Multi-item fabrication-shipment decision model featuring multi-delivery, postponement, quality assurance, and overtime

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A B S T R A C T

The study applies a postponement strategy to a multi-item fabrication-shipment decision making in a vendor-buyer coordinated environment with multi-delivery, quality reassurance, and overtime. To cope with the recent client demand trend asking for rapid response, quality, and diversified goods, today's manufacturers require a multi-item production-shipping scheme to satisfy customers' needs in cost-saving, quality, and timely matter. In our model, we first produce all needed mutual components and postpone manufacturing of finished goods in the second phase. To expedite mutual parts' fabrication time, overtime is used. Product quality is reassured through screening the defeats and reworking repairable defectives in both fabrication phases. To decide the optimal fabrication-shipment policy, we build a math model and apply the cost minimization technique to the problem. Upon deriving the optimal policy, we utilize an example to demonstrate how our model works and its capability in exposing various previously inaccessible information to the problem. These detailed results can facilitate managerial decision-making and boost the performance of such a specific multi-item postponement fabrication-shipment system in cost-saving, product quality, and timely response.

Nomenclature

The following are for finished products fabrication:

L	=	the number of finished goods,
λ_i	=	annual demand rate (where $i = 1, 2, \ldots, L$),
Q_i	=	batch size,
T_A	=	a decision variable (manufacturing cycle length),
$h_{1,i}$	=	unit holding cost,
$h_{2,i}$	=	unit holding cost in rework time,
$h_{4,i}$	=	unit safety cost,
C_i	=	unit fabrication cost,
K_i	=	setup cost,
$P_{1,i}$	=	annual production rate,
$P_{2,i}$	=	annual rework rate,
x_i	=	random defective proportion,
$\theta_{1,i}$	=	scrap proportion in uptime,
$\theta_{2,i}$	=	product i's scrap proportion in rework time,
φ_i	=	overall scrap proportion of end product i,
$d_{1,i}$	=	defective items' production rate,
$d_{2,i}$	=	scrapped items' production rate (where $d_{2,i} = P_{2,i}\theta_{2,i}$),

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$C_{R,i}$ = unit rework cost,
$C_{S,i}$ = unit disposal cost,
$t_{1,i}$ = uptime,
$t_{2,i}$ = rework time,
$t_{3,i}$ = shipping time,
$K_{D,i}$ = fixed shipping cost,
$C_{D,i}$ = unit shipping cost,
t_i^* = the sum of optimal uptime and rework times,
S_i = setup time,
$H_{1,i}$ = stock level when uptime ends,
$H_{2,i}$ = stock level when rework time ends.

The following are for mutual parts fabrication:

λ_0 = annual requirements,
Q_0 = batch size,
γ = completion rate as compared to the finished goods,
K_{T0} = setup cost with overtime implementation,
C_{T0} = unit cost with overtime implementation,
C_0 = regular unit cost,
K_0 = regular setup cost,
$t_{1,0}$ = ptime with overtime implementation,
$t_{2,0}$ = rework time with implementation of overtime,
$t_{3,0}$ = depletion time,
x_0 = random defective proportion,
$\theta_{1,0}$ = scrap proportion of defective mutual parts,
$d_{T1,0}$ = defective items' production rate (i.e., $d_{T1,,0} = P_{T1,0} x_0$),
$\theta_{2,0}$ = scrap proportion during rework time $t_{2,0}$,
$d_{T2,0}$ = scrapped items' production rate in $t_{2,0}$ (i.e., $d_{T2,0} = P_{T2,0}\theta_{2,0}$),
φ_0 = overall scrap proportion,
$H_{1,0}$ = stock level when uptime ends,
$H_{2,0}$ = stock level when rework ends,
H_i = stock level when each finished product i's uptime ends,
$P_{T1,0}$ = annual production rate with overtime implementation,
$P_{T2,0}$ = annual rework rate with overtime implemented,
$P_{1,0}$ = ordinary production rate,
$P_{2,0}$ = regular rework rate,
$\alpha_{1,0}$ = overtime added output-rate proportion,
$\alpha_{2,0}$ = the setup costs' linking factor,
$\alpha_{3,0}$ = the unit costs' linking factor,
$h_{1,0}$ = unit holding cost,
t_0^* = the sum of optimal uptime and rework time,
$C_{S,0}$ = unit disposal cost,
$C_{TR,0}$ = unit rework cost with overtime implemented,
$C_{R,0}$ = regular unit rework cost,
$h_{2,0}$ = reworked mutual n part's unit holding cost,
$h_{4,0}$ = safety mutual part's unit holding cost,
i_0 = unit holding cost's linking factor (i.e., $h_{1,i} = i_0 C_i$),
S_0 = setup time,

The following are general notation of this study:

n = shipment frequency (i.e., equal-size deliveries),
$t_{n,i}$ = interval of shipments (i.e., fixed time-interval between deliveries),
D_i = fixed shipping quantity,
I_i = number of finished goods left when $t_{n,i}$ ends,
$I(t)_i$ = stock level at time t,
$I_d(t)_i$ = defective stock level at time t,
$I_S(t)_i$ = scrapped items' stock level at time t,
$I_c(t)_i$ = finished goods' stock level at the customer side at time t,
$E[T_A]$ = the expected manufacturing cycle length,
$TC(T_A, n)$ = total system cost per cycle,
$E[TC(T_A, n)]$ = the expected total system cost per cycle,
$E[TCU(T_A, n)]$ = the expected system cost per unit time.

1. Introduction

The present work applies a postponement strategy to a multi-item production-delivery decision making in a vendor-buyer coordinated environment with a quality reassurance, multi-delivery, and overtime on mutual part's fabrication. We aim to help today's producers meet recent demand trends on rapid response, quality, and diversified goods. The postponement strategy delays the end items' differentiation point by making all needed mutual parts in the first phase. Hence, it targets speeding up the multi-item manufacturing time and lowering production costs. Graman and Magazine (2002) examined the influence of capacitated postponement on a single-period multiple-end products inventory system. Upon assembly, certain mutual parts items are completed to end products for meeting demand, and some common parts are postponed their commitment to finished goods. A numerical analysis shows that having a small portion of postponed capacity can be more beneficial than implementing non-postponed or altogether postponed policies. Guericke et al. (2012) developed stochastic global distribution networks incorporating the postponement strategies to deal with demand uncertainties regarding diversities and lead times. The researchers presented a stochastic two-stage model using mixed-integer linear programming. They applied it to apparel industry cases to facilitate production and distribution decision-making in uncertain environments. Mathematical programming software helped them demonstrate the potential benefits of their stochastic postponement model. Budiman and Rau (2019) explored a mixed-integer postponement model in a green supply chain environment aiming at responsive and more environmentally friendly global supply chain operations. Their model incorporated postponement, modularization, and processes for single- and multi-period planning horizons. In addition, it is associated with some speculating scenarios of carbon cap and tax in a green supply chain environment. Computational examples with comparison and extra analyses showed postponement strategies fitted in exploring eco-efficient supply-chain systems. Additional studies explored the impact of different postponement strategies on various manufacturing and supply-chain systems (Krishnan and Ulrich, 2001; Van Mieghem, 2004; Weber, 2008; Chiu et al., 2021).

In phase one of this study, we implement an overtime strategy to efficiently shorten mutual part's uptime and rework time in the proposed postponement multi-item fabrication-delivery model. Singh (2003) investigated the work efficiency for accelerating project-schedule by overmanning and overtime in the construction and architectural environment. The research claimed that the conventional judgment of overtime/overmanning relied on experienced superintendent or foreman, and their decision sometimes is subjective. The efficiency loss is often left out in their calculations. Hence, the research utilized the industry's commonly used standard charts in a sample problem to demonstrate how the superintendent or foreman can correctly and objectively calculate overtime and overmanning. Singer and Obach (2013) built an infinitely repeated game model to analyze overtime time and the needed adjustment of the workforce. Their model assumed workers could decide to work overtime, and the employer could adjust the amount of required personnel. The Nash equilibrium conditions are embedded in the game with the needs of workers-firm collaborative communication. They applied the model to the Chilean smelting plant for empirical tests and revealed the importance of communication in personnel decision-making. Jeunet and Bou Orm (2020) explored the relationship between workforce and quality in manufacturing projects, focusing on accelerated completion by implementing overtime or hiring temporary workers. The researchers specifically investigated retaining minimum quality level and productivity losses due to exceeding workforce in their study. They optimized overtime and short-term work usages using mixed-integer linear programming to simultaneously minimize the system cost, makespan, and overall quality losses for each activity. Their model was applied to real locomotive cases with justification via comparison with existing approaches from the literature. Additional works explored the influence of various overtime plans on planning and management of fabrication systems and supply chains (Lambooij et al., 2007; Conway & Sturges, 2014; Soriano et al., 2020; Keyvanshokooh et al., 2021).

Defeats are inevitable in most fabrication systems. Our model includes a careful screening of defectives and actions of scrapped and reworked to ensure the desired product quality. Glock and Jaber (2013) examined a multi-stage fabrication-inventory model featuring learning effects, scrap, and rework. Defeats exist at each stage of their serial manufacturing line. The rework processes help repair some defective items and items that fail after rework are discarded. The learning and forgetting effects are assumed in the manufacturing and remanufacturing stages. The researchers developed a multi-stage production-inventory system and used four different measures (i.e., uptime, in-process stock level, frequency of shipment, and process yield) to evaluate the aggregate performance of the studied system. When applying the model to real applications, the practitioners can weigh each measure according to its importance. Their results include the impact of learning/forgetting rates and weight of each measurement on system overall performance. Ullah and Sarkar (2020) considered the selection of recovery-channel in a hybrid production-reproduction model with product quality and radio frequency identification (RFID). The researchers first explored the reasons for the low return rate on disposed used electronic products and then developed a dual return channel hybrid production-reproduction model based on RFID to boost the recycling rate. Next, they built a math model incorporating the RFID recovery channel's implementation costs. Various selections of RFID return channels are studied and compared to offer in-depth information to help designers and managers make decisions. Additional studies focused on impact of different quality matters and reassurances on manufacturing planning and operations (Moussawi-Haidar et al., 2016; Polotski et al., 2019; Sahebi et al., 2019; Son & Van Hop, 2021; Abukhader & Onbaşıoğlu, 2021; Duan et al., 2021). In a vendor-buyer coordinated business environment, buyers often request their orders to be filled in a fixed quantity multi-delivery plan. Jha and Shanker (2013) considered an integrated fabrication-inventory model featuring single vendor, multi-buyer, service level constraints, and controllable lead time. Buyers' demands follow normal distribution independently,

and lead times can be shortened by adding the crash cost. Continuous review and backorder policies under services level constraints are used. The researchers built a model and applied the Lagrangian multiplier method to jointly find the optimal order size, lead time, and shipment frequency that minimizes combined expected vendor-buyers system cost. They also used an example to illustrate how their model works. Nogueira et al. (2020) examined the production-delivery problem featuring parallel batching machines to increase buyer satisfaction and maximize profits. Generic sizes and fabricating jobs are scheduled on identical parallel machines to meet order due time and boost total profits. The researchers proposed a math formulation under constraints to enhance the performance of the commercial solver. In addition, they developed two polynomial methods to obtain relaxed and feasible solutions separately. Finally, the researchers demonstrated that their algorithms' effective and efficient capability fit practitioner's daily usage. Additional studies focused on impact of various product shipping plans on supply-chain planning and operations (Mabrouk, 2020; Moin et al., 2020; Tran et al., 2020; Sumrit, 2020; Farmand et al., 2021; Singamsetty and Thenepalle, 2021). This work applies a postponement strategy to a multi-item fabrication-shipment decision making in a vendor-buyer coordinated environment with multi-delivery, quality reassurance, and overtime. Since little previous works focused on this area, we aim to fill the gap.

2. Problem statement and mathematical modeling

We propose a two-stage scheme to explore the problem with fabricating the necessary common parts in the 1st stage and making the client's requirement end items in the 2nd stage. We assume a constant mutual part's completion proportion γ and production rate $P_{1,0}$. We implement an overtime option to increase common parts' output rate by $\alpha_{1,0}$ to $P_{T1,0}$ to reduce its production uptime. The following formula explicitly explains its relationship:

$$P_{T1,0} = P_{1,0}\left(1+\alpha_{1,0}\right).\tag{1}$$

The consequent production unit and setup costs related to overtime implementation are given below.

$$C_{T0} = \left(1+\alpha_{3,0}\right)C_0,\tag{2}$$

$$K_{T0} = \left(1+\alpha_{2,0}\right)K_0,\tag{3}$$

where $\alpha_{3,0}$ and $\alpha_{2,0}$ are the factors linked to the regular unit and setup costs. L finished goods have constant demand rates λ_i (where $i = 1, 2, …, L$) and fabricating rates $P_{1,i}$ depending on the mutual part's completion rate γ. If $\gamma = 0.5$, for example, then $P_{1,0}$ and $P_{1,i}$ both are double their regular production rates as in a production scheme with single stage.

During the production processes, defective proportion x_0 and x_i are randomly produced. Among them, $\theta_{1,0}$ and $\theta_{1,i}$ proportion are scrapped. The other $(1 - \theta_{1,0})$ and $(1 - \theta_{1,i})$ items are rework-able. The annual rework rates respectively are $P_{T2,0}$ (also with overtime implemented) and $P_{2,i}$. The following formula explains $P_{T2,0}$'s relationship with the regular rework rate $P_{2,0}$:

$$P_{T2,0} = \left(1+\alpha_{1,0}\right)P_{2,0}.\tag{4}$$

The consequent unit rework cost relating to overtime implementation is given below:

$$C_{TR,0} = \left(1+\alpha_{3,0}\right)C_{R,0}.\tag{5}$$
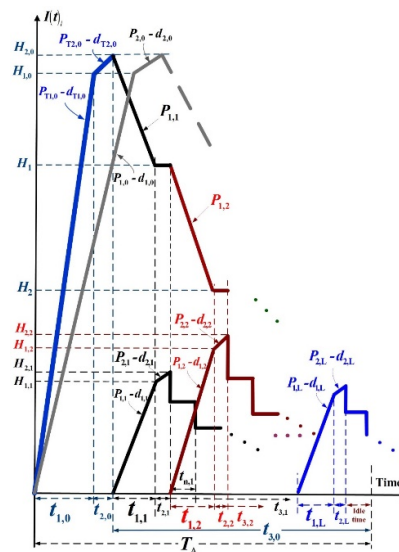
Fig. 1. Stock level of this study as compared to the same system with no overtime (in grey)

Assuming $\theta_{2,0}$ and $\theta_{2,i}$ proportions of the reworked items are scrapped for they fail the repair process. Fig. 1 depicts the stock level of this study. When stage one's uptime ends, the stock level rises to $H_{1,0}$. Then, it reaches $H_{2,0}$ when stage one's rework time ends. Starting stage two, mutual parts' stocks begin to deplete as the finished goods' fabrication starts. Meantime, when each finished goods' uptime ends, finished goods' stock rises to $H_{1,i}$ (see Fig. 1). Then, the stock increases to $H_{2,i}$ when the rework ends. Moreover, this study doesn't allow shortages, so we must have the following relationships $(P_{1,i} - d_{1,i} - \lambda_i > 0)$ and $(P_{T1,0} - d_{T1,0} - \lambda_0 > 0)$. Fig. 2 and Fig. 3 exhibit separately the defective and scrapped stock levels. Fig. 2 indicates that when $t_{1,0}$ and $t_{1,i}$ end, the defective mutual parts and finished goods rise to $(d_{T1,0} t_{1,0})$ and $(d_{1,i} t_{1,i})$. Then, upon removing the scraps, the common part's and end product's stock levels gradually deplete to zero in the rework processes. Fig. 3 illustrates that $[d_{1,i}t_{1,i}(\theta_{1,i}) + d_{2,i} t_{2,i}]$ and $[d_{T1,0}t_{1,0}(\theta_{1,0}) + d_{T2,,0} t_{2,0}]$ and are the maximal scrapped levels in stages 2 and 1 of this study.
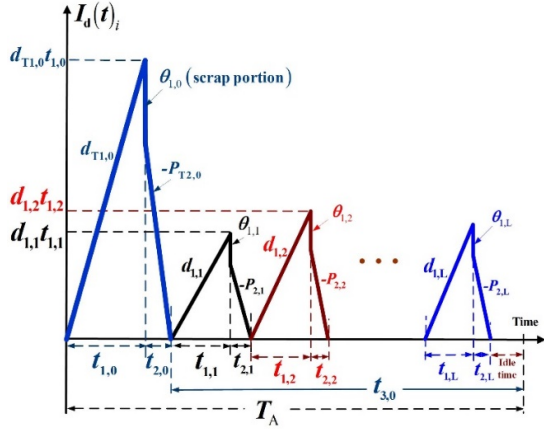

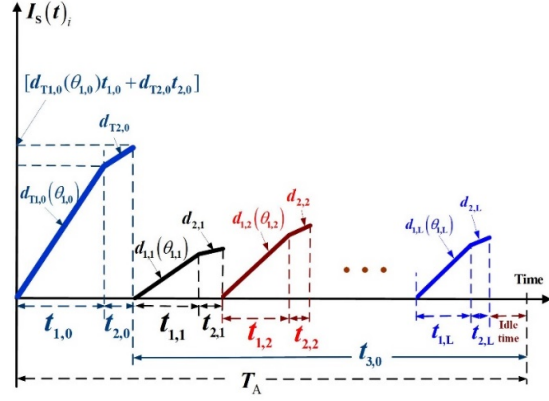**Fig. 2.** Defective stock levels of this study

Fig. 3. Stock level of scrapped items in the proposed problem

Fig. 1 shows that when each end product's rework time ends, its stock level reaches $H_{2,i}$, and end product's n equal-size shipments start in $t_{3,i}$ (see Fig. 4). The total inventories during $t_{3,i}$ is shown in Eq. (6):

$$\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right) H_{2,i}\left(t_{3,i}\right) = \left(\frac{1}{n^2}\right) H_{2,i}\left[\frac{n(n-1)}{2}\right]\left(t_{3,i}\right) = \left(\frac{n-1}{2n}\right) H_{2,i}\left(t_{3,i}\right). \tag{6}$$

Fig. 5 illustrates customer side's stock level. Each product i's total inventories at the customer side are expresses in Eq. (7):

$$\left[\frac{n\left(D_i - I_i\right)t_{n,i}}{2} + \frac{n\left(n+1\right)}{2}I_i t_{n,i} + \frac{nI_i\left(t_{1,i} + t_{2,i}\right)}{2}\right], \tag{7}$$

where

$$t_{n,i} = \frac{t_{3,i}}{n}, \tag{8}$$

$$D_i = \frac{H_{2,i}}{n}, \tag{9}$$

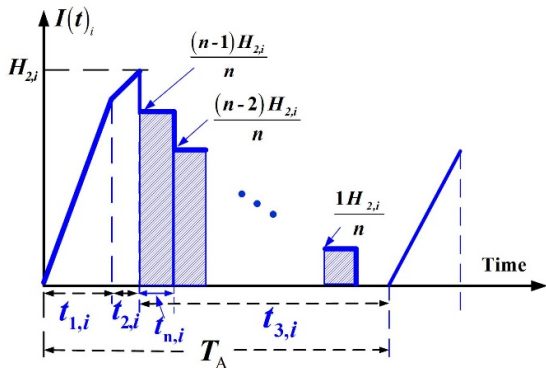$$I_i = D_i - \lambda_i\left(t_{n,i}\right), \tag{10}$$

Fig. 4. End product i's stock level in its delivery time $t_{3,i}$
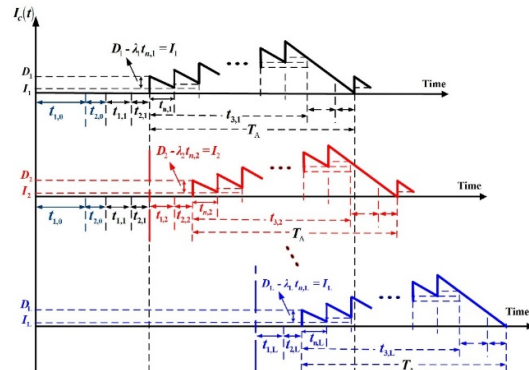

Fig. 5. Customer side's stock level of finished item i

2.1. Formulas in finished goods' fabrication

For $i = 1, 2, \ldots, L$, we observe the following equations according to the problem statement with figures above:

$$Q_i = \frac{\lambda_i T_A}{1 - \varphi_i x_i}, \tag{11}$$

$$t_{1,i} = \frac{H_{1,i}}{P_{1,i} - d_{1,i}} = \frac{Q_i}{P_{1,i}}, \tag{12}$$

$$H_{1,i} = t_{1,i}\left(P_{1,i} - d_{1,i}\right), \tag{13}$$

$$t_{2,i} = \frac{H_{2,i} - H_{1,i}}{P_{2,i} - d_{2,i}} = \frac{x_i Q_i}{P_{2,i}}, \tag{14}$$

$$H_{2,i} = \left(P_{2,i} - d_{2,i}\right)t_{2,i} + H_{1,i}, \tag{15}$$

$$T_A = \frac{Q_i\left(1 - x_i\varphi_i\right)}{\lambda_i} = t_{1,i} + t_{2,i} + t_{3,i}, \tag{16}$$

$$t_{3,i} = T_A - \left(t_{1,i} + t_{2,i}\right), \tag{17}$$

$$d_{1,i} = P_{1,i}x_i, \tag{18}$$

$$d_{2,i} = P_{2,i}\theta_{2,i}, \tag{19}$$

$$\varphi_i = \theta_{1,i} + \left(1 - \theta_{1,i}\right)\theta_{2,i}. \tag{20}$$

The total requirements of mutual parts are (refer to Eq. (11)):

$$H_{2,0} = \sum_{i=1}^{L} Q_i = \sum_{i=1}^{L} \frac{\lambda_i T_A}{1 - \varphi_i x_i}. \tag{21}$$

2.2. Formulas in mutual parts' fabrication

According to the problem statement, Eq. (21), and Figures shown above, the following formulas are directly gained:

$$Q_0 = \frac{H_{2,0}}{1 - \varphi_0 x_0}, \tag{22}$$

$$t_{1,0} = \frac{H_{1,0}}{P_{T1,0} - d_{T1,0}} = \frac{Q_0}{P_{T1,0}}, \tag{23}$$

$$H_{1,0} = t_{1,0}\left(P_{T1,0} - d_{T1,0}\right), \tag{24}$$

$$t_{2,0} = \frac{H_{2,0} - H_{1,0}}{P_{T2,0} - d_{T2,0}} = \frac{Q_0\left(1 - \theta_{1,0}\right)x_0}{P_{T2,0}}, \tag{25}$$

$$H_{2,0} = \left(P_{T2,0} - d_{T2,0}\right)t_{2,0} + H_{1,0}, \tag{26}$$

$$\lambda_0 = \frac{\sum_{i=1}^{L} Q_i}{T_A}, \tag{27}$$

$$d_{T1,0} = P_{T1,0}x_0, \tag{28}$$

$$d_{T2,0} = P_{T2,0}\theta_{2,0}, \tag{29}$$

$$\varphi_0 = \theta_{1,0} + \left(1 - \theta_{1,0}\right)\theta_{2,0}, \tag{30}$$

$$T_A = t_{1,0} + t_{2,0} + t_{3,0}, \tag{31}$$

$$H_1 = H_{2,0} - Q_1, \tag{32}$$

$$H_i = H_{(i-1)} - Q_i \ , \ for \ i = 2, \ 3, \ ..., L \tag{33}$$

$$H_L = H_{(L-1)} - Q_L = 0 \ . \tag{34}$$

3. System cost analysis and optimization process

3.1. Cost analysis

$TC(T_A, n)$, the total cost per cycle consists of both stages' cost relating to (1) setup, (2) variable, (3) rework/disposal, (4) end products' delivery, and (5) vendor's and buyer's stock holding. So, it includes the follows:

$$TC(T_A, \ n) = \left\{ \begin{array}{l} C_{T0}Q_0 + K_{T0} + C_{TR,0}\left(1-\theta_{1,0}\right)Q_0 x_0 + h_{2,0}\left(t_{2,0}\right)\left[\dfrac{d_{T1,0}\left(1-\theta_{1,0}\right)t_{1,0}}{2}\right] + h_{4,0}\varphi_0\left(x_0 Q_0\right)T_A \\ \\ +C_{S,0}\varphi_0\left(Q_0 x_0\right) + h_{1,0}\left[\dfrac{H_{1,0}t_{1,0}}{2} + \dfrac{d_{T1,0}t_{1,0}}{2}\left(t_{1,0}\right) + \dfrac{H_{2,0}+H_{1,0}}{2}\left(t_{2,0}\right) + \sum_{i=1}^{L}\left[H_i\left(t_{1,i}+t_{2,i}\right)+\dfrac{Q_i}{2}\left(t_{1,i}\right)\right]\right] \end{array} \right\}$$

$$+\sum_{i=1}^{L}\left\{ \begin{array}{l} C_i Q_i + K_i + C_{R,i}\left(1-\theta_{1,i}\right)Q_i x_i + C_{S,i}\varphi_i\left(Q_i x_i\right) + nK_{D,i} + C_{D,i}\left(1-\varphi_i x_i\right)Q_i \\ \\ +h_{1,i}\left[\dfrac{d_{1,i}t_{1,i}}{2}\left(t_{1,i}\right)+\dfrac{H_{1,i}t_{1,i}}{2}+\dfrac{H_{2,i}+H_{1,i}}{2}\left(t_{2,i}\right)+H_{2,i}\left(\dfrac{n-1}{2n}\right)\left(t_{3,i}\right)\right]+h_{4,i}\varphi_i\left(x_i Q_i\right)T_A \\ \\ +h_{2,i}\left(t_{2,i}\right)\left[\dfrac{d_{1,i}t_{1,i}\left(1-\theta_{1,i}\right)}{2}\right]+h_{3,i}\left[\dfrac{n(n+1)}{2}I_i t_{n,i}+\dfrac{n(D_i-I_i)t_{n,i}}{2}+\dfrac{nI_i\left(t_{1,i}+t_{2,i}\right)}{2}\right] \end{array} \right\} \tag{35}$$

After extra derivation (see Appendix A), the following $E[TCU(T_A, n))]$ is gained:

$$E\left[TCU\left(T_A, \ n\right)\right] = \lambda_0 E_{00} C_0\left(1+\alpha_{3,0}\right)+\dfrac{K_0\left(1+\alpha_{2,0}\right)}{T_A}+C_{R,0}E_{10}\left(1-\theta_{1,0}\right)\left(1+\alpha_{3,0}\right)\lambda_0$$

$$+C_{S,0}\lambda_0\varphi_0 E_{10}+h_{2,0}\left(E_{10}\right)^2\lambda_0^2 T_A\left[\dfrac{\left(1-\theta_{1,0}\right)^2}{2P_{2,0}\left(1+\alpha_{1,0}\right)}\right]+h_{4,0}\varphi_0 E_{10}\lambda_0 T_A+h_{1,0}\sum_{i=1}^{L}\left(\dfrac{1}{2P_{1,i}}\right)\left(E_{0i}\right)^2\lambda_i^2 T_A$$

$$+h_{1,0}\left\{\left(\dfrac{1}{2}\right)\lambda_0^2 E_{0P}T_A\left(E_{00}\right)^2+\sum_{i=1}^{L}\left(-\lambda_i T_A E_{iP}\right)\left[\sum_{j=1}^{i}\left(\lambda_j E_{0j}\right)\right]+\left[\sum_{i=1}^{L}\left(\lambda_i E_{0i}\right)\right]\cdot\sum_{i=1}^{L}\left(\lambda_i T_A E_{iP}\right)\right\} \tag{36}$$

$$+\sum_{i=1}^{L}\left\{ \begin{array}{l} C_i\left(E_{0i}\lambda_i\right)+\dfrac{K_i}{T_A}+C_{R,i}\left[\left(\lambda_i E_{1i}\right)\left(1-\theta_{1,i}\right)\right]+C_{S,i}\varphi_i\left(\lambda_i E_{1i}\right)+\dfrac{nK_{D,i}}{T_A}+C_{D,i}\lambda_i+\dfrac{h_{1,i}\lambda_i^2 E_{2i}}{2}T_A \\ \\ +h_{4,i}\varphi_i\lambda_i E_{1i}T_A+h_{2,i}\left[\dfrac{\lambda_i^2\left(1-\theta_{1,i}\right)^2\left(E_{1i}\right)^2 T_A}{2P_{2,i}}\right]+\dfrac{h_{3,i}\lambda_i^2 E_{iP}T_A}{2}+\dfrac{h_{3,i}-h_{1,i}}{2n}\left(\lambda_i^2 T_A\right)\left(\dfrac{1}{\lambda_i}-E_{iP}\right) \end{array} \right\}$$

3.2. The optimization process

Apply the Hessian Matrix Equations to $E[TCU(T_A, n)]$ (Rardin, 1998):

$$\left[T_A \quad n\right]\cdot\left(\begin{array}{cc} \dfrac{\partial^2 E\left[TCU\left(T_A, \ n\right)\right]}{\partial T_A^{\ 2}} & \dfrac{\partial^2 E\left[TCU\left(T_A, \ n\right)\right]}{\partial T_A \partial n} \\ \\ \dfrac{\partial^2 E\left[TCU\left(T_A, \ n\right)\right]}{\partial T_A \partial n} & \dfrac{\partial^2 E\left[TCU\left(T_A, \ n\right)\right]}{\partial n^2} \end{array}\right)\cdot\left[\begin{array}{c} T_A \\ n \end{array}\right]=\sum_{i=1}^{L}\left\{\dfrac{2K_i}{T_A}\right\}+\dfrac{2K_0\left(1+\alpha_{2,0}\right)}{T_A}>0 \tag{37}$$

Eq. (37) yields positive, since $(1 + \alpha_{2,0})$, K_0, K_i, and T_A are positive. It confirms that $E[TCU(T_A, n)]$ is strictly convex for all n and T_A values > 0. Hence, the minimum of $E[TCU(T_A, n)]$ exists. By setting first-derivatives of $E[TCU(T_A, n)]$ regarding n and T_A equal to zero to simultaneously decide T_A* and $n*$.

$$\dfrac{\partial E\left[TCU\left(T_A, \ n\right)\right]}{\partial n}=\sum_{i=1}^{L}\left[\dfrac{K_{D,i}}{T_A}-\dfrac{\left(h_{3,i}-h_{1,i}\right)}{2n^2}\left(\lambda_i^2 T_A\right)\left(\dfrac{1}{\lambda_i}-E_{iP}\right)\right]=0 \tag{38}$$

$$\frac{\partial E[TCU(T_A, n)]}{\partial T_A} = \left\{ \begin{array}{l} \dfrac{-K_0\left(1+\alpha_{2,0}\right)}{T_A^2} + h_{2,0}\left(E_{10}\right)^2 \lambda_0^{\,2}\left[\dfrac{\left(1-\theta_{1,0}\right)^2}{2P_{2,0}\left(1+\alpha_{1,0}\right)}\right] + h_{4,0}\varphi_0 E_{10}\lambda_0 \\[3mm] +h_{1,0}\sum\limits_{i=1}^{L}\left(\dfrac{1}{2P_{1,i}}\right)\left(E_{0i}\right)^2\lambda_i^{\,2} + \sum\limits_{i=1}^{L}\left\{\dfrac{-\left(K_i+nK_{D,i}\right)}{T_A^2}\right\} \\[3mm] +h_{1,0}\left\{\left(\dfrac{\lambda_0^{\,2}}{2}\right)E_{0P}\left(E_{00}\right)^2 - \sum\limits_{i=1}^{L}\left(\lambda_i E_{iP}\right)\left[\sum\limits_{j=1}^{i}\left(\lambda_j E_{0j}\right)\right] + \sum\limits_{i=1}^{L}\left(\lambda_i E_{iP}\right)\left[\sum\limits_{i=1}^{L}\left(\lambda_i E_{0i}\right)\right]\right\} \\[3mm] +\sum\limits_{i=1}^{L}\left\{\begin{array}{l}\dfrac{h_{1,i}\lambda_i^{\,2}E_{2i}}{2} + h_{4,i}\varphi_i\lambda_i E_{1i} + h_{2,i}\left[\dfrac{\left(1-\theta_{1,i}\right)^2\left(E_{1i}\right)^2\lambda_i^{\,2}}{2P_{2,i}}\right] \\[3mm] +\dfrac{h_{3,i}\lambda_i^{\,2}E_{iP}}{2} + \dfrac{h_{3,i}-h_{1,i}}{2n}\left(\lambda_i^{\,2}\right)\left(\dfrac{1}{\lambda_i}-E_{iP}\right)\end{array}\right\} \end{array}\right\} = 0. \tag{39}$$

Solving linear system of Eqs. (38) and (39)), we obtain optimal policies of $T_A{}^*$ and n^* as follows:

$$T_A^* = \sqrt{\dfrac{2\left[K_0\left(1+\alpha_{2,0}\right)+\sum\limits_{i=1}^{L}\left(K_i+nK_{D,i}\right)\right]}{\begin{array}{l}h_{2,0}\left(E_{10}\right)^2\lambda_0^{\,2}\left[\dfrac{\left(1-\theta_{1,0}\right)^2}{\left(1+\alpha_{1,0}\right)P_{2,0}}\right]+2h_{4,0}\varphi_0 E_{10}\lambda_0 + h_{1,0}\sum\limits_{i=1}^{L}\left(\dfrac{\lambda_i^{\,2}}{P_{1,i}}\right)\left(E_{0i}\right)^2 \\[3mm] +h_{1,0}\left\{\lambda_0^{\,2}E_{0P}\left(E_{00}\right)^2 - 2\sum\limits_{i=1}^{L}\left(E_{iP}\lambda_i\right)\left[\sum\limits_{j=1}^{i}\left(E_{0j}\lambda_j\right)\right] + 2\left[\sum\limits_{i=1}^{L}\left(E_{0i}\lambda_i\right)\right]\cdot\sum\limits_{i=1}^{L}\left(E_{iP}\lambda_i\right)\right\} \\[3mm] +\sum\limits_{i=1}^{L}\left\{h_{1,i}E_{2i}\lambda_i^{\,2} + 2h_{4,i}\lambda_i\varphi_i E_{1i} + \dfrac{h_{2,i}\lambda_i^{\,2}\left(1-\theta_{1,i}\right)^2\left(E_{1i}\right)^2}{P_{2,i}} + h_{3,i}E_{iP}\lambda_i^{\,2} + \left(\dfrac{1}{\lambda_i}-E_{iP}\right)\dfrac{\lambda_i^{\,2}\left(h_{3,i}-h_{1,i}\right)}{n}\right\}\end{array}}}, \tag{40}$$

and

$$n^* = \sqrt{\dfrac{\left[K_0\left(1+\alpha_{2,0}\right)+\sum\limits_{i=1}^{L}\left(K_i\right)\right]\cdot\sum\limits_{i=1}^{L}\left[\left(\dfrac{1}{\lambda_i}-E_{iP}\right)\left(\lambda_i^{\,2}\right)\left(h_{3,i}-h_{1,i}\right)\right]}{\left(\sum\limits_{i=1}^{L}K_{D,i}\right)\left\{\begin{array}{l}h_{2,0}\left(E_{10}\right)^2\lambda_0^{\,2}\left[\dfrac{\left(1-\theta_{1,0}\right)^2}{\left(1+\alpha_{1,0}\right)P_{2,0}}\right]+2h_{4,0}\varphi_0 E_{10}\lambda_0 + h_{1,0}\sum\limits_{i=1}^{L}\left(\dfrac{1}{P_{1,i}}\right)\left(E_{0i}\right)^2\lambda_i^{\,2} \\[3mm] +h_{1,0}\left\{\lambda_0^{\,2}\left(E_{00}\right)^2 E_{0P} - 2\sum\limits_{i=1}^{L}\left(E_{iP}\lambda_i\right)\left[\sum\limits_{j=1}^{i}\left(E_{0j}\lambda_j\right)\right] + 2\left[\sum\limits_{i=1}^{L}\left(E_{0i}\lambda_i\right)\right]\cdot\sum\limits_{i=1}^{L}\left(E_{iP}\lambda_i\right)\right\} \\[3mm] +\sum\limits_{i=1}^{L}\left\{h_{1,i}E_{2i}\lambda_i^{\,2} + 2h_{4,i}\lambda_i\varphi_i E_{1i} + \dfrac{h_{2,i}\lambda_i^{\,2}\left(1-\theta_{1,i}\right)^2\left(E_{1i}\right)^2}{P_{2,i}} + h_{3,i}E_{iP}\lambda_i^{\,2}\right\}\end{array}\right\}}}. \tag{41}$$

4. Numerical example

A simulated example demonstrates the capability and applicability of our model. Suppose 5 distinct client-required products must be satisfied by a vendor-buyer coordinated system with the postponement, overtime option, and quality reassurance. The relating system parameters' values are assumed as shown in Tables 1, 2(a), and 2(b). In comparison, Table B-1(a) and (b) (in Appendix B) show the parameters' values in its corresponding single-stage scheme.

Table 1
The parameters' values in stage one

C_0	$P_{1,0}$	λ_0	K_0	γ	$C_{S,0}$	δ	$\alpha_{2,0}$	$h_{1,0}$	$\alpha_{1,0}$
$40	120000	17406	$8500	0.5	$10	0.5	0.1	$8	0.5
$C_{R,0}$	$P_{2,0}$	$h_{4,0}$	x_0	$\theta_{1,0}$	i_0	$\theta_{2,0}$	φ_0	$h_{2,0}$	$\alpha_{3,0}$
$25	96000	$8	2.5%	0.046	0.2	0.046	0.09	$8	0.25

Table 2(a)

The assumed values for stage two's parameters (1 of 2)

Product i	C_i	$P_{1,i}$	K_i	$h_{4,i}$	$h_{1,i}$	λ_i	$P_{2,i}$	$h_{3,i}$	$h_{2,i}$
1	$40	112258	$8500	$16	$16	3000	89806	$70	$16
2	$50	116066	$9000	$18	$18	3200	92852	$75	$18
3	$60	120000	$9500	$20	$20	3400	96000	$80	$20
4	$70	124068	$10000	$22	$22	3600	99254	$85	$22
5	$80	128276	$10500	$24	$24	3800	102621	$90	$24

Table 2(b)

The assumed values for stage two's parameters (2 of 2)

Product i	$K_{D,i}$	x_i	$C_{S,i}$	$\theta_{1,i}$	$C_{D,i}$	$\theta_{2,i}$	φ_i	$C_{R,i}$
1	$1800	2.5%	$10	0.046	$0.1	0.046	0.09	$25
2	$1900	7.5%	$15	0.094	$0.2	0.094	0.18	$30
3	$2000	12.5%	$20	0.146	$0.3	0.146	0.27	$35
4	$2100	17.5%	$25	0.200	$0.4	0.200	0.36	$40
5	$2200	22.5%	$30	0.258	$0.5	0.258	0.45	$45

We apply Eqs. (40), (41), and (36) to obtain $T_A^* = 0.5299$, $n^* = 4$, and $E[TCU(T_A^*, n^*)] = \$2,364,584$. $E[TCU(T_A, n)]$'s convexity relating to T_A and n is explicitly demonstrated in Fig. 6. It illustrates $E[TCU(T_A, n)]$ remarkably increases as both n and T_A deviate from their optimal points.
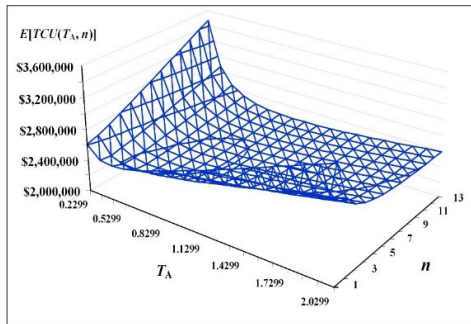

**Fig. 6.** Convexity of $E[TCU(T_A, n)]$ regarding T_A and n
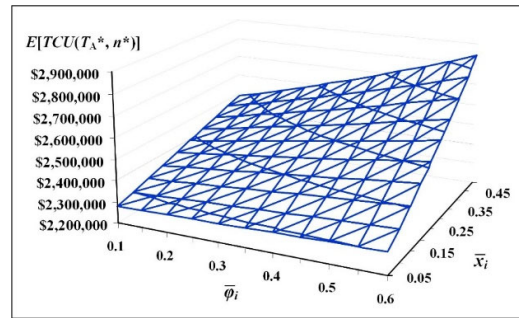
Fig. 7. Behavior of $E[TCU(T_A^*, n^*)]$ relating to the mean φ_i and x_i

4.1. Collective effect of key system factors on the problem

The collective effect of mean scrap and nonconforming proportions on $E[TCU(T_A^*, n^*)]$ is depicted in Fig. 7. As both mean φ_i and x_i rise, $E[TCU(T_A^*, n^*)]$ significantly surges. It discloses the average x_i value has greater influence than the average φ_i on $E[TCU(T_A^*, n^*)]$. Fig. 8 depicts the combined effect of mean nonconforming and scrap proportions on the optimal T_A^*. As both mean x_i and φ_i increase, T_A^* considerably declines. It exposes that the changing of n^* value causes T_A^* to have a sharp drop.
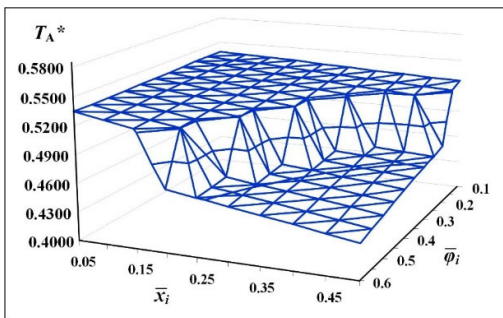
Fig. 8. The combined effect of mean x_i and φ_i on T_A^*
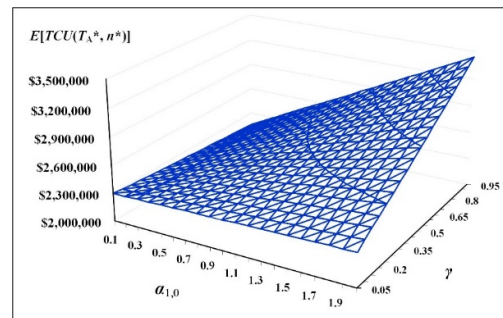
Fig. 9. The behavior of $E[TCU(T_A^*, n^*)]$ relating to $\alpha_{1,0}$ and γ

The analytical results of joint influence of overtime added proportion $\alpha_{1,0}$ and common component's completion rate γ on $E[TCU(T_A^*, n^*)]$ are exposed in Fig. 9. As both γ and $\alpha_{1,0}$ rise, $E[TCU(T_A^*, n^*)]$ noticeably surges. Fig. 10 discloses the collective effect of common component's completion rate γ and overtime added proportion $\alpha_{1,0}$ on the optimal cycle length T_A^*. As both γ and $\alpha_{1,0}$ rise, T_A^* remarkably drops. It also shows the changing of n^* value causes T_A^* to have a sharp
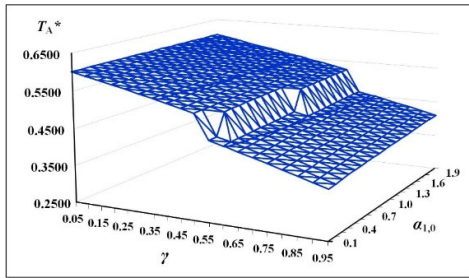
decline.

Fig. 10. Collective effect of γ and $\alpha_{1,0}$ on T_A^*
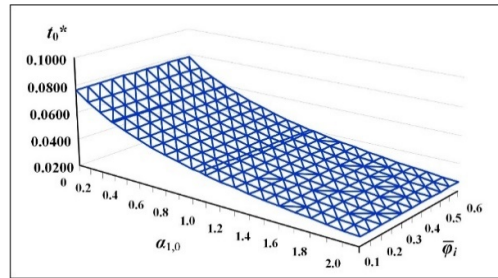

Fig. 11. The behavior of t_0^* relating to $\alpha_{1,0}$ and the mean φ_i

Fig. 11 discloses the combined impact of $\alpha_{1,0}$ and the mean φ_i on t_0^*. As $\alpha_{1,0}$ increases, t_0^* declines significantly, and as mean φ_i rises, t_0^* decreases slightly.

4.2. The effect of the key system feature on the problem

Our model can also analyze individual system feature on the problem. Fig. 12 disposes the impact of the shipment frequency on main contributors to $E[TCU(T_A^*, n^*)]$. It confirms that at $n^* = 4$, $E[TCU(T_A^*, n^*)] = \$2,364,584$. As n deviates from n^*, $E[TCU(T_A^*, n^*)]$ increases in both directions. As n rises, (i) the total delivery cost upsurges due to the number of fixed delivery cost increases; (ii) the number of stocks per shipment D_i decreases (refer to Eq. (9)); hence, the buyer's holding cost declines; and (iii) on the contrary, the vendor's holding cost surges (see Eq. (7)).
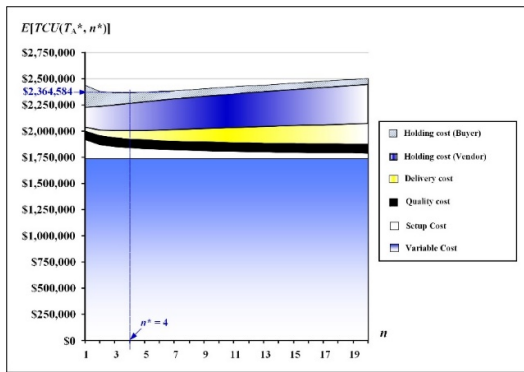

Fig. 12. The effect of frequency of delivery n on key cost contributors in $E[TCU(T_A^*, n^*)]$
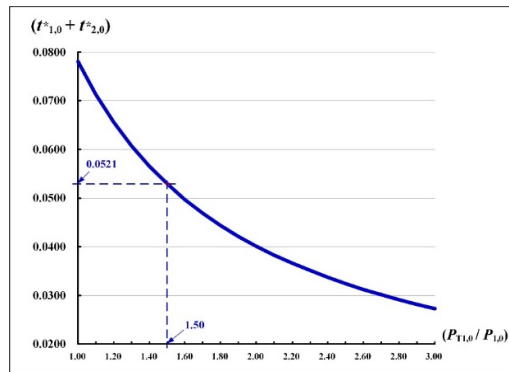

Fig. 13. The effect of ratio $(P_{T1,0} / P_{1,0})$ on $(t^*_{1,0} + t^*_{2,0})$

Fig. 13 shows the impact of $(P_{T1,0} / P_{1,0})$ ratio on $(t^*_{1,0} + t^*_{2,0})$. It tells that $(t^*_{1,0} + t^*_{2,0})$ declines to 0.0521 when overtime ratio sets at 1.5 (50% more outputs with overtime option). Fig. 14 exposes the effect of ratio $(P_{T1,0} / P_{1,0})$ on utilization. It exposes the utilization falls to 0.2521 from 0.3012. A 16.31% drop because of implementing an additional 50% overtime outputs.
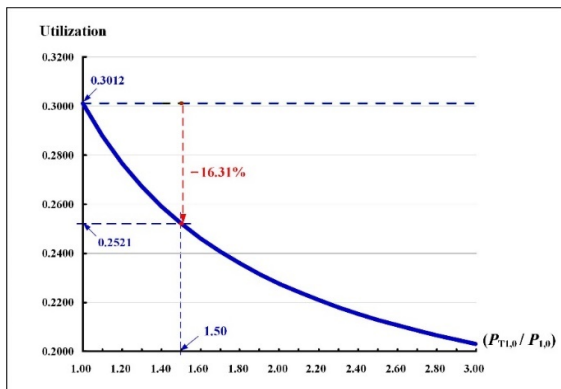

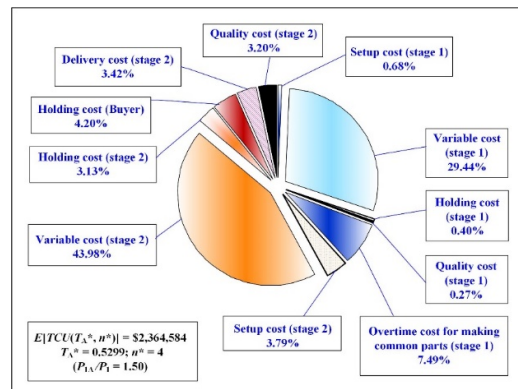**Fig. 14.** The effect of ratio $(P_{T1,0} / P_{1,0})$ on utilization

Fig. 15. Breakup of $E[TCU(T_A^*, n^*)]$

Fig. 15 exhibits the breakup of distinct contributors of $E[TCU(T_A{}^*, n^*)]$. It discloses two major cost components: the variable costs in stages two and one, each contributes 43.98% and 29.44%. The overtime relevant cost of 7.49% is the third large contributor. It follows that the buyer's holding cost 4.20%, the end-items' setup and delivery cost 3.79% and 3.42%. Then, the overall quality relevant cost 3.20%. Furthermore, our model can investigate other detailed influences of system features on the problem. Fig. 16 exhibits the impact of $(P_{T1,0}/P_{1,0})$ ratio on the stage two's variable costs for finished goods. It points out that $(P_{T1,0}/P_{1,0})$ has insignificant effect on variable cost of each finished goods.
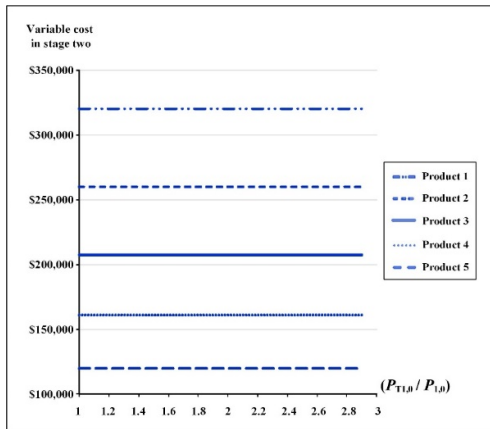

Fig. 16. The $(P_{T1,0}/P_{1,0})$ ratio's impact on variable costs of finished goods
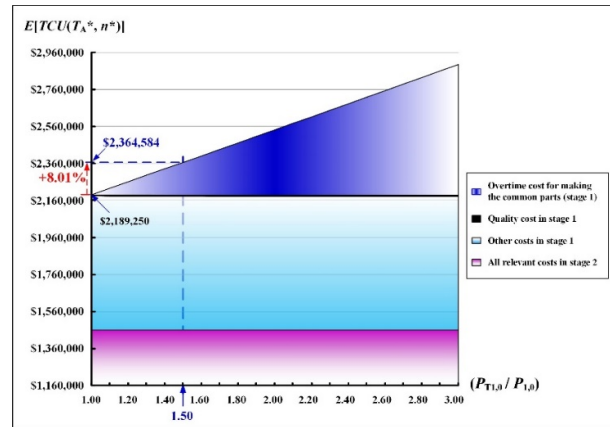

Fig. 17. The effect of $(P_{T1,0}/P_{1,0})$ on distinct cost contributors of $E[TCU(T_A{}^*, n^*)]$

Fig. 17 discloses the impact of $(P_{T1,0}/P_{1,0})$ ratio on major cost contributors in $E[TCU(T_A{}^*, n^*)]$. It reconfirms for $(P_{T1,0}/P_{1,0})$ at 1.5, our optimal $E[TCU(T_A{}^*, n^*)] = \$2,364,584$, and indicates that the significant impact $(P_{T1,0}/P_{1,0})$ ratio on the overtime cost for making mutual parts. Our postponement model allows production managers to analyze the influence of different relationships of γ and its associating values δ on $E[TCU(T_A{}^*, n^*)]$. The results are demonstrated in Fig. 18. It confirms for a linear relationship with $\gamma = 0.5$, $E[TCU(T_A{}^*, n^*)] = \$2,364,584$. The proposed model can always help reveal the optimal operating policy and system cost for any nonlinear relationships as may exist in real application systems.
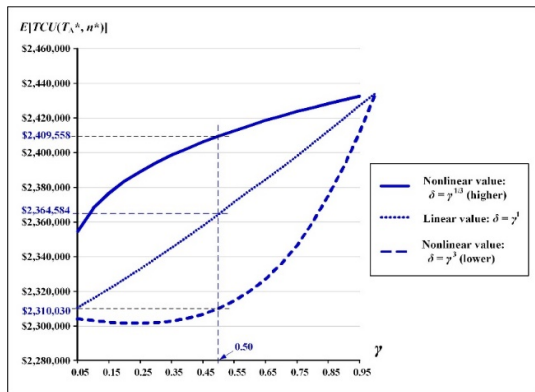

Fig. 18. The behavior of $E[TCU(T_A{}^*, n^*)]$ relating to different relationships of γ and δ
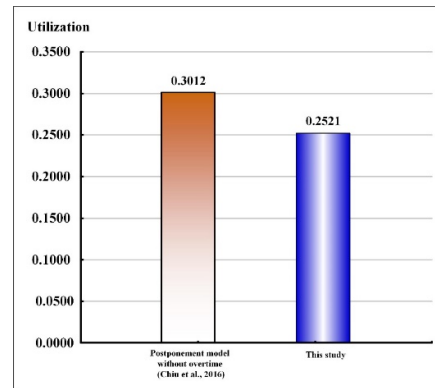
Fig. 19. Comparison of this study's utilization with that in an existing work without overtime

4.3. Comparison

Fig. 19 compares our utilization with that of a previous work (without considering overtime; Chiu et al., 2016). It indicates by implementing the overtime for making the common component, our utilization (i.e., $(t_0{}^* + t_i{}^*) / T_A{}^*$) declines to 0.2521, a 16.31% decrease from 0.3012. The price paying is a surge of 8.01% in $E[TCU(T_A{}^*, n^*)]$, that is from \$2,189,250 (Chiu et al., 2016) to our \$2,364,584.

5. Conclusions

To meet the recent client demand trend on rapid response, quality, and diversified goods, manufacturers today must seek a multi-item fabrication-delivery scheme to retain desirable quality, respond promptly to orders, boost machine utilization, and save operating costs. Motivated by helping producers, this study successfully built a postponement multi-item fabrication-shipment model with overtime and quality reassurance. In addition, we obtain the system cost through modeling, formulation, and analytical derivations (as shown in Sections 2 and 3). Lastly, by applying the differential calculus (i.e., the Hessian matrix equations) to minimize the system cost, we simultaneously determine the optimal fabricating cycle and shipping frequency.

Then, we utilize an example to numerically illustrate how our model works and its capability in exposing various previously inaccessible information to the problem. Examples of such decision-relevant information include (1) confirmation of the convexity of the system cost (see Fig. 6). (2) Exploring the collective influence of different system factors such as: average defective and scrap rates, or overtime add-up proportion and mutual part's completion proportion, and overtime add-up ratio and average scrap rate on system cost, optimal cycle, and optimal stage one's running time (i.e., uptime and rework time; see Figs. 7-11). (3) Disclosing the impact of individual system features such as the delivery frequency on the critical system cost contributors (Fig. 12); the overtime ratio on the sum of optimal mutual part's running time or on the utilization (Figs. 13-14). (4) Revealing the breakup of detailed cost contributors (Fig. 15) and the overtime ratio on variable cost of each finished goods (Fig. 16) and on significant system-cost contributors (Fig. 17). (5) Exposing various relationships of mutual part's completion rate and its associating values on the system cost (Fig. 18). (6) Exhibiting a comparison of our model against a related model in previous work (see Fig. 19). Finally, this work contributes to the existing literature in the following ways: (1) It proposes a postponement model to solve explicitly a multi-item fabrication-shipment problem featuring quality reassurance, multi-delivery, and overtime. (2) It exposes diverse, crucial in-depth system information for helping managerial decision making. Incorporating variable demand rates for finished products into this specific problem is worth exploring in the future study.

Acknowledgment

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Appendix – A

The derivations of Eq. (36).

This study substitutes Eqs. (1) to (34) in $TC(T_A, n)$, applies the expected values $E[x_0]$, and $E[x_i]$ to deal with random defeats proportions, calculates $E[TC(T_A, n)] / E[T_A]$ to obtain the following $E[TCU(T_A, n)]$:

$$
\begin{aligned}
E\left[TCU\left(T_A,n\right)\right]=\frac{E\left[TC\left(T_A,n\right)\right]}{E\left[T_A\right]}=&\frac{K_0\left(1+\alpha_{2,0}\right)}{T_A}+\frac{C_0\left(1+\alpha_{3,0}\right)\lambda_0}{1-E\left[x_0\right]\varphi_0}+\frac{C_{R,0}E\left[x_0\right]\lambda_0\left(1+\alpha_{3,0}\right)\left(1-\theta_{1,0}\right)}{1-E\left[x_0\right]\varphi_0}\\
&+\frac{C_{S,0}\varphi_0 E\left[x_0\right]\lambda_0}{1-E\left[x_0\right]\varphi_0}+\frac{h_{2,0}T_A\lambda_0^2}{\left(1-E\left[x_0\right]\varphi_0\right)^2}\left[\frac{\left(1-\theta_{1,0}\right)^2 E\left[x_0\right]^2}{2P_{2,0}\left(1+\alpha_{1,0}\right)}\right]+h_{4,0}\left(\frac{\varphi_0 E\left[x_0\right]\lambda_0 T_A}{1-E\left[x_0\right]\varphi_0}\right)+h_{1,0}\sum_{i=1}^{L}\left(\frac{1}{2P_{1,i}}\right)\frac{T_A\lambda_i^2}{\left(1-E\left[x_i\right]\varphi_i\right)^2}\\
+h_{1,0}&\left\{\begin{array}{l}\left(\frac{1}{2}\right)\left(\frac{T_A\lambda_0^2}{\left(1-E\left[x_0\right]\varphi_0\right)^2}\right)\left\{\left[\frac{1}{P_{1,0}\left(1+\alpha_{1,0}\right)}\right]+\left[\frac{\left(1-\theta_{1,0}\right)E\left[x_0\right]\left[2-\left(\varphi_0+1\right)E\left[x_0\right]\right]}{P_{2,0}\left(1+\alpha_{1,0}\right)}\right]\right\}\\ +\sum_{i=1}^{L}\left\{-\left[\frac{T_A\lambda_i}{1-E\left[x_i\right]\varphi_i}\right]\left[\frac{1}{P_{1,i}}+\frac{\left(1-\theta_{1,i}\right)E\left[x_i\right]}{P_{2,i}}\right]\right\}\left[\sum_{j=1}^{i}\left(\frac{\lambda_j}{1-E\left[x_j\right]\varphi_j}\right)\right]\\ +\left[\sum_{i=1}^{L}\left(\frac{\lambda_i}{1-E\left[x_i\right]\varphi_i}\right)\right]\cdot\sum_{i=1}^{L}\left[\frac{\lambda_i T_A}{1-E\left[x_i\right]\varphi_i}\right]\left[\frac{1}{P_{1,i}}+\frac{\left(1-\theta_{1,i}\right)E\left[x_i\right]}{P_{2,i}}\right]\end{array}\right\}\\
+\sum_{i=1}^{L}&\left\{\begin{array}{l}C_i\left(\frac{\lambda_i}{1-E\left[x_i\right]\varphi_i}\right)+\frac{K_i}{T_A}+C_{R,i}\left[\left(1-\theta_{1,i}\right)E\left[x_i\right]\left(\frac{\lambda_i}{1-E\left[x_i\right]\varphi_i}\right)\right]+C_{S,i}\left(\varphi_i E\left[x_i\right]\left(\frac{\lambda_i}{1-E\left[x_i\right]\varphi_i}\right)\right)\\ +\frac{nK_{D,i}}{T_A}+C_{D,i}\lambda_i+\frac{h_{1,i}}{2}\frac{\lambda_i^2 T_A}{\left(1-E\left[x_i\right]\varphi_i\right)^2}\left[\frac{E\left[x_i\right]\varphi_i}{P_{1,i}}+\frac{\left(1-E\left[x_i\right]\varphi_i\right)^2}{\lambda_i}+\frac{\left(1-\theta_{1,i}\right)E\left[x_i\right]\left(1-E\left[x_i\right]\right)}{P_{2,i}}\right]\\ +h_{4,i}\left[\frac{\varphi_i E\left[x_i\right]\lambda_i T_A}{1-E\left[x_i\right]\varphi_i}\right]+h_{2,i}\left[\left(\frac{\lambda_i^2 T_A}{\left(1-E\left[x_i\right]\varphi_i\right)^2}\right)\frac{\left(1-\theta_{1,i}\right)^2 E\left[x_i\right]^2}{2P_{2,i}}\right]+\frac{h_{3,i}}{2}\left[\frac{\lambda_i^2 T_A}{1-E\left[x_i\right]\varphi_i}\right]\left[\frac{1}{P_{1,i}}+\frac{\left(1-\theta_{1,i}\right)E\left[x_i\right]}{P_{2,i}}\right]\\ +\frac{h_{3,i}-h_{1,i}}{2n}\left(\frac{\lambda_i^2 T_A}{\left(1-E\left[x_i\right]\varphi_i\right)^2}\right)\left[\frac{\left(1-E\left[x_i\right]\varphi_i\right)^2}{\lambda_i}-\frac{\left(1-E\left[x_i\right]\varphi_i\right)}{P_{1,i}}-\frac{E\left[x_i\right]\left(1-\theta_{1,i}\right)\left(1-E\left[x_i\right]\varphi_i\right)}{P_{2,i}}\right]\end{array}\right\}
\end{aligned}
\quad \text{(A-1)}
$$

Let E_{1i}, E_{0i}, E_{10}, E_{00}, and E_{0j} be

$$E_{0i} = \frac{1}{\left(1 - \varphi_i E[x_i]\right)} \text{ and } E_{1i} = \frac{E[x_i]}{\left(1 - \varphi_i E[x_i]\right)} \text{ for } i = 1, 2, \dots, L;$$

$$E_{00} = \frac{1}{\left(1 - \varphi_0 E[x_0]\right)}; \ E_{10} = \frac{E[x_0]}{\left(1 - \varphi_0 E[x_0]\right)}; \ E_{0j} = \frac{1}{\left(1 - \varphi_j E[x_j]\right)} \text{ for } j = 1, 2, \dots, i. \tag{A-2}$$

Substitute Eq. (A-2) in Eq. (A-1), we have

$$E\left[TCU\left(T_A, n\right)\right] = C_0 \lambda_0 E_{00}\left(1 + \alpha_{3,0}\right) + \frac{K_0\left(1 + \alpha_{2,0}\right)}{T_A} + C_{R,0}\left(1 + \alpha_{3,0}\right)\lambda_0\left(1 - \theta_{1,0}\right)E_{10}$$

$$+ C_{S,0} E_{10} \lambda_0 \varphi_0 + h_{2,0}\left(E_{10}\right)^2 T_A \lambda_0^2 \left[\frac{\left(1 - \theta_{1,0}\right)^2}{2\left(1 + \alpha_{1,0}\right)P_{2,0}}\right] + h_{4,0} E_{10} \lambda_0 \varphi_0 T_A + h_{1,0} \sum_{i=1}^{L}\left(\frac{1}{2P_{1,i}}\right)T_A \lambda_i^2 \left(E_{0i}\right)^2$$

$$+ h_{1,0}\left\{\begin{array}{l}\left(\frac{1}{2}\right)\left(E_{00}\right)^2 \lambda_0^2 T_A \left\{\left[\frac{1}{\left(1 + \alpha_{1,0}\right)P_{1,0}}\right] + \left[\frac{\left[2 - E[x_0]\left(\varphi_0 + 1\right)\right]\left(1 - \theta_{1,0}\right)E[x_0]}{\left(1 + \alpha_{1,0}\right)P_{2,0}}\right]\right\} \\ + \sum_{i=1}^{L}\left\{-\lambda_i T_A\left[\frac{\left(1 - \theta_{1,i}\right)E_{1i}}{P_{2,i}} + \frac{E_{0i}}{P_{1,i}}\right]\right\}\left[\sum_{j=1}^{i}\left(\lambda_j E_{0j}\right)\right] + \sum_{i=1}^{L}\left(\lambda_i T_A\right)\left[\frac{E_{0i}}{P_{1,i}} + \frac{\left(1 - \theta_{1,i}\right)E_{1i}}{P_{2,i}}\right] \cdot \left[\sum_{i=1}^{L}\left(\lambda_i E_{0i}\right)\right]\end{array}\right\} \tag{A-3}$$

$$+ \sum_{i=1}^{L}\left\{\begin{array}{l}C_i\left(E_{0i}\lambda_i\right) + \frac{K_i}{T_A} + C_{R,i}\left(1 - \theta_{1,i}\right)\left(\lambda_i E_{1i}\right) + C_{S,i} E_{1i} \varphi_i \lambda_i + \frac{nK_{D,i}}{T_A} + C_{D,i}\lambda_i \\ + \frac{h_{1,i} T_A \lambda_i^2}{2}\left[\frac{1}{\lambda_i} + \frac{\varphi_i E_{0i} E_{1i}}{P_{1,i}} + \frac{\left(1 - \theta_{1,i}\right)E_{0i} E_{1i}\left(1 - E[x_i]\right)}{P_{2,i}}\right] + h_{4,i}\varphi_i \lambda_i E_{1i} T_A + h_{2,i}\left[\frac{\left(1 - \theta_{1,i}\right)^2 \lambda_i^2 \left(E_{1i}\right)^2 T_A}{2P_{2,i}}\right] \\ + \frac{h_{3,i}\lambda_i^2 T_A}{2}\left[\frac{E_{0i}}{P_{1,i}} + \frac{\left(1 - \theta_{1,i}\right)E_{1i}}{P_{2,i}}\right] + \frac{h_{3,i} - h_{1,i}}{2n}\left(\lambda_i^2 T_A\right)\left[\frac{1}{\lambda_i} - \frac{E_{1i}\left(1 - \theta_{1,i}\right)}{P_{2,i}} - \frac{E_{0i}}{P_{1,i}}\right]\end{array}\right\}$$

Let E_{2i}, E_{0P}, and E_{iP} be as follows:

$$E_{0P} = \left[\frac{1}{\left(1 + \alpha_{1,0}\right)P_{1,0}}\right] + \left[\frac{\left(1 - \theta_{1,0}\right)E[x_0]\left[2 - E[x_0]\left(\varphi_0 + 1\right)\right]}{P_{2,0}\left(1 + \alpha_{1,0}\right)}\right];$$

$$E_{2i} = \left[\frac{1}{\lambda_i} + \frac{\varphi_i E_{0i} E_{1i}}{P_{1,i}} + \frac{\left(1 - \theta_{1,i}\right)E_{0i} E_{1i}\left(1 - E[x_i]\right)}{P_{2,i}}\right]; \tag{A-4}$$

$$E_{iP} = \left[\frac{E_{0i}}{P_{1,i}} + \frac{\left(1 - \theta_{1,i}\right)E_{1i}}{P_{2,i}}\right] \text{ for } i = 1, 2, \dots, L.$$

Substitute Eq. (A-4) in Eq. (A-3), we obtain Eq. (36).

Appendix B

Table B-1(a)
The corresponding values of parameters in one-stage manufacturing scheme

Product i	$P_{1,i}$	$h_{4,i}$	C_i	$P_{2,i}$	$h_{1,i}$	λ_i	K_i	$h_{2,i}$	$h_{3,i}$
1	58000	$16	$80	46400	$16	3000	$17000	$16	$70
2	59000	$18	$90	47200	$18	3200	$17500	$18	$75
3	60000	$20	$100	48000	$20	3400	$18000	$20	$80
4	61000	$22	$110	48800	$22	3600	$18500	$22	$85
5	62000	$24	$120	49600	$24	3800	$19000	$24	$90

Table B-1(b)
The corresponding values of parameters in one-stage manufacturing scheme

Product i	$K_{D,i}$	$C_{R,i}$	$C_{D,i}$	$C_{S,i}$	$\theta_{1,i}$	x_i	$\theta_{2,i}$	φ_i
1	$1800	$50	$0.1	$20	0.094	5%	0.094	0.18
2	$1900	$55	$0.2	$25	0.146	10%	0.146	0.27
3	$2000	$60	$0.3	$30	0.200	15%	0.200	0.36
4	$2100	$65	$0.4	$35	0.258	20%	0.258	0.45
5	$2200	$70	$0.5	$40	0.322	25%	0.322	0.54