Multiproduct manufacturer-retailer coordinated supply chain with adjustable rate for common parts, delayed differentiation, and multi-shipment

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A B S T R A C T

Operating in today's turbulent and competitive world marketplaces, manufacturers must find the best production scheme and delivery policy to meet timely client's multiproduct requirements and minimize the total manufacturing-shipment expenses. This study proposes a two-stage delayed differentiation model for a multiproduct manufacturer-retailer coordinated supply chain featuring the adjustable-rate for making common parts and a multi-shipment policy for transporting finished goods. The aim is to help present-day manufacturers achieve their operational goals mentioned above. The mathematical techniques help us build a specific model to explicitly represent the problem and derive its overall operating expense. Then, the convexity of the total expense is verified by Hessian matrix equations. The differential calculus helps derive the cost-minimized fabrication-shipment decision. This study offers an example to demonstrate the applicability and capabilities of our proposed model numerically. The following crucial information has been made available to the managers to facilitate their operating decision makings: (1) the problem's best fabrication-shipment policy; (2) the collective influence of various common part's completion rates and values on the problem's total expenses and optimal fabrication-shipment policy; (3) the impact of various adjustable-rates in stage one on utilization and stage one's uptime; (4) the details of cost contributors to the problem; and (5) the collective impacts of critical features on the problem's performance.

1. Introduction

Operating in the stiff and turbulent world marketplaces, current manufacturers must find the most efficient and economical supply chain strategy in fabrication schemes and shipping plans to meet timely client multiproduct needs. Implementing a delayed differentiation strategy during the multiproduct making and expediting the time-consuming common parts' producing process helps the manufacturers gain a competitive advantage in system efficiency and overall cost-savings. Arranging the making of numerous products on a single machine can boost its utilization. Schneider and Rinks (1989) considered a periodic-review multi-item storage-space constraint inventory problem with the tradeoff of customer service level and the inventory policy's cost. The researchers derived the system's approximation solution based on asymptotic features from the renewal theorem to meet space constraints, system cost, and workload for arranging customer orders. Katayama (1998) proposed a hierarchical model to evaluate a joined management procedure of mixed lines multiproduct manufacturing system. The presented model considered skilled workers, capable facilities, and tools to allow the efficient production of various products. For the line balancing processes, the researcher proposed an algorithm to minimize the needed stations. For the job sequencing processes, a target chasing approach is used to enable effective and reliable production. The researcher proposed a two-stage hierarchical structure to incorporate the abovementioned procedures and evaluate their

performance with simulated experiments under different situations. Delayed differentiation is an effective manufacturing strategy in planning multi-item production. It makes the standard, intermediate components at one time and fabricates the customized final goods in the second phase to reduce overall manufacturing cost and uptime. Rajagopalan and Swaminathan (2001) presented a coordinated fabrication model to study the collaboration between discrete capacity acquisition and fabrication planning in a constant demand growing/fluctuating situation. The researchers proposed a model using mathematical programming attempting to derive the best acquisition policy of capacity and the optimal fabrication-inventory decision in finite time periods. The results' effectiveness and quality are justified thru computational examples. The influence of manufacturing capacity, capital cost, fabrication decision, and product variety on the optimal solution are investigated to gain crucial managerial insights into the problem. Graman (2010) built a cost minimization model to study the best replenishing levels for end items and postponement strategy in an order-up-to two-product manufacturing environment. The researcher used non-linear programming to solve the model's formulations and gain the optimal inventory and postponement strategies. The author also conducted the sensitivity analyses of the variations in holding, packaging, and postponement costs, fill rate, demand, and product value on the obtained solution. Weskamp et al. (2019) developed a model to investigate the optimal postponement strategy in stochastic-demand supply chains. Their two-stage stochastic model considered integration fabrication and shipment with postponement options, lead times, uncertain demand, shortage cost. The research applied mixed-integer linear programming and used an apparel industry's case to demonstrate their model's applicability. Both risk aversion and the benefits of their proposed model were examined and discussed. Other works (Davis and Sasser, 1995; Nahmias, 2009; Sheikh et al., 2019; Garcia et al., 2020; Chiu et al., 2021; Fatehi-Kivi et al., 2021; Ohmori and Yoshimoto, 2021; Quetschlich et al., 2021; Samuel et al., 2021) considered various features of optimizing manufacturing plans and operations on multiproduct and with postponement alternatives.

Expediting manufacturing/output rate is a commonly used approach to cut down the manufacturing needed time effectively. By adding shifts, applying overtime to an existing shift, or adjusting the machine fabrication rate, manufacturers could expedite their manufacturing/ output rate. Yang et al. (2000) determined the best fabrication and set up schedule for a single machine dual product system considering adjustable fabrication rate. Their study aimed at minimizing the system's setup, backlogging, and average stock-carrying costs. The obtained optimal result was verified by the Hamilton-Jacobi-Bellman equation plus the dynamic programming. Sharma (2008) reexamined an existing work that dealt with the impact of changes in flexible rates on a manufacturing system by adding variations in stock-carrying costs relating to a single or multi-product replenishing system. The research studied the influence of different carrying-cost scenarios on the system with flexible manufacturing rates. Dellagi et al. (2017) explored smoothing an integrated fabrication-maintenance plan via controlling the manufacturing rate to meet stochastic demand and the needed customer service level. The researchers developed a model to analyze and derive a near-optimal fabrication-maintenance policy considering variable fabrication and failure rates in a finite planning period. The numerical demonstration showed how the model worked and discussed various possible trade-offs consideration. Multiple discontinuous shipments are constantly used in real-world supply chains to move inventories from the manufacturer's warehouse to retailers. Pasandideh et al. (2010) examined a multiproduct economic manufacturing quantity problem featuring orders with discrete shipments, backlogging, and limited spaces. The researchers proposed a genetic algorithm using nonlinear integer programming to solve this problem. Numerical experiments with different variable values and problem sizes are conducted to illustrate the applicability and capability of their proposed model and solution methodology. İnkaya et al. (2018) explored the influence of competition, various costs, and coordination on product price and variety decisions in a producer-customer supply chain environment. The researchers considered costs incurred on the producer's end to be materials, production, storage, and shipping. They examined three scenarios in the supply chain's coordination. They compared them with uncoordinated one to expose (1) the relationship between materials cost and product differentiation strategy and (2) the competitive intensity issue. The authors also conducted numerical experiments to demonstrate the trade-offs of quality level and materials cost. Other works (Brahmi et al., 2020; Nogueira et al., 2020; Tolooie et al., 2020; Astuty et al., 2021; Ekin and Aktekin, 2021; Frank et al., 2021; Martins et al., 2021; Ongcunaruk et al., 2021; Pando et al., 2021) considered various features of adjustable manufacturing rates and discrete shipping plans to optimize fabrication-inventory systems and supply-chain operations. This study proposes a two-stage delayed differentiation model for a multiproduct manufacturer-retailer coordinated supply chain featuring the adjustable-rate for making common parts and a multi-shipment policy for transporting finished goods.

2. The proposed multiproduct manufacturer-retailer system

2.1. Nomenclature

Notation used in common parts' fabrication (i.e., in stage 1)

λ_0	=	annual requirement of common parts,
S_0	=	setup time,
Q_0	=	lot-size,
$P_{T1,0}$	=	adjustable rate per year,
$t_{1,0}$	=	uptime when implementing the adjustable-rate,
$\alpha_{1,0}$	=	adding proportion of adjustable-rate,
$P_{1,0}$	=	regular rate,

$t_{2,0}$ = depletion time,
t_0^* = optimal uptime,
K_{T0} = setup cost when implementing the adjustable-rate,
$\alpha_{2,0}$ = the connected factor between K_{T0} and K_0,
K_0 = regular setup cost,
C_{T0} = unit cost when implanting the adjustable-rate,
$\alpha_{3,0}$ = the connected factor between C_{T0} and C_0,
γ = completion rate (as compared to the finished product),
C_0 = standard unit cost,
$h_{1,0}$ = unit holding cost,
$H_{1,0}$ = stock level when its uptime ends;

Notation used in finished items' fabrication (i.e., in stage 2)
λ_i = item i's requirements per year (where $i = 1, 2, …, L$),
L = the number of different finished products,
Q_i = lot-size,
S_i = setup time,
T_A = rotation cycle length – decision variable one,
$P_{1,i}$ = annual manufacturing rate,
K_i = setup cost,
C_i = unit cost,
n = number of equal-size shipments – decision variable two,
$t_{n,i}$ = a fixed time-interval between two succeeding shipments,
$t_{1,i}$ = uptime,
$t_{2,i}$ = delivery time,
t_i^* = the sum of optimal uptimes (i.e., $\Sigma_i(t^*_{1,i})$),
$h_{1,i}$ = unit holding cost,
i_0 = holding cost ratio (i.e., $h_{1,i} = i_0 C_i$),
$H_{1,i}$ = stock level when its uptime ends,
$K_{D,i}$ = fixed transportation cost,
$C_{D,i}$ = unit tranportation cost,
H_i = inventory level of common partsl when end item i's uptime ends,
$I(t)_i$ = stock level at time t (where $i = 0, 1, 2, …, L$),
D_i = fixed-quantity per delivery,
$TC(T_A, n)$ = total system cost per cycle,
$I_c(t)_i$ = retailer's stock level at time t,
I_i = number of items left when $t_{n,i}$ ends,
$TCU(T_A, n)$ = the system cost per unit time.

2.2. Description and modeling

This study explores a two-stage delayed differentiation multiproduct manufacturer-retailer coordinated system with adjustable-rate for common parts and multi-shipment policy. It extends the multiproduct finite production rate (FPR) model to incorporate postponement strategy with a known constant γ, an adjustable-rate $P_{T1,0}$ for making the common parts in 1st stage, and a multi-shipment rule for transporting the finished goods. Both stages' regular manufacturing rates $P_{1,0}$ and $P_{1,i}$ depend on γ. For example, if $\gamma = 0.5$, then $P_{1,0}$ and $P_{1,i}$ double the single-stage system' normal rates. Additionally, the relationship of adjustable-rate $P_{T1,0}$ and regular-rate $P_{1,0}$ and the consequent cost-increase connection are exhibited in the following expressions:

$$P_{T1,0} = P_{1,0}\left(1+\alpha_{1,0}\right) \tag{1}$$

$$K_{T0} = K_0\left(1+\alpha_{2,0}\right) \tag{2}$$

$$C_{T0} = C_0\left(1+\alpha_{3,0}\right) \tag{3}$$

Fig. 1 depicts the proposed study's stock level. It indicates the stock level upsurges to $H_{1,0}$ as uptime ends. Then, stage two begins to fabricate the finished product i, so the common intermediate parts start to deplete. Each finished product i's inventory level rises to $H_{1,i}$ when their uptime complete (see Figure 1). The proposed system doesn't allow any stock-out occurrences, so formulas $(P_{1,0} - \lambda_0 > 0)$ and $(P_{1,i} - \lambda_i > 0)$ must hold. Based on the assumption of stage two, we observe the following expressions for $i = 1, 2, …, L$:
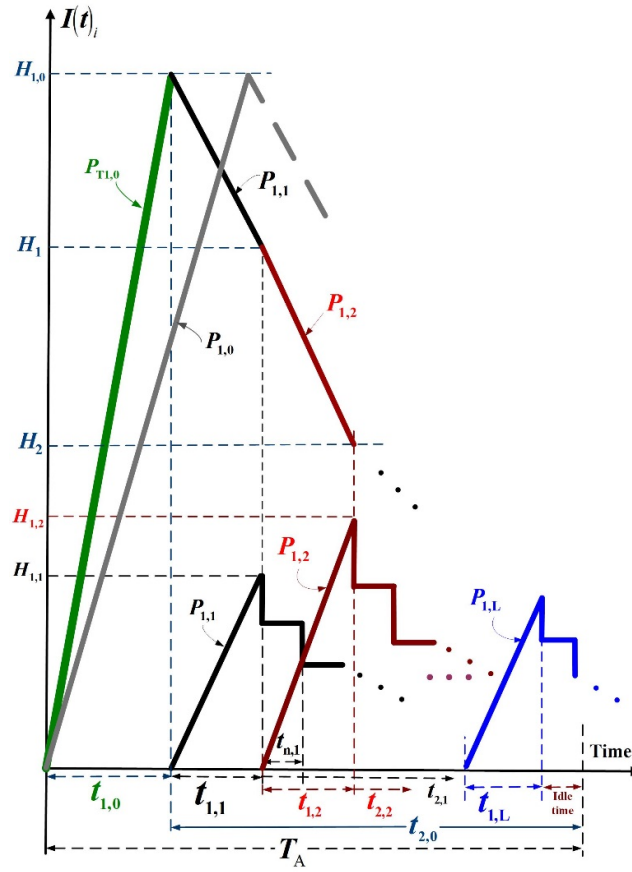
Fig. 1. Stock level of the proposed two-stage multiproduct manufacturer-retailer coordinated system with adjustable-rate for common parts, delayed differentiation, and multi-shipment compared to the same system without adjustable rate (in grey)

$$T_{A} = \frac{Q_i}{\lambda_i} = t_{1,i} + t_{2,i} \tag{4}$$

$$Q_i = \lambda_i T_{A} \tag{5}$$

$$t_{1,i} = \frac{Q_i}{P_{1,i}} = \frac{H_{1,i}}{P_{1,i}} \tag{6}$$

$$t_{2,i} = T_{A} - t_{1,i} \tag{7}$$

$$H_{1,i} = Q_i = P_{1,i} t_{1,i} \tag{8}$$

According to the assumption of stage one, we know that the common intermediate parts must meet the need for fabricating finished products (as shown in Eq. (9), also refer to Eq. (5)), and the following additional expressions are observed:

$$H_{1,0} = \sum_{i=1}^{L} Q_i = \sum_{i=1}^{L} \lambda_i T_{A} \tag{9}$$

$$T_{A} = t_{1,0} + t_{2,0} \tag{10}$$

$$t_{1,0} = \frac{Q_0}{P_{T1,0}} = \frac{H_{1,0}}{P_{T1,0}} \tag{11}$$

$$\lambda_0 = \frac{\sum_{i=1}^{L} Q_i}{T_{A}} \tag{12}$$

$$H_1 = H_{1,0} - Q_1 \tag{13}$$

$$H_i = H_{(i-1)} - Q_i \; , \; \; where \; i = 2, \; 3, \; ..., L \tag{14}$$

$$H_L = H_{(L-1)} - Q_L = 0 \tag{15}$$

Fig. 2 illustrates the manufacturer's inventory level in each finished item i's transporting time, and they are shipped to the retailer under n equal-size shipments in each interval $t_{n,i}$ time. The total stocks in $t_{2,i}$ is given in Eq. (16):

$$\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right)H_{1,i}\left(t_{2,i}\right) = \left(\frac{1}{n^2}\right)H_{1,i}\left(t_{2,i}\right)\left[\frac{n(n-1)}{2}\right] = \left(\frac{n-1}{2n}\right)H_{1,i}\left(t_{2,i}\right) \tag{16}$$
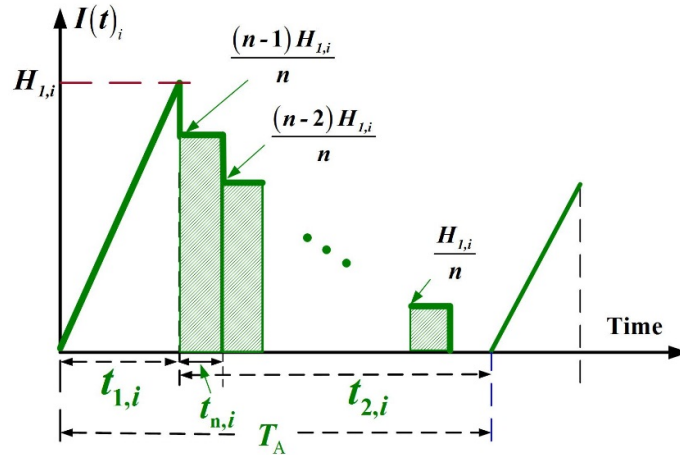
Fig. 2. Manufacturer's finished product i level in shipping time $t_{2,i}$

Fig. 3. demonstrates the retailer's inventory level of each item i. The retailer's total stocks are given in Eq. (17):

$$\left[\frac{n\left(D_i - I_i\right)t_{n,i}}{2} + \frac{n\left(n+1\right)}{2}I_i t_{n,i} + \frac{nI_i\left(t_{1,i}\right)}{2}\right] \tag{17}$$

where

$$D_i = \frac{H_{1,i}}{n} \tag{18}$$

$$t_{n,i} = \frac{t_{2,i}}{n} \tag{19}$$

$$I_i = D_i - \lambda_i\left(t_{n,i}\right) \tag{20}$$
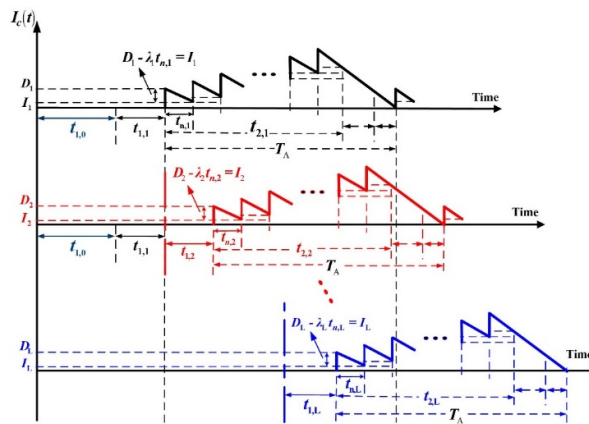

Fig. 3. Retailer's product i level

3. Overall system cost and the manufacturing-shipment policy

3.1. Overall system cost

The overall system cost per cycle $TC(T_A, n)$ includes the following manufacturing-shipment related expenses: (1) variable, (2) setup, (3) finished items' transportation, and (4) inventory holding at both manufacturer and retailer ends:

$$
TC(T_A,\,n) = \left\{ C_{T0}Q_0 + K_{T0} + h_{1,0}\left[\frac{H_{1,0}t_{1,0}}{2} + \sum_{i=1}^{L}\left[\frac{Q_i}{2}(t_{1,i}) + H_i(t_{1,i})\right]\right]\right\}
$$

$$
+ \sum_{i=1}^{L}\left\{
\begin{array}{l}
C_iQ_i + K_i + nK_{D,i} + C_{D,i}Q_i + h_{1,i}\left[\dfrac{H_{1,i}t_{1,i}}{2} + \left(\dfrac{n-1}{2n}\right)H_{1,i}(t_{2,i})\right] \\[2ex]
+h_{3,i}\left[\dfrac{n(D_i - I_i)t_{n,i}}{2} + \dfrac{n(n+1)}{2}I_i t_{n,i} + \dfrac{nI_i(t_{1,i})}{2}\right]
\end{array}
\right\}
\tag{21}
$$

Substituting Eqs. (1) to (20) in Eq. (21) and computing $TC(T_A, n)\,/\,T_A$, we gain the following $TCU(T_A, n)$:

$$
TCU(T_A,\,n) = \frac{TC(T_A,\,n)}{T_A} = (1+\alpha_{3,0})C_0\lambda_0 + \frac{(1+\alpha_{2,0})K_0}{T_A} + h_{1,0}\sum_{i=1}^{L}\left(\frac{\lambda_i^2 T_A}{2P_{1,i}}\right)
$$

$$
+h_{1,0}\left\{\left[\frac{\lambda_0^2 T_A}{2(1+\alpha_{1,0})P_{1,0}}\right] + \sum_{i=1}^{L}\left\{-\left(\frac{\lambda_i T_A}{P_{1,i}}\right)\right\}\left[\sum_{j=1}^{i}(\lambda_j)\right] + \left[\sum_{i=1}^{L}(\lambda_i)\right]\cdot\sum_{i=1}^{L}\left(\frac{\lambda_i T_A}{P_{1,i}}\right)\right\}
\tag{22}
$$

$$
+\sum_{i=1}^{L}\left\{C_i\lambda_i + \frac{K_i}{T_A} + \frac{nK_{D,i}}{T_A} + C_{D,i}\lambda_i + \frac{h_{1,i}\lambda_i T_A}{2} + \frac{h_{3,i}\lambda_i^2 T_A}{2P_{1,i}} + \frac{(h_{3,i}-h_{1,i})\lambda_i^2 T_A}{2n}\left(\frac{1}{\lambda_i} - \frac{1}{P_{1,i}}\right)\right\}
$$

3.2. The optimal manufacturing-shipment policy

Applying the Hessian Matrix Equations $TCU(T_A, n)$ (Rardin, 1998), one obtains the following:

$$
\begin{bmatrix} T_A & n \end{bmatrix} \cdot
\begin{pmatrix}
\dfrac{\partial^2 TCU(T_A,\,n)}{\partial T_A^2} & \dfrac{\partial^2 TCU(T_A,\,n)}{\partial T_A \partial n} \\[2ex]
\dfrac{\partial^2 TCU(T_A,\,n)}{\partial T_A \partial n} & \dfrac{\partial^2 TCU(T_A,\,n)}{\partial n^2}
\end{pmatrix} \cdot
\begin{bmatrix} T_A \\ n \end{bmatrix} =
\left\{\frac{2K_0(1+\alpha_{2,0})}{T_A} + \sum_{i=1}^{L}\left(\frac{2K_i}{T_A}\right)\right\} > 0
\tag{23}
$$

Eq. (23) is positive, for the outcomes of K_0, $(1 + \alpha_{2,0})$, T_A, and K_i, are positive. Hence, we prove that $TCU(T_A, n)$ is strictly c onvex for all T_A and $n > 0$. Then, by solving the linear system of $TCU(T_A, n)$'s first-derivatives (i.e., Eqs. (24) & (25)), one derives T_A^* and n^*:

$$
\frac{\partial TCU(T_A,\,n)}{\partial n} = \sum_{i=1}^{L}\left[\frac{K_{D,i}}{T_A} - \frac{(h_{3,i}-h_{1,i})\lambda_i^2 T_A}{2n^2}\left(\frac{1}{\lambda_i} - \frac{1}{P_{1,i}}\right)\right] = 0
\tag{24}
$$

$$
\frac{\partial TCU(T_A,\,n)}{\partial T_A} =
\left\{
\begin{array}{l}
\dfrac{-(1+\alpha_{2,0})K_0}{T_A^2} + h_{1,0}\sum_{i=1}^{L}\left(\dfrac{\lambda_i^2}{2P_{1,i}}\right) + h_{1,0}\left[\dfrac{\lambda_0^2}{2(1+\alpha_{1,0})P_{1,0}}\right] \\[2ex]
+h_{1,0}\left\{\sum_{i=1}^{L}\left\{-\left(\dfrac{\lambda_i}{P_{1,i}}\right)\right\}\left[\sum_{j=1}^{i}(\lambda_j)\right] + \left[\sum_{i=1}^{L}(\lambda_i)\right]\cdot\sum_{i=1}^{L}\left(\dfrac{\lambda_i}{P_{1,i}}\right)\right\} \\[2ex]
+\sum_{i=1}^{L}\left\{\dfrac{-(K_i + nK_{D,i})}{T_A^2} + \dfrac{h_{1,i}\lambda_i}{2} + \dfrac{h_{3,i}\lambda_i^2}{2P_{1,i}} + \dfrac{(h_{3,i}-h_{1,i})\lambda_i^2}{2n}\left(\dfrac{1}{\lambda_i}-\dfrac{1}{P_{1,i}}\right)\right\}
\end{array}
\right\} = 0
\tag{25}
$$

Therefore, the optimal manufacturing-shipment policy (T_A^*, n^*) are found:

$$
T_A^* = \sqrt{\frac{2\left[(1+\alpha_{2,0})K_0 + \sum_{i=1}^{L}(K_i + nK_{D,i})\right]}{
\begin{array}{l}
h_{1,0}\sum_{i=1}^{L}\left(\dfrac{\lambda_i^2}{P_{1,i}}\right) + \sum_{i=1}^{L}\left\{h_{1,i}\lambda_i + \dfrac{h_{3,i}\lambda_i^2}{P_{1,i}} + \dfrac{(h_{3,i}-h_{1,i})\lambda_i^2}{n}\left(\dfrac{1}{\lambda_i}-\dfrac{1}{P_{1,i}}\right)\right\} \\[2ex]
+h_{1,0}\left[\dfrac{\lambda_0^2}{(1+\alpha_{1,0})P_{1,0}}\right] + h_{1,0}\left\{-2\sum_{i=1}^{L}\left(\dfrac{\lambda_i}{P_{1,i}}\right)\left[\sum_{j=1}^{i}(\lambda_j)\right] + 2\left[\sum_{i=1}^{L}(\lambda_i)\right]\cdot\sum_{i=1}^{L}\left(\dfrac{\lambda_i}{P_{1,i}}\right)\right\}
\end{array}
}}
\tag{26}
$$

and

$$n^* = \sqrt{\dfrac{\left[\left(1+\alpha_{2,0}\right)K_0 + \sum_{i=1}^{L}(K_i)\right]\cdot\sum_{i=1}^{L}\left[\left(h_{3,i}-h_{1,i}\right)\lambda_i^2\left(\dfrac{1}{\lambda_i}-\dfrac{1}{P_{1,i}}\right)\right]}{\left(\sum_{i=1}^{L}K_{D,i}\right)\left\{\begin{array}{l}h_{1,0}\sum_{i=1}^{L}\left(\dfrac{\lambda_i^2}{P_{1,i}}\right)+\sum_{i=1}^{L}\left\{h_{1,i}\lambda_i+\dfrac{h_{3,i}\lambda_i^2}{P_{1,i}}\right\}+h_{1,0}\left[\dfrac{\lambda_0^2}{\left(1+\alpha_{1,0}\right)P_{1,0}}\right]\\[2ex]+h_{1,0}\left\{-2\sum_{i=1}^{L}\left(\dfrac{\lambda_i}{P_{1,i}}\right)\left[\sum_{j=1}^{i}(\lambda_j)\right]+2\left[\sum_{i=1}^{L}(\lambda_i)\right]\cdot\sum_{i=1}^{L}\left(\dfrac{\lambda_i}{P_{1,i}}\right)\right\}\end{array}\right\}}}$$

(27)

3.3. Discussion on setup times

In most cases, the total setup times S_i is insignificant to the operating cycle. However, to be on the safe side, one should compute $T_{\min}$ (Nahmias, 2009, also as shown in Eq. (28)) and make sure that $T_{\min} < T_A{}^*$. If not, select $\max(T_{\min}, T_A{}^*)$ as the operating cycle time.

$$T_{\min} = \frac{\sum_{i=0}^{L} S_i}{1-\left[\dfrac{\lambda_0}{P_{T1,0}}+\sum_{i=1}^{L}\left(\dfrac{\lambda_i}{P_{1,i}}\right)\right]}$$

(28)

3.4. Discussion on the machine capacity

Moreover, for a two-stage multiproduct manufacturing plan, managers should make sure that they have adequate capacity to perform the task in both stages. That is, expressions (29) and (30) must hold (Nahmias, 2009):

$$\left[\left(t_{1,0}\right)+\sum_{i=1}^{L}\left(t_{1,i}\right)\right]<T_A \text{ or } \left[\left(\dfrac{Q_0}{P_{T1,0}}\right)+\sum_{i=1}^{L}\left(\dfrac{Q_i}{P_{1,i}}\right)\right]<T_A$$

(29)

or

$$\left[\dfrac{\lambda_0}{P_{T1,0}}+\sum_{i=1}^{L}\left(\dfrac{\lambda_i}{P_{1,i}}\right)\right]<1$$

(30)

4. Numerical illustration

The following example illustrates the proposed multiproduct manufacturer-retailer coordinated system's capability and applicability numerically. Tables 1 and 2 show the assumed parameters' values in stages one and two of our numerical example. Table A-1 (in Appendix A) exhibits the similar system's assumed parameters' values which using a one-stage fabrication scheme for comparison purposes.

Table 1
Parameters' values assumed in stage 1 of our example

$P_{1,0}$	C_0	δ	K_0	$h_{1,0}$	γ	$\alpha_{1,0}$	λ_0	i_0	$\alpha_{2,0}$	$\alpha_{3,0}$
120000	$40	0.5	$8500	$8	0.5	0.5	17000	0.2	0.1	0.25

Table 2
Parameters' values assumed in stage one of our example

Product i	$K_{D,i}$	λ_i	$h_{1,i}$	$C_{D,i}$	C_i	$h_{3,i}$	$P_{1,i}$	K_i
1	$1800	3000	$16	$0.1	$40	$70	112258	$8500
2	$1900	3200	$18	$0.2	$50	$75	116066	$9000
3	$2000	3400	$20	$0.3	$60	$80	120000	$9500
4	$2100	3600	$22	$0.4	$70	$85	124068	$10000
5	$2200	3800	$24	$0.5	$80	$90	128276	$10500

4.1. The convexity of $TCU(T_A{}^*, n^*)$ and optimal policies

Apply formulas (26), (27), and (22) to derive $T_A{}^* = 0.5398$, $n^* = 4$, and $TCU(T_A{}^*, n^*) = \$2,254,128$. The behavior of $TCU(T_A,$

n) concerning T_A and n is depicted in Fig. 4. It not only demonstrates the convexity of $TCU(T_A, n)$. $E[TCU(T_A^*, n^*)]$ noticeably upsurges as both T_A and n and T_A depart from the optimal values.
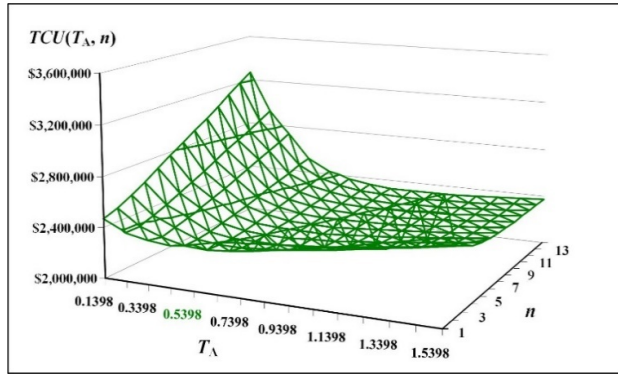

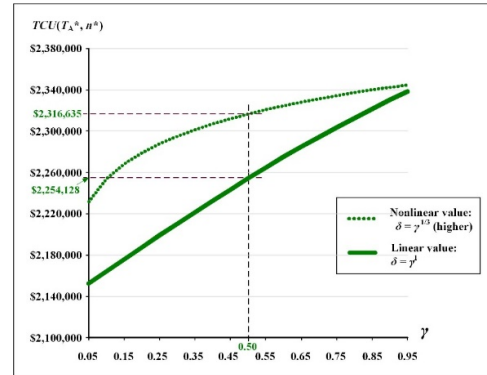**Fig. 4.** The behavior of $TCU(T_A, n)$ concerning to T_A and n

Fig. 5. The behavior $E[TCU(T_A^*, n^*)]$ regarding nonlinear and linear relationships of δ and γ

4.2. The influence of main system's features

Fig. 5 reveals the impact of nonlinear and linear and relationships of the value δ and completion rate γ on $TCU(T_A^*, n^*)$. For the nonlinear case with higher component value (i.e., $\delta = \gamma^{1/3}$), at $\gamma = 0.5$, we find $TCU(T_A^*, n^*) = \$2,316,635$. For the linear case (i.e., $\delta = \gamma^1$), at $\gamma = 0.5$, we reconfirm $TCU(T_A^*, n^*) = \$2,254,128$. Fig. 6 exposes the behavior of T_A^* concerning nonlinear and linear and relationships of δ and γ. For the nonlinear case with higher component value, at $\gamma = 0.5$, we find $T_A^* = 0.4192$. For the linear case, at $\gamma = 0.5$, we reconfirm $T_A^* = 0.5398$.
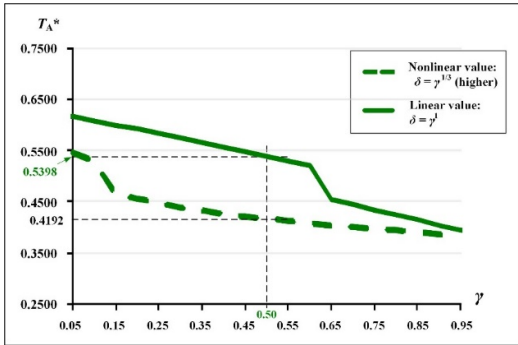

Fig. 6. The behavior T_A^* concerning nonlinear and linear relationships of δ and γ
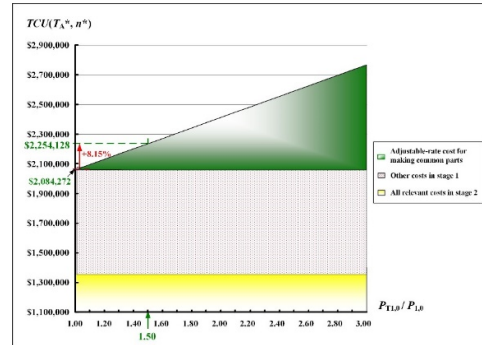

Fig. 7. The impact of $(P_{T1,0}/P_{1,0})$ on key cost contributors in $TCU(T_A^*, n^*)$

Fig. 7 shows the influence of $(P_{T1,0}/P_{1,0})$ on key cost contributors in $TCU(T_A^*, n^*)$. As $(P_{T1,0}/P_{1,0})$ rises, the most influence component is the common parts' adjustable-rate cost. At $(P_{T1,0}/P_{1,0}) = 1.5$, it reconfirms $TCU(T_A^*, n^*) = \$2,254,128$; a 8.15% increase in $TCU(T_A^*, n^*)$ (i.e., from \$2,084,272 for the model without implementing adjustable-rate). Fig. 8 investigates the details of $TCU(T_A^*, n^*)$'s cost contributors. The following are the major cost contributors to $TCU(T_A^*, n^*)$ in order: (i) variable cost for multiproduct; (ii) the common parts' variable cost; (iii) the common parts' adjustable-rate cost; (iv) the retailer holding cost; (v) setup cost for multiproduct; and (vi) cost of transporting multiproduct; etc.
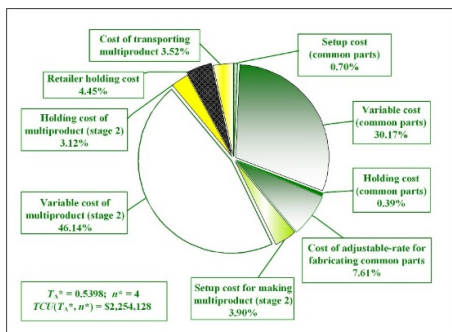

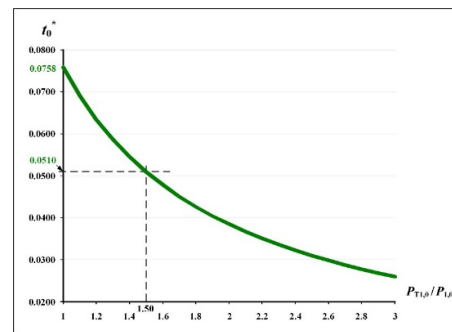**Fig. 8.** The details of $TCU(T_A^*, n^*)$'s cost contributors

Fig. 9. The ratio $(P_{T1,0}/P_{1,0})$ effects on t_0^*

Fig. 9 depicts $(P_{T1,0}/P_{1,0})$ impact on t_0^*. At $(P_{T1,0}/P_{1,0}) = 1.5$, t_0^* drops to 0.0510 (years) from 0.0758, a 32.72% decline. Fig. 10 explores the $(P_{T1,0}/P_{1,0})$ influence on system utilization. At $(P_{T1,0}/P_{1,0}) = 1.5$, utilization declines to 0.2357 from 0.2829, a 16.69% drop.
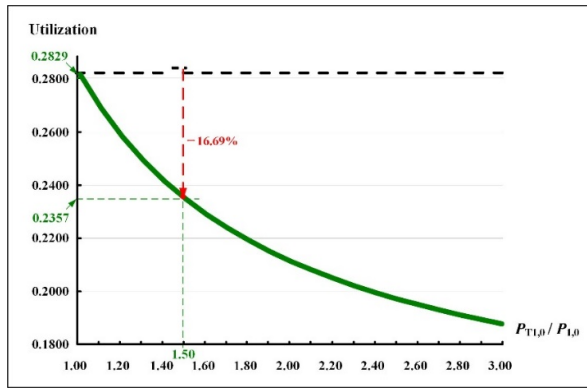

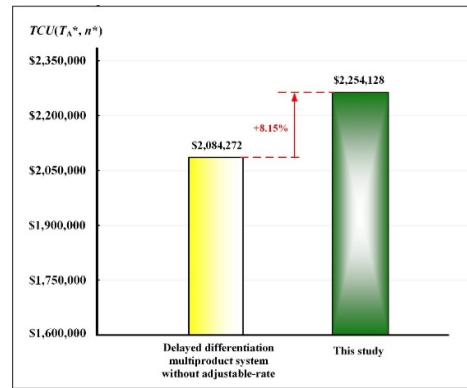**Fig. 10.** The influence on utilization concerning $(P_{T1,0}/P_{1,0})$

Fig. 11. Comparing our study's $E[TCU(T_A^*, n^*)]$ with that of a closely related model

For a 16.69% reduction in utilization, this study pays a price of 8.15% rise in $TCU(T_A^*, n^*)$ (i.e., increasing from \$2,084,272 to \$2,254,128; as exhibited in Fig. 11). Furthermore, the proposed model can explore other detailed system feature effects on the problem. Fig. 12 displays the adjustable-rate unit cost ratio (C_{T0}/C_0) impact on end products' variable cost. It indicates (C_{T0}/C_0) ratio has an insignificant influence on each end product's variable cost.
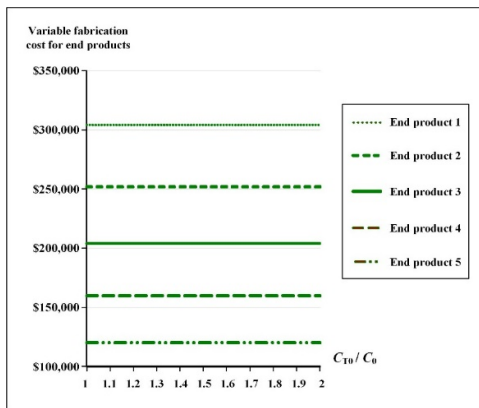

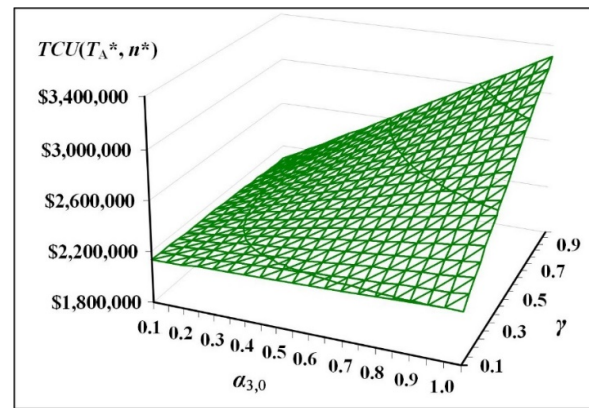**Fig. 12.** The ratio (C_{T0}/C_0) impact on each end product's variable cost

Fig. 13. Behavior of $TCU(T_A^*, n^*)$ relating to $\alpha_{3,0}$ and γ

4.3. Collective impact of main system features

Fig. 13 investigates the collective impact of the adjustable-rate added portion of unit cost $\alpha_{3,0}$ and γ on $TCU(T_A^*, n^*)$. It shows $TCU(T_A^*, n^*)$ varies insignificantly as both $\alpha_{3,0}$ and γ stay at the lower values, and $TCU(T_A^*, n^*)$ severely surges as both $\alpha_{3,0}$ and γ rise higher. Fig. 14 discloses the combined influence of γ and the adjustable-rate added output proportion $\alpha_{1,0}$ on T_A^*. As γ rises, T_A^* radically declines, and as $\alpha_{1,0}$ increases, T_A^* slightly changes. It exposes that γ has more impact on T_A^* than $\alpha_{1,0}$. n^* value changes as γ value increases from 0.5 to 0.6, and the change in n^* has caused T_A^* to decline sharply.
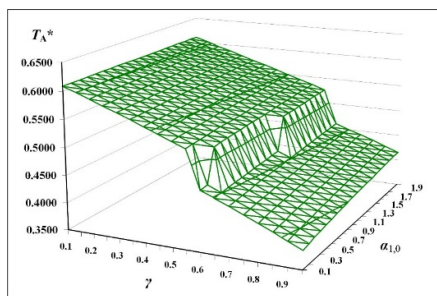

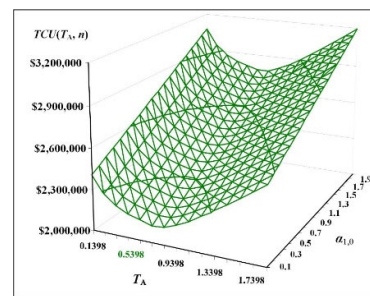**Fig. 14.** The combined influence on T_A^* regarding γ and $\alpha_{1,0}$

Fig. 15. The behavior of $TCU(T_A, n)$ relating to T_A and $\alpha_{1,0}$

Moreover, Figure 15 illustrates $TCU(T_A, n)$'s behavior relating to T_A and n. As the adjustable-rate added output proportion $\alpha_{1,0}$ rises, $TCU(T_A, n)$ upsurges significantly. As T_A departs from the optimal value of T_A*, $TCU(T_A, n)$ rises in both directions.

5. Conclusions

The present study applies the mathematical method to construct a delayed differentiation two-stage model for exploring a multiproduct manufacturer-retailer coordinated supply chain system with an adjustable rate for making common parts and a multi-shipment plan for shipping the finished goods. The Hessian matrix equations assist us in optimizing the problem's cost-minimization fabrication-shipment decision. An example shows the applicability and capability of our model numerically. This study's significant contribution includes the following:
(1) It develops an exact model to represent and explore a delayed differentiation multiproduct manufacturer-retailer coordinated system with real-life characteristics;
(2) It numerically demonstrates the proposed model's applicability and capability by disclosing various crucial system-relating characteristics that can facilitate managerial decision-making.

The impact of incorporating an adjustable rate in stage two for making finished products in the system will be a worthy investigating work.

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Appendix – A

Table A-1
The assumed values of corresponding variables in a single-stage scheme

Product i	$K_{D,i}$	λ_i	$h_{1,i}$	$C_{D,i}$	C_i	$h_{3,i}$	$P_{1,i}$	K_i
1	$1800	3000	$16	$0.1	$80	$70	58000	$17000
2	$1900	3200	$18	$0.2	$90	$75	59000	$17500
3	$2000	3400	$20	$0.3	$100	$80	60000	$18000
4	$2100	3600	$22	$0.4	$110	$85	61000	$18500
5	$2200	3800	$24	$0.5	$120	$90	62000	$19000