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A multi-objective hub covering location problem under congestion using simulated annealing algorithm

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A B S T R A C T

Hub covering problem is one of the most popular areas of research due to wide ranges of applications in different service or manufacturing industries. This paper considers a multi-objective hub covering location problem under congestion. The proposed study of this paper considers two objectives where the first one minimizes total transportation cost and the second one minimizes total waiting time for all hobs. The resulted multi-objective decision making problem is formulated as mixed integer programming. Simulated annealing is used to solve the resulted model and the performance of the proposed model is compared against two other alternative methods, particle sward optimization and NSGA-II. The results are compared in terms of four criteria including quality metric, mean ideal distance, diversification metric and spacing metric. The results indicate that the proposed model could perform better than the other two alternative methods in terms of quality metric but the results are somehow mix in terms of other three criteria.

1. Introduction

Hubs are normally referred to facilities that serve as switching, transshipment and sorting points from different distribution systems to various demand nodes. The primary objective of hub location is to locate hub facilities and allocate demand nodes to hubs to route the traffic between origin–destination pairs. Alumur and Kara (2008) presented a good classification on hub location models, discussed trends on hub location, and provided a synthesis of the literature. One of the most popular forms of Hub location is the one with single assignment, which is the problem of locating hubs and assigning the terminal nodes to hubs to minimize the cost of hub installation and the cost of routing the traffic in the network. There are normally some capacity restrictions on the amount of traffic, which could transit by hubs. Labbé et al. (2005) investigated polyhedral properties of this kind of problem and developed a branch-and-cut algorithm.

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Ebery et al. (2000) presented formulations and solution techniques for the capacitated multi-location-allocation hub problem. They developed a new mixed integer linear programming formulation and constructed a heuristic algorithm, using shortest paths and provided an upper bound from this heuristic in a linear-programming-based branch-and-bound solution procedure. In addition, they presented the results of extensive computational experience with both the heuristic and the exact proposed technique. Ernst and Krishnamoorthy (1999) presented a method to solve capacitated single allocation hub location using a modified mixed integer linear programming formulation for p-hub median problems. They developed some heuristic algorithms for its solution based on simulated annealing (SA) and random descent (RDH), implemented an upper bound to develop an LP-based branch and bound solution technique. The problem tried to find applications in the design of postal delivery networks, specifically in the location of capacitated mail sorting and distribution centers. They examined the algorithms on data obtained from some application and reported some promising results.

Boland et al. (2004) considered formulations and solution techniques for multiple allocation hub location problems. They presented a number of results, which helps us develop preprocessing procedures and tightening constraints for existing mathematical programming formulations. They used flow cover constraints for capacitated problems to improve performance and presented the results of their computational experience, which demonstrate that all of these steps could effectively reduce the computational effort needed to obtain optimal solutions.

Labbé et al. (2005) presented tight integer linear programming formulations for hub location problem along with some properties of the optimal solutions, which can be used to speed up the resolution. They reported that computational instances of medium size could be solved very efficiently using the new proposed method. Sasaki and Fukushima (2003) proposed a new formulation of one-stop capacitated hub-and-spoke framework as a natural extension of the un-capacitated one-stop model. The proposed model involved arc capacity constraints as well as hub capacity constraints, which helped incorporate some practical factors into the model. They also presented a branch-and-bound based exact solution technique with Lagrangian relaxation bounding strategy, and reported some results of numerical experiments based on real aviation data. Their computational results indicated that the proposed capacitated model could provide some efficient results.

Marín (2005) presented a problem formulation to solve splittable capacitated multiple allocation hub location problems. de Camargo et al. (2008) presented an efficient Benders decomposition algorithms based on a well-known formulation to tackle the uncapacitated multiple allocation hub location problem and solved some large instances, considered 'out of reach' of other exact methods in reasonable amount of time. Contreras et al. (2009) considered the capacitated hub location problem with single assignment and proposed a Lagrangean relaxation to compute tight upper and lower bounds. The Lagrangean function that they used could exploit the structure of the problem and it could be decomposed into smaller sub-problems, which could be solved, efficiently. Besides, they presented some simple reduction instances, based on the Lagrangean relaxation bounds, which helps reduce the size of the formulation and reduces the computational effort, significantly. Aykin (1995) introduced a framework for the design of some distribution network with networking policies and models together with exact and heuristic solution methods. Pirkul and Schilling (1998) presented an efficient procedure for designing single allocation hub and spoke systems. Abdinnour-Helm (2001) used simulated annealing (Brooks & Morgan, 1995) to solve the p-hub median problem.

2. The proposed model

In this paper, we consider a classical hub problem by considering queuing theory. The following summarizes the necessary assumptions.

2.1 Assumptions

1. All locations are precisely determined and can be considered as hub location.
2. The number of hubs allowed to build the transportation network has a certain value.
3. The input flow is determined based on a pre-specified regulation and time is independent from flow.
4. In case the inflow is more than service time, there is queue in the system and there is a waiting time.
5. There is a capacity for each inflow.
6. The capacity between different locations is unlimited.

2.2 Objectives

1. The first objective minimizes total transportation cost.
2. The second objective minimizes total waiting time.

2.3 Notations

N	A set of all transportation network points,
P	Maximum number of permitted hubs,
W_{ij}	Maximum flow between node *i* and node *j*,
C_{ij}	The cost of transportation between node *i* and node *j*,
F_k	Fixed cost of establishing hub on point *k*,
r_k	Radius of coverage for hub *k*,
P_k	Setup time for hub *k*,
T_k	Total time consumed on hub *k*,
O_i	The output of node *i*,
D_i	Demand for node *i*,
X_{ik}	A binary variable, which is one when point *i* is assigned to hub *k* and zero, otherwise,
Y_{kl}^{i}	A binary variable, which is one when point *i* has an outflow from hub *k* to hub *l*,
$\alpha \in [0,1]$	Discount factor of transportation cost between two hubs
$\delta \in [0,1]$	Discount factor of transportation cost between a demand point and hub,
M	A big number.

2.4 Problem statement

The following summarizes the problem statement of the proposed method.

$$\min z_1 = \sum_{i=1}^{n}\sum_{k=1}^{n} \delta C_{ij}\left(O_i + D_i\right)X_{ik} + \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} \alpha C_{kl} Y_{kl}^{i} + \sum_{k=1}^{n} F_k X_{kk} \tag{1}$$

$$\min z_2 = \sum_{k=1}^{n} \frac{P_k}{2}\left(\left[\sum_{i=1}^{n}\left(O_i + D_i\right)X_{ik}\right]^2 + \sum_{i=1}^{n}\left(O_i + D_i\right)X_{ik}\right) \tag{2}$$

subject to

$$\sum_{i=1}^{n} X_{ik} = 1; \forall i = \{1,2,\ldots,n\} \tag{3}$$

$$\sum_{k=1}^{n} X_{kk} = P \tag{4}$$

$$C_{ik} X_{ik} \le r_k X_{kk}; \forall i = \{1,2,...,n\}, k = \{1,2,...,n\} \tag{5}$$

$$\sum_{l=1}^{n} Y_{kl}^{i} + \sum_{j=1}^{n} W_{ij} X_{jk} = O_i X_{ik} + \sum_{l=1}^{n} Y_{lk}^{i}; \quad \forall i = \{1,2,...,n\}, k = \{1,2,...,n\} \tag{6}$$

$$Y_{kl}^{i} \le M X_{kk}; \quad \forall i = \{1,2,...,n\}, k = \{1,2,...,n\}, l = \{1,2,...,n\} \tag{7}$$

$$Y_{kl}^{i} \le M X_{ll}; \quad \forall i = \{1,2,...,n\}, k = \{1,2,...,n\}, l = \{1,2,...,n\} \tag{8}$$

$$Y_{kl}^{i} \le M X_{ik}; \quad \forall i = \{1,2,...,n\}, k = \{1,2,...,n\}, l = \{1,2,...,n\} \tag{9}$$

$$X_{ik} \in \{0,1\}; \quad \forall i = \{1,2,...,n\}, k = \{1,2,...,n\} \tag{10}$$

$$Y_{kl}^{i} \ge 0; \quad \forall i = \{1,2,...,n\}, k = \{1,2,...,n\}, l = \{1,2,...,n\} \tag{11}$$

The first objective function, stated in Eq. (1), minimizes total cost of transportation, which consists of two parts of fixed and variable costs. Eq. (2) presents the second objective function, which is associated with the minimization of total waiting time in hub. Eq. (3) specifies that only one point can be assigned to only one hub. Eq. (4) specifies the number of predetermined hubs. Eq. (5) specifies how far a hub can give services. Eq. (6) determines that all inputs must be equal to all outputs. According to Eqs. (7-9), a point can get a point only when l and k denote hubs and point i is assigned to hub k. Finally, Eq. (10) and Eq. (11) determines that the variables are binary.

The proposed model of this paper uses simulated annealing (SA) approach introduced originally by Karimi et al. (2012) and we extend it for a more general form of multi-objective problem. The chromosome we choose for the proposed study of this paper consists of two parts. The first part determines how we should distribute the goods and the section part allocate hubs to their positions.

2.5. Arrangement of chromosome

The proposed model of this paper uses a string, size $2\times(n+p)$, with two parts where the first part contains n real numbers representing the points and the second part includes p binary numbers denoting the hubs. For instance, if there are 10 points and 4 hubs we first arrange the following string,

0.23	0.98	0.03	0.56	0.72	0.10	0.41	0.69	0.49	0.26	2	5	7	10

To fill the second row, we first copy the last four cells into the second row and assign a rank number to the remaining cells as follows,

0.23	0.98	0.03	0.56	0.72	0.10	0.41	0.69	0.49	0.26	2	5	7	10
8	1	10	4	2	9	6	3	5	7	2	5	7	10

Now, we select four points randomly as follows,

0.23	0.98	0.03	0.56	0.72	0.10	0.41	0.69	0.49	0.26	2	5	7	10
8	1	10	4	2	9	6	3	5	7	2	5	7	10

Here, 1, 2, 6 and 7 are hubs and the other numbers are points which are assigned to hubs from left to right. For instance, point 8 is assigned to hub 1, points 10 and 4 are assigned to hub 2, point 9 is assigned to hub 6 and finally point 3 and 5 are assigned to hub 7.

2.6. Mutation

The proposed multi-objective SA uses two positions in each chromosome and randomly changes their addresses for neighborhood search. Fig. 1 to Fig. 3 show details of our strategy,

Initial solution	0.10	0.41	0.69	0.49	0.26
Mutated solution	0.10	0.26	0.69	0.49	0.41

Fig. 1. A simple mutation operation

The second operation selects a chunk of chromosome a reverse the order. Fig. 2 shows details of this operation.

Initial solution	0.10	0.41	0.69	0.49	0.26
Mutated solution	0.10	0.26	0.49	0.69	0.41

Fig. 2. A simple mutation operation

Finally, the third strategy is to generate a position randomly and replace it with a newly generated random number as we show in Fig. 3.

Initial solution	0.10	0.41	0.69	0.49	0.26
	Random number =65.0				
Mutated solution	0.10	0.65	0.49	0.69	0.41

Fig. 3. A simple mutation operation

2.7. Crossover

The proposed model of this paper uses three methods for crossover namely, one-point, two-point and uniform. The first method, one-point, divides the region into two sections and exchanges the information of two chromosomes as shown in Fig. 4 as follows,

First selected	23.0	98.0	03.0	56.0	72.0	10.0	41.0	69.0	49.0	26.0
Second selected	0.53	0.21	0.43	0.16	0.27	***0.68***	**0.30**	**0.89**	**0.97**	**0.06**
First new	0.23	0.98	0.03	0.56	*0.72*	**0.68**	**0.30**	**0.89**	**0.97**	**0.06**
Second new	0.53	*0.21*	0.43	0.16	0.27	**0.10**	**0.41**	***0.69***	**0.49**	**0.26**

Fig. 4. The point crossover operation

The two-point strategy selects two regions of the chromosomes to exchange the information as shown in Fig. 5 as follows,

First selected	0.23	0.98	0.03	**0.56**	**0.72**	**0.10**	**0.41**	0.69	0.49	0.26
Second selected	0.53	0.21	0.43	**0.16**	***0.27***	***0.68***	**0.30**	0.89	0.97	0.06
The first new	0.23	0.98	0.03	**0.16**	**0.27**	**0.68**	**0.30**	0.69	0.49	0.26
The second new	0.53	0.21	0.43	**0.56**	**0.72**	**0.10**	**0.41**	0.89	0.97	0.06

Fig. 5. Two point strategy

The last strategy uses a uniform vector and arranges entities based on that vector.

First selected	0.23	0.98	0.03	0.56	0.72	0.10	0.41	0.69	0.49	0.26
Coverage vector	1	0	0	1	1	0	1	0	0	1
Second selected	0.53	0.21	0.43	0.16	0.27	0.68	0.30	0.89	0.97	0.06
First new	0.53	0.98	0.03	0.16	0.27	0.10	0.30	0.69	0.49	0.06
Second new	0.23	0.21	0.43	0.56	0.72	*0.68*	0.41	0.89	0.97	0.26

Fig. 6. The uniform strategy

As we can observe from the results of Fig. 6, when a cell inside the coverage vector receives a value one, the method exchanges two cells.

2.8 Annealing operation

The proposed simulated annealing strategy uses a probability density function to accept new solutions as follows,

$$P = e^{-\Delta f / T}, \tag{1}$$

where Δf is the difference between the objective functions of new and old solutions and T is the annealing temperature. Since we have two objective functions, we extend Eq. (1) to the following,

$$\Delta = \left| \frac{f_1(x) - f_1(y)}{f_1(x)} + \frac{f_2(x) - f_2(y)}{f_2(x)} \right|. \tag{12}$$

The annealing operations start at temperature T_0 and the temperature is reduced by a factor α until it reaches T_f. The proposed method of this paper uses different criteria to terminate the algorithm such as having a limitation on CPU time, no improvement on objective function, etc.

3. The results

In this section, before we implement the proposed method for some standard problems, we use parameter tuning based on response surface methodology to adjust parameters. In addition, in order to evaluate the performance of the proposed method, we compare the results with NSGA-II and multi-objective particle swarm optimization (MOPSO). Table 1 shows details of our parameters used for the proposed method.

Table 1
The summary of the parameters used for SA

Factors	*Optimal real value*	
	S	*L*
T_0	*10*	*13*
α	*0..84*	*0.91*
nMove	*10*	*16*
nPop	*5*	*6*
pCrossover	*0.5*	*0.7*
β	*1.8*	*2*

In Table 1, S and L represent parameter values used for small and large size problems. For NSGA-II, the number of initial population are 200 and 300 for small and large problems, respectively. Mutation and crossover rates are 0.2 and 0.8, respectively. In addition, termination criteria are set to 30NFC and 100,000 for small and large-scale problems. In addition, Table 2 shows details of parameters used for MOPSO.

Table 2
The summary of parameters setting for MOPSO

	Problem size									
	S	*L*	*S*	*L*	*S*	*L*	*S*	*L*	*S*	*L*
Factor			c_1		c_2		*PopSize*		*MaxItr*	
Tuned	0.62	0.84	1.2	1.4	1.5	1.8	50	120	200	500

We use four criteria to measure the quality of solutions namely; Quality Metric (QM), Mean Ideal Distance (MID), Diversification Metric (MD) and Spacing Metric (SM). These criteria are calculated as follows,

$$MID = \frac{\sum_{i=1}^{n} \sqrt{\left(\frac{f_{1i} - f_1^{best}}{f_{1,total}^{max} - f_{1,total}^{min}} \right)^2 + \left(\frac{f_{2i} - f_2^{best}}{f_{2,total}^{max} - f_{2,total}^{min}} \right)^2}}{n} \tag{13}$$

$$DM = \sqrt{\left(\frac{\max f_{1i} - \min f_{1i}}{f_{1.total}^{max} - f_{1.total}^{min}} \right)^2 + \left(\frac{\max f_{2i} - \min f_{2i}}{f_{2total}^{max} - f_{2total}^{min}} \right)^2} \tag{14}$$

$$SM = \frac{\sum_{i=1}^{n-1} \left| \bar{d} - d_i \right|}{(n-1)\bar{d}}, \tag{15}$$

where d represents deviation between two Pareto solutions, which is calculated based on Euclidian norm and $\bar{d}$ is the average of all d_i . We compare our results on 67 different benchmark problems. Table 3 and Table 4 show the results of our computations for some small instances.

Table 3
The summary of the performance of the proposed method versus NSGA-II and MOPSO for small instances

Problem No.	Quality Metric (QM)			Spacing Metric (SM)		
	NSGA-II	MOPSO	MOPSA	NSGA-II	MOPSO	MOPSA
10	0.235	0	0.765	0.827	0.625	0.741
10	0.105	0	0.895	0.661	0.495	0.778
15	0.250	0	0.750	0.791	0.788	0.920
15	0	0	1	0.693	0.785	0.868
15	0	0	1	0.571	1.092	0.634
20	0	0	1	1.184	0.999	0.881
20	0.235	0	0.765	0.778	1.257	0.973
20	0.434	0	0.565	0.560	1.036	0.874

Problem No.	Diversity Metric (DM)			Mean Ideal Distance (MID)		
	NSGA-II	MOPSO	MOPSA	NSGA-II	MOPSO	MOPSA
10	1.397	0.996	1.574	0.633	0.632	0.518
10	1.102	1.067	1.414	0.704	0.597	0.581
15	0.652	0.208	1.414	0.873	0.608	0.242
15	0.404	1.125	0.664	0.712	0.872	0.348
15	0.203	1.232	0.444	0.339	0.708	0.230
20	1.323	1.270	1.087	0.776	0.696	0.523
20	0.714	1.268	0.896	0.440	0.674	0.399
20	0.958	1.029	1.169	0.518	0.845	0.538

As we can observe from Table 3, the proposed method provides better quality results in terms quality metric for small instances. In terms of Spacing, the proposed method provides some mix results. In terms of diversity, all four instances provide results that are more diverse.

Table 4
The summary of the performance of the proposed method versus NSGA-II and MOPSO for small instances

Problem	Diversity Metric (DM)			Mean Ideal Distance (MID)		
No.	NSGA-II	MOPSO	MOPSA	NSGA-II	MOPSO	MOPSA
10	1.397	0.996	1.574	0.633	0.632	0.518
10	1.102	1.067	1.414	0.704	0.597	0.581
15	0.652	0.208	1.414	0.873	0.608	0.242
15	0.404	1.125	0.664	0.712	0.872	0.348
15	0.203	1.232	0.444	0.339	0.708	0.230
20	1.323	1.270	1.087	0.776	0.696	0.523
20	0.714	1.268	0.896	0.440	0.674	0.399
20	0.958	1.029	1.169	0.518	0.845	0.538
20	1.063	1.075	1.161	0.575	0.751	0.621
25	1.295	0.436	1.381	0.601	0.437	0.247
25	0.960	0.943	1.297	0.663	0.762	0.718
25	1.105	0.775	1.314	0.536	0.577	0.511
25	0.559	0.911	1.414	0.482	0.576	0.287
30	0.566	1.012	1.279	0.697	0.731	0.632
30	1.160	0.954	1.178	0.762	0.547	0.485
30	1.103	0.860	1.478	0.781	0.846	0.500
30	0.484	1.021	1.010	0.297	0.481	0.379
30	0.733	1.267	0.947	0.479	0.646	0.276
30	0.699	0.696	1.184	0.579	0.860	0.554

In addition, we have used the proposed model of this paper and tested against other alternative methods and the results are summarized in Tables 5-8.

Table 5
The results of the proposed model for larger sizes

Problem	Quality Metric (QM)			Spacing Metric (SM)		
No.	NSGA-II	MOPSO	MOPSA	NSGA-II	MOPSO	MOPSA
40	0.071	0	0.928	1.302	1.417	1.390
40	0.3634	0.272	0.364	1.019	1.167	1.267
40	0.357	0	0.642	1.628	1.295	1.649
40	0.318	0.182	0.500	1.059	1.272	1.321
40	0.370	0	0.630	0.984	1.374	1.372
40	0.0416	0	0.958	1.037	1.296	0.935
40	0.240	0	0.760	1.2678	1.049	1.410
40	0.111	0.111	0.778	0.979	1.422	0.928

Problem	Diversity Metric (DM)			Mean Ideal Distance (MID)		
No.	NSGA-II	MOPSO	MOPSA	NSGA-II	MOPSO	MOPSA
40	1.244	1.105	1.267	0.707	0.776	0.430
40	1.019	1.225	1.175	0.609	0.750	0.452
40	0.986	1.264	1.176	0.554	0.813	0.377
40	1.032	1.105	1.179	0.664	0.677	0.408
40	0.702	1.192	1.064	0.731	0.765	0.563
40	0.458	1.170	0.827	0.430	0.767	0.322
40	1.203	0.802	1.132	0.506	0.601	0.431
40	1.036	1.349	1.350	0.692	0.823	0.360

As we can observe from the results of Table 5, in terms of quality of solutions, in most cases, the proposed model of this paper performs better than alternative methods. In terms of other three criteria, the proposed model seems to be comparable.

Table 6
The summary of the performance of the proposed method for some large instances

Problem No.	Quality Metric (QM)			Spacing Metric (SM)		
	NSGA-II	MOPSO	MOPSA	NSGA-II	MOPSO	MOPSA
50	0.434	0.086	0.478	1.339	1.461	1.479
50	0.105	0	0.895	1.342	1.181	1.292
50	0.238	0	0.762	1.465	1.548	1.466
50	0	0	1	1.076	1.346	0.764
50	0	0	1	1.042	1.249	1.446
50	0	0	1	1.469	1.212	0.893
50	0	0	1	0.753	1.202	1.432
50	0	0.107	0.892	1.127	1.043	0.795
50	0.160	0	0.840	0.952	0.998	0.863
50	0	0	1	1.001	1.020	1.033

Problem No.	Diversity Metric (DM)			Mean Ideal Distance (MID)		
	NSGA-II	MOPSO	MOPSA	NSGA-II	MOPSO	MOPSA
50	0.722	1.289	1.340	0.641	0.731	0.658
50	0.633	0.826	1.145	0.526	0.679	0.492
50	1.342	1.084	1.175	0.603	0.601	0.443
50	0.799	1.266	0.920	0.554	0.715	0.482
50	0.443	0.985	1.388	0.490	0.517	0.383
50	1.137	1.174	0.752	0.457	0.608	0.281
50	0.875	0.961	1.155	0.504	0.734	0.369
50	1.077	0.500	1.232	0.702	0.526	0.373
50	0.904	1.222	1.414	0.585	0.668	0.640
50	0.918	0.887	1.112	0.680	0.551	0.230

Table 7
The summary of the performance of the proposed method for some large instances

Problem No.	Quality Metric (QM)			Spacing Metric (SM)		
	NSGA-II	MOPSO	MOPSA	NSGA-II	MOPSO	MOPSA
70	0	0	1	0.672	0.501	0.905
70	0	0	1	0.517	1.299	0.811
70	0	0	1	0.586	0.593	0.878
70	0	0	1	0.737	0.402	0.752
70	0.200	0	0.800	0.826	0.514	0.953
70	0	0	1	0.495	1.032	0.427
70	0	0.076	0.924	1.230	0.559	0.893
70	0	0	1	0.994	0.789	0.806
70	0	0	1	0.726	1.119	0.850
70	0	0	1	0.632	0.904	0.608
70	0	0	1	1.019	1.069	1.071
70	0.352	0	0.647	0.721	1.024	0.721
70	0.273	0	0.727	0.491	1.151	0.993
70	0	0	1	1.039	0.550	0.673

Problem No.	Diversity Metric (DM)			Mean Ideal Distance (MID)		
	NSGA-II	MOPSO	MOPSA	NSGA-II	MOPSO	MOPSA
70	0.470	0.122	1.00	0.492	0.992	0.125
70	0.110	1.095	0.417	0.519	0.743	0.250
70	0.362	0.249	1.043	0.959	0.806	0.365
70	0.815	0.443	1.367	0.672	0.519	0.174
70	1.066	0.484	0.808	0.692	0.832	0.336
70	0.709	1.189	0.749	0.509	0.846	0.364
70	1.203	0.445	1.041	0.643	0.563	0.257
70	1.081	0.605	0.600	0.696	0.758	0.222
70	0.321	1.184	0.891	0.275	0.668	0.250
70	0.561	1.109	0.723	0.643	0.832	0.127
70	0.604	1.246	0.820	0.518	0.737	0.220
70	0.918	1.136	0.751	0.491	0.753	0.459
70	0.173	1.009	1.051	0.456	0.724	0.256
70	0.724	0.552	0.912	0.847	1.021	0.210

Table 8
The summary of the performance of the proposed method for some large instances

Problem No.	Quality Metric (QM)			Spacing Metric (SM)		
	NSGA-II	MOPSO	MOPSA	NSGA-II	MOPSO	MOPSA
100	0	0	1	0.468	1.231	0.653
100	0.5	0	0.5	1.037	0.509	1.364
100	0	0	1	0.013	0.199	1.177
100	0	0.250	0.750	0.499	0.298	0.568
100	0.333	0.0833	0.583	0.659	0.823	1.044
100	0	0	1	1.704	0.285	0.547
100	0	0	1	0.892	1.487	0.454
100	0	0	1	1.035	0.841	0.963
100	0	0	1	0.7051	1.000	0.580
100	0	0	1	1.069	0.062	0.711
100	0	0	1	0.633	0.968	0.392
100	0	0	1	1.240	0.357	1.028
100	0	0	1	0.911	0.861	1.355
100	0	0	1	0.628	1.076	1.114
100	0	0	1	1.105	0.901	0.765
100	0	0	1	0.977	1.106	1.036

Problem No.	Diversity Metric (DM)			Mean Ideal Distance (MID)		
	NSGA-II	MOPSO	MOPSA	NSGA-II	MOPSO	MOPSA
100	0.231	0.649	0.656	0.832	1.349	0.243
100	1.039	0.512	1.080	0.449	0.749	0.440
100	1.042	0.422	1.164	0.798	0.852	0.099
100	0.869	1.290	0.834	0.727	0.761	0.397
100	0.442	1.042	1.016	0.310	0.364	0.230
100	1.174	0.255	0.367	0.673	0.606	0.147
100	0.187	1.100	0.562	0.394	0.872	0.027
100	1.131	0.855	0.955	0.653	0.590	0.275
100	0.289	1.146	0.680	0.467	0.830	0.275
100	1.047	0.232	1.191	0.590	0.207	0.340
100	0.585	0.958	0.657	0.571	0.880	0.175
100	1.058	0.724	0.895	0.624	0.523	0.218
100	1.211	0.559	1.121	0.692	0.734	0.482
100	0.871	0.760	1.156	0.963	0.794	0.392
100	0.660	1.064	0.891	0.758	0.916	0.179
100	0.447	1.187	0.943	0.815	0.765	0.280

Note that as the size of the problem increases, it is getting difficult to solve the problem directly using mathematical programming. In fact, for large size, problems the only practical choice is to use metaheuristics methods. Fig. 7 to Fig. 10 show the summary of the performance of the proposed method in terms of four criteria.

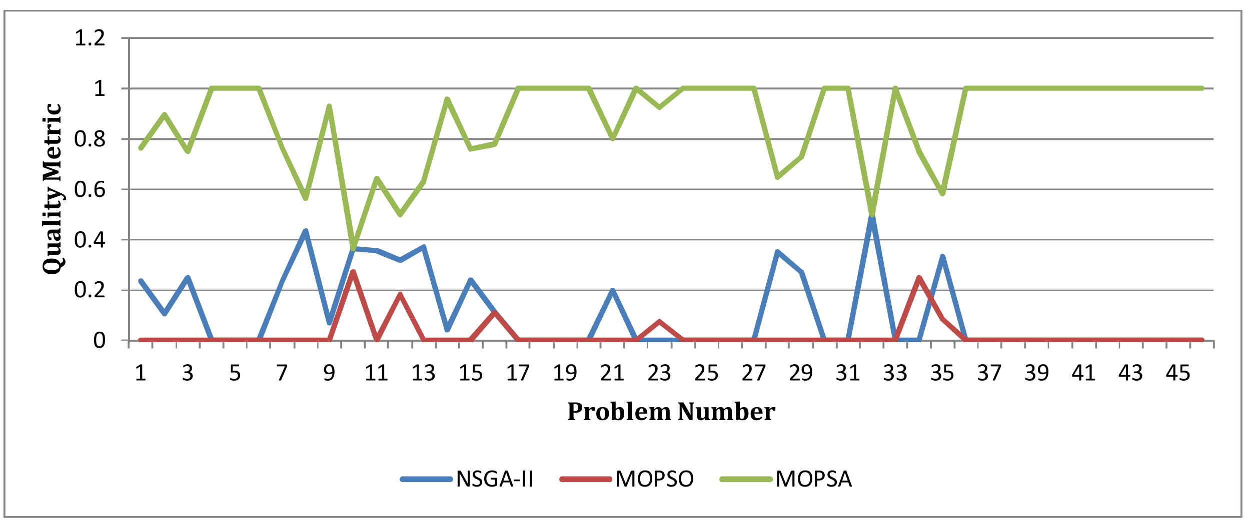

Fig. 7. The summary of the performance of the proposed method in terms quality metric

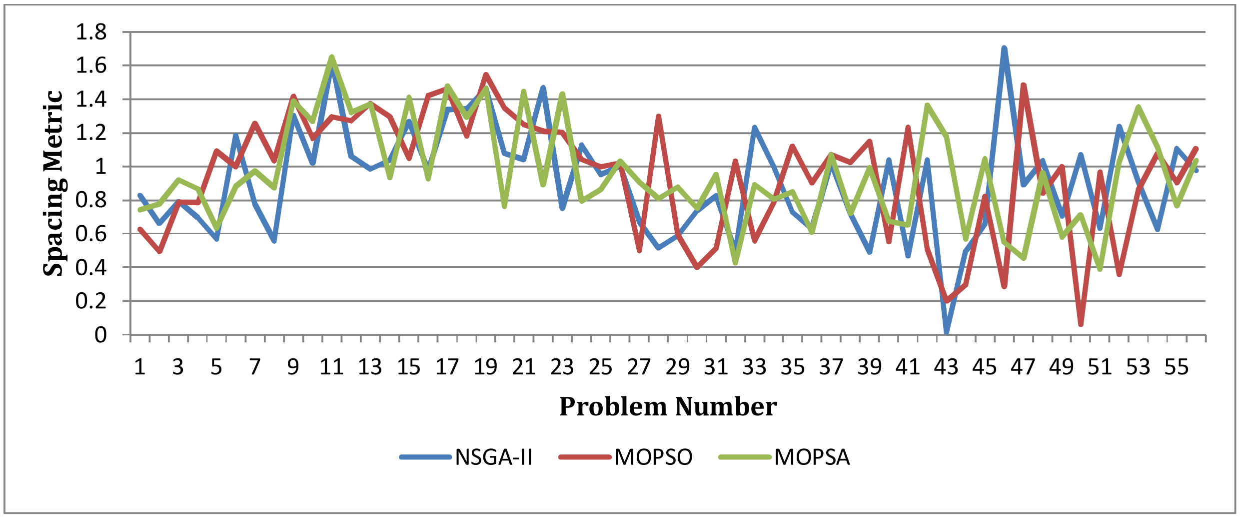

Fig. 8. The summary of the performance of the proposed method in terms spacing metric

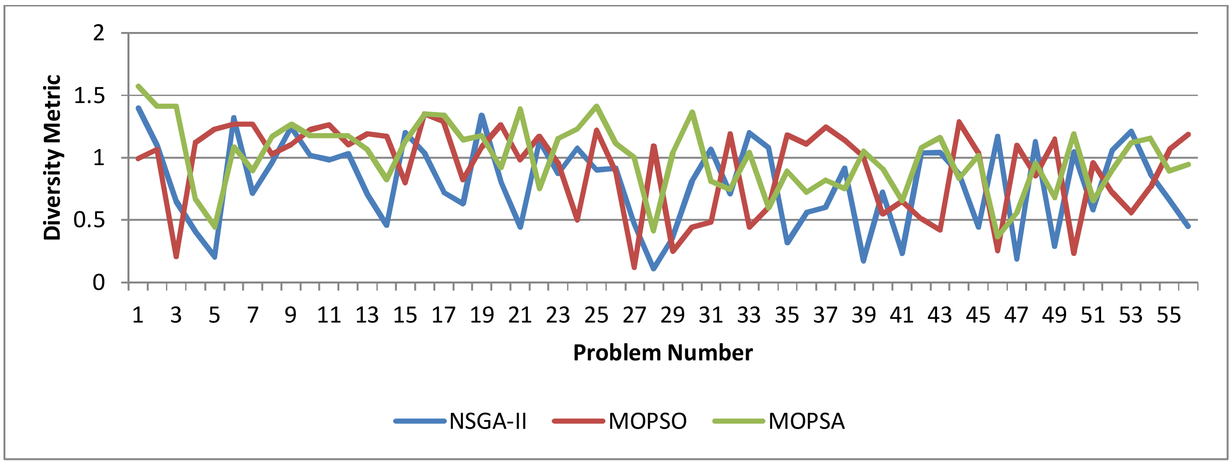

Fig. 9. The summary of the performance of the proposed method in terms diversity metric

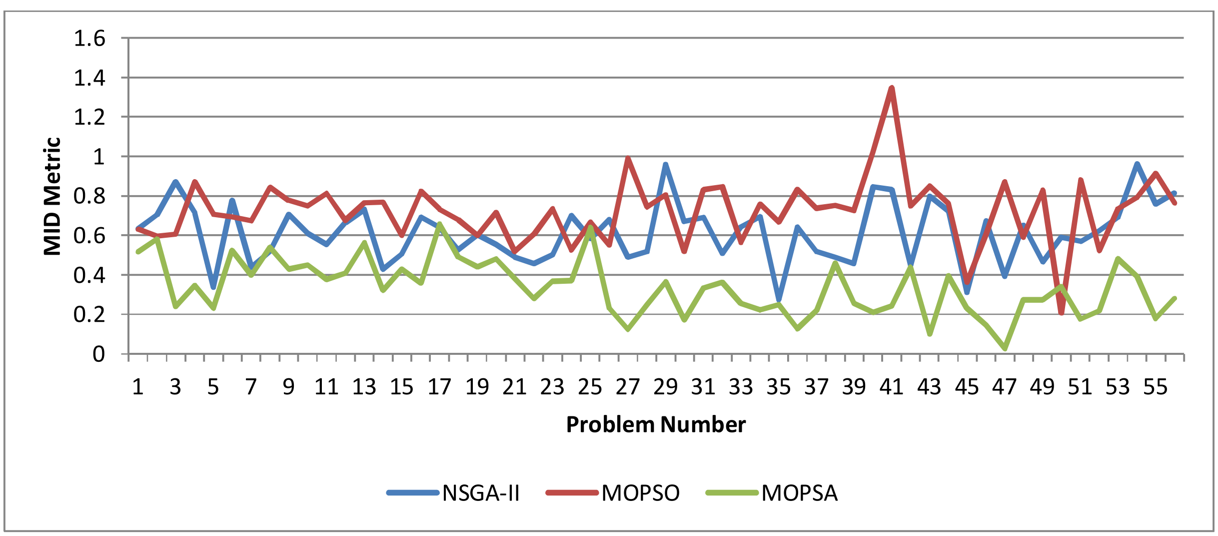

Fig. 10. The summary of the performance of the proposed method in terms MD metric

According to the results of Fig. 7, the proposed model of this paper provides better quality Pareto solutions but the results of Fig. 8, Fig. 9 and Fig. 10 indicate some mix results.

3. Conclusion

In this paper, we have presented a multi-objective decision making problem to assign hubs in different places by data concession. The proposed study considers two objectives: the first one was the minimization of total transportation cost and the second one was associated with minimization of total waiting times in all hubs. We have presented simulated annealing as an alternative solution to generate Pareto solution strategies and compared the performance of the proposed model with two alternative solution strategies. The results of testing various benchmark problems indicate that simulated annealing may provide better quality solutions but it may not necessarily produce better solutions in terms of other criteria.

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