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Contribution of robust optimization on handling agricultural processed products supply chain problem during Covid-19 pandemic

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A B S T R A C T

This research aims to show how decision sciences can make a significant contribution on handling the supply chain problem during Covid-19 Pandemic. The paper discusses how robust optimization handles uncertain demand in agricultural processed products supply chain problems within two scenarios during the pandemic situation, i.e., the large-scale social distancing and partial social distancing. The study assumes that demand and production capacity are uncertain during a pandemic situation. Robust counterpart methodology is employed to obtain the robust optimal solution. To this end, the uncertain data is assumed to lie within a polyhedral uncertainty set. The result shows that the robust counterpart model is a computationally tractable through linear programming problem. Numerical experiment is presented for the Bandung area with a case on sugar and cooking oil that is the most influential agricultural processed products besides the main staple food of the Indonesian people, rice.

1. Introduction

Since the emergence of the 2019 novel coronavirus (2019-nCoV) in Wuhan, China, many countries around the world have been infected with the virus with very serious cases (Lu et al., 2020). On February 11, 2020, the World Health Organization (WHO) announced the new name for the virus to be 2019-nCoV: Coronavirus Desease (Covid-19). The International Committee on Taxonomy decided that Covid-19 is a Severe Acute Respiratory Syndrome (SARS) that has a very high speed of spread. Until July 2021, the total number of Covid-19 cases in the world reached 199.022.248 and continues to increase with a death total of 4.240.374 since 17 November 2019 (Worldometers, 2021). This has resulted in a weakening of the economy due to a decrease in household consumption or purchasing power (Irawan & Alamsyah, 2021), especially for Indonesia as a country with a lower middle class of 115 million people or 45% of the total population. The rapid spread of the virus through droplets (liquid that comes out of a person's nose or mouth when sneezing, coughing, or talking) has resulted in the Indonesian government implementing a lockdown system called Pembatasan Sosial Berskala Besar (PSBB) or we known as Large-scale Social Distancing (Andriani, 2020). This system is an attempt to break the chain of viruses (Allen, 2021). The government also launched a health protocol for the public, such as maintaining hand hygiene, not touching faces, implementing coughing and sneezing ethics, wearing masks, maintaining distance, and increasing the body's immune strength (Sari et al., 2020). Unfortunately, this system has an impact on economic growth (Muhyiddin & Nugroho, 2021), changes in consumer behavior (Mehta et al., 2020), and decreased food buying and selling, causing losses for people in various sectors such as tourism, education, social, economic, health, and food needs which must be properly distributed (Andriani, 2020).

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Bandung is the capital city of West Java Province and is one of the major cities in Indonesia that has been heavily affected by the Covid-19 pandemic. Apart from being a big city, Bandung benefits from its position because it acts as the first and main point of food distribution, including agricultural processed products, where about 90% of the food needs are supplied from outside the region. The supply of agricultural processed products obtained by Bandung is very diverse, such as cooking oil, sugar, wheat flour, and rice. These processed products are very important and must always be available and well distributed to the public. However, government regulation during pandemic situations limits the movement of consumers and producers, so that they only move by 30%. Therefore, the supply chain management of agricultural processed products in Bandung is disrupted. A study of the food supply chain system, especially agricultural processed products, must be carried out. An effective strategy to ensure product distribution runs well during a pandemic is to develop a food hub, which is part of the local food system that helps farmers to develop their business through offering a combination of production, distribution, and marketing services. The food hub concentrates on the relationship between producers (farmers and breeders) and food consumers (restaurants, hospitals, schools, etc.) on a local and regional scale (Matson, 2013). Food hubs benefit farmers in providing additional markets on a larger scale, serve as a single pick-up point for distributors and customers, and can benefit consumers as well as the public by creating new jobs. Local Food Hubs (LFH) are food hubs that operate on a local scale. In the case of Bandung as a local scale, it must be ensured that the LFH is built in each district or at least it is built in the right district and works properly. This study also aims to show how decision science such as Operations Research and Optimization Modeling can make a significant contribution on handling the supply chain problem during Covid-19 pandemic. With considering the supply chain problem, in this paper the objective is to maximize product suppliers so that all demands are fulfilled. The best scenario is determined at the end of the study. There are two uncertainty assumptions that apply to this research model, the uncertainty of demand and the production capacity of agricultural processed products that are in the polyhedral uncertainty set. In this paper, robust optimization modeling is used to model the problem. Previous research that discusses Robust optimization models related to supply chain problems, namely Pishvaee et al. (2011), which discusses Robust optimization models to deal with data uncertainty in closed-loop supply chain design problems by assuming parameters of uncertainty in the amount of product returns. Second, Li's research (2016) which discusses the Robust optimization model with one producer and many consumers to correct inaccurate supply chain demand by assuming uncertainty parameters in demand. Third, the research by Delkhosh and Sadjadi (2019), which discusses the Robust optimization model to develop a micro-algae organic fuel supply chain with the aim of maximizing the benefits of the organic fuel supply chain and minimizing greenhouse gas emissions. Fourth, the research of Perdana et al. (2020) which discusses the Robust optimization model of food supply chain problems in the form of vegetables, eggs, and rice in West Java Province during the Covid-19 period, the study assumes uncertainty in the parameters of demand, production capacity, and the selling price of food which is in the set of box uncertainty. Differs to Perdana et al. (2020), in this paper, a robust optimization model for agricultural processed products is presented.

2. Materials and Methods

2.1. Robust Optimization

Optimization problems in real life often use data that cannot be known precisely. This kind of data is named uncertainty. The methodology for dealing with uncertainty data in optimization is the Robust Counterpart proposed by Ben-Tal and Nemirovski (2002). This uncertainty can be caused by errors in data measurements such as measurement of dimensions and temperature of an object, errors in estimating data, and errors in rounding numbers (Perdana, et al., 2020). This uncertainty problem can be solved using Robust optimization. A general model of the linear optimization problem is as follows (Ben-Tal and Nemirovski, 2002):

$$\min_{\mathbf{x}} \mathbf{c}^{\mathrm{T}}\mathbf{x}: \mathbf{A}\mathbf{x} \leq \mathbf{b}, \tag{1}$$

where $\mathbf{c} \in \mathbb{R}^n, \mathbf{x} \in \mathbb{R}^n, \mathbf{A} \in \mathrm{M}_{\mathrm{m,n}}(\mathbb{R}), \mathbf{b} \in \mathbb{R}^m$.

The general uncertain form of the linear optimization is obtained by assuming the parameter $(\mathbf{c}, \mathbf{A}, \mathbf{b})$ is uncertain. The general model of the uncertain linear optimization problem is as follows (Ben-Tal, et al., 2009):

$$\min_{\mathrm{x}} \mathbf{c}^{\mathrm{T}}\mathbf{x}: \mathbf{A}\mathbf{x} \leq \mathbf{b} \,|\, (\mathbf{c}, \mathbf{A}, \mathbf{b}) \in \mathcal{U}. \tag{2}$$

The uncertain linear optimization problem model can always be formed into an uncertain linear optimization problem which only contains uncertainty in the constraint function (Yanikoglu et al., 2018). The Robust Counterpart is a single deterministic problem that has its uncertainty removed. All of the uncertainties in the model can be collected in the constraint matrix **A** with $\mathbf{A} \in \mathcal{U}$, so that all Robust optimization problems become:

$$\min_{\mathrm{x}} \mathbf{c}^{\mathrm{T}}\mathbf{x}: \mathbf{A}\mathbf{x} \leq \mathbf{b} \,|\, \mathbf{A} \in \mathcal{U}, \tag{3}$$

where $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{b} \in \mathbb{R}^m$, $\mathcal{U}$ is a primitive uncertain set, and **A** is a matrix size $(\mathrm{m} \times \mathrm{n})$. Furthermore, the constraint matrix **A** is expressed in terms of a primitive uncertain parameter $\zeta \in \mathcal{Z}$, where $\mathcal{Z} \subset \mathbb{R}^L$ is the uncertain set of primitives, so we get:

$$\min_{\mathrm{x}} \mathbf{c}^{\mathrm{T}}\mathbf{x}: \mathbf{A}(\zeta)\mathbf{x} \leq \mathbf{b} \,|\, \zeta \in \mathcal{Z}, \tag{4}$$

$$\min_{\mathbf{x}} \mathbf{c}^{\mathrm{T}}\mathbf{x}: \mathbf{a}_{\mathrm{i}}^{\mathrm{T}}\left(\zeta\right)\mathbf{x} \leq \mathbf{b}_{\mathrm{i}}, \tag{5}$$

$$\mathrm{i} = 1, .., \mathrm{m}, \forall \zeta \in \mathcal{Z}. \tag{6}$$

The solution $\mathcal{Z} \in \mathbb{R}^{\mathrm{L}}$ is called robust feasible if it satisfies all uncertain constraints $[\mathbf{A}(\zeta)\mathbf{x} \leq \mathbf{b}]$ for all realizations of $\zeta \in \mathcal{Z}$. Given the uncertainty on the Robust optimization assumptions that are constraint-wise, the model problem (5) can be focused on a single constraint:

$$\left(\bar{\mathbf{a}} + \mathbf{P}\zeta\right)^{\mathrm{T}}\mathbf{x} \leq \mathbf{b}, \forall \zeta \in \mathcal{Z}, \tag{7}$$

which $(\bar{\mathbf{a}} + \mathbf{P}\zeta)$ is a affine function of the primitive uncertain parameter $\zeta \in \mathcal{Z}$, $\mathbf{a} \in \mathbb{R}^{\mathrm{n}}$, and $\mathbf{P} \in \mathrm{M}_{\mathrm{n,L}}(\mathbb{R})$. Next is the determination of ζ which depends on the use of the uncertain set $\mathcal{U}$ by assuming a polyhedral uncertainty set. Polyhedral uncertainty set defined as follows (Gorissen, BL, et al., 2015):

$$\mathrm{Z} = \left\{\zeta : \mathbf{d} - \mathbf{D}\zeta \geq 0\right\}, \tag{8}$$

where $\mathbf{D} \in \mathrm{M}_{\mathrm{m,L}}(\mathbb{R})$, $\zeta \in \mathbb{R}^{\mathrm{L}}$, and $\mathbf{d} \in \mathbb{R}^{\mathrm{m}}$. Uncertain set $\mathcal{U}$ can be defined as:

$$\mathcal{U} = \left\{\mathbf{a} \middle| \mathbf{a} = \bar{\mathbf{a}} + \mathbf{P}\zeta, \mathbf{d} - \mathbf{D}\zeta \geq 0\right\} \tag{9}$$

To obtain a formulation of Robust Counterpart with the uncertainties present in the polyhedral uncertainties, then applied the definition of the set of polyhedral uncertainties to inequality (7) as follows:

$$\left(\bar{\mathbf{a}} + \mathbf{P}\zeta\right)^{\mathrm{T}}\mathbf{x} \leq \mathbf{b}, \forall \zeta : \mathbf{d} - \mathbf{D}\zeta \geq 0. \tag{10}$$

Gorissen, BL, et al. (2015) stated that a Robust Counterpart formulation with a set of polyhedral uncertainties was computationally tractable obtained through the following three steps:

1. Reformulation of the left side inequality constraint in (10) so that it is equivalent to the worst-case formulation,
$$\max_{\zeta : \mathbf{d} - \mathbf{D}\zeta \geq 0} \left(\bar{\mathbf{a}} + \mathbf{P}\zeta\right)^{\mathrm{T}}\mathbf{x}, \tag{11}$$
which is equivalent to
$$\bar{\mathbf{a}}^{\mathrm{T}}\mathbf{x} + \max_{\zeta : \mathbf{d} - \mathbf{D}\zeta \geq 0} (\mathbf{P}^{\mathrm{T}}\mathbf{x})^{\mathrm{T}}\zeta. \tag{12}$$
2. Formulation of the dual form of the maximization problem on inequality (12). The primal form of equation (12):
$$\max_{\zeta}\left\{(\mathbf{P}^{\mathrm{T}}\mathbf{x})^{\mathrm{T}}\zeta : \mathbf{d} - \mathbf{D}\zeta \geq 0\right\}. \tag{13}$$
Next, the primal form is changed to the dual form. The dual form of (13) is as follows:
$$\min\{\mathbf{d}^{\mathrm{T}}\mathbf{y}: \mathbf{D}^{\mathrm{T}}\mathbf{y} = \mathbf{P}^{\mathrm{T}}\mathbf{x}, \mathbf{y} \geq \mathbf{0}\}. \tag{14}$$
Based on the Strong Duality Theorem, the value of the objective function of the inequality (13) and its dual form in (14) have the same value, so that equation (12) is equivalent to:
$$\bar{\mathbf{a}}^{\mathrm{T}}\mathbf{x} + \min_{\mathbf{y}}\{\mathbf{d}^{\mathrm{T}}\mathbf{y}: \mathbf{D}^{\mathrm{T}}\mathbf{y} = \mathbf{P}^{\mathrm{T}}\mathbf{x}, \mathbf{y} \geq \mathbf{0}\} \leq \mathbf{b}. \tag{15}$$
3. The constraint function in (15) is fulfilled for a feasible solution contained in the feasible set $\mathcal{F} = \{\mathrm{y} : \mathbf{D}^{\mathrm{T}}\mathrm{y} = \mathbf{P}^{\mathrm{T}}\mathbf{x}, \mathbf{y} \geq 0\}$, then the constraint function can be ascertained to be fulfilled for the upper minimum value of $\mathbf{y}$. The final formulation of Robust Counterpart:
$$\exists\mathbf{y}: \bar{\mathbf{a}}^{\mathrm{T}}\mathbf{x} + \mathbf{d}^{\mathrm{T}}\mathbf{y} \leq \mathbf{b}, \mathbf{D}^{\mathrm{T}}\mathbf{y} = \mathbf{P}^{\mathrm{T}}\mathbf{x}, \mathbf{y} \geq \mathbf{0}. \tag{16}$$

Constraints in Eq. (16) is in the form of Linear Programming (LP), so that referring to Ben-Tal and Nemirovski (2002), the Robust Counterpart is guaranteed to be computationally tractable.

2.2. Optimization Model for Supply Chain Problem

Perdana et al. (2020) developed a optimization model for supply chain problem with two objective functions that is minimizing logistics costs and maximizing product suppliers for all demand. Perdana, et al. (2020) have three addition scenarios that differentiate the type of distribution between producers and consumers, between Regional Food Hubs (RFH) and consumers, and their aims. The supply chain optimization model is related to the distribution of vegetables, rice, and eggs between producers, RFH, and consumers in 37 cities affected by Covid-19 in West Java Province. The objective function of the model is formulated as follows:

$$\max\left\{\sum_{c\in C}\sum_{i\in I}\sum_{j\in J} v_{ci}w_{cji}\right\}, \tag{17}$$

$$\min\ \{\mathrm{a}+\mathrm{b}+\mathrm{c}+\mathrm{d}\}, \tag{18}$$

where:

$$a = h\sum_{j\in J} x_j\, , b = \sum_{c\in C}\sum_{j\in J}\sum_{i\in I} b_{ji}d_{ci}w_{cji}\, , c = q\sum_{c\in C}\sum_{j\in J} p_{cj}\, , d = \sum_{c\in C}\sum_{k\in K}\sum_{j\in J} b_{kj}f_{ck}y_{ckj},$$

with six general constrains:

$$\sum_{k\in K} f_{ck}y_{ckj} = p_{cj}\, , \forall c \in C, j \in J, \tag{19}$$

$$\sum_{i\in I} d_{ci}w_{cji} = p_{cj}\, , \forall c \in C, j \in J, \tag{20}$$

$$\sum_{j\in J} y_{ckj} \le 1, \forall\ c \in C, k \in K, \tag{21}$$

$$\sum_{j\in J} w_{cji} \le 1, \forall c \in C, i \in I, \tag{22}$$

$$y_{ckj} \le x_j, \forall\ c \in C, k \in K, j \in J, \tag{23}$$

$$w_{cji} \le x_j, \forall c \in C, j \in J, i \in I. \tag{24}$$

Additional constraints for scenarios 1:

$$y_{ckr} = 0, \forall k \in K - \{r\}, c \in C, r \in R, \tag{25}$$

$$w_{cjr} = 0, j \in J - \{r\}, c \in C, r \in R, \tag{26}$$

$$y_{cjr} = 0, \forall j \in J - \{r\}, c \in C, r \in R, \tag{27}$$

$$w_{cri} = 0, \forall i \in I - \{r\}, c \in C, r \in R. \tag{28}$$

Additional constraints for scenarios 2 are (27) and (28) with the addition of two other constraints, namely:

$$\sum_{k\in K-\{r\}} y_{ckr} \le m_{2r}n_c, \forall\ c \in C, r \in R, \tag{29}$$

$$\sum_{j\in J-\{r\}} w_{cjr} \le m_{4r}n_c, \forall\ c \in C, r \in R. \tag{30}$$

Additional constraints for scenarios 3 are (29) and (30) with the addition of two other constraints, namely:

$$\sum_{j\in J-\{r\}} y_{crj}f_{cr} \le m_{1r}n_c, \forall\ c \in C, r \in R, \tag{31}$$

$$\sum_{i\in I-\{r\}} y_{cri}d_{ci} \le m_{3r}n_c, \forall\ c \in C, r \in R, \tag{32}$$

where: $x_j \in [0,1], \forall j \in$; $P_{cj} \in \mathbb{R}, \forall c \in c, j \in J$; $y_{ckj}, y_{ckr}, y_{cjr} \in [0,1]\, , \forall\ c \in C, k \in K,\ \ j \in J$; $w_{cji}, w_{cjr}, w_{cri} \in [0,1], , \forall\ c \in C, j \in\ J, i \in I$.

The description of the set, parameters, and objective variables used in the model is as follows.

- Sets
  - I : Demand/consumer zone
  - J : Regional Food Hubs (RFH) zone
  - K : Production zone/producers zone
  - C : Commodity
  - R : Red zone (the biggest center of the spread of Covid-19)
- Parameters
  - d_{ci} : Demand for products c in City i (tonnes)
  - v_{ci} : Selling prices of products c in City i (Rp/ton)

f_{ck} : Production capacity of products c in City Producers k (tonnes)
b_{ji} : Distribution costs between City j and City i to transport the product (Rp/tonnes)
b_{jk} : Distribution costs between City j and City k to transport the product (Rp/tonnes)
q : Product handling cost based on health protocol (Rp/tonnes)
h : RFH handling cost (Rp/hub)
n_c : Maximum amount of product c that can be distributed in one line (tonnes/distribution)

- Decision Variables

$x_j = 1$ if RFH is will be built in City j and $x_j = 0$ if RFH will not be built in City j ($\forall j \in J$).
p_{cj} : RFH capacity for products c in City j (tonnes/day), $P_{cj} \in \mathbb{R}, \forall c \in C, j \in J$.
y_{ckj} : The number of products c in City producers k that are sent to RFH in City j, $y_{ckj} \in \mathbb{R}[0,1], \forall\, c \in C, k \in K, j \in J$.
y_{ckr} : The number of products c in City producers k that are sent to red zone RFH in City r, $y_{ckr} \in \mathbb{R}[0,1], \forall\, c \in C, k \in K, j \in J$.
y_{cjr} : The number of products c in City producers j that are sent to red zone RFH in City r, $y_{cjr} \in \mathbb{R}[0,1], \forall\, c \in C, k \in K, j \in J$.
w_{cji} :The number of requests for products c in City i that are fulfilled by RFH in City j, $w_{cji} \in \mathbb{R}[0,1], \forall\, c \in C, j \in J, i \in I$.
w_{cjr} : The number of requests for products c in City j that are fulfilled by red zone RFH in City r, $w_{cjr} \in \mathbb{R}[0,1], \forall\, c \in C, j \in J, i \in I$
w_{cri} : The number of requests for products c in red zone City r that are fulfilled by RFH in City i, $w_{cri} \in \mathbb{R}[0,1], \forall\, c \in C, j \in J, i \in I$.
m_{2r} : Variable that determines how many times the distribution of products c must be done from producers in the green zone to RFH in the red zone r.
m_{4r} : Variables that determine how many times the distribution of products c must be done from RFH in the green zone to consumers in the red zone r.
m_{1r} :Variable that determines the number of times the distribution of product c must be completed from producers in the red zone to RFH in the green zone r.
m_{3r} :Variable that determines the number of times the distribution of product c must be completed from RFH in the red zone to consumers in the green zone r.

3. Results and discussion

3.1. Robust Optimization Model of Agricultural Processed Product Supply Chain Problem

This study uses a nominal optimization model sourced from Perdana et al. (2020) as in (2.2) with some changes. Then, the model is changed to a Robust optimization model with uncertainty parameters which are discussed later.

There are four differences between this research and Perdana, et al. (2020). First, the commodities used are agricultural processed products, namely sugar and cooking oil, while the reference article uses rice, eggs and vegetables. This is because sugar and cooking oil are the most influential agricultural processed products besides the main staple food of Indonesian people, rice. Second, using the Linear Programming method of solving because it has one objective function, that is the first objective function that maximizes the demand for the product, while the reference article uses the lexicographic method because it has two objective functions.

Third, the research was carried out on a smaller scale, namely the districts in Bandung, while in the reference article, the research was carried out on a larger scale, namely cities in West Java Province, so that the naming of Regional Food Hubs (RFH) was changed to Local Food Hubs (LFH). Fourth, this study uses two scenarios, large-scale social distancing and partial social distancing as shown in Table 1. Therefore, the secondary data used in this study are clearly different, namely data on agricultural processed products of sugar and cooking oil obtained from various sources.

In this problem, in changing the nominal optimization model with to a Robust optimization model, the parameters that are uncertain are the demand for products in the consumer zone and the production capacity in the production zone because the amount of demand and capacity of a product will change depending on the needs of consumers and because of the limited movement of consumers in the production zone. Limited distribution of agricultural processed products, the slowdown in the production process, and changes in the supply chain system caused by the Covid-19 pandemic are also causes for the uncertainty of the amount of capacity and demand for a product. Therefore, the uncertainty parameter in this model are the demand of product c in Districts i (ton) symbolized by d_{ci} dan production capacity of product c in the producer Districts k (ton) symbolized by f_{ck}, so it can be assumed that $d_{ci}, f_{ck} \in \mathcal{U}$.

Table 1
Differences between scenario 1 and scenario 2

DISTRIBUTION TYPE	SCENARIO 1 (LARGE SCALE SOCIAL DISTANCING)	SCENARIO 2 (PARTIAL SOCIAL DISTANCING)
BETWEEN PRODUCER AND LFH	Only applies product distribution in each green and red zone	The distribution of products from producers in the green zone to LFH in the red zone is permitted, otherwise it is not permitted
BETWEEN LFH AND CONSUMER	Only applies product distribution in each green and red zone	The distribution of products from LFH in the green zone to consumers in the red zone is permitted, otherwise it is not permitted
AIM	Reduce the spread of Covid-19 and see if each zone can meet its own needs or not	Reducing the spread of Covid-19 from the red zone to the green zone, but the green zone can still support the distribution process to the red zone

To reformulate the uncertainty parameter in the constraint function, based on the definition of the uncertainty set as in (9), the f_{ck} and d_{ci} parameter can be written as:

$$f_{ck} = \overline{f_{ck}} + \mathbf{P1}_{\mathrm{ck}} \zeta, \forall \zeta \in \mathrm{Z}, \tag{33}$$

$$d_{ci} = \overline{d_{ci}} + \mathbf{P2}_{\mathrm{ci}} \zeta, \forall \zeta \in \mathrm{Z}, \tag{34}$$

with $\overline{f_{ck}}$, $\overline{d_{ci}} \in \mathbb{R}^n$ is the nominal value vector of production capacity c in District k and production demand c in District i, $\mathbf{P1}_{\mathrm{ck}}, \mathbf{P2}_{\mathrm{ci}} \in \mathbb{R}^{n\times L}$ is the confounding matrix, and $\zeta \in \mathbb{R}^L$ is a primitive uncertainty vector. Next, substitute (33) to constrains that have an uncertainty $\overline{f_{ck}}$ in (19) and (34) to constrains that have a uncertainty $\overline{d_{ci}}$ in (20):

$$\sum_{k\in K} (\overline{f_{ck}} + \mathbf{P1}_{\mathrm{ck}} \zeta) y_{ckj} = p_{cj}, \forall c \in C, j \in J, \zeta \in Z, \tag{35}$$

$$\sum_{i\in I} (\overline{d_{ci}} + \mathbf{P2}_{\mathrm{ci}} \zeta) w_{cji} = p_{cj}, \forall c \in C, j \in J, \zeta \in Z, \tag{36}$$

Next, based on 2.1, Robust Counterpart formulation with the assumption that the uncertain parameters d_{ci} and f_{ck} are in the set the polyhedral uncertainty is computationally tractable and is obtained in the following three steps.

Step 1: **Reformulate the inequality constraint containing the uncertainty ζ, so that it is equivalent to the worst-case formulation.**
Constraints (35) and (36) can be expressed in vector form:

$$\left(\bar{f} + \mathbf{P1} \zeta\right)^T y = p, \tag{37}$$

that equivalent to:

$$\bar{f}^T y + \left(\mathbf{P1} \zeta\right)^T y = p, \tag{38}$$

and

$$\left(\bar{d} + \mathbf{P2} \zeta\right)^T w = p, \tag{39}$$

that equivalent to:

$$\bar{d}^T w + \left(\mathbf{P2} \zeta\right)^T w = p. \tag{40}$$

Constraints (38) and (40) reformulation is performed worst-case, obtained:

$$\bar{f}^T y + \max_{\zeta} \left(\mathbf{P1} \zeta\right)^T y = p, \tag{41}$$

that equivalent to:

$$\bar{f}^T y + \max_{\zeta} (\mathbf{P1}^T y)^T \zeta = p, \tag{42}$$

and

$$\bar{d}^T w + \max_{\zeta} \left(\mathbf{P2} \zeta\right)^T w = p, \tag{43}$$

that equivalent to:

$$\bar{d}^T w + \max_{\zeta}(\mathbf{P2}^{\mathrm{T}} w)^T \zeta = p. \tag{44}$$

Step 2: Dual form formulation based on Duality Theory.

The set of polyhedral uncertainties is defined as $Z = \left\{ \zeta : \mathbf{d} - \mathbf{D}\zeta \geq 0 \right\}$, where $\mathbf{D} \in M_{m,L}(\mathbb{R})$, $\zeta \in \mathbb{R}^L$, and $\mathbf{d} \in \mathbb{R}^m$, so that (42) and (44) change to:

$$\bar{f}^T y + \max_{\zeta : \mathbf{d1} - \mathbf{D1}\zeta \geq 0} (\mathbf{P1}^T y)^T \zeta = p, \tag{45}$$

And

$$\bar{d}^T w + \max_{\zeta : \mathbf{d2} - \mathbf{D2}\zeta \geq 0} (\mathbf{P2}^T w)^T \zeta = p. \tag{46}$$

Next, focused on the second term of the left-hand side in Eq. (45) and Eq. (46):

$$\max_{\zeta : \mathbf{d1} - \mathbf{D1}\zeta \geq 0} (\mathbf{P1}^{\mathrm{T}} y)^T \zeta , \tag{47}$$

which can be expressed as a primal problem as:

$$\max(\mathbf{P1}^T y)^T \zeta , \tag{48}$$

$$s.t\ \mathbf{D1}\zeta \leq \mathbf{d1}, \forall \zeta \in Z, \tag{49}$$

and

$$\max_{\zeta : \mathbf{d2} - \mathbf{D2}\zeta \geq 0} (\mathbf{P2}^T w)^T \zeta , \tag{50}$$

which can be expressed as a primal problem as:

$$\max(\mathbf{P2}^T w)^T \zeta , \tag{51}$$

$$s.t\ \mathbf{D2}\zeta \leq \mathbf{d2}, \forall \zeta \in Z. \tag{52}$$

The dual problem of primal problem Eq. (48), Eq. (49) and Eq. (51), Eq. (52) are:

$$\begin{aligned} &\min \mathbf{d1}^T \gamma, \\ &s.t. \mathbf{D1}^{\mathrm{T}} \gamma 1 = \mathbf{P1}^T y, \\ &\gamma 1 \geq 0, \end{aligned} \tag{53}$$

and

$$\begin{aligned} &\min \mathbf{d1}^T \gamma 2 \\ &s.t. \mathbf{D2}^T \gamma 2 = \mathbf{P2}^{\mathrm{T}} w, \\ &\gamma 2 \geq 0. \end{aligned} \tag{54}$$

The primal-dual relationship used is strong duality, so that the optimum values for primal problems are same as the optimum values for dual problems, so that:

$$\max_{\zeta} \left\{ (\mathbf{P1}^{\mathrm{T}} \mathrm{y})^{\mathrm{T}} \zeta : \mathbf{D1}\zeta \leq \mathbf{d1} \right\} = \min_{\gamma 1} \{ \mathbf{d1}^{\mathrm{T}} \gamma 1 : \mathbf{D1}^{\mathrm{T}} \gamma 1 = \mathbf{P1}^{\mathrm{T}} \mathrm{y}, \gamma 1 \geq 0 \}, \tag{55}$$

and

$$\max_{\zeta} \left\{ (\mathbf{P2}^{\mathrm{T}} \mathrm{w})^{\mathrm{T}} \zeta : \mathbf{D2}\zeta \leq \mathbf{d2} \right\} = \min_{\gamma 2} \{ \mathbf{d2}^{\mathrm{T}} \gamma 2 : \mathbf{D2}^{\mathrm{T}} \gamma 2 = \mathbf{P2}^{\mathrm{T}} \mathrm{w}, \gamma 2 \geq 0 \}. \tag{56}$$

Substitute Eq. (55) to Eq. (31) and Eq. (45) to Eq. (46), so that:

$$\bar{f}^T y + \min_{\gamma 1} \{ \mathbf{d1}^T \gamma 1 : \mathbf{D1}^{\mathrm{T}} \gamma 1 = \mathbf{P1}^T y, \gamma 1 \geq 0 \} = p, \tag{57}$$

and

$$\bar{d}^T w + \min_{\gamma 2} \{ \mathbf{d2}^{\mathrm{T}} \gamma 2 : \mathbf{D2}^{\mathrm{T}} \gamma 2 = \mathbf{P2}^{\mathrm{T}} \mathrm{w}, \gamma 2 \geq 0 \} = p. \tag{58}$$

Step 3: **The dual formulations are fulfilled for a feasible solution**.

The dual formulations in (57) and (58) are fulfilled for a feasible solution $\gamma 1$ and $\gamma 2$ that contained in the feasible set $\mathcal{F}1 = \{ \gamma 1 | \mathbf{D1}^{\mathrm{T}} \gamma 1 = \mathbf{P1}^T y, \gamma 1 \geq 0 \}$, so $\exists \gamma 1 \geq 0 \ni \mathbf{D1}^T \gamma 1 = \mathbf{P1}^T y$ and $\mathcal{F}2 = \{ \gamma 2 | \mathbf{D2}^{\mathrm{T}} \gamma 2 = \mathbf{P2}^T w, \gamma 2 \geq 0 \}$, so $\exists \gamma 2 \geq 0 \ni \mathbf{D2}^T \gamma 2 = \mathbf{P2}^T w$. Due to the existence of the guaranteed solution, then the constraints in (57) and (58) can be written as:

$$\begin{aligned} &\bar{f}^T y + \mathbf{d1}^{\mathrm{T}} \gamma 1 = p, \\ &\mathbf{D1}^T \gamma 1 = \mathbf{P1}^{\mathrm{T}} y, \\ &\gamma 1 \geq 0, \end{aligned} \tag{59}$$

And

$$\bar{d}^T w + \mathbf{d2}^T \gamma 2 = p,$$
$$\mathbf{D2}^T \gamma 2 = \mathbf{P2}^{\mathrm{T}} w, \tag{60}$$
$$\gamma 2 \geq 0.$$

The form of constraints (59) and (60) can be restated in sigma and index form as:

$$\sum_{k \in K} \overline{f_{ck}}\, y_{ckj} + \sum_{h \in H} \mathbf{d1}_{\mathrm{h}} \gamma 1_h = p_{cj}, \forall c \in C, j \in J,$$

$$\sum_{h \in H} \mathbf{D1}_{zh} \gamma 1_h = \sum_{k \in K} \sum_{j \in J} \mathbf{P1}_{zj} y_{ckj}, \forall c \in C\,, \tag{61}$$
$$\forall z = 1,2,3, \dots, L, \gamma 1_h \geq 0, \forall h \in H,$$

and

$$\sum_{i \in I} \overline{d_{ci}} w_{cji} + \sum_{h \in H} \mathbf{d2}_h \gamma 2_h = p_{cj}, \forall c \in C, j \in J,$$
$$\sum_{h \in H} \mathbf{D2}_{zh} \gamma 2_h = \sum_{j \in J} \sum_{i \in I} \mathbf{P2}_{zj} w_{cji}\,, \forall c \in C, \tag{62}$$
$$\forall z = 1,2,3, \dots, L, \gamma 2_h \geq 0, \forall h \in H.$$

It can be seen that the constraints in (61) and (62) are linear and for all $\gamma 1_h$ and $\gamma 2_h$ are nonnegative. The Robust Counterpart model of the supply chain problem for agricultural processed products with a polyhedral uncertainty set is as follows:

$$\max \left\{ \sum_{c \in C} \sum_{i \in I} \sum_{j \in J} v_{ci} w_{cji} \right\} \tag{63}$$

with general constraints:

$$\sum_{j \in J} y_{ckj} \leq 1, \forall\, c \in C, k \in K, \tag{64}$$

$$\sum_{j \in J} w_{cji} \leq 1, \forall c \in C, i \in I, \tag{65}$$

$$y_{ckj} \leq x_j, \forall\, c \in C, k \in K, j \in J, \tag{66}$$

$$w_{cji} \leq x_j, \forall c \in C, j \in J, i \in I, \tag{67}$$

$$\sum_{k \in K} \overline{f_{ck}}\, y_{ckj} + \sum_{h \in H} \mathbf{d1}_{\mathrm{h}} \gamma 1_h = p_{cj}, \forall c \in C, j \in J, \tag{68}$$

$$\sum_{h \in H} \mathbf{D1}_{zh} \gamma 1_h = \sum_{k \in K} \sum_{j \in J} \mathbf{P1}_{zj} y_{ckj}\,, \forall c \in C, \tag{69}$$

$$\sum_{i \in I} \overline{d_{ci}} w_{cji} + \sum_{h \in H} \boldsymbol{d2}_h \gamma 2_h = p_{cj}, \forall c \in C, j \in J, \tag{70}$$

$$\sum_{h \in H} \mathbf{D2}_{zh} \gamma 2_h = \sum_{j \in J} \sum_{i \in I} \mathbf{P2}_{zj} w_{cji}\,, \forall c \in C, z = 1,2,3, \dots, L, \tag{71}$$

and additional constraints in scenarios 1 and 2 as in (25) to (30), where:
$x_j \in [0,1], \forall j \in;\ P_{cj} \in \mathbb{R}, \forall c \in c, j \in J;\ y_{ckj}, y_{ckr}, y_{cjr} \in [0,1]\,, \forall\, c \in C, k \in K, j \in J,\ w_{cji}, w_{cjr}, w_{cri} \in [0,1], \forall\, c \in C,\ j \in J, i \in I;\quad \gamma 1_h \geq 0, \gamma 2_h \geq 0, \forall h \in H.$

3.2. Case Study

This numerical experiment uses secondary data obtained from various sources. The data used is data in 2020 because it is the year that the Covid-19 pandemic began in Indonesia, especially in Bandung. There are 30 districts in Bandung that act as a producer zone, a potential zone for LFH development, a consumer zone, and a red zone which are determined based on the high number of Covid-19 cases. Five districts in Bandung that have the highest active cases confirmed by Covid-19 can be seen in Table 2 (Covid-19 Information Center/Pusicov Bandung, 2020). The 30 districts can be symbolized based on the required index, where the sequence of numbers is based on the order of the districts in Table 3. There are two types of agricultural processed products used, sugar and cooking oil, so we have $c = \{1,2\}$, where index 1 is for sugar and index 2 is for cooking oil. Consumer demand for commodities can be calculated from the average per capita consumption multiplied by the number of residents in each district in Bandung, where all data uses in 2020. Average sugar per capita consumption of people in Bandung is predicted 13.6 kg/capita/year, while for cooking oil is 11.38 liters/capita/year or equivalent to 9.11

kg/capita year (Sabarella et al., 2018). It is assumed that the sugar production capacity is 46% more than the total demand, while the cooking oil production capacity is 42% more than the total demand (Sabarella, et al., 2018). Data on consumer demand and production capacity for commodities can be seen in Table 3.

Table 2
Red zone districts in Bandung.

NO	DISTRICT	ACTIVE CASE (PEOPLE)
1	Antapani	52
2	Arcamanik	56
3	Bandung Kulon	25
4	Batununggal	36
5	Bojongloa Kaler	35

The selling price of sugar and cooking oil in each district in Bandung is assumed to be the same, Rp12,833/kg or equivalent to Rp12,833,000/ton and Rp15,333/liter or equivalent to Rp17,016,630/ton (Food Price Information Forum, 2021). The maximum amount of agricultural processed products that can be distributed in one lane is assumed to be 20 tons.

Table 3
Demand and capacity of sugar and cooking oil

Index	District	Demand		Capacity	
		Sugar (tonnes/year)	Cooking Oil	Sugar (tonnes/year)	Cooking Oil (tonnes/year)
1	Andir	1.350	905	1.971	1.284
2	Antapani	1.081	724	1.578	1.028
3	Arcamanik	1.057	708	1.544	1.006
4	Astana Anyar	4.451	2.982	6.499	4.234
5	Babakan Ciparay	1.920	1.286	2.804	1.827
6	Bandung Kidul	824	552	1.203	784
7	Bandung Kulon	1.838	1.231	2.683	1,748
8	Bandung Wetan	390	261	570	371
9	Batunuggal	1.642	1.100	2.397	1.562
10	Bojongloa Kaler	1.683	1.127	2.457	1.601
11	Bojongloa Kidul	1.181	791	1.725	1.124
12	Buah Batu	1.394	934	2.036	1.326
13	Cibeunying Kaler	956	640	1.395	909
14	Cibeunying Kidul	1.531	1.026	2.235	1.456
15	Cibiru	1.009	676	1.472	959
16	Cicendo	1.303	873	1.903	1.240
17	Cidadap	734	492	1.072	698
18	Cinambo	344	231	503	328
19	Coblong	54	36	78	51
20	Gedebage	556	372	812	529
21	Kiara Condong	1.773	1.188	2.589	1.687
22	Lengkong	967	648	1.411	920
23	Mandala Jati	4.452	2.982	6.499	4.234
24	Panyileukan	544	365	795	518
25	Ranca Sari	1.153	772	1.683	1.097
26	Regol	1.095	734	1.599	1.042
27	Sukajadi	1.392	932	2.032	1.324
28	Sukasari	1.052	705	1.537	1.001
29	Sumur Bandung	510	341	744	485
30	Ujungberung	1.196	801	1.747	1.138

3.2.1. Determination of Convex Hull, Uncertainty Martix, and Uncertainty Vector

This study uses data on demand and capacity for processed agricultural products, sugar and cooking oil which have been assumed to contain uncertainty. To ensure that the optimal solution for the Robust Counterpart model is definitely obtained, according to Ben-Tal and Nemirovskri (1998), the convex hull theory can be used. This theory can connect and contain all available indefinite data without including unnecessary data and provides a "boundedness assumption" that if there is a convex and compact set $Q \subset \mathbb{R}^n$, then the set must contain a set of feasible solutions to all problems $(P) \in \wp$. Therefore, this section determines the convex hull of each uncertainty parameter with the help of software R. The resulting convex hull image shows the comparison between the x-axis as the index of the 30 districts and the y-axis as the number of requests and capacities of sugar and cooking oil, to obtain four types of convex hull, namely convex hull demand for sugar as in Figure 1, convex hull capacity of sugar in Fig. 2, convex hull demand for cooking oil in Fig. 3, and convex hull capacity for cooking oil in Fig. 4.

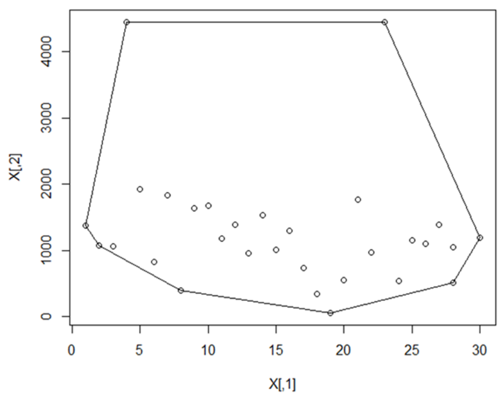

Fig. 1. Convex hull demand for sugar

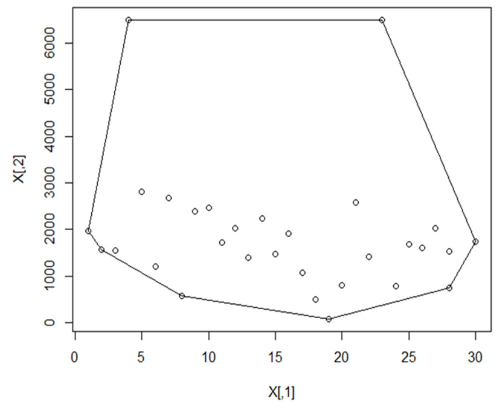

Fig. 2. Convex hull capacity of sugar

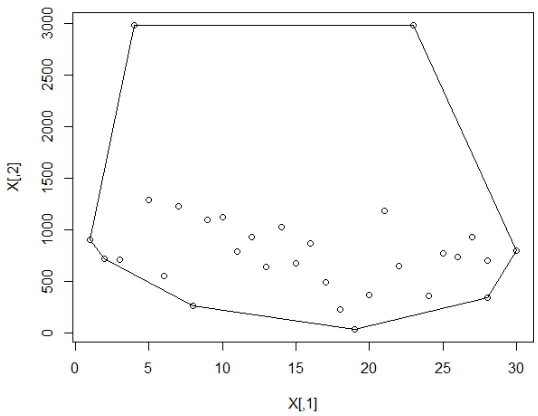

Fig. 3. Convex hull demand of cooking oil

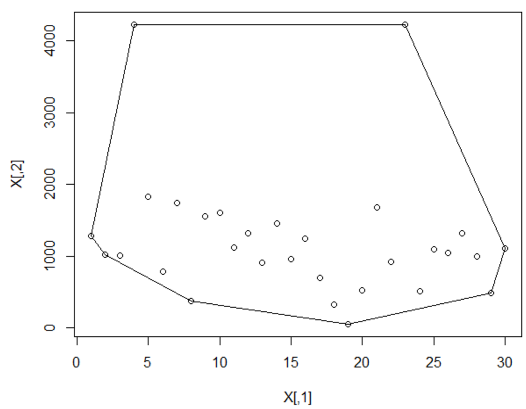

Fig. 4. Convex hull capacity of cooking oil

Departing from the acquisition of four types of convex hull, next is the determination as uncertainty matrix and uncertainty vector for each parameter using the help of the polyhedral uncertainty set definition. These uncertainty matrices and vectors are required and used in the Robust Counterpart calculation using R. Determination of uncertainty matrix and vector can be seen in Table 4 and Table 5 by assuming x as ζ_1 and y as ζ_2 .

Table 4
Determination of uncertainty matrix on sugar commodities

Demand of Sugar	**Line equations:** $\boldsymbol{y - 1.023,67x \geq 356,333}$ $\boldsymbol{y - 0,0526316x \geq 4.450,79}$ $\boldsymbol{y + 465,143x \geq 15,150.3}$ $\boldsymbol{y - 686x \geq -19.384}$ $\boldsymbol{y - 45,6x \geq -812,4}$ $\boldsymbol{y + 30,5455x \geq 634,364}$ $\boldsymbol{y - 0x \geq 4.451}$ $\boldsymbol{y + 299x \geq 1.679}$	**Matrix form:** $\boldsymbol{\begin{bmatrix} -1 & 1.023,6 \\ -1 & 0,0526316 \\ -1 & -465,143 \\ -1 & 686 \\ -1 & 45,6 \\ -1 & -30,5455 \\ -1 & 0 \\ -1 & -299 \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix} \leq \begin{bmatrix} -356,333 \\ -4.450,79 \\ -15.150,3 \\ 19.384 \\ 812,4 \\ -634,364 \\ -4.451 \\ -1.679 \end{bmatrix}}$ Obtained: $\mathbf{D1} = \begin{bmatrix} -1 & 1.023,6 \\ -1 & 0,0526316 \\ -1 & -465,143 \\ -1 & 686 \\ -1 & 45,6 \\ -1 & -30,5455 \\ -1 & 0 \\ -1 & -299 \end{bmatrix} \quad \mathbf{d1} = \begin{bmatrix} -356,333 \\ -4.450,79 \\ -15.150,3 \\ 19.384 \\ 812,4 \\ -634,364 \\ -4.451 \\ -1.679 \end{bmatrix}$

Capacity of Sugar	Line equations: $y + 393x \geq 2.364$ $y + 168x \geq 1.914$ $y + 44{,}7273x \geq 927{,}818$ $y - 66{,}6x \geq -1.187{,}4$ $y - 1.003x \geq -28{,}343$ $y + 678{,}857x \geq 22.122{,}7$ $y - 0x \geq 6.499$ $y - 1.509{,}33x \geq 461{,}667$	Matrix form: $\begin{bmatrix} -1 & -393 \\ -1 & -168 \\ -1 & -44{,}7273 \\ -1 & 66{,}6 \\ -1 & 1.003 \\ -1 & -678{,}857 \\ -1 & 0 \\ -1 & 1.509{,}33 \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix} \leq \begin{bmatrix} -2.364 \\ -1.914 \\ -927{,}818 \\ 1.187{,}4 \\ 28{,}343 \\ -22.122{,}7 \\ -6.499 \\ -461{,}667 \end{bmatrix}$ Obtained: $\mathbf{D1'} = \begin{bmatrix} -1 & -393 \\ -1 & -168 \\ -1 & -44{,}7273 \\ -1 & 66{,}6 \\ -1 & 1.003 \\ -1 & -678{,}857 \\ -1 & 0 \\ -1 & 1.509{,}33 \end{bmatrix} \quad \mathbf{d1'} = \begin{bmatrix} -2.364 \\ -1.914 \\ -927{,}818 \\ 1.187{,}4 \\ 28{,}343 \\ -22.122{,}7 \\ -6.499 \\ -461{,}667 \end{bmatrix}$

Table 5
Determination of the uncertainty matrix on cooking oil commodities

Demand of Cooking Oil	**Line equations:** $\boldsymbol{y + 181x \geq 1.086}$ $\boldsymbol{y + 77{,}1667x \geq 878{,}333}$ $\boldsymbol{y + 20{,}4545x \geq 424{,}636}$ $\boldsymbol{y - 30{,}5x + \geq -543{,}5}$ $\boldsymbol{y - 460x \geq -12{,}999}$ $\boldsymbol{y + 311{,}571x \geq 10.148{,}1}$ $\boldsymbol{y - 0x \geq 2.982}$ $\boldsymbol{y - 692{,}333x \geq 212{,}667}$	**Matrix form:** $\begin{bmatrix} \mathbf{-1} & \mathbf{-181} \\ \mathbf{-1} & \mathbf{-77{,}1667} \\ \mathbf{-1} & \mathbf{-20{,}4545} \\ \mathbf{-1} & \mathbf{30{,}5} \\ \mathbf{-1} & \mathbf{460} \\ \mathbf{-1} & \mathbf{-311{,}571} \\ \mathbf{-1} & \mathbf{0} \\ \mathbf{-1} & \mathbf{692{,}333} \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix} \leq \begin{bmatrix} \mathbf{-1.086} \\ \mathbf{-878{,}333} \\ \mathbf{-424{,}636} \\ \mathbf{543{,}5} \\ \mathbf{12{,}999} \\ \mathbf{-10.148{,}1} \\ \mathbf{-2.982} \\ \mathbf{-212{,}667} \end{bmatrix}$ Obtained: $\mathbf{D2} = \begin{bmatrix} -1 & -181 \\ -1 & -77{,}1667 \\ -1 & -20{,}4545 \\ -1 & 30{,}5 \\ -1 & 460 \\ -1 & -311{,}571 \\ -1 & 0 \\ -1 & 692{,}333 \end{bmatrix} \quad \mathbf{d2} = \begin{bmatrix} -1.086 \\ -878{,}333 \\ -424{,}636 \\ 543{,}5 \\ 12{,}999 \\ -10.148{,}1 \\ -2.982 \\ -212{,}667 \end{bmatrix}$
Capacity of Cooking Oil	Line equations: $y + 256x \geq 1.540$ $y + 109{,}5x \geq 1.247$ $y + 29{,}0909x \geq 603{,}727$ $y - 43{,}3x \geq -773{,}6$ $y - 628x \geq -17{,}727$ $y + 45{,}857x \geq 14.488{,}7$ $y - 0x \geq 4.234$ $y - 983{,}333x \geq 300{,}667$	Matrix form: $\begin{bmatrix} -1 & -256 \\ -1 & -109{,}5 \\ -1 & -29{,}0909 \\ -1 & 43{,}3 \\ -1 & 628 \\ -1 & -45{,}857 \\ -1 & 0 \\ -1 & 983{,}333 \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix} \geq \begin{bmatrix} -1.540 \\ -1.247 \\ -603{,}727 \\ 773{,}6 \\ 17{,}727 \\ -14.488{,}7 \\ -4.234 \\ -300{,}667 \end{bmatrix}$ Obtained: $\mathbf{D2'} = \begin{bmatrix} -1 & -256 \\ -1 & -109{,}5 \\ -1 & -29{,}0909 \\ -1 & 43{,}3 \\ -1 & 628 \\ -1 & -45{,}857 \\ -1 & 0 \\ -1 & 983{,}333 \end{bmatrix} \quad \mathbf{d2'} = \begin{bmatrix} -1.540 \\ -1.247 \\ -603{,}727 \\ 773{,}6 \\ 17{,}727 \\ -14.488{,}7 \\ -4.234 \\ -300{,}667 \end{bmatrix}$

3.2.2 Numerical Experiments

Numerical Experiments of Agricultural Processed Products Supply Chain Problem in Robust Optimization Model with Polyhedral Uncertainty Sets Scenario 1

This section discusses the results of numerical experiments that are applied to the Robust optimization model for the agricultural processed products supply chain problem Scenario 1. The uncertainty parameters that are demand and product capacity are in the polyhedral uncertainty set. Scenario 1 is an additional constraint to the nominal model in Perdana, et al. (2020) when large-scale social distancing is applied as described in Table 1 with the types of agricultural processed products in the form of sugar and cooking oil. This numerical experiment requires an uncertainty matrix and vector $\mathbf{d1}, \mathbf{d1'}, \mathbf{d2}, \mathbf{d2'}, \mathbf{D1}, \mathbf{D1'}, \mathbf{D2},$ and $\mathbf{D2'}$ that has been obtained in Table 4 and Table 5.

The total maximum demand for sugar is Rp5.000.000.000, while for cooking oil is Rp5.000.000.000. The supply chain for the distribution of sugar and cooking oil from the producer zone to LFH scenario 1 is shown in Fig. 5 and Fig. 7, while the distribution from LFH to the consumer zone is shown in Fig. 6 and Fig. 8. The Fulfillment Ratio for the two products is shown in Table 6.

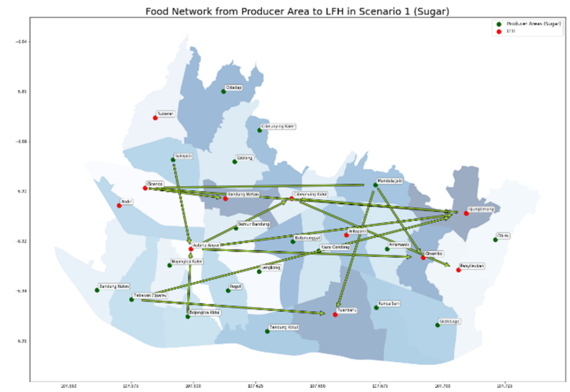

Fig. 5. Sugar distribution flow from producers to LFH in robust optimization model scenario 1

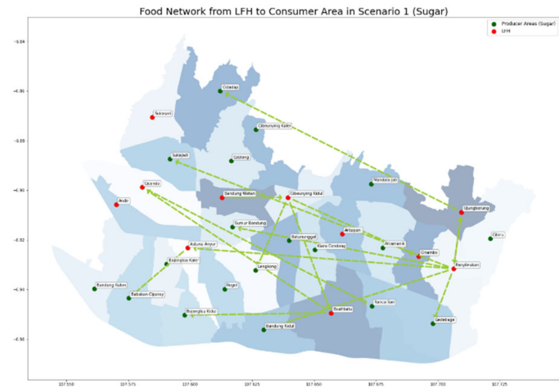

Fig. 6. Sugar distribution flow from LFH to consumers in robust optimization model scenario 1

Fig. 5 shows the distribution flow of sugar from the producer zone to the LFH, from the figure there are 11 out of a total of 30 districts that are the optimal locations for LFH which is symbolized by the red dot, namely Andir, Antapani, Astana Anyar, Bandung Wetan, Buah Batu, Cibeunying Kidul, Cicendo, Cinambo, Panyileukan, Sukasari, and Ujungberung. There are 19 other districts that are not an optimal location. This is due to the constraints of the first scenario which limit the movement between the red zone and the green zone. In this study, all distribution does not consider costs as described by Ritonga (2021) in the previous sub-chapter, so that to meet and maximize demand in each LFH district, the distribution is carried out among various districts from the production zone. Fig. 6 shows the next distribution flow of sugar, namely from the selected optimal LFH district to the final zone, the consumer zone marked with a green dot. There are still 11 consumer zones that have not been distributed by LFH. This can also be seen in Table 6. The fulfillment ratio is the ratio that determines the percentage of the number of requests that are fulfilled in each consumer district. This ratio is determined by adding up the number of distributions to the last location (consumers) in each district. For sugar, there are still 11 districts that have not been fulfilled with a 0% ratio, namely Arcamanik, Bandung Kulon, Bandung Wetan, Batununggal, Bojongloa Kaler, Cibeunying Kaler, Coblong, Kiara Condong, Mandala Jati, and Regol. This is because there is no distribution to these districts due to the exhaustion of capacity to be distributed to other districts.

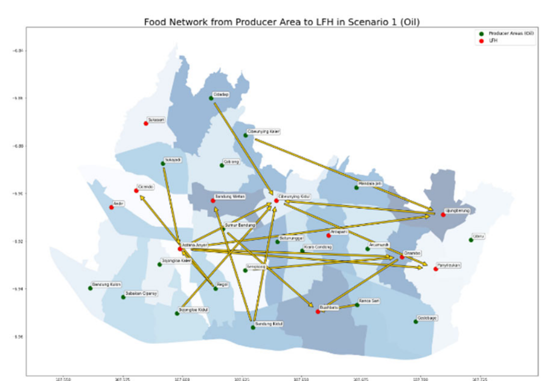

Fig. 7. Cooking oil distribution flow from producers to LFH in robust optimization model scenario 1

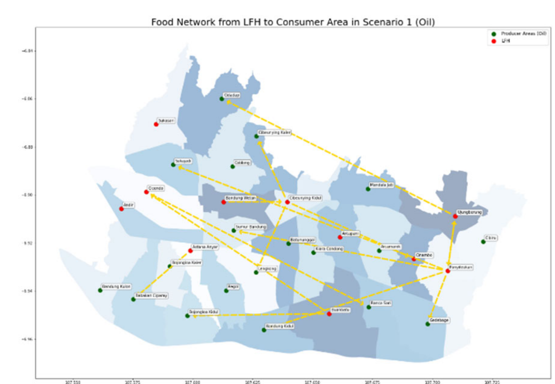

Fig. 8. Cooking oil distribution flow from LFH to consumers in robust optimization model scenario 1

With the same explanation as for sugar agricultural processed products, the first oil distribution flow from the producer zone to LFH is shown in Fig. 7, then continued to the second distribution from LFH to the consumer zone is shown in Fig. 8. For the oil fulfillment ratio, there is one more zone consumer who are not distributed, namely Astana Anyar District.

Numerical Experiments of Agricultural Processed Products Supply Chain Problem in Robust Optimization Model with Polyhedral Uncertainty Sets Scenario 2

This section discusses the results of numerical experiments applied to the Robust optimization model for supply chain problems of processed agricultural products. Scenario 2. Parameters of uncertainty on demand and product capacity are in a polyhedral indeterminate set. Scenario 2 is an additional obstacle to the nominal model when partial social distancing is applied as described in Table 1 with the types of processed agricultural products in the form of sugar and cooking oil. This numerical experiment requires an uncertainty matrix and vector $\mathbf{d1}, \mathbf{d1'}, \mathbf{d2}, \mathbf{d2'}, \mathbf{D1}, \mathbf{D1'}, \mathbf{D2}$, and $\mathbf{D2'}$ that has been obtained in Table 4 and Table 5. The total maximum demand for sugar is Rp5.000.000.000, while for cooking oil is Rp5.000.000.000. The supply chain for the distribution of sugar and cooking oil from the producer zone to LFH scenario 1 is shown in Fig. 9 and Fig. 11, while the distribution from LFH to the consumer zone is shown in Fig. 10 and Fig. 12. The Fulfillment Ratio for the two products is shown in Table 7.

Table 6
Fulfillment ratio of sugar and cooking oil in robust optimization model scenario 1

Consumer District	Demand (Ton)		Demand Fulfilled (Ton)		Fulfilment Ratio	
	Sugar	Cooking Oil	Sugar	Cooking Oil	Sugar	Cooking Oil
Andir	1.350	905	1.350	905	100%	100%
Antapani	1.081	724	1.081	724	100%	100%
Arcamanik	1.057	708	0	0	0%	0%
Astana Anyar	4.451	2.982	4.451	0	100%	0%
Babakan Ciparay	1.920	1.286	1.920	1.286	100%	0%
Bandung Kidul	824	552	824	552	100%	100%
Bandung Kulon	1.838	1.231	0	0	0%	0%
Bandung Wetan	390	261	0	0	0%	0%
Batunuggal	1.642	1.100	0	0	0%	0%
Bojongloa Kaler	1.683	1.127	0	0	0%	0%
Bojongloa Kidul	1.181	791	1.181	791	100%	100%
Buah Batu	1.394	934	1.394	0	100%	0%
Cibeunying Kaler	956	640	0	640	0%	100%
Cibeunying Kidul	1.531	1.026	1.531	1.026	100%	100%
Cibiru	1.009	676	0	0	0%	0%
Cicendo	1.303	873	1.303	873	100%	100%
Cidadap	734	492	734	492	100%	100%
Cinambo	344	231	344	231	100%	100%
Coblong	54	36	0	0	0%	0%
Gedebage	556	372	556	372	100%	100%
Kiara Condong	1.773	1.188	0	0	0%	0%
Lengkong	967	648	967	648	100%	100%
Mandala Jati	4.452	2.982	0	0	0%	0%
Panyileukan	544	365	544	365	100%	100%
Ranca Sari	1.153	772	1.153	772	100%	100%
Regol	1.095	734	0	0	0%	0%
Sukajadi	1.392	932	1.392	932	100%	100%
Sukasari	1.052	705	1.052	705	100%	100%
Sumur Bandung	510	341	510	341	100%	100%
Ujung Berung	1.196	801	1.196	801	100%	100%

Fig. 9 shows the distribution flow of sugar from the producer zone to LFH, from the figure there are 14 out of a total of 30 districts that are the optimal locations for LFH which is symbolized by the red dot, namely Arcamanik, Astana Anyar, Batununggal, Bojongloa Kaler, Bojongloa Kidul, Buah Batu, Cibeunying Kidul, Cidadap, Cinambo, Gedebage, Lengkong, Panyileukan, Regol, and Sukajadi. There are 16 other districts that are not optimal. However, scenario 2 gives the optimal number of districts for the construction of more LFH than scenario 1. This is due to the constraints of the second scenario where the tightness of the distribution is reduced, namely that it is permitted the distribution of products from producers in the green zone to LFH in the red zone and the distribution of products from LFH in the green zone to consumers in the red zone.

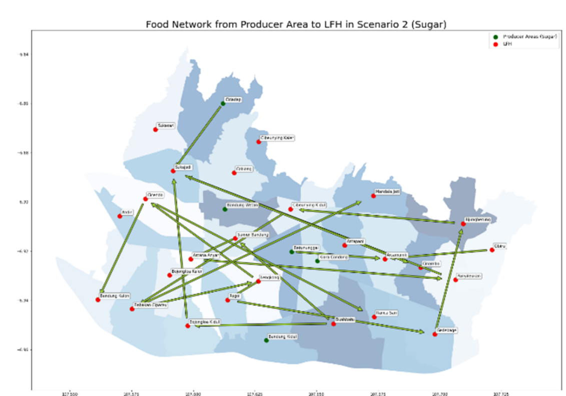

Fig. 9. Sugar distribution flow from producers to LFH in robust optimization model scenario 2

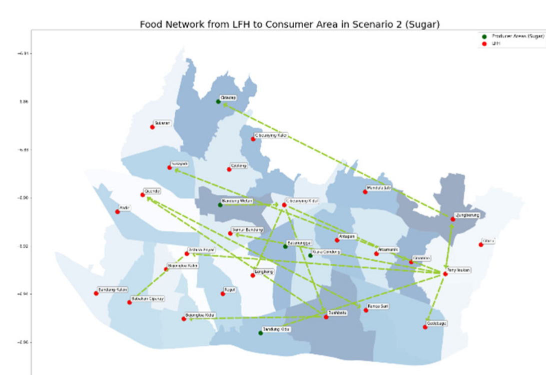

Fig. 10. Sugar distribution flow from LFH to consumers in robust optimization model scenario 2

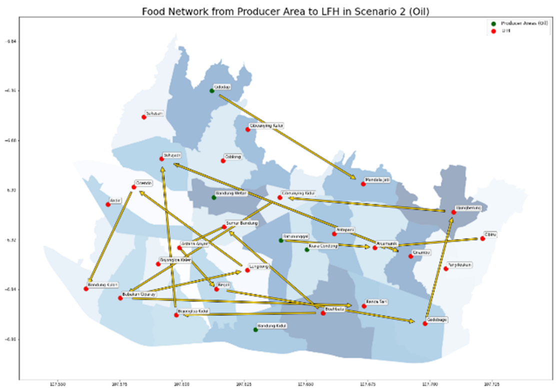

Fig. 11. Cooking oil distribution flow from producers to LFH in robust optimization model scenario 2

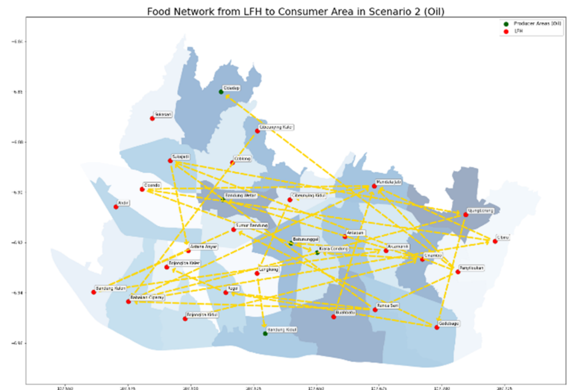

Fig. 12. Cooking oil distribution flow from LFH to consumers in robust optimization model scenario 2

Since there are more LFH locations and the influence of scenario 2 constraints, then in Figure 10 and Figure 12, which are images showing the distribution flow of sugar and cooking oil from LFH to the end zone (consumer) marked with a green dot, demand in the consumer zone is more fulfilled than scenario 1. In scenario 2, for sugar, there are 8 districts of consumer zones that are not distributed, namely Bandung Wetan, Bojongloa Kidul, Buah Batu, Cibeunying Kidul, Cinambo, Panyileukan, Sukasari, and Sumur Bandung districts. As for cooking oil, there are only 2 districts of the consumer zone that are not fulfilled, namely Buah Batu District, and Cibeunying Kaler. This can also be seen in Table 7.

Table 7
Fulfillment ratio of sugar and cooking oil in robust optimization model scenario 2

Consumer District	Demand (Tonnes)		Fulfilled Demand (Tonnes)		Fulfillment Ratio	
	Gula Pasir	Minyak Goreng	Gula Pasir	Minyak Goreng	Gula Pasir	Minyak Goreng
Andir	1.350	905	1.350	905	100%	100%
Antapani	1.081	724	1.081	724	100%	100%
Arcamanik	1.057	708	1.057	708	100%	100%
Astana Anyar	4.451	2.982	4.451	2.982	100%	100%
Babakan Ciparay	1.920	1.286	1.920	1.286	100%	100%
Bandung Kidul	824	552	824	552	100%	100%
Bandung Kulon	1.838	1.231	1.838	1.231	100%	100%
Bandung Wetan	390	261	0	261	0%	100%
Batunuggal	1.642	1.100	1.642	1.100	100%	100%
Bojongloa Kaler	1.683	1.127	1.683	1.127	100%	100%
Bojongloa Kidul	1.181	791	0	791	0%	100%
Buah Batu	1.394	934	0	0	0%	0%
Cibeunying Kaler	956	640	956	0	100%	0%
Cibeunying Kidul	1.531	1.026	0	1.026	0%	100%
Cibiru	1.009	676	1.009	676	100%	100%
Cicendo	1.303	873	1.303	873	100%	100%
Cidadap	734	492	734	492	100%	100%
Cinambo	344	231	0	231	0%	100%
Coblong	54	36	54	36	100%	100%
Gedebage	556	372	556	372	100%	100%
Kiara Condong	1.773	1.188	1.773	1.188	100%	100%
Lengkong	967	648	967	648	100%	100%
Mandala Jati	4.452	2.982	4.452	2.982	100%	100%
Panyileukan	544	365	0	365	0%	100%
Ranca Sari	1.153	772	1.153	772	100%	100%
Regol	1.095	734	1.095	734	100%	100%
Sukajadi	1.392	932	1.392	932	100%	100%
Sukasari	1.052	705	0	705	0%	100%
Sumur Bandung	510	341	0	341	0%	100%
Ujung Berung	1.196	801	1.196	801	100%	100%

Furthermore, in Table 8 and Table 9, the order based on the index of the optimal district and its capacity in tons for the distribution of sugar and cooking oil in scenario 1 and scenario 2.

Table 8
List of LFH optimal location and its optimal capacity for sugar (S1 for scenario 1 and S2 for scenario 2)

Index	District	S1 (Tonnes)	S2 (Tonnes)		Index	District	S1 (Tonnes)	S2 (Tonnes)
1	Andir	1.255,56	-		16	Cicendo	1.153	-
2	Antapani	1.081	-		17	Cidadap	-	570
3	Arcamanik	-	812		18	Cinambo	1.683	1.980
4	Astana Anyar	4.619	4.659		20	Gedebage	-	4.531
8	Bandung Wetan	1.531	-		22	Lengkong	-	3.521
9	Batunuggal	-	1.723,9158		24	Panyileukan	4.478	822,0842
10	Bojongloa Kaler	-	872		26	Regol	-	5.010
11	Bojongloa Kidul	-	503		27	Sukajadi	-	744
12	Buah Batu	2.484	3.684		28	Sukasari	1.052	-
14	Cibeunying Kidul	2.267	2.073		30	Ujungberung	3.116	-

Table 9
List of LFH optimal location and its optimal capacity for cooking oil (S1 for scenario 1 and S2 for scenario 2)

Index	District	S1 (Tonnes)	S2 (Tonnes)		Index	District	S1 (Tonnes)	S2 (Tonnes)
1	Andir	500,959	403,759		16	Cicendo	772	1.295,242
2	Antapani	724	724		18	Cinambo	1.127	261
4	Astana Anyar	3.094	3.204		20	Gedebage	-	4.531
5	Babakan Ciparay	-	5,64		22	Lengkong	-	3.912,242
8	Bandung Wetan	1.026	-		24	Panyileukan	2.998	1.629
9	Batunuggal	-	1.723,9158		26	Regol	-	4.384,759
10	Bojongloa Kaler	-	670,364		27	Sukajadi	-	744
12	Buah Batu	1.664	5.108		28	Sukasari	705	705
14	Cibeunying Kidul	1.519	1.827		30	Ujungberung	2.088	979

Based on the previous explanation, the Robust optimization model for the supply chain problem of agricultural processed products in the form of sugar and cooking oil scenario 1 gives the results of the maximization objective function is smaller than scenario 2. Scenario 1 provides lower costs because of the larger distribution restrictions. However, this study does not consider the cost of fulfilling demand, so with the aim of maximizing consumer demand in order to be fulfilled, scenario 2 is better to do. In addition, scenario 2 also provides more optimal LFH locations than scenario 1, so that demand fulfillment in the consumer zone will be greater and distribution will run optimally. Therefore, scenario 2 is still better to do.

4. Conclusions

One of the contributions of Operations Research and Optimization Modeling during Covid-19 Pandemic is in a role of decision support system, such as solving the uncertain supply chain problem for agricultural processed products with Robust optimization methodology. Taking into account the uncertainty parameter is assumed to be the demand and capacity of agricultural processed products. Robust optimization model of the supply chain problem for agricultural processed products is solved using a polyhedral uncertainty set approach, then a model is obtained computationally tractable, it means that the problem can be solved computationally in polynomial time, thus the robust optimal solution is obtained. The result of this study can be considered as scientific proof for the government to regulate the processed product supply chain problem, especially for sugar and cooking oil in Bandung Area.

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