Management Science Letters 5 (2015) 29–38

Contents lists available at GrowingScience

Management Science Letters

homepage: www.GrowingScience.com/msl

Reliability measures of a computer system with priority to PM over the H/W repair activities subject to MOT and MRT

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C H R O N I C L E

Article history:
Received September 18, 2014
Accepted 15 December 2014
Available online
December 16 2014

Keywords:
Reliability Measures
Computer System
Preventive Maintenance
Maximum Operation and Repair Times
Priority and Replacement

A B S T R A C T

This paper concentrates on the evaluation of reliability measures of a computer system of two-identical units having independent failure of h/w and s/w components. Initially one unit is operative and the other is kept as spare in cold standby. There is a single server visiting the system immediately whenever needed. The server conducts preventive maintenance of the unit after a maximum operation time. If server is unable to repair the h/w components in maximum repair time, then components in the unit are replaced immediately by new one. However, only replacement of the s/w components has been made at their failure. The priority is given to the preventive maintenance over repair activities of the h/w. The time to failure of the components follows negative exponential distribution whereas the distribution of preventive maintenance, repair and replacement time are taken as arbitrary. The expressions for some important reliability measures of system effectiveness have been derived using semi-Markov process and regenerative point technique. The graphical behavior of the results has also been shown for a particular case.

1. Introduction

Over the past few decades, the demand of reliable h/w and s/w components has increased manifolds due to their applications in every sphere of life, particularly in industrial management. Therefore, importance of reliable computer systems has been desired for the successful operation and to protect the integrity of stored information. The failure of computer system causes organizations several hours or days of downtime. Therefore, a major challenge to the industrialists is to provide a high reliability computer system for the customers. For this purpose, they are exploring new techniques for the improvement of reliability of their products. In spite of these efforts, a little work has been carried out for the reliability modeling of computer systems. In addition, most of the research work carried out so far in the subject of s/w and h/w reliability has been limited to the consideration of either h/w subsystem alone or s/w subsystem alone. However, there are many complex systems in which h/w and s/w

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doi: 10.5267/j.msl.2014.12.010

components work together to provide computer functionality. Friedman et al. (1992) and Welke et al. (1995) tried to develop a combined reliability model for the whole system in which hardware and software components work together. Lai et al. (2002) proposed a model for availability analysis of distributed hardware/software systems. Recently, many researchers such as Malik and Anand (2010), Kumar and Malik (2011), Malik et al. (2009) and Malik (2013) studied reliability models of a computer system with different repair policies. Barak and Barak (2013) discussed a reliability model of a cloud under the concept of maximum operation and repair times.

Further, it is a common knowledge that the continued operation and ageing of operating systems gradually reduce their performance, reliability and safety. Moreover, a breakdown of such systems is costly, dangerous and may create confusion in our society. It is, therefore, of great importance to maintain the reliability up to a certain level of such systems with high reliability. It is also proved that preventive maintenance can slow the deterioration process of a repairable system and restore the system in a younger age or state. Thus, the method of preventive maintenance can be used to improve the reliability and profit of system. The concept of preventive maintenance has been used by Malik and Nandal (2010) while analyzing a redundant system with maximum operation time. Kumar et al. (2012) and Malik and Kumar (2012) proposed a reliability model for computer system introducing the concept of preventive maintenance of the unit after a maximum operation time and repair time. Further, the reliability of a system can be enhanced by making replacement of the components by new one in case repair time is too long i.e., if it extends to a pre-specific time. Singh and Agrafiotis (1995) analyzed stochastically a two-unit cold standby system subject to maximum operation and repair time. Kumar and Malik (2012) developed a reliability model for a computer system with priority to s/w replacement over h/w replacement under the assumption of maximum operation time. Sureria et al. (2012) established a reliability model of a computer system with priority to s/w replacement over h/w repair under the assumptions of independent h/w and s/w failures. Anand and Malik (2012) suggested a reliability model of a computer system with arbitrary distributions for h/w and s/w replacement time.

Keeping in view of the above facts and to fill up the gap, a stochastic model for computer system of two-identical units having independent failure of h/w and s/w components has been designed. Initially one unit is operative and other is kept as spare in cold standby. There is a single server who visits the system immediately whenever needed. The server conducts PM of the unit after a maximum operation time. If the server is unable to repair the h/w components in the unit in maximum repair time then components are replaced immediately by new one. However, only replacement of the s/w components has been made at their failure. The priority is given to the preventive maintenance over repair activities of the h/w. The time to failure of the components follows negative exponential distribution whereas the distribution of preventive maintenance, repair and replacement time are taken as arbitrary. The expressions for some important reliability measures of system effectiveness such as mean time to system failure (MTSF), availability, busy period of the server due to PM, busy period of the server due to h/w repair, busy period of the server due to h/w replacement, busy period of the server due to s/w replacement, expected number of h/w replacements, expected number of s/w replacements, expected number of visits of the server and profit function are obtained using semi-Markov and regenerative point technique. The graphical behavior of MTSF, availability and profit function has also been observed for a particular case.

2. Notations

E	The set of regenerative states
NO	The unit is operative and in normal mode
Cs	The unit is in cold standby
a/b	Probability that the system has hardware / software failure
λ_1/λ_2	Constant hardware / software failure rate
α_0	Maximum constant rate of Operation Time

β_0	Maximum constant rate of Repair Time,
Pm/PM	The unit is under preventive Maintenance/ under preventive maintenance continuously from previous state
WPm/WPM	The unit is waiting for PM / waiting for preventive maintenance continuously from previous state
HFur/HFUR	The unit is failed due to hardware and is under repair / under repair continuously from previous state
HFurp/HFURP	The unit is failed due to h/w and is under replacement / under replacement continuously from previous state
HFwr / HFWR	The unit is failed due to h/w and is waiting for repair/waiting for repair continuously from previous state
SFurp/SFURP	The unit is failed due to the s/w and is under replacement/under replacement continuously from previous state
SFwrp/SFWRP	The unit is failed due to the software and is waiting for replacement / waiting for replacement continuously from previous state
h(t) / H(t)	pdf / cdf of replacement time of unit due to software
g(t) / G(t)	pdf / cdf of repair time of the hardware
m(t)/ M(t)	pdf / cdf of replacement time of the hardware
f(t) / F(t)	pdf / cdf of the time for PM of the unit
q_{ij} (t)/ Q_{ij}(t)	pdf / cdf of passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0, t]
pdf / cdf	Probability density function/ Cumulative density function
$q_{ij.kr}$ (t)/$Q_{ij.kr}$(t)	pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k, r once in (0, t]
μ_i(t)	Probability that the system up initially in state $S_i \in E$ is up at time t without visiting to any regenerative state
W_i(t)	Probability that the server is busy in the state S_i upto time 't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.
m_{ij}	Contribution to mean sojourn time (μ_i) in state S_i when system transit directly to state S_j so that $\mu_i = \sum_j m_{ij}$ and $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*\prime}(0)$
®/©	Symbol for Laplace-Stieltjes convolution/Laplace convolution
~ / *	Symbol for Laplace Steiltjes Transform (LST) / Laplace Transform (LT)

$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt$ as (1)

3. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{01}=\frac{\alpha_0}{A},\ p_{02}=\frac{a\lambda_1}{A},\ p_{03}=\frac{b\lambda_2}{A},\ p_{10}=f^*(A),\ p_{16}=\frac{a\lambda_1}{A}[1-f^*(A)]=p_{12.6},\ p_{18}=\frac{b\lambda_2}{A}[1-f^*(A)]=p_{13.8},\ p_{1.13}=\frac{\alpha_0}{A}[1-f^*(A)]=p_{11.13},\ p_{20}=g^*(B),\ p_{24}=\frac{\beta_0}{B}[1-g^*(B)],\ p_{25}=\frac{\alpha_0}{B}[1-g^*(B)],\ p_{2.11}=\frac{b\lambda_2}{B}[1-g^*(B)],\ p_{2.12}=\frac{a\lambda_1}{B}[1-g^*(B)],\ p_{30}=h^*(A),\ p_{37}=\frac{a\lambda_1}{A}[1-h^*(A)]=p_{32.7},\ p_{39}=\frac{\alpha_0}{A}[1-h^*(A)]=p_{3,1.9},\quad p_{40}=m^*(A),\ p_{3,10}=\frac{b\lambda_2}{A}[1-h^*(A)]=p_{33.10},\quad p_{52}=f^*(0), \tag{2}$$

$p_{4.16} = \frac{\alpha_0}{A}[1- m^*(A)]$, $p_{16,4} = f^*(0)$, $p_{72} = h^*(0)$, $p_{83} = f^*(0)$, $p_{93} = f^*(0)$, $p_{10.3} = h^*(0)$, $p_{11.3} = g^*(\beta_0)$, $p_{11.14} = 1- g^*(\beta_0)$, $p_{4,17} = \frac{b\lambda_2}{A}[1- m^*(A)] = p_{43.17}$, $p_{12.2} = g^*(\beta_0)$, $p_{12.15} = 1- g^*(\beta_0)$, $p_{13.1} = f^*(0)$, $p_{14.3} = m^*(0)$, $p_{4.18} = \frac{a\lambda_1}{A}[1- m^*(A)] = p_{42.18}$, $p_{15.2} = m^*(0)$, $p_{17.3} = m^*(0)$, $p_{18.2} = m^*(0)$, $p_{23.11} = \frac{b\lambda_2}{B}[1- g^*(B)][g^*(\beta_0)]$, $p_{23.11,14} = \frac{b\lambda_2}{B}[1- g^*(B)][1- g^*(\beta_0)]$, $p_{22.12} = \frac{a\lambda_1}{B}[1- g^*(B)]\, g^*(\beta_0)$, $p_{22.12,15} = \frac{a\lambda_1}{B}[1- g^*(B)][1- g^*(\beta_0)]$,

where $A= a\lambda_1+b\lambda_2+\alpha_0$ and $B= a\lambda_1+b\lambda_2+\alpha_0+\beta_0$.

It can be easily verified that

$$p_{01}+p_{02}+p_{03} = p_{10}+p_{16}+p_{18}+p_{1.13} = p_{20}+p_{24}+p_{25}+ p_{2,11}+p_{2.12} = p_{30}+p_{37}+p_{39}+p_{3,10} = p_{40}+p_{4.16}+p_{4.17}+ p_{4.18} = p_{52} = p_{72} = p_{83} = p_{91} = p_{10.3} = p_{11.3} + p_{11.14} = p_{12.2} + p_{12.15} = p_{13.1} = p_{14.1} = p_{15.2} = p_{16.4} = p_{17.3} = p_{18.2} = p_{10} + p_{1.6} + p_{11.13} + p_{13.8} = p_{20} + p_{24} + p_{25} + p_{23,11} + p_{23.11,14} + p_{22,12} + p_{22.12,15} = p_{30}+p_{31.9}+p_{32.7}+p_{33.10} = p_{40} + p_{4.16} + p_{42.18} + p_{43.17} = 1. \quad (3)$$

The mean sojourn times (μ_i) is the state S_i are

$$\mu_0 = \frac{1}{A}\ \mu_1 = \frac{1}{A+\alpha},\ \mu_2 = \frac{1}{\theta+B},\ \mu_3 = \frac{1}{A+\beta},\ \mu_4 = \frac{1}{A+\gamma},$$

$$\mu_4' = \frac{A\gamma^2+(a\lambda_1+b\lambda_2)\gamma^2+\gamma\alpha_0 A-(a\lambda_1+b\lambda_2)(A+\gamma)^2}{A\gamma(\gamma+A)^2},\ \mu_3' = \frac{1}{\beta},\ \mu_1' = \frac{1}{\alpha},\ \mu_5 = \frac{1}{\alpha},\ \mu_{16} = \frac{1}{\alpha} \quad (4)$$

$$\mu_2' = \frac{\begin{array}{l}(\theta+B)^3(\theta+\beta_0)^3+(a\lambda_1+b\lambda_2)B^2\{\theta(\theta+B)B+\theta(\theta+B)(\theta+\beta_0)-\theta\beta_0(\theta+\beta_0)\}+(a\lambda_1+b\lambda_2)(\theta+B)\\ \{(B)^3(\theta+\beta_0)^3+(B)^3(\theta+\beta_0)^3+(B)^3+(\theta+\beta_0)-\theta(\theta+B)(\theta+\beta_0)\}-\alpha_0(\theta+B)^2(\theta+\beta_0)^3B+\theta\alpha_0 B\\ (\theta+B)(\theta+\beta_0)-\theta B^2(\theta+\beta_0)^2\alpha_0\end{array}}{B^2(\theta+B)^2(\theta+\beta_0)^2}$$

Also

$$m_{01}+m_{02}+m_{03} = \mu_0 \qquad m_{10}+m_{16}+m_{18}+m_{1.13} = \mu_1$$
$$m_{20}+m_{24}+m_{25}+m_{2.11}+m_{2.12} = \mu_2 \qquad m_{40}+m_{4.17}+m_{4.18}+m_{4.16} = \mu_4$$
$$m_{51}+m_{5.16} = \mu_5 \quad m_{11.14}+m_{11.3} = \mu_{11} \quad m_{12.15}+m_{12.2} = \mu_{12} \quad m_{62} = \mu_6\ \ m_{72} = \mu_7,\ \ m_{83} = \mu_8, \quad (5)$$
$$m_{91} = \mu_9,\ m_{10.3} = \mu_{10}, \qquad m_{10}+m_{16}+m_{13.8}+m_{11.13} = \mu_1'\text{(say)}$$
$$m_{20}+m_{24}+m_{25}+m_{22.12}+m_{22.12,15}+m_{23.11}+m_{23.11,14} = \mu_2'\text{(say)}$$
$$m_{30}+m_{39}+m_{32.7}+m_{33.10} = \mu_3'\text{(say)}, \quad m_{40}+m_{42.18}+m_{43.17}+m_{4.16} = \mu_4'(say),$$

4. Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the c.d.f of first passage time from the regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relation for ϕ_i (t):

$$\phi_i(t)=\sum_j Q_{i,j}(t)®\phi_j(t)+\sum_k Q_{i,k}(t), \tag{6}$$

where *j* is an un-failed regenerative state to which the given regenerative state *i* can transit and *k* is a failed state to which the state *i* can transit directly.

Taking LST of Eq. (6) and solving for $\tilde{\phi}_0(s)$ yields,

$$R^*(s)=\frac{1-\tilde{\phi}_0(s)}{s}. \tag{7}$$

The reliability of the system model can be obtained by taking Laplace inverse transform of (7). The mean time to system failure (MTSF) is given by

$$\text{MTSF}=\lim_{s\to o}\frac{1-\tilde{\phi}_0(s)}{s} = \frac{N_1}{D_1} \quad \text{where} \tag{8}$$

$N_1 = \mu_0+p_{01}\mu_1+p_{02}\mu_2+p_{03}\mu_3+p_{24}p_{02}\mu_4$ and $D_1 = 1-p_{01}p_{10}-p_{02}p_{20}-p_{03}p_{30}-p_{02}p_{24}p_{40}$.

5. Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state i at t = 0. The recursive relations for $A_i(t)$ are given as

$$A_i(t)=M_i(t)+\sum_j q_{i,j}^{(n)}(t)©A_j(t), \tag{9}$$

where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions. $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$$M_0(t)=e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}, M_1(t)=e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{F(t)}, M_2(t)=e^{-(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)t}\overline{G(t)} \tag{10}$$
$$M_3(t)=e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{H(t)}, \quad M_4(t)=e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{M(t)}.$$

Taking LT of above relations (9) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty)=\lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2}, \tag{11}$$

where

$N_2=(-p_{24})\{\mu_0[(1-p_{11.13})(p_{12.6}p_{43.17}-p_{42.18}p_{13.8})+p_{12.6}p_{31.9}p_{43.17}-p_{13.8}p_{31.9}p_{43.18})]-\mu_1[p_{01}\{p_{32.7}p_{43.17}$
$+p_{42.18}(1-p_{11.13})\}+p_{31.9}p_{01}p_{43.17}-p_{03}p_{42.18}p_{31.9}]\}+\mu_3\{-p_{01}(p_{12.6}p_{43.17}-p_{42.18}p_{13.8})-(1-p_{11.13})p_{32.7}p_{02}+p_{42.18}p_{03}$
$(1-p_{11.13})\}-\mu_4[p_{01}\{(1-p_{33.10})p_{12.6}+p_{13.8}p_{32.7}\}+p_{02}\{(1-p_{11.13})(1-p_{33.10})-p_{13.8}p_{31.9}\}+p_{03}\{p_{32.7}(1-p_{11.13})+$
$p_{31.9}p_{12.6}\}]+(1-p_{4.16}p_{16.4})\{\mu_0[(1-p_{11.13})\{(1-p_{33.10})(1-p_{22.12}-p_{22.12,15}-p_{52}p_{25})-(p_{23.11}+p_{23.11,14})p_{32.7}\}-(p_{23.11}+$
$p_{23.11,14})p_{31.9}p_{12.6}-(1-p_{22.12}-p_{22.12,15}-p_{52}p_{25})p_{13.8}p_{31.9}]+\mu_1[p_{01}\{(1-p_{33.10})(1-p_{22.12}-p_{22.12,15}-p_{52}p_{25})-(p_{23.11}+$
$p_{23.11,14})p_{32.7}\}-(p_{23.11}+p_{23.11,14})p_{31.9}p_{12.6}-(1-p_{22.12}-p_{22.12,15}-p_{52}p_{25})p_{13.8}p_{31.9}]+(\mu_2)[p_{01}\{(1-p_{33.10})p_{12.6}$
$+p_{13.8}p_{32.7}\}+p_{02}\{(1-p_{11.13})(1-p_{33.10})-p_{13.8}p_{31.9}\}+p_{03}\{p_{32.7}(1-p_{11.13})+p_{31.9}p_{12.6}\}]+\mu_3[p_{01}\{p_{13.8}(1-p_{22.12}-$
$p_{22.12,15}-p_{52}p_{25})+(p_{23.11}+p_{23.11,14})p_{12.6}\}+(1-p_{11.13})(p_{23.11}+p_{23.11,14})p_{02}+(1-p_{22.12}-p_{22.12,15}-p_{52}p_{25})p_{03}(1-$
$p_{11.13})]\}$ and

$D_2=(-p_{24})\{\mu_0[(1-p_{11.13})(p_{12.6}p_{43.17}-p_{42.18}p_{13.8})+p_{12.6}p_{31.9}p_{43.17}-p_{13.8}p_{31.9}p_{43.18})]-\mu_1'[p_{01}\{p_{32.7}p_{43.17}$
$+p_{42.18}(1-p_{11.13})\}+p_{31.9}p_{01}p_{43.17}-p_{03}p_{42.18}p_{31.9}]\}+\mu_3'\{-p_{01}(p_{12.6}p_{43.17}-p_{42.18}p_{13.8})-(1-p_{11.13})p_{32.7}p_{02}+p_{42.18}p_{03}$
$(1-p_{11.13})\}-(\mu_4'+p_{4.16}\mu_{16})[p_{01}\{(1-p_{33.10})p_{12.6}+p_{13.8}p_{32.7}\}+p_{02}\{(1-p_{11.13})(1-p_{33.10})-$

$p_{13.8}p_{31.9}\}+p_{03}\{p_{32.7}(1-p_{11.13})+ p_{31.9}p_{12.6}\}]+(1-p_{4.16}p_{16.4})\{\mu_0[(1-p_{11.13})\{(1-p_{33.10})(1-p_{22.12}-p_{22.12,15}-p_{52}p_{25})-(p_{23.11}+p_{23.11,14})p_{32.7}\}-(p_{23.11}+p_{23.11,14})p_{31.9}p_{12.6}-(1-p_{22.12}-p_{22.12,15}-p_{52}p_{25})p_{13.8}p_{31.9}]+\mu_1'[p_{01}\{(1-p_{33.10})(1-p_{22.12}-p_{22.12,15}-p_{52}p_{25})-(p_{23.11}+p_{23.11,14})p_{32.7}\}-(p_{23.11}+p_{23.11,14})p_{31.9}p_{12.6}-(1-p_{22.12}-p_{22.12,15}-p_{52}p_{25})p_{13.8}p_{31.9}]+(\mu_2'+p_{25}\mu_5)[p_{01}\{(1-p_{33.10})p_{12.6}+p_{13.8}p_{32.7}\}+p_{02}\{(1-p_{11.13})(1-p_{33.10})-p_{13.8}p_{31.9}\}+p_{03}\{p_{32.7}(1-p_{11.13})+p_{31.9}p_{12.6}\}]+\mu_3'[p_{01}\{p_{13.8}(1-p_{22.12}-p_{22.12,15}-p_{52}p_{25})+(p_{23.11}+p_{23.11,14})p_{12.6}\}+(1-p_{11.13})(p_{23.11}+p_{23.11,14})p_{02}+(1-p_{22.12}-p_{22.12,15}-p_{52}p_{25})p_{03}(1-p_{11.13})]\}$.

6. Busy Period Analysis for Server

Let $B_i^P(t)$ $B_i^R(t)$ $B_i^S(t)$ *and* $B_i^{HRp}(t)$ be the probabilities that the server is busy in Preventive maintenance of the system, repairing the unit due to hardware failure, replacement of the software and hardware components at an instant 't' given that the system entered state i at t = 0. The recursive relations for $B_i^P(t)$ $B_i^R(t)$ $B_i^S(t)$ *and* $B_i^{HRp}(t)$ are as follows:

$$B_i^p(t)=W_i(t)+\sum_j q_{i,j}^{(n)}(t)©B_j^p(t), \qquad B_i^R(t)=W_i(t)+\sum_j q_{i,j}^{(n)}(t)©B_j^R(t),$$

$$B_i^S(t)=W_i(t)+\sum_j q_{i,j}^{(n)}(t)©B_j^S(t), \quad B_i^{HRp}(t)=W_i(t)+\sum_j q_{i,j}^{(n)}(t)©B_j^{HRp}(t), \tag{12}$$

where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions. *$W_i(t)$* be the probability that the server is busy in state S_i due to preventive maintenance, hardware and software failure up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_1=e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{F}(t)+(\alpha_0e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}© 1)\overline{F}(t)+(a\lambda_1e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}© 1)\overline{F}(t)+(b\lambda_2e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}© 1)\overline{F}(t),$$

$$W_2=e^{-(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)t}\overline{G}(t)+(\alpha_0e^{-(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)t})\overline{G}(t)+(a\lambda_1e^{-(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)t}© 1)\overline{G}(t)+(b\lambda_2e^{-(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)t}© 1)\overline{G}(t),$$

$$W_3=e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{H}(t)+(\alpha_0e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}© 1)\overline{H}(t)+(a\lambda_1e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}© 1)\overline{H}(t)+(b\lambda_2e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}© 1)\overline{H}(t),$$

$$W_4=e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{M}(t)+(\alpha_0e^{-(a\lambda_1+b\lambda_2+\alpha_0)t})\overline{M}(t)+(a\lambda_1e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}© 1)\overline{M}(t)+(b\lambda_2e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}© 1)\overline{M}(t), \quad W_5=\overline{F}(t)\, W_{16}=\overline{F}(t)\cdot$$

Taking LT of above relations (12) and solving for $B_i^P(t)$ $B_i^R(t)$ $B_i^S(t)$ *and* $B_i^{HRp}(t)$ the time for which server is busy due to PM, h/w repair and h/w and s/w replacements respectively is given by

$$B_0^H=\lim_{s\to 0} sB_0^{*H}(s) = \frac{N_3^H}{D_2}, \quad B_0^S=\lim_{s\to 0} sB_0^{*S}(s) = \frac{N_3^S}{D_2}, \quad B_0^R=\lim_{s\to 0} sB_0^{*R}(S)=\frac{N_S^R}{D_2} \quad \text{and} \tag{13}$$

$$B_0^{HRp}=\lim_{s\to 0} sB_0^{*HRp}(S)=\frac{N_S^{HRp}}{D_2}.$$

where

$N_3^P(t)=(p_{24})[W_1^*(0)[-p_{01}\{p_{32.7}p_{43.17}+p_{42.18}(1-p_{33.10})\}+p_{31.9}p_{02}p_{43.17}-p_{03}p_{42.18}p_{31.9}]+W_{16}^*(0)\ p_{4.16}[p_{01}\{(1-p_{33.10})p_{12.6}+p_{13.8}p_{32.7}\}+p_{02}\{(1-p_{11.13})(1-p_{33.10})-p_{13.8}p_{31.9}\}+p_{03}\{p_{32.7}(1-p_{11.13})+p_{31.9}p_{12.6}\}]]+(1-p_{4.16}p_{16.4})[W_1^*(0)\{p_{01}\{(1-p_{33.10})(1-p_{22.12}-p_{22.12,15}-p_{52}p_{25})-(p_{23.11}+p_{23.11,14})p_{32.7}\}+(p_{23.11}+p_{23.11,14})p_{02}p_{31.9}+p_{03}(1-p_{22.12}-p_{22.12,15}-p_{52}p_{25})p_{31.9}\}+W_5^*(0)\ p_{25}[p_{01}\{(1-p_{33.10})p_{12.6}+p_{13.8}p_{32.7}\}+p_{02}\{(1-p_{11.13})(1-p_{33.10})-p_{13.8}p_{31.9}\}+p_{03}\{p_{32.7}(1-p_{11.13})+p_{31.9}p_{12.6}\}]]$

$N_3^R(t)=W_2^*(0)[p_{01}\{(1-p_{33.10})p_{12.6}+p_{13.8}p_{32.7}\}+p_{02}\{(1-p_{11.13})(1-p_{33.10})-p_{13.8}p_{31.9}\}+p_{03}\{p_{32.7}(1-p_{11.13})+p_{31.9}p_{12.6}\}]$.

$N_3^S(t)=W_3^*(0)\{p_{24}[p_{01}(p_{12.6}p_{43.17}-p_{42.18}p_{13.8})+(1-p_{11.13})p_{32.7}p_{02}-p_{42.18}p_{03}(1-p_{11.13})]+(1-p_{4.16}p_{16.4})[p_{01}\{p_{13.8}(1-p_{22.12}-p_{22.12,15}-p_{52}p_{25})+(p_{23.11}+p_{23.11,14})p_{12.6}\}+(1-p_{11.13})(p_{23.11}+p_{23.11,14})p_{02}+(1-p_{22.12}-p_{22.12,15}-p_{52}p_{25})p_{03}(1-p_{11.13})]\}$

$N_3^{HRp}(t)=$ $p_{24}W_4^*(0)$ [p_{01}\{(1-$p_{33.10}$) $p_{12.6}$ +$p_{13.8}p_{32.7}$\}+ p_{02}\{(1-$p_{11.13}$) (1-$p_{33.10}$)-$p_{13.8}p_{31.9}$\}+p_{03}\{$p_{32.7}$(1-$p_{11.13}$)+ $p_{31.9}p_{12.6}$\}].

7. Expected Number of Replacements of the Units

Let $R_i^H(t)$ and $R_i^S(t)$ the expected number of replacements of the failed hardware and software components by the server in (0, t] given that the system entered the regenerative state i at t = 0.
The recursive relations for $R_i^H(t)$ and $R_i^S(t)$ are given as

$$R_i^H(t)=\sum_j Q_{i,j}^{(n)}(t)®\left[\delta_j+R_j^H(t)\right] \quad , \qquad R_i^S(t)=\sum_j Q_{i,j}^{(n)}(t)®\left[\delta_j+R_j^S(t)\right] \ , \tag{15}$$

where j is any regenerative state to which the given regenerative state *i* transits and δj=1, if *j* is the regenerative state where the server does job afresh, otherwise δj= 0. Taking LT of relations and, solving for $\tilde{R}_0^H(s)$ and $\tilde{R}_0^S(s)$. The expected numbers of replacements per unit time to the hardware and software failures are respectively of given by

$$R_0^H(\infty)=\lim_{s\to 0} s\tilde{R}_0^H(s) = \frac{N_4^H}{D_2} \text{ and } R_0^S(\infty)=\lim_{s\to 0} s\tilde{R}_0^S(s)= \frac{N_4^S}{D_2}, \tag{16}$$

where

$N_4^H(t)=$ (1-$p_{4.16}p_{16.4}$) ($p_{22,\ 12.15}$+ $p_{23,\ 11.14}$) [p_{01}\{(1-$p_{33.10}$) $p_{12.6}$ +$p_{13.8}p_{32.7}$\}+ p_{02}\{(1-$p_{11.13}$) (1-$p_{33.10}$)-$p_{13.8}p_{31.9}$\}+p_{03}\{$p_{32.7}$(1-$p_{11.13}$)+ $p_{31.9}p_{12.6}$\}]+(p_{40}+ $p_{42.18}$+ $p_{43.17}$) p_{24} [p_{01}\{(1-$p_{33.10}$) $p_{12.6}$ +$p_{13.8}p_{32.7}$\}+ p_{02}\{(1-$p_{11.13}$) (1-$p_{33.10}$)-$p_{13.8}p_{31.9}$\}+p_{03}\{$p_{32.7}$(1-$p_{11.13}$)+ $p_{31.9}p_{12.6}$\}]
$N_4^S(t)=$ p_{01}[$p_{12.6}$\{($p_{23.11}$+ $p_{23.11,14}$) (1-$p_{4.16}p_{16.4}$)+ $p_{24}p_{43.17}$\}+$p_{13.8}$\{(1-$p_{22.12}$-$p_{22.12,15}$-$p_{52}p_{25}$) (1-$p_{4.16}p_{16.4}$)-$p_{24}p_{42.18}$\}]+(1-$p_{11.13}$)[p_{02}\{($p_{23.11}$+ $p_{23.11,14}$) (1-$p_{4.16}p_{16.4}$)+ $p_{24}p_{43.17}$\}+p_{03}\{(1-$p_{22.12}$-$p_{22.12,15}$-$p_{52}p_{25}$) (1-$p_{4.16}p_{16.4}$)- $p_{24}p_{42.18}$\}] and D_2 is already mentioned.

8. Expected Number of Visits by the Server

Let $N_i(t)$ be the expected number of visits by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $N_i(t)$ are given as

$$N_i(t)=\sum_j Q_{i,j}^{(n)}(t)®\left[\delta_j+N_j(t)\right] \ , \tag{17}$$

where *j* is any regenerative state to which the given regenerative state *i* transits and δj=1, if *j* is the regenerative state where the server does job afresh, otherwise δj= 0. Taking LT of relation (20) and solving for $\tilde{N}_0(s)$. The expected number of visit per unit time by the server are given by

$$N_0(\infty)=\lim_{s\to 0} s\tilde{N}_0(s)= \frac{N_5}{D_2}, \tag{18}$$

where
N_5 ==(-p_{24})\{ (1-$p_{11.13}$) [$p_{32.7}p_{43.17}$+$p_{42.18}$ (1-$p_{33.10}$)]+$p_{12.6}p_{31.9}p_{43.17}$ –$p_{13.8}$ $p_{31.9}p_{42.18}$)\}+ (1-$p_{4.16}p_{16.4}$)\{ (1-$p_{11.13}$)[(1-$p_{33.10}$)(1-$p_{22.12}$-$p_{22.12,15}$-$p_{52}p_{25}$)-($p_{23.11}$+ $p_{23.11,14}$)$p_{32.7}$\}-$p_{12.6}$($p_{23.11}$+ $p_{23.11,14}$)$p_{31.9}$ -(1-$p_{22.12}$-$p_{22.12,15}$-$p_{52}p_{25}$) $p_{13.8}p_{13.8}$]\}

9. Cost-Benefit Analysis

The profit incurred to the system model in steady state can be obtained as

$$P=K_0A_0-K_1B_0^P-K_2B_0^R-K_3B_0^S-K_4B_0^{HRp}-K_5R_0^H-K_6R_0^S-K_7N_0 \tag{19}$$

K_0 = Revenue per unit up-time of the system
K_1 = Cost per unit time for which server is busy due preventive maintenance
K_2 = Cost per unit time for which server is busy due to hardware failure
K_3 = Cost per unit replacement of the failed software component
K_4 = Cost per unit replacement of the failed hardware component
K_5 = Cost per unit replacement of the failed hardware
K_6 =. Cost per unit replacement of the failed software
K_7 = Cost per unit visit by the server

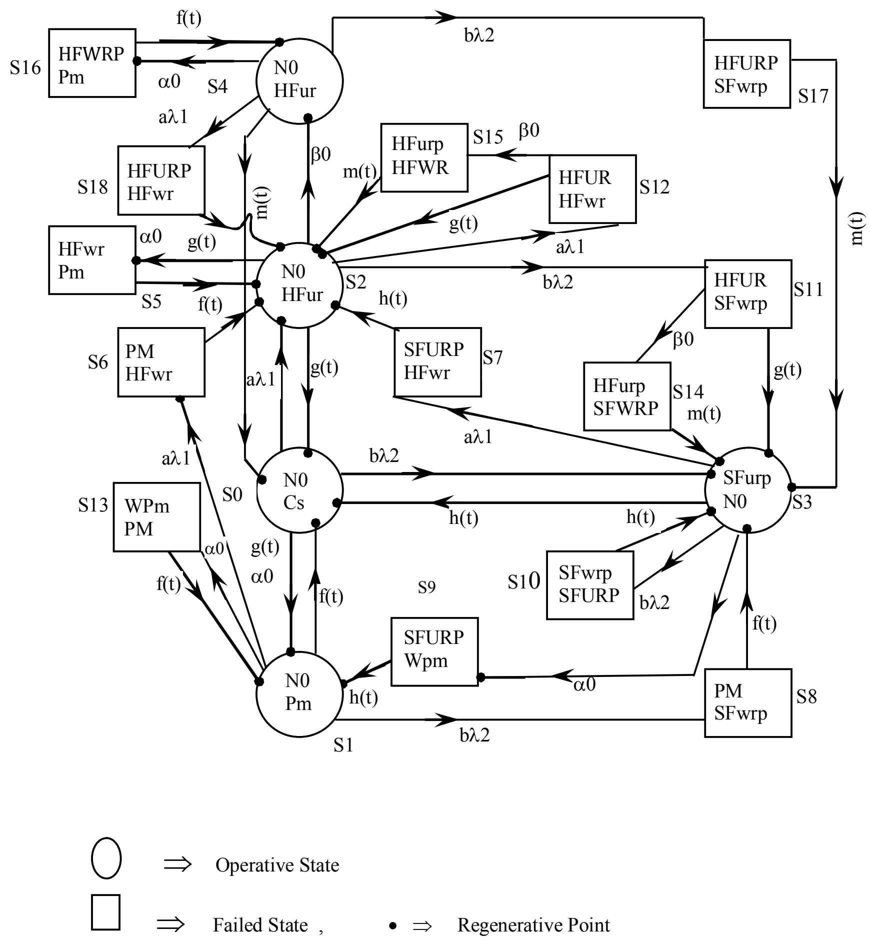

Fig. 1. State Transition Diagram

10. Conclusion

For the particular case, $g(t) = \theta e^{-\theta t}$, $h(t) = \beta e^{-\beta t}$, $f(t) = \alpha e^{-\alpha t}$ and $m(t) = \gamma e^{-\gamma t}$, the graphs for mean time to system failure (MTSF), availability and profit are drawn with respect to preventive maintenance rate (α) for fixed values of other parameters as shown respectively in Figs. 2 to 4. These figures indicate that MTSF, Availability and profit increase with the increase of PM rate (α) and repair rate (θ) of the hardware components. But the value of these measures decrease with the increase of maximum operation time (α_0). However, if we increase maximum constant rate of repair time (β_0), then the value of MTSF increases while availability and profit follow a decline trend. It is also observed that availability and profit decrease by interchanging the values of a and b i.e. a=.3 and b=.7. Hence, it is suggested that a computer system of two identical- units having independent failure of h/w and s/w components can be made more profitable

(i) By reducing the maximum repair time of the h/w components.
(ii) Making replacement of the hardware components by new one in case repair time is too long.
(iii) By controlling the failure rate of the software.

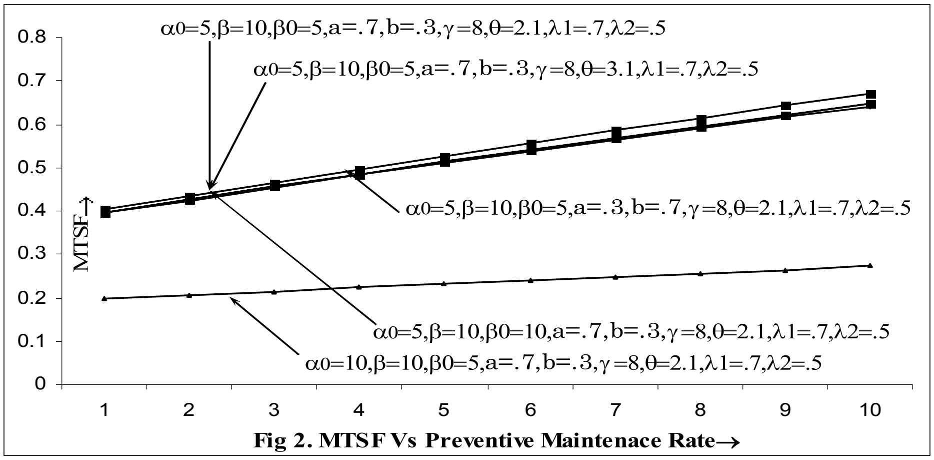

Fig. 2. MTSF Vs. Preventive Maintenance Rate

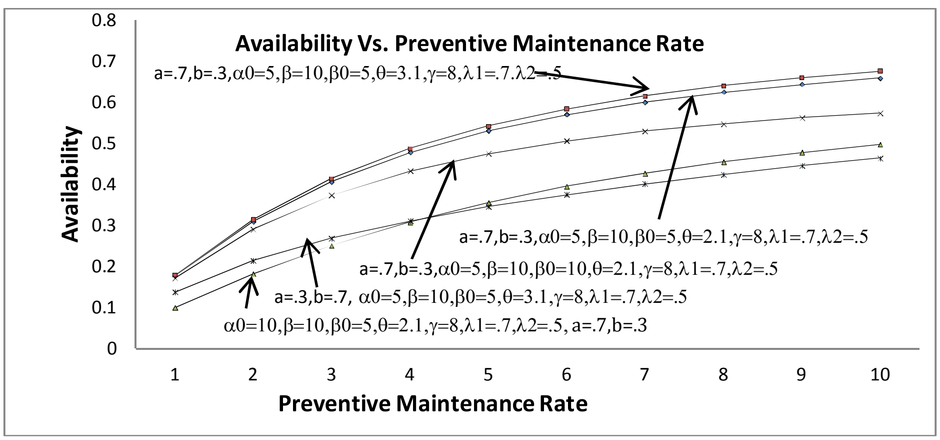

Fig. 3. Availability Vs. Preventive Maintenance Rate

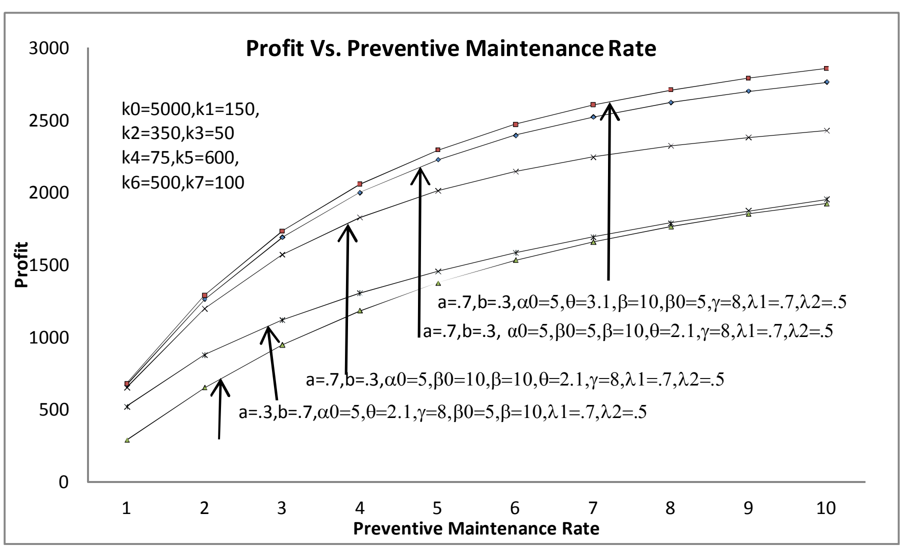

Fig.4. Profit Vs. Preventive Maintenance Rate

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