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A fuzzy multi-objective multi-follower linear Bi-level programming problem to supply chain optimization

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A B S T R A C T

In today's world, many planning problems include a hierarchical decision structure with independent and often conflicting objectives. Therefore, the optimization of supply chains with hierarchical structure is essential. In this paper, we investigate a fuzzy multi-objective multi-products supply chain optimization problem in a bi-level structure with one level corresponding to a manufacturer planning problem, while the other to K distribution centers problem. In our model, customer demand and supply chain costs are considered uncertain and will be modeled with use of fuzzy sets. We first describe how different kinds of problems can be modeled as bi-level programming problems. Then, this fuzzy model is first converted into an equivalent crisp model by using α-cut method in each level, and then by applying extended Kuhn–Tucker approach, we have a linear multi-objective programming problem. Fuzzy goal programming technique is applied to solve the multi-objective linear programming problem to obtain a set of Pareto-optimal solutions. Finally, a numerical example is illustrated to demonstrate the feasibility of the proposed approach.

1. Introduction

In today's global competition, various products must be produced according to customer's demand. There is also an increasing trend for high quality products and many firms prefer to outsource part of their needs. In today’s competitive market, enterprises and manufacturers in addition to considering organization and internal resources, need to manage and control resources and related elements outside of their organizations because they want to achieve a competitive advantage or advantages with the purpose of more market share. Accordingly, activities such as supply and demand planning, material procurement, production and product planning, inventory control, distribution, delivery and customer service executed in company level previously now have been moved to supply chain level. Controlling and coordinating of all activities plays essential role on supply chain management. Supply chain management (SCM) is a phenomenon that does this in a way that customers can receive fast and reliable services and quality products in reasonable amount expenses. In this regard, optimization of the supply chain has become a very important issue for many companies. Supply chain optimization is the application of processes and tools to ensure the optimal operation of a

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manufacturing and distribution supply chain. This includes the optimal placement of inventory within the supply chain, minimizing operating costs (including manufacturing costs, transportation costs, and distribution costs). Implementation of appropriate strategy for minimizing costs and increasing flexibility in a competitive and complex market are some of the challenges for supply chain optimization. On the other hand, many planning problems require the decisions synthesis of several, interacting individuals or agencies. Often, these groups are arranged within a hierarchical administrative structure, each with independent and perhaps conflicting objectives. Multi-level mathematical programming (MLP) is identified as mathematical programming, which solves decentralized planning problems with multiple executors in a multi-level or hierarchical organization. In many real-world supply chain optimization problems, we are faced with multiple objectives, often are also in conflict. Traditionally, most studies have been focused on revenue maximization or costs minimization as a single-objective optimization problem. While, in a real supply chain, managers may be looking for optimizing multiple conflicting objectives such as reducing costs and increasing customer satisfaction, simultaneously. In addition, uncertainty is the main factor, which influences the efficiency and coordination of supply chain and tends to spread up and down the chain that leads to significantly influence on the chain performance. This uncertainty may occur in several instances, such as uncertainties in demand, supply (delivery time, etc.), costs and supply chain and so on.

1.1. Literature review

1.1.1. Supply chain optimization

Optimization is the most useful analytical tool, which helps decision makers make rational decisions. Optimization models allow managers to explore the space of decision options and constraints and ability to measure the tradeoffs among cost, service, quality and time. It can be discussed from different aspects like supply chain design (Sabri & Beamon, 2000; Altiparmak et al., 2006; Wang et al., 2010), supply chain planning (Schulz et al., 2005; Liang & Cheng, 2009; Peidro et al., 2009), supply chain scheduling (Chen & Lee, 2004; Jung et al., 2004; Yao Liu, 2009), vendor selection (Mokashi & Kokossis, 2003; Kumar et al., 2006).

In supply chains performance measures, there are two classifications of qualitative and quantitative. Customer satisfaction, flexibility, and effective risk management can be categorized as qualitative factors. Quantitative factors are also categorized by: (1) objectives that are based on cost or profit such as cost minimization, profit maximization, etc. and (2) objectives that are based on some measure(s) of customer responsiveness such as customer response time minimization, lead time minimization, etc. (Alborzi et al., 2011).

Cost is usually the most important measure for the performance of the supply chain. Companies' benefits are affected by the cost of their operations, directly. Therefore, the importance and impact of cost on the overall performance are quite clear. On the other hand, responsiveness is also considered as an important performance measure of supply chain in a competitive market environment that is rapidly changing. A company with responsive supply chain is enabled to meet market demand in shorter delivery times and faster reactions. Some researchers also studied supply chain optimization in other performance aspects like environmental, financial and social issues (Zhou et al., 2000; Guillén et al., 2005; Azaron et al., 2008; Wang et al., 2010; Akgul et al., 2010; Pinto-Varela et al., 2011; Gebreslassie et al., 2012) in addition to the above criteria. The environmental aspects are shown with parameters such as the minimizing the amount of hazardous waste produced, maximizing the amount of material and energy recovered (Zhou & et al., 2000), environmental impact (Akgul et al., 2010; Pinto-Varela et al., 2011) and the level of environmental protection (Wang et al., 2010). The financial aspects are shown by financial risk in different models (Guillén et al., 2005; Azaron et al., 2008; Gebreslassie et al., 2012). Zhou et al. (2000), considered supply chain optimization from social aspect in addition to environmental and financial aspect. For social sustainability, products should ensure that the needs of population are met.

The SCM optimization may be considered at different levels depending on the strategic, tactical and operational variables involved in decision-making (Mele et al., 2007). The strategic level concerns those decisions that would have a long-lasting effect on the firm such as size and location of production, warehouse and distribution departments, technology selection process, production-distribution planning and collaboration with suppliers. The tactical level includes long and medium-term management decisions updated at a rate ranging between once every quarter and once every year. These include overall purchasing and production decisions, inventory policies, and transport strategies. The operational level refers to day-to-day decisions such as implementing production plans, receiving customer order, handling material from warehouses and purchasing from suppliers. Many researchers have considered supply chain optimization in one level. Lakhal et al. (2000), Yadav et al. (2009), Jula and Leachman (2011) and Paksoy and Pehlivan (2012) investigated it in strategic level, Chan and Chung (2004), Chen and Lee (2004) and Peidro et al. (2009) in tactical level and Wada et al. (2005) and Akgul et al. (2010) proposed models in operational level. On the other hand, some papers have presented supply chain optimization at two or three levels. Jung et al. (2004) and Schulz et al. (2005) modeled optimization in tactical and operational level and Altiparmak (2006) and Pinto-Varela et al. (2011) presented it at three decision levels.

1.1.2. Bilevel programming

The standard mathematical programming problem deals with finding an optimal solution for just one decision maker. Nevertheless, many planning problems contain a hierarchical decision structure, each with independent and often conflicting objectives. These types of problems can be modeled using a multi-level mathematical programming technique. Bi-level programming problem, which is a special case of multiple levels programming problem, involves two optimization problems where the feasible region of the upper level problem is determined implicitly by the solution set of the lower level problem. The decision maker at the upper level (the leader) optimizes his/her objective function independently and is influenced by the reaction of the decision maker at the lower level (the follower) who makes his/her decision after the leader. Briefly, each decision maker independently seeks his/her own interest, but is influenced by the action of the other decision maker. This hierarchical decision process arises in many fields, including decentralized resource planning, highway pricing, the power market, logistics, economic, manufacturing, and road network management that are referred to as multilevel decision problems (Zhang et al., 2010).

It has been proved that solving the bi-level linear programming is an NP-hard problem and even it is an NP-hard problem to find local optimal solution of the bi-level linear programming (Wang et al., 2010). A linear bi-level programming problem has an important property that at least one global optimal solution is attained at an extreme point of constraint region (Gao et al., 2010). Based on this property, many algorithms have been proposed for solving Bi-level linear programming problems (Shi et al., 2005-a; Shi et al., 2005-b; Gao et al., 2010; Zhang et al., 2010; Zheng et al., 2011). These algorithms can be roughly classified into three categories: the vertex enumeration based approaches (Zhang et al., 2010; Ansari & Zhiani Rezai, 2011; Calvete & Gale, 2012), the Kuhn-tucker approaches (Calvet Gale, 2004; Lu et al. 2006; Shi et al., 2005-b; Mishra et al., 2007) in which a bi-level programming problem is transformed into a single level problem that solves the leader's problem while including the follower's optimality conditions as extra constraints; and the heuristics (Gao & Liu, 2005; Wang et al., 2008; Kuo & Hang, 2009; Lin et al., 2008; Jiang et al. 2013), which are known as global optimization techniques based on convergence analysis.

The Kuhn-tucker is the best known method for solving bi-level programming approach. The main strategy of Kuhn-Tucker approach is to replace the follower's problem with its Kuhn-tucker's conditions and adds them as a constraint o leader's problem. Reformulation of linear bi-level programming problem is a standard mathematical program that is relatively easy to solve, since all the constraints are linear and with the elimination or reduction of that constraint, a standard linear programming is obtained that can be solved by simplex algorithm.

In a multi-level decision making problem, decision makers in one level may consider multiple objectives, simultaneously and these objectivs are often in conflict. Many papers have been addressed bi-level single objective programming problems, but not many papers exist, which tackle bi-level multi-objective problems (Zheng et al., 2011). Osman et al. (2004) presented an approach via using fuzzy set theory for solving bi-level and multiple level multi-objective problem. Baky (2010) studied FGP algorithm for solving decentralized bi-level multi-objective programming problems. The fuzzy goals of the objectives are determined by determining individual optimal solutions. The fuzzy goals are then characterized by the associated membership functions transformed into fuzzy flexible membership goals by means of introducing over- and under-deviational variables and assigning highest membership value (unity) as aspiration level to each of them. To elicit the membership functions of the decision vectors controlled by the upper level DM, the optimal solution of the upper level MOLP problem is separately determined.

The original bi-level programming technique mainly deals with one leader and one follower decision problems. In real word applications, multiple followers that are multiple decision units at the lower level may be involved. So the leader's decision will be affected not only by those followers' individual reactions but also by the relationships among them. For each possible solution of the leader, those followers may have their different reactions. These followers may or may not share their decision variables. They may have their individual objectives and constraints but work with others cooperatively, or may have their common objectives or common constraints (Lu et al., 2006). Shi et al. (2007), proposed an extended Kth-best approach for linear bi-level multi-follower programming problems with partial shared variables among followers. Wang et al. (2009) studied a class of bi-level multi-followers programming in which there are partial shared variables among followers. A fuzzy interactive decision making approach is proposed to derive a satisfactory solution for decision makers not only by considering the dominated action of the leader but also by treating (treats) the ratios of satisfaction between the leader and the followers. Zhang and Lu (2010) considered a multi-objective multi-follower linear bi-level programming problem with fuzzy uncertainty in parameters and cooperative relationship between followers. They solve the problem by using of kth-best method. From other studies in this area, we can point out Ansari and Zhiani Rezai (2011) to consider a multi-follower problem with uncooperative relationship.

This paper is organized as follows. In Section 2, the model is formulated. Then the procedure for solving the model is presented in section 3. An example of a supply chain with one manufacturer in first level and 2 distribution centers in second level is illustrated in Section 4. Discussions, further remarks, and future research plans are concluded in Section 5.

2. Problem formulation

2.1. Problem description, assumptions, and notations

The bi-level problem with multi-products, multi-objective and multi-follower in supply chains are examined here. Consider a supply chain with two levels with a manufacturer as a leader at high level and K distribution centers as K followers in low level. Manufacturer has been hegemonic power in the chain and distribution centers as followers should adopt the best decisions with regard to decisions of manufacturer. The manufacturer has produced m products and distributed them to distribution centers that sale products in a same market. Assume that the market demand and costs is normally fuzzy/ imprecise due to incomplete and/or unobtainable information. Fig.1. illustrates the supply chain structure.

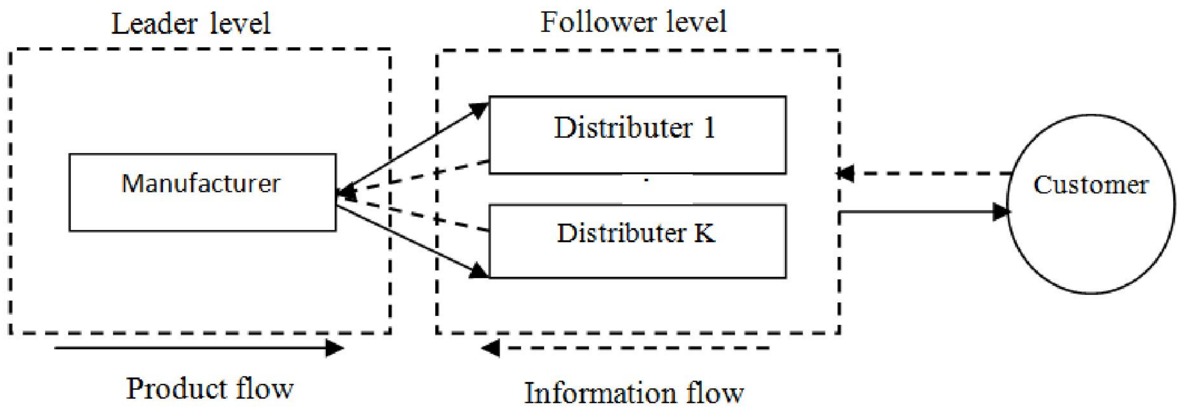


Fig.1. Supply chain structure

The fuzzy mathematical programming model designed here is based on the following assumptions:

(1) Demand and cost functions are fuzzy with imprecise aspiration levels.

(2) All objective functions and constraints are linear equations.

(3) The production costs and distribution cost/ time at manufacturer level and distribution cost/time on a distribution centers are directly proportional to the units manufactured and shipped capacity per truck, respectively.

(4) The pattern of triangular distribution is adopted to represent all of the fuzzy/imprecise numbers and the linear membership functions are specified for all of the fuzzy numbers involved in the proposed model.

(5) Shortage is not allowed in any of levels.

(6) Each distribution center has determined minimum inventory level.

(7)There is no collaboration between followers.

- *Index sets*:

i: Indexes for product type ($i= 1, 2,\ldots, m$)

j: Indexes for distribution centers ($j=1, 2,\ldots, K$)

- *Decision variables:*

X_{ij}: Production volume of product i for distribution center j

Y_{ij}: Inventory level of ith product in distribution center j

- *Parameters:*

a_i: Production cost per unit for ith product

h_{ij}: Inventory carrying cost per unit for ith product in distribution center j

d_{ij}: Transportation cost per unit for ith product from manufacturer to distribution center j

k_{ij}: Transportation cost per unit for ith product from distribution center j to customer

u_{ij}: Delivery time per unit for ith product from manufacturer to distribution center j

c_i: Capacity per unit for ith product of manufacturer

r_i: Resource per unit for ith product of manufacturer

dc_{ij}: Capacity per unit for ith product of distribution center j

rf_{ij}: Resource per unit for ith product of distribution center j

s_{ij}: Capacity per truck delivered for ith product from manufacturer to distribution center j

C: Production capacity of manufacturer

DC_j: Capacity of distribution center j

R: Resources available to manufacturer for all products

RF_j: Resources available to distribution center j for all products

m_{ij}: Minimum inventory level of ith product for distribution center j

D_i: Demand for ith product

2.2. Fuzzy multi-objective multi- follower linear Bi-level model

2.2.1. Leader level:

2.2.1.1. Objective functions

Minimize total cost

An operating objective of production parts is minimizing their costs, which typically consists of its manufacturing cost and distribution cost between manufacturer and distribution centers. This objective is normally fuzzy owing to incomplete and/or unavailable cost information.

$$min\, Z_{m1} = \sum_{j=1}^{2}\sum_{i=1}^{I} \widetilde{a_i}\, X_{ij} + \sum_{j=1}^{2}\sum_{i=1}^{I} \widetilde{b_i}\, X_{ij} \tag{1}$$

- *Minimize total delivery time*

$$min\, Z_{m2} = \sum_{J=1}^{2}\sum_{i=1}^{I} \left[\frac{u_{ij}}{s_{ij}}\right] X_{ij} \tag{2}$$

2.2.1.2. Constraints

- Production amounts from the manufacturer should meet the levels required at the distribution centers.

$$X_{ij} \geq Y_{ij} \quad \forall\, i, j \tag{3}$$

- Production levels at the manufacturer are limited by production capacity.

$$\sum_{j=1}^{2}\sum_{i=1}^{I} c_i\, X_{ij} \leq C \qquad \forall\, i, j \tag{4}$$

- Production levels at the manufacturer are limited by resource available in manufacturer.

$$\sum_{j=1}^{2}\sum_{i=1}^{I} r_i\, X_{ij} \leq R \tag{5}$$

2.2.2. Follower level

2.2.2.1. Objective functions

- *Minimize total cost*

Total costs for distribution center j are included inventory carrying cost and transportation cost of products from distribution center j to customer.

$$min\, Z_{DCj} = \sum_{i=1}^{I} \widetilde{h_{ij}}\, Y_{ij} + \sum_{i=1}^{I} \widetilde{d_{ij}}\, Y_{ij} \quad (6)$$

- *Minimize total resources*

$$min\, Z_{DCj} = \sum_{i=1}^{I} rf_{ij}\, Y_{ij} \quad (7)$$

2.2.2.2. Constraints

- Sums of individual distribution centers holding should meet demands in markets.

$$\sum_{j=1}^{2} Y_{ij} \geq \widetilde{D}_i \quad (8)$$

- Inventory levels at the distribution center j is limited by its capacity

$$\sum_{i=1}^{I} dc_{ij}\, Y_{ij} \leq DC_j \quad (9)$$

- Inventory levels at the distribution center j are limited by resource available in it.

$$\sum_{i=1}^{I} rf_{ij}\, Y_{ij} \leq RF_j \quad (10)$$

- Inventory levels at the distribution center j should be greater than minimum inventory level

$$Y_{ij} \geq m_{ij} \quad (11)$$

3. Solution methodology

In this paper, we assume that the DM has already adopted triangular fuzzy numbers to represent the fuzzy market demand and supply chain costs. In practice, the DM are familiar with estimating optimistic, pessimistic and most likely parameters and the pattern of triangular distribution is commonly adopted due to ease in defining the maximum and minimum limit of deviation of the fuzzy number from its central value. The primary advantages of the triangular fuzzy number are the simplicity and flexibility of the fuzzy arithmetic operations (Liang, 2008).

In the process of defuzzification, there are many important measures to compare two fuzzy numbers, such as Hausdorff distance (Chaudhuri & Rosenfeld, 1999), Hamming distance (Diamond & Kloeden, 1994) and Jimenez (Jimenez, 2006). In this paper $\alpha - \text{cut}$ method will be used to approximate a fuzzy number.

Definition 1: let Eq. (12) be a fuzzy multi-objective function. According to Zhang and Lu (2010), crisp equation of Eq. (12) will be Eq. (13), wherein each of objective functions are transformed into two left and right limits objective functions:

$$min_{x\epsilon X}\,\tilde{F}(x,y)=\left(\tilde{F}_1(x,y),\dots,\tilde{F}_s(x,y)\right)^T \tag{12}$$

$$min_{x\epsilon X}(F(x,y))_{\alpha}^{L(R)}=(\left(F_1(x,y)\right)_{\alpha}^{L},\left(F_1(x,y)\right)_{\alpha}^{R},\dots,\left(F_s(x,y)\right)_{\alpha}^{L},(F_s(x,y))_{\alpha}^{R})^T \tag{13}$$

About constraints, if the problem constrains are to the form of Eq. (14), according to Zhang and Lu (2010), each constraint is transformed into two left (Eq. 15) and right (Eq. 16) limits:

$$\tilde{A}^1x+\sum_{j=1}^{k}\tilde{B}_j^1y_j\le\tilde{b}^1 \tag{14}$$

$$\tilde{A}^{1}{}_{\alpha}^{L}x+\sum_{j=1}^{k}\tilde{B}_j^{1}{}_{\alpha}^{L}y_j\le\tilde{b}^{1}{}_{\alpha}^{L} \tag{15}$$

$$\tilde{A}^{1}{}_{\alpha}^{R}x+\sum_{j=1}^{k}\tilde{B}_j^{1}{}_{\alpha}^{R}y_j\le\tilde{b}^{1}{}_{\alpha}^{R} \tag{16}$$

After defuzzification, α as a problem input will be substituted and will be obtained a multi-objective multi-followers linear bi-level problem. In this paper for solving multi-followers bi-level problem, the Kuhn- Tucker method will be utilized. For this purpose, the multi-objective functions in follower's level should be transformed into single objective function.

There are various solution approaches for solving the multi-objective problem like sequential optimization, e-constraint method, weighting method, goal programming, goal attainment, distance-based method direction-based method and fuzzy goal programming. Recently, GA has been successfully applied to obtain Pareto-optimal solutions for multi-objective optimization problems, too. Here the weighting method is used for solving multi-objective issue on the followers' level. Since the objective functions are not from the same kinds, so that one of them are cost function and the next objective is a resource function. So before using weighting method, the normalization of objectives will be used for homogenizing them. Since each of the followers problem contain two objective functions (cost and resource minimization), thus according to Eqs. (17) and (18), objective functions in followers level will be transformed into single objective.

$$min_{y\epsilon Y}\,DC_j=w_{1j}DC_{1j}+w_{2j}\left(\left[Z_{DC2j}\right]_{\alpha}^{L}+\left[Z_{DC2j}\right]_{\alpha}^{R}\right) \tag{17}$$

$$w_{1j}+w_{2j}=1 \tag{18}$$

Definition 2: Assume that the multi-followers linear bi-level programming problem is like Eq. (19),

$$min_{x\in X}\quad F(x,y_1,\dots,y_k)=cx+\textstyle\sum_{s=1}^{k}d_s\,Y_s \tag{19}$$

$$s.t.\quad Ax+\textstyle\sum_{s=1}^{k}B_t\,y_t\le b$$

$$\min_{y_i\in Y_i}f_i(x,y_i)=c_ix+e_iy_i$$

$$A_ix+C_iy_i\le b_i$$

Where $c\in R^n$, $c_i\in R^n$ $d_i\in R^{m_i}$, $e_i\in R^{m_i}$,$b\in R^p$,$b_i\in R^{q_i}$,$A\in R^{p\times n}$, $B_i\in R^{p\times m_i}$, $A_i\in R^{q_i\times n}$, $C_i\in R^{q_i\times m_i}$,$i=1,2,\dots,K$.

Theorem 1: A necessary and sufficient condition that $(x^*,y_1^*,\dots,y_k^*)$ solves the linear Bi-level multi-follower problem Eq. (19) is that there exist (row) vectors $u_1^*,u_2^*\dots,y_k^*,v_1^*,v_2^*\dots,v_k^*$ and $w_1^*,w_2^*\dots,w_k^*$such that $(x^*,y_1^*,\dots,y_k^*,u_1^*,u_2^*\dots,y_k^*,v_1^*,v_2^*\dots,v_k^*,w_1^*,w_2^*\dots,w_k^*)$ solve (Lu et al., 2006):

$$min\ F(x, y_1, \dots, y_k) = cx + \sum_{s=1}^{k} d_s Y_s \tag{20}$$

$$s.t.\ \ Ax + \sum_{s=1}^{k} B_t\, y_t \le b$$
$$A_i x + C_i y_i \le b_i$$
$$u_i B_i + v_i C_i - w_i = -e_i$$
$$u_i \left(b - \ Ax - \sum_{s=1}^{k} B_t\, y_t \right) + v_i (b_i - A_i x - C_i y_i) + w_i y_i = 0$$
$$x \ge 0, y_i \ge 0, u_i \ge 0, v_i \ge 0, w_i \ge 0, \qquad i = 1, 2, \dots, K$$

After the transformation of bi-level problem to single level, a multi-objective linear programming will be obtained. For solving of this problem, fuzzy goal programming will be used.
In conventional GP models, decision makers are required to specify a precise aspiration level for each of the objectives. In general, especially in large scale problems, this can be quite a difficult task for decision makers. Applying fuzzy set theory to GP has the advantage that decision makers are allowed to specify imprecise aspiration levels. Here, an objective with an imprecise aspiration level can be treated as a fuzzy goal. In this paper, we will consider the following FGP problem, which contains m fuzzy goals $G_k(x)$:

$$G_k(x) \gtrsim g_k \ \ (or \ \ G_k(x) \lesssim g_k \ ) \ \ k = 1, 2, \dots, m \tag{21}$$
$$s.t. Ax \le b; \quad x \ge 0$$

where $G_k(x) \gtrsim (\lesssim) g_k$ indicates the fuzzy goal approximately greater than or equal to (approximately less than or equal to) the aspiration level g_k. Furthermore, we take the following assumption:

(A) $G_k(x)$, $k = 1, 2, \dots, m$, are continuous functions;

(B) Let $S = \{x \mid Ax \le b, x \ge 0\}$ be a non-empty compact set.

The fuzzy goals can be identified as fuzzy sets defined over a feasible set with membership functions. The linear membership function μ_k for the kth fuzzy goal $G_k(x) \gtrsim g_k$ can be expressed as:

$$\mu_k(x) = \begin{cases} 1, & G_k(x) \ge g_k; \\ 1 - \frac{g_k - G_k(x)}{\Delta_{kL}}, & g_k - \Delta_{kL} \le G_k(x) \le g_k \\ 0, & G_k(x) \le g_k - \Delta_{kL} \end{cases} \tag{22}$$

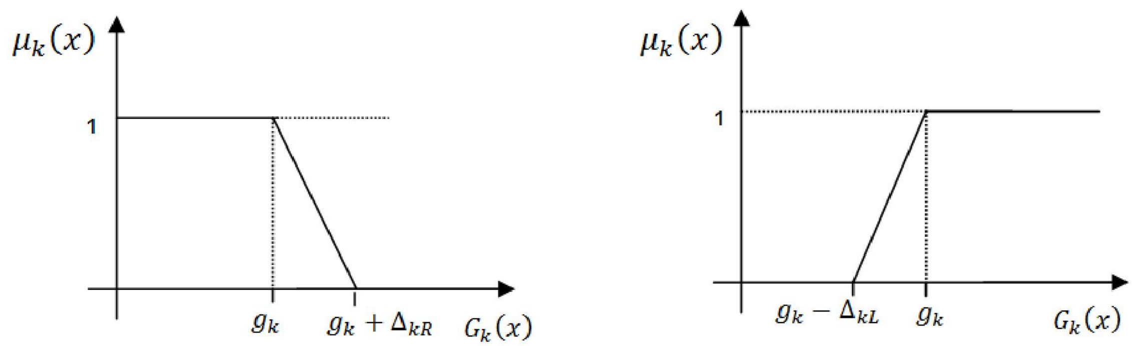
(See Fig. 2.a).

where Δ_{kL} is the lower maximum admissible violation from the aspiration level g_k.In case of the fuzzy goal $G_k(x) \lesssim g_k$, the membership function is defined as:

$$\mu_k(x) = \begin{cases} 1, & G_k(x) \le g_k; \\ 1 - \frac{G_k(x) - g_k}{\Delta_{kR}}, & g_k \le G_k(x) \le g_k + \Delta_{kR} \\ 0, & G_k(x) \ge g_k + \Delta_{kR} \end{cases} \tag{23}$$

(See Fig. 2.b).

where Δ_{kR} are chosen constants of the upper maximum admissible violations from the aspiration level g_k. The constants Δ_{kL} and Δ_{kR} are either subjectively chosen by decision makers or are tolerances in a technical process (Li, 2012).



$a. G_k(x) \lesssim g_k$ $\quad$ $b.\ G_k(x) \gtrsim g_k$

Fig. 2. Distribution functions of goals

Thus, the problem (1) can be transformed into the multi-objective model:

$$\begin{aligned} & max \quad \lambda \\ s.t. \quad & x \in S = \{Ax \leq b; \quad x \geq 0\} \\ & \lambda \leq \mu_k(x) \leq 1, k = 1,2, \dots, m \\ & \lambda \geq 0 \end{aligned} \tag{24}$$

The model above uses the min-operator for aggregating goals to determine the decision set and then to find the element with the highest membership degree.

$$\lambda = \max_{x} \min_{1 \leq k \leq m} (\mu_1(x), \mu_2(x), \dots, \mu_k(x)) \tag{25}$$

4. Numerical example

To illustrate the solution approach, computational experiments are presented in this section. We consider a supply chain with two levels. In the higher level, we have a manufacturer as a leader and in the lower level we have two distribution centers as two followers. Product 1 and product 2 are produced by the manufacturer, and then distributed them to two distribution centers. The numerical values of model parameters for manufacturer are given in Table 2. The numerical values of model parameters for distribution center 1 and 2 are given in Table 3 and 4.

Table 2
Data for the manufacturer

Parameters	value	Parameters	value	Parameters	value
a_1	$\widetilde{50} = (35, 50, 60)$	c_2	3	s_{11}	20
a_2	$\widetilde{30} = (24, 30, 50)$	u_{11}	30	s_{21}	10
b_{11}	$\widetilde{15} = (12, 15, 20)$	u_{21}	30	s_{12}	20
b_{21}	$\widetilde{10} = (9, 10, 20)$	u_{12}	20	s_{22}	10
b_{12}	$\widetilde{25} = (20, 25, 30)$	u_{22}	20	R	5000
b_{22}	$\widetilde{20} = (16, 20, 25)$	r_1	2	C	6000

c_1	4	r_2	3		

Table 3
Data for the distribution center 1

Parameters	value	Parameters	value	Parameters	value
h_{11}	$\widetilde{15} = (12, 15, 20)$	dc_{11}	2	m_{11}	150
h_{21}	$\widetilde{12} = (9, 12, 18)$	dc_{21}	3	m_{21}	90
d_{11}	$\widetilde{25} = (20, 25, 30)$	rf_{11}	2	DC_1	1500
d_{21}	$\widetilde{20} = (16, 20, 25)$	rf_{21}	4	RF_1	2500

Table 4
Data for the distribution center 2

Parameters	value	Parameters	value	Parameters	value
h_{12}	$\widetilde{10} = (9, 10, 20)$	dc_{12}	2	m_{12}	120
h_{22}	$\tilde{8} = (4, 8, 10)$	dc_{22}	3	m_{22}	85
d_{12}	$\widetilde{15} = (12, 15, 20)$	rf_{12}	1	DC_2	2500
d_{22}	$\widetilde{10} = (9, 10, 20)$	rf_{22}	3	RF_2	2000

The customer demand value for product 1 and 2 are according to Table 5.

Table 5
Demand value for product 1 and 2.

parameters	value
D_1	$\widetilde{820} = (750, 820, 890)$
D_2	$\widetilde{450} = (400, 450, 550)$

4.1. Solution by proposed approach

The problem can be solved according to the procedure set out above. First, the fuzzy objective functions and constraints are converted into crisp ones using Eqs. (12-16) at $\alpha = 0, 0.1, 0.2, \ldots, 1$. The crisp problem is transformed into multi-objective linear programming according to Eqs. (17-20). In this paper, the aspiration level (g_k) of each goal is obtained as the best possible solution by having the same constraints and ignoring other goals. Δ_{kR} are subjectively chosen by decision makers. So, two maximum admissible violation from the aspiration level for cost goal (Δ_{1R}) are considered to be equal to 5000 and 8000. Also two maximum admissible violation from the aspiration level for delivery time goal (Δ_{2R}) are considered to be equal to 100 and 200. Thus, we will have four different combinations with respect to the deviations from goals. By introducing the auxiliary variable λ, the multi-objective linear programming problem can be transformed into an equivalent ordinary single-goal linear programming problem using the minimum operator to aggregate all fuzzy sets (Eq. 24).

The auxiliary variable λ $(0 \leq \lambda \leq 1)$ represents overall DM satisfaction with the determined goal values in a multi-objective programming problem. Finally, the crisp single-goal linear programming model is run with LINGO computer software. Fig. 3 illustrates the changes of DM satisfaction with determined goal values (λ) with respect to α value. As we can see in Fig. 3, with an increase of α, λ is reduced. Also with decrease of Δ_{kR} value, λ is reduced. Fig. 4 depicts the changes for the objective values with respect to α value. Z_{m2} behaves differently than $[Z_{m1}]_\alpha^L$ and $[Z_{m1}]_\alpha^R$ with changes in maximum admissible violation from the aspiration level (Δ_{kR}) . The decision maker can choose his/her acceptable solution according to Fig. 3 and Fig. 4 that are the solution sets of the problem.

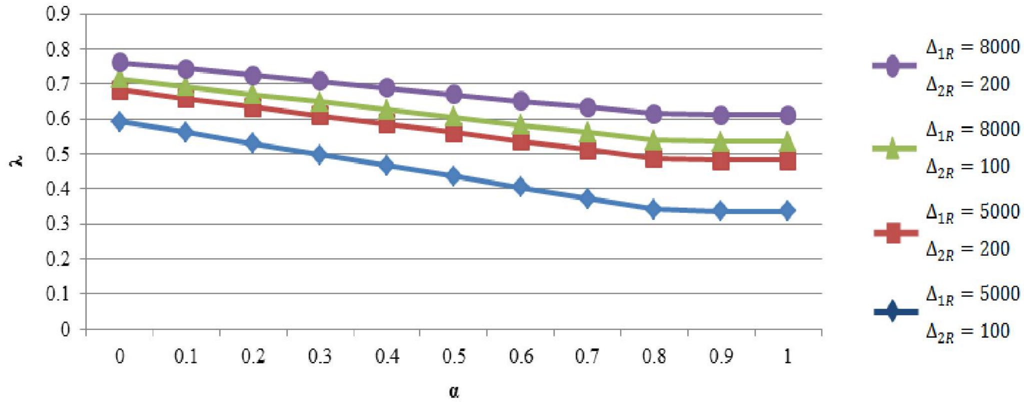


Fig. 3. Changes of λ vs. α

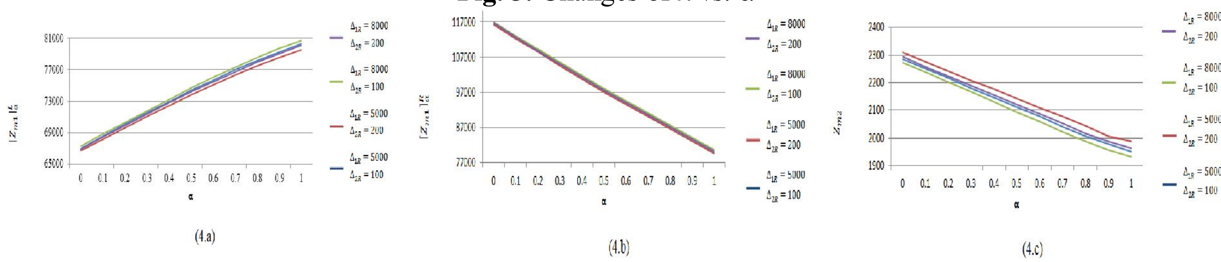


Fig. 4. Objective values with respect to changes in α

5. Conclusion

With respect to the importance of supply chain optimization with hierarchical structures, in this paper, supply chain optimization in a multi-objective multi-follower linear bi-level problem with uncertain customer demand and costs was discussed. For this purpose, a supply chain with a manufacturer as a leader and K distribution centers as K followers are considered which manufacturer produces various products and distribute them to distribution centers that sale products in a same market. Fuzzy objective functions and constraints are converted into crisp ones using $\alpha -$ cut method. With using extended Kuhn–Tucker approach, bi-level problem is transformed into single level problem. Finally, we develop a fuzzy goal programming model to solve obtained multi-objective linear programming problem. A numerical example is presented to demonstrate the effectiveness of the model and solution method. According to predefined solution method, the example was solved and a set of Pareto-optimal solutions were obtained for choice of decision makers for choices that they maid.

Finally, there are some possible directions for future research. In this paper, it is assumed that there isn't any shortage in any of supply chain levels. While we need to examine this issue in some real-world problems and inter costs associated with shortages and other constraints relevant to the problem. Also we have assumed that there is an uncooperative relationship between followers in the lower level where there is no sharing of decision variables among the followers. In such a situation, there are obviously neither shared objectives nor shared constraints among the followers. While different relationships among these followers except uncooperative relationship like cooperative and partial cooperative could cause multiple different processes for deriving an optimal solution for the upper level's decision-making. So it can be considered according to model conditions.

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