Management Science Letters 4 (2014) 899–904

Contents lists available at GrowingScience

Management Science Letters

homepage: www.GrowingScience.com/msl

An application of Markowitz theorem on Tehran Stock Exchange

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CHRONICLE

Article history:
Received December 28, 2013
Accepted 24 March 2014
Available online
March 29 2014

Keywords:
Tehran Stock Exchange
Investment
Markowitz Theorem
Cardinality constraint

ABSTRACT

During the past 65 years, there have been tremendous efforts on portfolio selection problem. The standard Markowitz mean–variance model to portfolio selection includes tracing out an efficient frontier, a continuous curve demonstrating the tradeoff between return and risk. This frontier can be often detected via standard quadratic programming, categorized in convex optimization. Traditional Markowitz problem has been recently extended into a new form of mixed integer nonlinear problems by considering various constraints such as cardinality constraints, industry limitation, etc. This paper proposes a mixed integer nonlinear programming to determine optimal asset allocation on Tehran Stock Exchange. The results have indicated that a petrochemical firm named Farabi has gained 44% of the portfolio followed by a drug firm named Kosar Pharmacy gaining 28%. In addition, banking sector was the third winning firm where Eghtesad Novin bank gained nearly 10% of the portfolio. Minerals and mining firms were the next sector in our portfolio where Gol Gohar Iron Ore and Tehran Cement collected 0.73% and 0.57% of the portfolio, respectively. In our survey, auto industry gained only 0.26% of the portfolio, which belonged to Saipa group.

1. Introduction

For years, Markowitz theorem (Markowitz, 1952, 1970) has been widely used to determine optimal investment strategies. The theory has been well studies under various conditions (Fabozzi et al., 2007). The standard Markowitz mean–variance model to portfolio selection includes tracing out an efficient frontier, a continuous curve demonstrating the tradeoff between return and risk. This frontier can be often detected via standard quadratic programming, categorized in convex optimization. Chang et al. (2000) considered the problem of locating the efficient frontier associated with the standard mean–variance portfolio optimization model. They extended the original model by considering cardinality constraints, which limited a portfolio to be limited to a specified number of assets, and to consider limits on the proportion of the portfolio held in a given asset. They also showed the differences arising in the shape of this efficient frontier when such constraints imposed

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doi: 10.5267/j.msl.2014.3.027

and solved the resulted model using three heuristic algorithms based upon genetic algorithms, tabu search and simulated annealing for locating the cardinality constrained efficient frontier.

Streichert et al. (2004) also solved the same portfolio optimization problem using evolutionary algorithms by considering the cardinality constrained. Maringer and Kellerer (2003) considered the same optimization of cardinality constrained portfolios with a hybrid local search algorithm. Soleimani et al. (2009) extended the problem by adding three options to the original model, which would lead Markowitz's model to a more practical one. They considered the minimum transaction lots, cardinality constraints and sector capitalization, which was proposed in this research for the first time as a constraint for Markowitz model. The explained that the new model could be formulated as an Np-Hard problem and they proposed a genetic algorithm to solve the resulted model. Branke et al. (2009) proposed to combined an active set algorithm optimized for portfolio selection into a multi-objective evolutionary algorithm (MOEA). The idea was to let the MOEA come up with some convex subsets of the set of all possible portfolios, solve a critical line algorithm for each subset, and then merge the partial solutions into a solution of the original non-convex problem. They showed that the resulting envelope-based MOEA substantially outperforms existing MOEAs. Anagnostopoulos and Mamanis (2010) considered the portfolio optimization model with three objectives and discrete variables. Skolpadungket et al. (2007) applied different techniques of multi-objective genetic algorithms to solve portfolio optimization by considering some realistic constraints, namely cardinality constraints, floor constraints and round-lot constraints. Fernández and Gómez (2007) considered the same portfolio selection using neural networks.

2. The proposed study

In this paper, we proposed an extended Markowitz model by considering different real-world limits on the original cardinality model including bound constraints, sector limitation, etc.

2.1. Variables and notations

X_i: is the number of shares purchased from the share i,
Z_i: is the binary variable from the share i, if selected is equal to one and zero, otherwise
Y_j: j industry binary variables, if selected is equal to one, zero, otherwise
W_i: is weight of i share in portfolio
g_i: is the weight of j industry in portfolio
i: stock index
j: Industry Index
Com: The fixed fee deals
N: number of selected stocks
σ: covariance between industry or stock
P_{i}: i free float shares
F: The percentage of minimum commission rate of buying shares
C_i: the price of selective stock
φ: volume of transactions
P: Minimum number of free floating shares
R: optimal return level
r_i: rate of expected return
M: A big number
IN: Total amount of investment
S: Total number of industries
B_{upperi} : The maximum amount of investment in the share of the i
B_{loweri} : The minimum investment amount in the share of the i

2.2. Mathematical model

The mathematical model is formulated as follows,

$$Z=\min \sum_{i=1}^{N} \sum_{j=1}^{n} w_i w_j \sigma_{ij} + \sum_{i=1}^{n} \sum_{j=1}^{n} g_j g_x \sigma_{jx} \quad (1)$$

subject to

$$\sum_{i=1}^{n} x_i c_i v_i \geq (IN - com)R, \quad (2)$$

$$\sum_{i=1}^{n} x_i c_i \leq IN - com \quad (3)$$

$$w_i = \frac{C_i X_i}{\sum_{i=1}^{N} X_i C_i}, i = (1,2,\ldots,N) \quad (4)$$

$$B_{lower_i} - M(1-z_i) \leq x_i c_i \leq B_{upper_i} + M(1-z_i), \quad i = (1,2,\ldots,N) \quad (5)$$

$$\sum_{i}^{n} w_i p_i \geq p \quad (6)$$

$$f \sum_{i=1}^{n} x_i c_i - com \geq = \leq 0 \quad (7)$$

$$\frac{x_i}{M} \leq z_i \leq x_i i = (1,2,\ldots N) \quad (8)$$

$$\frac{\sum_{i \in j} z_i}{M} \leq y_j \leq M \sum_{i \in j} z_i, j = (1,2,\ldots,s) \quad (9)$$

$$\sum_{i \in |j|} w_i + (1-y_i) \geq \sum_{i \in |j+1|} w_i, j = (1,2,\ldots,s-1) \quad (10)$$

$$\sum_{i=1}^{n} x_i \geq = \leq \varphi \quad (11)$$

$$g_j = \sum_{i=j} w_i \quad (12)$$

The proposed model determines the amount of shares invested in each firm. In addition, parameters include monthly stock returns, monthly returns of covariance and industry are between returns of stock and industry and finally limitations include the budget, expected returns, volume of transactions, etc. The objective function minimizes the expected return by considering budget constrain. For more details, please see Chang et al. (2000), Branke et al. (2009) and Soleimani et al. (2009). The proposed model has been applied on monthly information gathered from Tehran Stock Exchange by considering Covariance between stock returns and mentioned industries, budget, investor optimum efficiency, free float stock, etc. The proposed model has been investigated in four different stages.

The first stage: Stocks returns of selected research companies were collected in three-year timeframe and Covariance of stock returns were calculated by Excel software.

The second stage: Returns of selected research industries were collected in three-year timeframe and covariance between industry returns was calculated by Excel software.

The third stage: Return of per share for a period of 3 years "36 months" has been calculated and its arithmetic mean has been used as the coefficients in limitations of the model. Accordingly, the return of each company has been calculated over a period of 3 years with taken into account.

The fourth stage: Based on information obtained, limitations of the model were defined including range of the asset, the minimum and the maximum choice of industry, transaction costs of buying shares, free float stock restrictions, etc.

3. The results

In this section, we present details of our findings on the implementation of the proposed model in four different scenarios. Table 1 shows details of our findings.

Table 1
The summary of the results of our survey

Firm\industry	Transaction volume	The relative weight	The relative weight of industry
Khsapa (Automotive)	0.879032	0.026904	0.026904
Dkosar (Drug)	4.963364	0.281042	0.304890
Dalbr (Drug)	2.455367	0.023848	0.304890
Stran (cement)	1.453312	0.057312	0.057312
Shfara (NPC)	5.288052	0.443267	0.443267
Vnovyn (Banking)	3.254099	0.104431	0.104431
Non-metallic mineral	0.767893	0.073195	0.073195
Sum	19.06	1.000000	1.000000

As we can observe from the results of Table 1, during the time schedule of the study, drug industry has been the most attractive industry on Tehran Stock Exchange followed by petroleum industry, banking, non-metal as well as cements industry. We have performed sensitivity analysis on the proposed study under four different scenarios and Table 2 shows details of our findings.

4. Discussion and conclusion

In order to understand the behavior of the proposed model, we have applied it under four different stages. To implement this model in the first stage, collection of performance data was accomplished by RAH'AVARD NOVIN software. In the second stage, we have calculated the covariance between industry and stock. These values were used as the objective function coefficients of the decision variables. During the third stage, expected return measures of stock and industry, investment restrictions, the minimum and maximum choice of industry, transaction costs of buying stock and free float stock restrictions were calculated as constraints and parameters of the model. Then the values and parameters were used in the model. The results model has been coded in a commercial optimization software package and it was solved and its optimal solution obtained is as follows.

In our survey, a petrochemical firm named Farabi has gained 44% of the portfolio followed by a drug firm named Kosar Pharmacy gaining 28%. In addition, banking sector was the third winning firm where Eghtesad Novin bank gained nearly 10% of the portfolio. Minerals and mining firms were the next sector in our portfolio where Gol Gohar Iron Ore and Tehran Cement collected 0.73% and

0.57% of the portfolio, respectively. In our survey, auto industry gained only 0.26% of the portfolio, which belonged to Saipa group. We have discussed the results of the proposed model with some experts who were involved in Tehran Stock Exchange and they confirmed our survey result.

Table 2
The summary of portfolio optimization under four different scenarios

Description of variables	First stage	Second stage	Third stage	Fourth stage
Percentage of utility return	0.2	0.1	0.3	0.5
Percentage of free float stock	0.2	0.1	0.3	0.2
Objective function value	-0.1159559	-0.1191729	-0.1166649	-0.1170700
W1	0.000000	0.000000	0.000000	0.000000
W2	0.026904	0.028835	0.030754	0.029999
W3	0.023848	0.468473	0.123848	0.353848
W4	0.281042	0.238483	0.529411	0.381978
W5	0.000000	0.000000	0.000000	0.000000
W6	0.057312	0.061317	0.059116	0.049886
W7	0.000000	0.000000	0.000000	0.000000
W8	0.524524	0.507678	0.470588	0.526678
W9	0.000000	0.000000	0.000000	0.000000
W10	0.194432	0.131431	0.105775	0.104431
W11	0.000000	0.000000	0.000000	0.000000
W12	0.073195	0.068695	0.077996	0.070991
G1	0.026904	0.028835	0.030754	0.029999
G2	0.304890	0.4923220	0.529411	0.731254
G3	0.057312	0.061317	0.059116	0.049886
G4	0.443267	0.5076780	0.470588	0.526678
G5	0.104431	0.131431	0.105775	0.104431
G6	0.073195	0.068695	0.077996	0.070991
K1	0.000000	0.000000	0.1534477E-08	0.000000
K2	0.879032	1.003055	0.923632	0.899731
D1	2.455367	0.3377640E-01	0.5598630E-01	2.657898
D2	0.4550486E-01	0.8019910E-02	0.5627297E-01	0.6528135E-04
S1	0.000000	0.000000	0.000000	0.000000
S2	1.453312	1.754219	2.253411	1.894567
P1	0.000000	0.000000	0.000000	0.000000
P2	0.9048380E-01	0.1818947	0.5329260E-01	0.6955185E-01
B1	0.000000	0.000000	0.000000	0.000000
B2	0.2885965E-01	2.554125	3.574229	3.754111
V1	0.000000	0.000000	0.000000	0.000000
V2	0.767893	0.794894	0.729955	0.827893
IN	456.6111	941.2463	297.5072	365.4352
COM	5.679742	4.682818	4.480136	3.1818085
FINAL	0.6444784E-02	0.6808126E-02	0.5161765E-02	0.5520000E-02

Acknowledgement

The authors would like to thank the anonymous referees for constructive comments on earlier version of this paper.

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