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#### Design of a facility layout problem in cellular manufacturing systems with stochastic demands

# Amir-Mohammad Golmohammadi<sup>a\*</sup>, Arezoo Asadi<sup>b</sup>, Zaynab Akhoundpour Amiri<sup>c</sup> and Matineh Behzad<sup>d</sup>

<sup>a</sup>Ph.D. Student, Department of Industrial Engineering, University of Yazd, Yazd, Iran. <sup>b</sup>Msc Graduated Student of Industrial Engineering, Islamic Azad University, Karaj Branch, Alborz, Iran <sup>c</sup>Msc Graduated Student of Industrial Engineering, System Management and Productivity, University of Alghadir, Tabriz, Iran <sup>d</sup>Graduated Student of Science Computer, University of Golestan, Golestan, Iran **CHRONICLE ABSTRACT** 

Article history: Received: July 9, 2018 Received in revised format: July 9, 2018 Accepted: August 19, 2018 Available online: August 20, 2018	Cell formation and layout design are two important steps for the implementation of the production systems. The existing models majorly have focused on the cell formation issue and the layout design of machines, but have paid little attention to the placement of cells in dynamic environment. In addition, in most of the available papers, binary variables have been used for cell formation, and other information such as volume of production, operation sequences and production costs have played unimportant roles in the structure of the existing models. In this paper, a nonlinear program-
Keywords: Cellular manufacturing sys- tems Cell formation Exceptional elements Stochastic demand Dynamic cellular manufactur-	ming model under potentially dynamic conditions is proposed which minimizes the cost associated with the difference between the estimated demands from its expected value. The other purpose of the proposed model includes minimization of the total costs of inter/intra cellular movements of elements (forward and backward movements), the existence of exceptional elements, intercellular displacement of machines and cellular reconfiguration and operational costs and constant cost of machineries. The problem is solved via GAMS and a Genetic Algorithm (GA) is employed to solve the large sized problems and the results are analyzed.
ing system Layout formation	© 2018 by the authors; licensee Growing Science, Canada

#### 1. Introduction

In today's world, due to the increased power of the customer's choice and expansion of the competitive markets, different service or production companies are forced to make structural corrections. The group technology is one of the most important production philosophies aimed to determine, categorize, and allocate elements to group the element family as well as to assign machines to the cells to produce the family of elements (Jolai et al., 2012; Hadian et al., 2019). The cellular productive systems are considered the most important applications of the group technology as an efficient tool to improve the productivity and flexibility of the production. Reduction of transportation costs may be one of the most important reasons for development of cellular manufacturing systems (CMS) (Delgoshaei et al., 2016a). The benefits of implementing CMS are only achievable when the cell configuration and scheduling are executed, correctly. One of the fundamental steps in the design of CMS is the cell formation (CF), which has been

\* Corresponding author. E-mail address: <u>amir.m.golmohammadi@yazd.ac.ir</u> (A. M. Golmohammadi)

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widely studied. The cell formation involves two basic tasks: formation of the family of elements and allocation of machines to the cells, in such a way that inter/intra cellular displacement and material flow will be minimized. In this context, the exceptional elements that are common in designing the productive systems are known as a major barrier in cell designing and programming (Wang et al., 2001; Wang & Sarker, 2002; Chan et al., 2006, 2008). The exceptional elements refer to the elements required to be made in more than one cell leading to increased cellular displacement and consequently significant increasing of transportation costs. Thus, decreasing the exceptional elements is one of the most important purposes in designing CMS.

Facility layout is also another key issue in designing CMS including the intercellular machine layout and cell layout within the workshop. An efficient cell designing can reduce the costs of materials' moving, the work in process production, and the throughput rates (Benjaafar, 2002). A comprehensive layout not only increases the system performance, but also decreases about 40-50% of the production cost (Balakrishnan & Cheng, 2009). Therefore, considering the cell layout in designing CMS is very important. According to Wang and Sarker (2002), and Chosh et al. (2016), most of the existing papers have only addressed the cell formation issue, while the cell layout has not been significantly paid attention to. However, cell layout issue and cell formation affects each other and it is said that the simultaneous consideration of both of them is very essential for the successful design of CMS (Alfa et al., 1992). Although, each of the issues of cell formation and cell layout (intercellular and intracellular) is a complex one and simultaneous addressing of both of them is a tedious task. Thus, most of the existing studies either paid attention to one of these issues or addressed them respectively and not simultaneously (Bayram & Şahin, 2016). On the other hand, due to the short life cycle of products and fast changes in demands and consequently the requirement of the change in cells from one period to another, considering dynamic environment is very essential in designing the CMS. Cell reconfiguration includes costly activities like moving machines, adding new machines to the cell, and removing the existing machines from the cells (Safaei et al., 2008). The dynamic cellular production systems was first presented by Rheault et al. (1995) to increase the flexibility including the cell reconfiguration decisions from a period to another and mainly focused on changing the demand and composition of the products. The dynamic cellular production systems are generally more efficient than the traditional cellular production systems; especially with respect to the functional indices such as the work-in-process product, tardiness, throughput time, and all marginal costs (Drolet et al., 2008). On the other hand, due to the fact that the demand for products in each period is not deterministic, by considering demand as a random variable, designing the dynamic cellular production system is more real and functional. In a study on 32 production cycles in 15 factories, Marsh et al. (1997) found out that the structural changes could happen in 6 months after the start of the production cycle. Thus, when the productive cells are created, uncertainty in demands and the expected changes in products and their compositions should be taken into account. Applying these changes in designing the dynamic cellular production systems is more important (Co & Araar, 1988; Arkat et al., 2012).

Even though, several models and solving methods have been presented for designing the cellular manufacturing systems, but very few studies have been conducted simultaneously by considering the cell formation and layout problem under the multi-periodic conditions. The main purpose of this study is to investigate the issue of cell formation and layout in the potential dynamic conditions by considering the intercellular and intracellular displacement (forward and backward movements) and the exceptional elements. In the following, we first comprehensively review the relevant literature in Section 2. The mathematical model for simultaneous solving the problem of cell formation and cell layout in dynamic conditions is presented in Section 3. Then the proposed genetic algorithm (GA) to solve the model is introduced in Section 4. In Section 5, the results are compared with the exact solution of GAMS. Finally, conclusion and remarks for further research are outlined in Section 6.

## 2. State of art

The problem of cell formation and cell layout are two fundamental decisions in designing the CMS that have been addressed in recent studies in sequential or simultaneous forms. On the other hand, addressing

these problems in dynamic environment as well as considering the uncertainty in some parameters makes the system more complex and more real as well. Balakrishnan and Cheng (2007) proposed a comprehensive literature review of the studies conducted in the field of the dynamic cellular production systems especially under the uncertainty conditions. Delgoshaei et al. (2016a) comprehensively reviewed the material transfer methods, their relevant techniques and their effects of the dynamic cellular production systems on production. In the following, we present the review of papers related to the discussions presented in this study that are separately provided in three following parts regarding the significance of subjects:

## 2.1 The Sequential Approaches of cell formation and layout

Most papers that have addressed the solution of both of cell formation and cell layout problems have used the sequential approach in this field (Bayram & Şahin, 2016) so that the problems of elements facility formation, grouping the machines, cell formation, machine layout within the cells, and cell layout within the workshop have been sequentially investigated in several steps. In the context of the sequential approaches, Chen et al. (2006) proposed a two-step approach for the problems of cell formation and layout. In the first step, cell of machines and the element family have been marked with a mathematical model. In the second step, by using a macro approach, we have addressed the cell formation problems and reduction of the intracellular costs. In this study, the cell layout problem has been formulated as a linear layout and in the form of a quadratic assignment problem (QAP) and has been solved via genetic algorithm. Tavakoli-Moqadam et al. (2007) proposed a linear mathematical programming model for solving the inter/intra cellular layout problem under the probabilistic demand condition. In this study, the cell formation was a predefined problem as the input data to determine the inter/intra cellular layout.

Krishnan et al. (2012) used an optimization technique by using from-to chart under the title of input data for cell formation. Then, a modified grouping efficiency measure was determined to determine the efficiency of the grouping. In the following, a genetic algorithm-based approach was used for locating the machine cells in the layout matrix. Chang et al. (2009) proposed a two-step approach based on Tabu search or the problem of cell formation and cell layout by considering the alternative processing paths. In their approach, the problem of cell formation and intracellular layout was simultaneously solved in the first step. Then, the intercellular layout was determined according to the cell formation problem done in the first step.

#### 2.2 Simultaneous approaches of cell formation and layout

Dixit and Mishra (2009) developed CLASS algorithm by Mahdavi and Mahadevan (2008). Their study simultaneously models three problems of cell formation, machines layout within the cell and determination of cell layout in the workshop. Considering important parameters of production like producing operations, production volume, and the size of the elements category makes the problem of cellular production more complex but more real. Paydar et al. (2008) used a fuzzy goal programming based approach to solve the two problems of cell formation and cell layout simultaneously in designing CMS. In the model proposed by these authors, the cost of machineries, intercellular and intracellular displacements, operation production and the capacity of machineries was considered. Paydar et al. (2010) simultaneously addressed the elements' family and cell formation problems by considering the intracellular layout problem. The authors first modeled the problem of cellular production system as multiple departures single destination multiple travelling salesman problem (MDmTSP) and, they used a solving method based on simulated Annealing (SA) to solve the cell formation problems and intracellular layout design. Mahdavi et al. (2013) proposed a combinational approach for simultaneous solving the problems of cell formation and cell layout. In this study, the machines' cells are established in a set of predefined points and machines are located in a linear form within each cell. Bagheri and Bashiri (2014) proposed a comprehensive mathematical model for simultaneous solving the problems of cell formation and assignment of operator and the problem of intracellular layout problem. They minimized the intercellular and intracellular movements, the cost of shipping materials and items related to the operator. The preferred solutions of the problem were obtained by using the LP-metric approach. Mohammadi and Forghani (2014) presented a GA for simultaneous solving the problems of cell formation and cell layout. Researchers calculated the materials transfer costs based on the real condition of the machine within the cell by considering the machine's dimensions and the aisle widths. In their approach, the assigned machines to a cell were ordered along a direct path. Moreover, the machine's cells were established top-down and as a multi-row layout in the workshop. Some years later, these researchers proposed a double-purpose model to the problems of simultaneous cell formation and cell layout (Mohamadi and Forqani, 2016). The first target function aimed to minimize the total intercellular and intracellular transfer and the second target function was intended to maximize the total similarity among the machines. These authors also introduced a new layout which was the improved version of the multi-row layout called S-shaped layout and used a combinational method based on the simulated analog and dynamic programming to solve the model (Mohamadi & Forqani, 2014, 2016; Chang et al., 2013).

#### 2.3 Dynamicity and uncertainty

In many issues of the real world, demand is different from one period to another one. Such a condition is known as the dynamic demand of elements. Changes in market, and product design, and producing new products are some reasons resulting in change in the demand in different periods. These changes may lead to imbalance in the parts routing and creation of bottleneck (Delgoshaei et al., 2016a). The dynamic cellular production systems was first proposed by Rheault et al. (1995) generally consider changes in demands and composition of products and they were more efficient that the traditional cellular production systems.

Tavakoli-Moqadam et al. (2007) proposed a mixed integer linear programming for designing CMS with fuzzy demand under the dynamic conditions. Minimizing the constant and variable costs of the operation, machine displacement and the intercellular displacement costs of the work-in-process products were considered as the purposes of the model.

Zhang (2011) considered planning issue while minimizing the inventory, operation and transportation costs. The solutions of reconstructing cell to cope with the market changes have been successfully used in this paper. Balakrishnan and Hung Cheng (2005) proposed a two-step approach for minimizing the costs of transportation and machine displacement costs in the cell reconfiguration by considering uncertainty in demands. Egilmez et al. (2012) concentrated on the uncertainty of operation times in a dynamic CMS. They also addressed the risk level in process of designing the CMS in dynamic environment. A few years later, the authors measured the impact of risk level in an integrated system of cell formation and layout by using Monte Carlo simulation. Renna and Ambrico (2015) proposed three models for designing, configuring, and programming the cells in dynamic conditions. In the proposed models, the researchers minimized the system's costs including the intercellular displacement, costs of machining and reconfiguration of cell and maximizing the net profit. Delgoshaei ad Gomes (2016) used the artificial networks for programming the cell layout when repairs and preventive maintenance were considered. Deep and Singh (2016) employed a mathematical combinational model for the problems of dynamic cell formation and part operation tradeoff in the dynamic environment through considering multiple part process rout. Their approach simultaneously determined the machines' cell, elements' family, and the optimal operation path. In the end, a simulated annealing based genetic algorithm has been used to solve the problem.

Previous studies and researches in the field of CMS show that although numerous models and solving methods have been focused for the problem of cell formation as the most important subject in designing CMS, very few studies have been conducted to simultaneously address the problems of cell formation and designing layout under the probabilistic dynamic conditions. Hence, in this paper, a comprehensive nonlinear mixed-integer programming model is presented for simultaneous consideration of cell formation and cell layout with the uncertain demand in the dynamic environment.

# 3. Mathematical model

## 3.1 Model's assumptions

For the development of the model, the following assumptions have been considered:

- Each part has a certain number of required operations and the operation time for all elements for each machine is shown.
- Type of elements and the probabilistic distribution function of demand of each part in each period are determined.
- Operational capacity and the cost of each type of machine are constant.
- Parts move among the cells in batch form and the cost of the intercellular and intracellular movements of the groups is constant.
- The size of each group for each product is clear and constant for all periods.
- The minimum number of cells is determined and constant during all periods.
- Displacement of machines from a cell to another cell is performed among the periods.
- The cost of intercellular displacement of each type of machine depends on the cells' sequence.
- The cost of intercellular transportation of the parts depends on the cell sequence and is independent of distance.
- Each type of machine can do one operation.
- The extra inventory between the periods is zero and the delayed order is not allowed.
- The demands of each period should be supplied in the same period.
- The efficiency of machines and production is considered 100%.
- The type of transportation system and the facility layout way is in U form and single-line.
- Machines are rented and the availability time of the machines in each period is specified.
- The fixed cost of each machine is specified and independent from the workload and only depends on the type of facility. It includes the costs related to repair and maintenance and the rent costs of the facility for each period.
- The desired places for machineries' layout within the cell have been previously determined.
- The similar (from the same type) machines are assigned to the same place in each period.

# 3.2 Symbols

*i*: index of machine *m*, i=1, 2, ... *j*: index of element *n*, j=1, 2, ... *k*: index of cell *C*, k=1, 2, ... *p*: index of machine status *mp*, p=1, 2, ...*h*: index of the programming period *H*, h=1, 2, ...

# 3.3 Parameters

 $B_j$ : the largeness of batch for the for transferring part j

 $C_{ja}$ : the cost of an intracellular displacement of part j

 $C_{je}$ : the cost of an intercellular displacement of part j

 $C_{jb}$ : the cost of an intracellular backward unit of part *j* 

Ci: the cost of an intercellular displacement unit of machine *i* 

 $C_{ii'h}^{j}$ : the cost of displacement of element j between the machine i and i in the period h

 $\dot{M}_{ih}$ : the position number of machine *i* (arrangement of machine *i* for layout) in the period *h* 

Nc: minimum number of cells formed in each period

NM: maximum number of machines located in a cell

 $R_{ij}$ : the number of the conducted operation on the element *j* by machine *i* 

 $f_{iib}^{j}$ : the number of travels for transferring the element j between the machines i and i in the period h

 $f_{ii'h}^{j} = \begin{cases} D_j/B_j & R_{i'j} - R_{ij} = 1 \text{ if } \\ 0 & R_{i'j} - R_{ij} \neq 1 \text{ if } \end{cases}$   $IC_{kh} = \begin{cases} 1 & \text{if the cell k is formed in the period } h \\ 0 & otherwise \end{cases}$   $T_i: \text{ the capacity of machine } i \text{ in each period}$   $T_{ij}: \text{ the processing time of element } j \text{ by the machine } i$   $SP: \text{ a set of } (i, j) \text{ pairs when } a_{ij} > 1$   $A_i: \text{ the fixed cost of machine } i \text{ in each period}$   $B_i: \text{ the cost of adding or reducing the machine } i$   $ED_{jh}: \text{ the standard deviation of demand of the element } j \text{ in the period } h$ 

#### 3.4 Introducing decision variables

$$\begin{split} X_{ikh} &= \begin{cases} 1 & \text{if the machine } i \text{ is assigned to the cell k in the period } h \\ 0 & \text{otherwise} \\ Y_{jkh} &= \begin{cases} 1 & \text{if the element j is assigned to the cell k in the period } h \\ 0 & \text{otherwise} \\ \end{cases} \\ Z_{iph} &= \begin{cases} 1 & \text{if the machine i is assigned to the position p in teh period } h \\ 0 & \text{otherwise} \\ \end{cases} \\ U_{ijkh} &= \begin{cases} 1 & X_{ikh} = 1, \ Y_{jkh} = 0 \text{ if} \\ 0 & \text{otherwise} \\ \end{cases} \\ V_{ijkh} &= \begin{cases} 1 & X_{ikh} = 0, \ Y_{jkh} = 1 \text{ if} \\ 0 & \text{otherwise} \\ \end{cases} \end{split}$$

 $X_{kk'h}^{i} = \begin{cases} 1 & k \neq k', \text{ if the machine } i \text{ is assigned to the cell } k \text{ in the period } h \text{ and being assigned to the cell } k' \text{ in the period } (h+1) \\ otherwise \\ \text{KA}_{ih}: \text{ the number of } i\text{-type machine added at the beginning of the period } p \end{cases}$ 

KR<sub>ih</sub>: the number of *i*-type machines eliminated at the beginning of the period h D<sub>jh</sub>: the production amount of element *j* in the period h

N<sub>ih</sub>: the number of type i machines required in the period h

Consequently, the cost of transferring the element j between the machines i and i' in period h is:

$$C_{ii'h}^{j} = \begin{cases} (M_{i'h} - M_{ih})C_{ja} & if & X_{ikh}, X_{i'kh} > 0, M_{i'h} > M_{ih} \\ (M_{ih} - M_{i'h})C_{jb} & if & X_{ikh}, X_{i'kh} > 0, M_{i'h} < M_{ih} \\ |k - k'|C_{je} & if & X_{ikh}X_{i'kh} = 0, X_{ikh}X_{i'k'h} > 0 \end{cases}$$
(1)

$$N_{ih}^* = Min\left\{N_{ih}, N_{i(h+1)}\right\} \qquad \forall i, h$$
<sup>(2)</sup>

According to the parameters and variables, the proposed nonlinear modeling for this problem is as follows:

$$\min \sum_{h=1}^{H} \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{i'=1}^{m} f_{ii'h}^{j} C_{ii'h}^{j} + \sum_{h=1}^{H} \sum_{k} \sum_{(i,j)\in sp} \frac{(U_{ijkh} + V_{ijkh})}{2} + \sum_{h=1}^{H-1} \sum_{i=1}^{m} \sum_{k=1}^{c} \sum_{k'=1}^{c} |k' - k| C_i X_{kk'h}^{i} N_{ih}^{*} + \sum_{h=2}^{H} \sum_{i=1}^{m} B_i (KA_{ih} + KR_{ih}) + \sum_{h=1}^{H} \sum_{i=1}^{m} A_i N_{ih} + \sum_{h=1}^{H} \sum_{i=1}^{m} \sum_{j=1}^{n} T_{ij} B_j D_{jh} + \sum_{h=1}^{H} \sum_{j=1}^{n} |D_{jh} - ED_{jh}|$$

$$(3)$$

subject to

~

$$\sum_{k=1}^{C} X_{ikh} = 1 , i = 1, 2, ..., m , \forall h$$
<sup>(4)</sup>

$$\sum_{\substack{k=1\\m}}^{C} Y_{jkh} = 1 , j = 1, 2, ..., n , \forall h$$
(5)

$$\sum_{k=1}^{m} X_{ikh} \le NM \times IC_{kh} \quad , \ k = 1, 2, \dots, C \quad , \ \forall h$$
<sup>(6)</sup>

$$\sum_{c}^{i=1} IC_{kh} \ge NC \qquad \forall h$$
<sup>(7)</sup>

$$\sum_{m=1}^{k=1} Z_{inh} = 1 , p = 1, 2, ..., mp , \forall h$$
(8)

$$\sum_{n=1}^{n=1} Z_{n} = 1, \quad i = 1, 2, \quad m \quad \forall h$$

$$\tag{9}$$

$$\sum_{\substack{p=1\\H}} Z_{iph} = 1, \quad i = 1, 2, ..., m, , \forall n$$
(10)

$$\sum_{h=1}^{N} N_{ih} < UB \quad , \quad \forall i$$

$$D_{jh} \le ED_{jh} + 1/96ST_{jh} \quad \forall j,h \tag{11}$$

$$\sum_{i=1}^{H} D_{jh} t_{ij} / N_{ih} \leq T_i$$

$$(12)$$

$$(13)$$

$$N_{ih}^{n-1} + KA_{i(h+1)} - KR_{i(h+1)} = N_{i(h+1)} \qquad \forall i,h$$
(14)

$$N_{ih}$$
,  $KA_{ih}$ ,  $KR_{ih}$ ,  $D_{ih} \ge 0$  and integer (15)

$$X_{ikh}, Y_{jkh}, Z_{iph}, U_{ijkh}, V_{ijkh}, X^{i}_{kk'h} = 0 \quad or \quad 1$$
 (16)

#### 3.5 Objectives

In this paper, a nonlinear integer model has been presented for simultaneous decision making about determination of cells, part family and facility layout in a probabilistic dynamic condition. The purposes of this model include minimizing the cost of intercellular and intracellular displacement of elements, moving machines, the costs of adding and removing machines, the fixed cost of machines, the cost of exceptional elements, the operational cost and the cost of estimated demand difference from its expected value as expressed in the following.

## 3.5.1 The cost of intercellular and intracellular displacement of parts

The first part of the function includes the costs of intercellular and intracellular displacement elements. The intracellular displacement where the machineries layout in the cell is determined is related to the intracellular moves of parts when the element is transferred to another machine for conducting the next operation. The aim is the optimal machine layout within the cell to minimize the displacement of machines. As shown in Eq. (1), the cost  $C_{ii'h}^{j}$  consists of the forward costs of part transferring ( $C_{ja}$ ) from the machine with lower position to the machine with higher position as well as the backward costs ( $C_{jb}$ ) from the higher position machine to the lower position one. It is natural that the rate of cost coefficient in the backward state must be considered more than the forward state.

Moreover, the  $C_{ii'h}^{j}$  calculates the cost of part transferring from a cell to another as shown in the third part of the Eq. (1). This cost is occurred when all operations related to a cell are not finished within the cell and are transferred to another cell to be completed. For example, if an intercellular movement for the

element *j* occurs between the cell *k* and *k'*, in this case, this element includes the intercellular displacement costs that are calculated regarding the difference in sequences of the layout of two cells *K* and *k'*. The intercellular displacement costs are in direct relationship with the distance of the two cells. By applying this target function, when the intercellular displacements of two cells are more than the other cells, these two cells will be placed next to each other. This cost is calculated by multiplying the absolute value of the distance between two cells (k - k') in the intercellular displacement cost coefficient of elements  $(C_{ie})$ . It is clear that  $(C_{ie})$  has the highest value compared to the two intracellular costs.

# 3.5.2 Cost of the exceptional elements

The second part of the target function implies the minimization of the exceptional elements. The exceptional elements refer to the elements that are required to be made in more than one cell and they lead to increase the intercellular displacement and consequently significantly increase the transportation cost. These elements (parts) that are conventional in the productive systems are known as a major barrier in designing and programming the cell. Thus, one of the most important purposes in designing CMS is the decrease in the exceptional elements number. Since the displacements are counted for two times, the coefficient 1/2 is considered in this function.

# 3.5.3 Costs of displacement of machines and cell reconfiguration

The third part of the target function has addressed minimization of the cost of moving machines because of the cell reconfiguration. As mentioned earlier, because of multi period assumption, the cost of moving a machine from a cell to another cell has been considered in this model. In a production model under dynamic conditions, the best cell production method may not be the best answer during all periods. Thus, with demand changes, inevitably change the arrangement of the extant machines in the cells will be occurred. This displacement will stop production and causes additional cost which is obtained based on the number of machines (of one type) transferred from a cell to another cell, the cost of a cellular displacement unit, and the difference in sequence number of the source cell to the destination cell.

# 3.5.4 Cost of adding and reducing machineries

The fourth part of the target function includes the costs of adding or reducing machines in the system during the programming periods that consists of the costs of transferring machines between the rented places.

# 3.5.5 Fixed costs of machines

The fifth part of the objective function has been proposed to minimize the fixed cost of machines. This cost includes that of renting machines and maintenance and overhead costs in each period which are different regarding the type of machines.

# 3.5.6 Operational cost

The sixth part of the objective function is intended to minimize the operational costs. In calculation of this cost, a buffer is considered for transferring. Also, the processing time of elements and the amount of production in different periods is considered.

# 3.5.7 The cost related to the difference between the estimated demands from its expected value

The seventh part of the objective function implies minimizing the difference in estimated demand from its expected value. In this model, it is assumed that the demand for each element is specified based on a statistical distribution according to the previous experiences and data. That is, in each period, the demand for each element is predicated in the way that the objective function be minimized. Consequently, the demand for elements enters the model as a probabilistic variable. To achieve this goal, we must determine

a confidence interval for the variable of demand for that element in each period regarding the distribution parameters of each element. As the demand for an element is lower than its expected value, the operational costs and the material transferring may decrease and its absolute value of standard deviation will increase. In the current study, it is assumed that the demand distribution of elements in each period follows the normal distribution. If Ed<sub>jh</sub> and ST<sub>jh</sub> are, respectively, considered as the mean and the standard deviation of normal distribution related to the element *j* in period *h*, a confidence interval of  $(1 - \alpha)$ % for the production level of the element *j* in the period *h* (D<sub>jh</sub>) can be expressed as follows:

$$D_{jh} \ge ED_{jh} - Z\alpha_{/2}ST_{jh}, \qquad D_{jh} \le ED_{jh} + Z\alpha_{/2}ST_{jh}.$$

According to the above confidence interval, to prevent facing shortage with the probability of  $(1 - \alpha)$ %, we should produce maximum  $ED_{jh} + Z\alpha_{/2}ST_{jh}$ . If the confidence level is considered equal to  $(1 - \alpha)$ = 0.95 and it is fixed during the programming periods, the element *j* production rate in the period *h* is obtained from the following equation:

$$(1 - \alpha) = 0/95 \rightarrow \alpha = 0.05 \rightarrow \alpha/2 = 0.025 \rightarrow Z\alpha/2 = 1.96$$

If the demand function is continuous and of beta distribution type, we should consider three values of optimistic (H), pessimistic (L), and probable (M) for demand. In this case, the values  $Ed_{jh}$  and  $ST_{jh}$  are achieved from the following equations:

$$ED_{jh} = \frac{L+4M+H}{6} \quad , \quad ST_{jh} = \frac{H-L}{6}$$

#### 3.6 Model constraints

The Constraint (4) ensures that each type of machine be assigned to one cell. The Constraint (5) is for assignment of each element to a cell. The limitation given in Eq. (6) prevents from the assignment of more than NM types of machine to each cell as well as the assignment of all machineries to a cell. The Constraint (7) ensures that at least NC cells are formed in each period. The Constraint (8) ensures that each position accepts only one type of machines in each period. With the limitation given in Eq. (9) in each period implies that, each machine is only assigned to one position. The Constraint (10) ensures that in each period, the number of machineries of the same type does not exceed a certain value. The Constraints (11) and (12) show that the demand rate estimated by the mode should not exceed the high and low limits of the confidence level of 95%. This confidence level has been determined by the relevant distribution function. Due to the Constraint (13), the loading rate on each machine in each period does not exceed its capacity. The Constraint (14) established the balance of the extant machines in each period regarding the added and eliminated machines.

### 4. Solving the model

### 4.1. Proposed genetic algorithm

Several algorithms have been applied in the context of a DCMS design to approach the appropriate design. One of the most popular forms of these designs is the Genetic Algorithm (GA). This section attempts to examine some aspects of this algorithm and demonstrates of its application in a DCMS design. The GA is known as the most popular meta-heuristic algorithm and is a component of evolutionary calculation and it is a subset of artificial intelligence. The primary idea of this algorithm is derived from the Darwin's evolutionary theory and its application is based on natural genetics (Ghosh et al., 2016).

In a way that the GA searches the solution space, not only the better quality solutions are acceptable, but also the solutions with lower fitness are acceptable, that leads the algorithm to escape from local optimum points. The GA varies in many ways with traditional optimization methods. In this algorithm, design space should be converted to genetic space. Therefore, we deal with series of coded variables. Another major difference between the GA and other optimization methods is that the GA works with a population or a set of points at a certain moment, while traditional optimization methods operate only in a particular point. A distinguishing feature of the GA is that principle of processing in this algorithm is random and it is guided to optimum place. Generally, the differences between the GA and other optimization methods can be expressed as follows:

- $\checkmark$  The GA does not search the solution in a single point and searches the solution in parallel.
- ✓ The GA does not use the deterministic rules and uses probabilistic rules.
- ✓ The GA is based on coded variables. Unless in cases which variables are illustrated as real numbers.
- ✓ The GA does not require backup information. It only determines the members of objective function and the fitness of path in search space.

Applying the GA, the following steps are necessary:

- Representing an appropriate solution structure,
- Obtaining appropriate initial solutions in a population size,
- Employing appropriate genetic operators (i.e., mutation and crossover) to obtain new solutions,
- Selecting population of the next generation from parent and offspring chromosomes,
- Chromosome evaluation measure (i.e., fitness function),
- Specifying the stopping criteria.

The algorithm in two phases is used to solve the model of this problem. The first phase is related to machine assignment to manufacturing cells and layout determination, and the second phase is about determination of part assignment to part families.

## 4.2. Parameter tuning

In this paper, we have utilized the Taguchi experimental design method to calibrate the parameters of meta-heuristic algorithms. This method has been introduced by Taguchi in early 1960s, and it is applicable in the designing of processes. The orthogonal arrays of the method are employed for the evaluation of a large number of factors with a few experiments. In the current problem, the L9 design of the Taguchi method is performed for the algorithms by using the Minitab 16.2 software. The Taguchi method strives to minimize variances of quality characteristics obtained from S/N ratio. Quality characteristic of this paper is considered as relative percentage deviation (RPD), which is employed to change objective function values to non-scale. Accordingly we prefer "the smaller-the better" type. RPD is determined as follows (Hsu & Su, 1998):

$$RPD = \frac{|Obj_i - Obj_{best}|}{|Obj_{best}|} \times 100,$$
<sup>(17)</sup>

where  $Obj_{best}$  and  $Obj_i$  are the best obtained objective values for a particular instance and the objective value obtained for the *i*th trial, respectively. Also the signal-to-noise for "the smaller-the better" characteristic is calculated as follows:

$$S/N = -10 \log \left(\frac{1}{n} \sum_{i=1}^{n} y_i^2\right),$$
(18)

where  $y_i$  represents the response value in the *i*th replication and *n* denotes the number of replications in experiments replications. We considered three factors that can have salient effects on the proposed evo-

lutionary algorithms. The considered levels of the parameters are presented in Table 1.

## Table 1

Considered Levels of parameters of genetic algorithms

	Parameter	Level 1	Level 2	1 Level 3
Pure GA	Crossover percentage	0.60	0.70	0.80
	Mutation percentage	0.05	0.10	0.15
	Population size	50	70	90

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The mean S/N ratio is calculated for each level of control factors. Fig. 1 depicts the level of control factors versus control factors. A larger value of the S/N ratio in the graphs is more desirable.

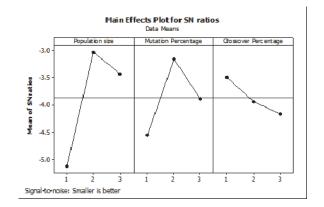


Fig. 1. Mean S/N ratios for the proposed GA

### 4.3. Solution view

One chromosome coding is required for the GA, so that we can illustrate the solutions of a problem. The way that chromosomes are viewed determines how a problem is formulated in a form of an algorithm and what genetic operators are applied. Each chromosome is formed from genes that can be shown as binary and integer numbers or combination of characters that is a coded form of a feasible solution (appropriate or inappropriate) from the problem. The considered chromosome for the first step of this problem includes a matrix with *H* rows and *M* columns that can be divided into the following sub-matrix.

- Sub-matrix of Z is related to assignment of machines to manufacturing cells. This sub-matrix consists of H (i.e., number of periods) rows and M (i.e., number of machines) columns. Each element of this matrix is a number between 1 and C (i.e., number of cells) and the element  $Z_{ih}$  represents the number of cell that includes machine type i in period h.
- Sub-matrix X is related to the horizontal component of machines' location. This sub-matrix also consists of H rows and M columns. With respect to the machines' dimension  $(1 \times 1)$ , one integer is sufficient to familiarize per horizontal and vertical components of the machines. Per element of this matrix is a number between 1 and E (i.e., length of the job shop) and the element  $x_{ih}$  represents the horizontal component of location that includes machine *i* in period *h*.
- Sub-matrix Y that is related to the vertical component of machines' location. This sub-matrix also consists of H rows and M columns. Each element of this matrix is a number between 1 and F (i.e., width of the job shop) and the element  $y_{ih}$  represents the vertical component of location that includes machine *i* in period *h*.

Fig. 2 and Fig. 3 illustrate the general and detailed views of the chromosome structure related to machines alignment to manufacturing cells, respectively.

Fig. 2. General view of the chromosome structure

$\begin{bmatrix} z_{11}z_{12} \\ \vdots & \vdots \\ z_{h1}z_{h2} \end{bmatrix}$		$\begin{array}{c} x_{11}x_{12} \\ \vdots \\ \vdots \end{array}$		$\begin{array}{c} y_{11}y_{12} \\ \vdots \\ \end{array}$	 $\begin{bmatrix} y_{1M} \\ \vdots \end{bmatrix}$
$  z_{h1}z_{h2} $	$Z_{hM}$	$x_{h1}x_{h2}$	$x_{hM}$	$y_{h1}y_{h2}$	<i>у<sub>ћМ</sub></i> ]]

Fig. 3. Detailed view of the chromosome structure.

The considered chromosome for the problem includes a matrix with *H* rows and *N* columns. Its detailed structure related to parts alignment to part families is shown in Fig. 4.

$$\begin{bmatrix} z'_{11}z'_{12} & z'_{1N} \\ \vdots & \vdots & \dots & \vdots \\ z'_{h1}z'_{h1} & z'_{HN} \end{bmatrix}$$

Fig. 4. Detailed view of the chromosome structure

## 4.4 Obtaining the primary solutions

The elements of matrix Z is obtained randomly from 1 to C. The elements of matrices X and Y are selected in a way that machines are not overlapped; in a way that numbers of a column from X and Y are not simultaneously equal to another column of X and Y. The elements of matrix z' are also obtained randomly from within the numbers of 1 to C.

# 4.5. Application of genetic operators

A genetic operator is used to produce a new generation of offspring.

# 4.5.1 Mutation

In the proposed GA, in sections of machine assignment to manufacturing cells, five types of the operator are used simultaneously. These operators are as follow:

- *Machine relocation in cells:* This operator is on matrix *Z* that is substituting two numbers from two columns in a row of matrix *Z*.
- *Relocation of two machines:* This operator is on matrix *X* and *Y* simultaneously. Selecting two columns from a row of matrices *X* and *Y* and substituting the numbers of these columns in the same row, the location of those machines in considered period will change.
- *Relocation and cell change of two machines:* This operator is on matrix *Z*, *X* and that is two previous operators simultaneously.
- *Approaching one machine to another:* This operator is on matrix *X* or *Y*. One of the columns from matrix *X* or *Y* is selected and it changes to the numbers of another column from the same matrix with one unit difference.
- Assigning the machines with more flow to a single cell: This operator is on matrix Z. With respect to the numbers of flow matrix, machines with more relations are assigned to a single cell.

In the section about part assignment to part families in this algorithm, we use substitution of two numbers from two columns of a row in matrix Z'.

## 4.5.2 Crossover

In both parts of the solution view in this algorithm, the selected crossover means substitution of a part of a row from parent with the same part of the same row of another parent and generating two offspring similar to the two parents.

## 4.6 Selecting the next generation

Selecting the parents for producing next generation plays a major rule in the genetic algorithm. The aim is to select the best chromosomes (i.e., the solutions that are better than others) for entering to the next generation or producing new generation. Generally, each chromosome with a particular probability has an opportunity to produce or enter to the next generation. Therefore, chromosomes with high fitness should have more probability to be selected. Several methods have been proposed for selecting the next generation that are in two categories: probabilistic and non-probabilistic. Probabilistic methods contain

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roulette wheel selection, scaling and grouping. Non-probabilistic methods include competitive selection and elite models. Roulette wheel mechanism is used in this study. In this method, members are selected based on their relative consistencies. In other words, a roulette wheel method selects the next generation's members to the number of population, giving more probabilities to more appropriate chromosomes and generating the random number between zero and one.

## 4.7 Criterion for evaluating chromosomes (Fitness function)

The fitness function is an implication of the objective function. In the GA, the fitness value for each chromosome is equivalent to the value of the objective function for a solution. For instance, if the objective in a cellular manufacturing problem is to minimize the sum of costs according to the problem model, an offspring will be acceptable when it minimizes the cost function relative to its parents. Also in this problem, fitness for each chromosome is calculated based on an objective function.

## 4.8 Stopping condition

To stop the GA and present a final solution, stopping criterion should be considered. The stopping criteria that are mainly used are as follows:

- The maximum specified numbers of generation: if number of generations passes the maximum specified numbers of generation, algorithm will end.
- Convergence of population: in broad terms, GA attempts to converge the population to a single population. If the current population converges to a single solution, algorithm will end.
- Reaching to specified solving time.

The criterion of the maximum specified numbers of the generation is used here.

## 5. Solving the problem and comparing the results

The proposed GA is coded by Visual Basic (VB) and run by a Pentium4 personal computer (PC). The number of populations, number of replications for generations, mutation rate and crossover rate for the proposed GA are considered 20, 100, 0.25 and 0.85, respectively. In order to validate the proposed model and verify its quality, a linear model of dynamic cell formation for problems is solved by GAMS. Two hours are considered as the maximum elapse time to solve the problem. After two hours, the GA is stopped and the best obtained solution is reported. The problem is resolved by the proposed GA and compared with the results obtained from GAMS. After 10 times running the program with VB, the solution obtained from this process is exactly similar to the obtained solution in GAMS. The results obtained from 8 problems with various dimensions that are solved with the GA and also using GAMS is shown in Table 2. The number of periods is 2, and the number of cells for 3 to 7 and 8 to 20 machines is considered 2 and 3, respectively.

## Table 2

Dimensions of problem	Average of 10 genetics	Optimum genetics	Time of genetic	Solution of GAMS	Time of GAMS
2×3	24283	24283	12	24283	42
4×6	28641	28641	17	28641	6180
5×8	85637	84180	22	84253	7200
6×9	120903	116046	32	121203	7200
7×11	224095	215470	42	248907	7200
8×13	497882	456124	65	706036	7200
10×12	706813	664902	79	771098	7200
12×20	920627	891287	121	-	-

Comparing the obtained results from GA and GAMS

## 6. Conclusion

In this study, a nonlinear mixed integer programming model has been proposed that has simultaneously addressed the problems of cell formation and inter/intra cellular layout. The purposes of the model include minimizing the total cost, operational cost, the fixed cost of machineries, the cost of intercellular and intracellular displacement of elements, machine displacement and cell reconfiguration, the cost related to the difference between the estimated demands from its expected value, and the cost related to the exceptional elements. Moreover, both intercellular and intracellular displacements have been considered by regarding the category size for transferring elements and applying both backward and forward movements and by considering more cost coefficient for the backward movements in this model. Finally, GA was proposed for solving the problem and its performance has been compared with the results of GAMS for several problems with small, medium and large dimensions regarding the target function value and time. The results show that the proposed GA algorithm can obtain the solution close to optimal value in less time. For further research, entering the production data such as production volume, preparation times, and inventory keeping among different periods can make the proposed model more realistic.

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