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A new memetic algorithm for solving split delivery vehicle routing problem

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A B S T R A C T

Split delivery vehicle routing problem is one of the traditional types of routing problems in which the demand of different points can be divided among vehicles and the objective is to minimize the path length, which vehicles travel. In this paper, fuel cost of vehicles which is assumed to be dependent on their traveled path and load is considered as the objective functions. Namely, the cost of the consumed fuel is proportionate to the unit of load carried per unit of distance. In order to solve the proposed model a new memetic algorithm is developed which has two rows. The performance of the proposed algorithm for 21 standard problems is compared with the optimum solutions obtained from mathematical programming standard solver and the solutions of the same algorithm with single row solution representation. The results express the efficiency of developed algorithm.

1. Introduction

Split delivery vehicle routing problem is one of the traditional types of routing problems in which the demand of different points can be divided among vehicles and the objective is to minimize the path length, which vehicles travel (Archetti et al., 2006; Anh, 2014). By splitting demand among different vehicles, the cost of transportation could be reduced significantly. The idea of split delivery was first introduced by Dror and Trudeau (1989, 1990) and despite simplicity there appears to be many applications for this problem (Chen et al., 2007). There are also several solution strategies for solving this problem. Jin et al. (2008), for instance, presented a column generation approach for the Split delivery vehicle routing problem (SDVRP). Stålhane et al. (2012) offered a branch-price-and-cut method for a ship routing and scheduling problem with split loads. Belenguer et al. (2000) presented a lower bound for the split delivery vehicle routing problem and Moreno et al. (2010) improved the lower bounds for the split delivery vehicle routing problem. Archetti et al. (2006) presented a tabu search algorithm for the split delivery vehicle routing problem. Archetti and Speranza (2012) considered vehicle routing problems with split deliveries. Archetti et al. (2014) presented two exact branch-and-cut algorithms for the SDVRP based on two relaxed formulations that provide lower bounds to the optimum. Boudia et al. (2007) offered an effective memetic algorithm with population management for

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the split delivery vehicle routing problem. Mota et al. (2007) proposed a metaheuristic for the vehicle routing problem with split demand, which was for the special case of the Vehicle Routing Problem in which the demands of the clients was split, i.e., any client can be serviced by more than one vehicle. Archetti et al. (2008) developed an optimization-based heuristic for the SDVRP and Aleman (2010) introduced an adaptive memory algorithm for this problem. Aleman and Hill (2010) offered a tabu search with vocabulary building method for the vehicle routing problem with split demands. Derigs et al. (2010) and Wen et al. (2015) presented a local search-based metaheuristics for the SDVRP. Wilck IV et al. (2012) offered a genetic algorithm for the split delivery vehicle routing problem while Khmelev and Kochetov (2015) built a hybrid VND method for this problem. Zhang et al. (2012) studied SDVRP for emergency logistics and proposed an evolutionary heuristic approach to solve such problem. Wang et al. (2013) performed a survey on vehicle routing problem by considering simultaneous deliveries and pickups with split loads and time windows. Wang et al. (2014) considered SDVRP for another application and using two-stage heuristic methods, they investigated vehicle routing problem with split deliveries and pickups. Yuyan et al. (2013) considered different perspective of SDVRP problem. Chen et al. (2014) performed a comprehensive survey on model and algorithm for an unpaired pickup and delivery vehicle routing problem with split loads. Zeng et al. (2014) presented an ant colony algorithm with memory grouping list for multi-depot vehicle routing problem. Zhang et al. (2015) presented an efficient forest-based tabu search algorithm for the SDVRP problem.

2. The proposed study

Split delivery vehicle routing problem is one of the traditional types of routing problems in which the demand of different points can be divided among vehicles and the objective is to minimize the path length, which vehicles travel. In this paper, fuel cost of vehicles which is assumed to be dependent on their traveled path and load is considered as the objective functions. Namely, the cost of the consumed fuel is proportionate to the unit of load carried per unit of distance. In order to solve the proposed model a new memetic algorithm is developed which has two rows to guarantee the optimality of solutions. Split delivery vehicle routing problem (SDVRP) can be defined as a graph of G = (V, E) where $V = \{0, 1, \ldots, n\}$ represents the nodes and E represents the arcs. Node 0 represents the origin and the other nodes represent demands. D_{ij} denotes the length of arc $(i, j) \in E, C_1$ and C_2 represent cost of travelling empty and full load vehicles, respectively. Therefore, a half load vehicles cost a linear combination of C_1 and C_2. Moreover, d_i is the distance for demand $i \in V - \{0\}$, and there are K vehicles with a capacity of Q.

Indices

i, j	demands and depot point
v	vehicles with $v \in \{1, 2, \ldots, K\}$
D_{ij}	distance between two point $(i, j) \in E$
d_i	demand for node i

Variables

x_{ij}^v	1 if vehicle v moves from node i to node j and 0, otherwise,
y_{iv}	the amount of demand j, which is covered by vehicle v,
z_{ij}^v	the amount of load v for path i-j

The mathematical model can be represented as follows,

$$\min \quad C_1 \sum_{v=1}^{K} \sum_{i,j \in V} D_{ij} x_{ij}^{v} + \frac{C_2 - C_1}{Q} \sum_{v=1}^{K} \sum_{i,j \in V} D_{ij} z_{ij}^{v} \tag{1}$$

subject to

$$\sum_{i \in V} \sum_{v=1}^{K} x_{ij}^{v} \geq 1, \qquad j \in V \tag{2}$$

$$\sum_{i \in V} x_{ij}^{v} - \sum_{i \in V} x_{ji}^{v} = 0, \qquad j \in V;\ v = 1, 2, \dots, K \tag{3}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ij}^{v} \leq |S| - 1, \qquad v = 1, 2, \dots, K;\ S \subseteq V - \{0\} \tag{4}$$

$$y_{iv} \leq d_i \sum_{j \in V} x_{ij}^{v}, \qquad i \in V - \{0\};\ v = 1, 2, \dots, K \tag{5}$$

$$\sum_{v=1}^{K} y_{iv} = d_i, \qquad i \in V - \{0\} \tag{6}$$

$$z_{ij}^{v} \leq Q x_{ij}^{v}, \qquad i \in V, j \in V - \{0\};\ v = 1, 2, \dots, K \tag{7}$$

$$\sum_{j \in V} z_{ji}^{v} - \sum_{j \in V} z_{ij}^{v} = y_{iv}, \qquad i \in V - \{0\};\ v = 1, 2, \dots, K \tag{8}$$

$$x_{ij}^{v} \in \{0, 1\}, \qquad i, j \in V;\ v = 1, 2, \dots, K \tag{9}$$

$$y_{iv} \geq 0, z_{ij}^{v} \geq 0 \qquad i, j \in V;\ v = 1, 2, \dots, K \tag{10}$$

The objective function of this model minimizes total cost of transportation. Eq. (2) guarantees that each node is visited at least once. Eq. (3) is associated with the flow between two nodes. Eq. (4) prevents the possibility of having circle among nodes. Eq. (5) is to ensure that each node is visited by a vehicle when there is a service delivered. Eq. (6) and Eq. (7) represent demand and supply constraints, respectively. Eq. (8) is associated with the amount of demand delivered by each vehicle. Finally, other variables determine the type of variables.

The proposed study of this paper uses memetic algorithm (Golberg, 1989; Back et al., 1997; Merz & Freisleben, 2001) for determine the near optimal solution for the proposed study of this paper. Fig. 1 demonstrates the Chromosome used for this study.

1	2	3	…	$n-1$	n
0.45827	0.79874	0.87947	…	0.48258	0.63179
0.28666	0.74006	0.38395	…	0.19846	0.82163

Fig. 1. The structure of the chromosome

According to Fig. 1, there are two columns where the first column demonstrates the priority of assigning demands for different routes and the second row shows the priority of meeting various demand points. The proposed study uses random numbers, which eliminates the possibility of having infeasible solutions. The local search for the proposed study of this paper is as follows,

Algorithm 1

1. Setup parameters,
2. Generate the initial population randomly,
3. Choose a member of population with the best fitness,
4. Until termination criteria is reached do the following,
   4.1 Generate parents
   4.2 Generate children using mutation operation on parents,
   4.3 Choose 10% of the population and do a local search,
   4.4 Evaluate the population,
   4.5 Choose the members with the best fitness,
   4.6 Choose the next generation.

3. The results

In this section, we present the implementation of the proposed method of this paper using some benchmark problems introduced by Chen et al. (2007). The method has been coded in MATLAB software. The results are shown when we use one column or two columns method to represent the initial solution. In addition, the mathematical model has been coded in GAMS software package and it was run for small scale problems and the solutions were compared with near-optimal solutions obtained using memetic method. Table 1 shows the results of our implementation.

Table 1
The summary of the implementation of memetic method versus GAMS method

Problem	One column			Two column			Optimal solution		Difference (%)
	Mean	Best	Time	Mean	Best	Time	Obj- func.	Time Cpu	
1	261.6	261.6	0.0571	261.6	**261.6**	0.129	261.6	0.34	0
2	353.4	353.4	0.0393	347.8	**347.8**	1.4731	347.8	0.59	0
3	278.2	278.2	0.375	278.2	**278.2**	1.7874	278.2	0.56	0
4	333.4	333.4	0.6375	328.6	**328.6**	2.0253	328.6	0.75	0
5	407.5	407.5	0.4449	410.5	**403.9**	4.256	403.9	0.94	0
6	328.5	327.6	0.8593	319.2	**319.2**	7.3423	319.2	0.89	0
7	372.6	372.6	0.7904	380.2	**372.6**	1.9636	372.6	0.94	0
8	514.7	510.6	0.7682	512.4	**500.4**	5.4767	500.4	1.66	0
9	417.4	389.35	0.7688	386.3	**384.71**	5.6282	384.71	3.12	0
10	438.13	437.8	1.0168	435.93	**426.2**	3.3591	426.2	3.54	0
11	391.95	381.85	1.8452	390.35	**381.85**	3.7663	381.85	1.43	0
12	482.17	475.1	2.8398	475.63	**455.9**	4.151	455.9	6.26	0
13	459.03	450.3	2.8022	457.3	451.7	6.5858	435.1	9.38	3.82
14	515.47	499.2	2.7051	502.27	**497.2**	7.6222	497.2	25.62	0
15	430.03	421.7	2.408	423.77	**408.5**	8.0459	408.5	40.67	0
16	489.2	484.7	3.0733	484.6	472.1	8.0318	459.65	184.92	2.71
17	502.52	498.83	2.3698	497.22	495.05	7.0091	471.53	233.78	4.99
18	563.4	543.1	3.229	2136.33	516.9	6.5164	513.9	348.76	0.58
19	604.26	593.31	4.0662	593.92	584.21	8.0763	581.79	1738.04	0.42
20	695.69	662.2	5.1335	682.61	656.14	13.9904	630.2	2514.91	4.12
21	831.82	793.79	5.1035	817.9	776.77	14.552	738.67*	7200	5.16
Mean	460.52	451.24	1.9682	529.65	443.86	5.7994	422.94	586.53	1.05

Note that the best solution obtained in 7200 seconds could be non-optimal. In addition, Fig. 1 shows the results of our proposed method for the best and average solutions.

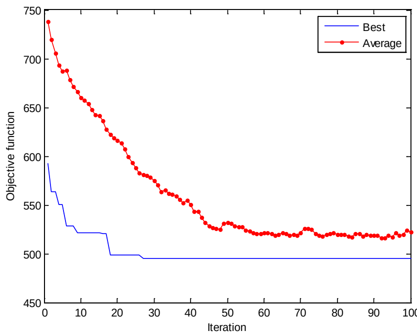

Fig. 1. The best versus average solution

4. Conclusion

In this paper, we have presented a memetic problem for SDVRP problem where the fuel cost of vehicles which were assumed to be dependent on their traveled path and load was considered as the objective functions. Namely, the cost of the consumed fuel is proportionate to the unit of load carried per unit of distance. In order to solve the proposed model a new memetic algorithm has been developed which contained two rows to guarantee the optimality of solutions. The performance of the proposed algorithm for 21 standard problems is compared with the optimum solutions obtained from mathematical programming standard solver and the solutions of the same algorithm with single row solution representation. The results have stated the efficiency of developed algorithm.

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