,462	27.01%	\$259,810	\$1,860	\$43,834
0.70	4	\$2,321,257	8.74%	\$78,390	\$102,410	\$898,282	\$671,059	28.91%	\$225,629	\$1,594	\$43,834
0.75	4	\$2,334,442	9.36%	\$78,360	\$102,452	\$911,461	\$718,658	30.78%	\$191,476	\$1,328	\$43,834
0.80	4	\$2,347,654	9.97%	\$78,335	\$102,487	\$924,668	\$766,256	32.64%	\$157,350	\$1,063	\$43,834
0.85	4	\$2,360,893	10.59%	\$78,315	\$102,514	\$937,904	\$813,855	34.47%	\$123,252	\$797	\$43,834
0.90	4	\$2,374,159	11.22%	\$78,302	\$102,534	\$951,167	\$861,454	36.28%	\$89,182	\$531	\$43,834
0.95	4	\$2,387,453	11.84%	\$78,293	\$102,545	\$964,459	\$909,053	38.08%	\$55,140	\$266	\$43,834
1.00	4	\$2,384,913	11.72%	-	-	\$962,236	\$956,868	40.12%	\$5,368	\$0	\$43,834