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Technical and economic efficiency measurement of African commercial banks using data envelopment analysis (DEA)

Evans Darko[a*], Nadia Saghi-Zedek[a] and Gervais Thenet[a]

[a]*Université de Rennes, CNRS, CREM - UMR 6211, F-35000 Rennes, France*

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A B S T R A C T

The paper aims to analyze the Technical Efficiency of 70 Commercial banks from 19 African countries from 2009-2020. Using the Data Envelopment Analysis (DEA) method of the two main approaches, Variable Return to Scale (VRS) and Constant Return to Scale (CRS) technique on a Panel Data. We find that African banks have a higher efficacy assessment with the VRS than the CRS technique, thus, with a Pure Technical Efficiency (PTE) score than Technical Efficiency (TE) . Our findings show that the majority of the banks are operating at very low levels of efficiency (not technically efficient), and inability to optimize the conversion of bank assets and liabilities into loan production for customers. Furthermore, the banks are operating inefficiently in scale, economic, and allocative manner due to mismatches in scale of production. Considering these findings, the implications of these inefficiencies extend to the overall economic development and financial stability of the region.

1. Introduction

The banking industry is an important player in every country's socio-economic and sustainable development such as the provision of financial services. Hence, evaluation and assigning performance metrics of banks are relevant in alignment with national policy landscapes. Traditionally, the overall performance of commercial banks is measured using ratios assessment. However, some studies have pointed out challenges using ratios assessment as a performance measure, Stainer (1997) asserted fundamental problems in the computation of the ratios due to external factors and non-associations to efficient resources utilized. Whilst Yeh (1996) argues that the reliance on standard ratios could be a misleading venture, Sherman and Gold (1985) argue that these ratios do not take into account long-term performance and other characteristics notably operations, marketing, financing, etc. The actual measurement of a commercial bank's performance is often expressed in its Efficiency levels, the efficiency is defined as the difference between observed input and output levels that correspond to their optimal values (Wheelock & Wilson, 1999). Again, Rao and Lakew (2012) contended that ratio assessment often misleads in measuring efficiency partially.

The efficiency of the African banking system is very crucial for her financial market development and also presents potential growth as an emerging market that requires stability to stimulate. For example, in terms of banking, Africa is the second most profitable region after Latin America with a return on equity (ROE) of 14.9% compared to a 9% global average (McKinsey Report, 2018), and Flamini et al. (2009) find bank profits are high in Sub-Saharan Africa (SSA) compared to other regions irrespective of the profit measure used. Concerning retail banking, Africa's retail banking penetration stands at 38% of the

* Corresponding author. Tel.: +33 49 18 63 13

E-mail address: evans.darko@univ-rennes1.fr (E. Darko)

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continent's GDP, with a projected 70% of the growth in Africa's retail banking revenue by 2025 (McKinsey Report,2018). Moreover, the Mobile banking revolution in sub-Saharan Africa accounts for 21% of the adult market (World Bank Report). In addition, since the efficient banking industry offers stabilization tools and also fosters the effectiveness of national monetary policy (Yilmaz, 2013). Consequently, the efficiency scores of commercial banks are a barometer of bank performance and the entire banking system and could be used to scrutinize the potential effects of government policies on efficiency (Wheelock and Wilson, 1999).

The multifaceted complexities one encounters in evaluating firm performance due to the dynamics of both the internal and external factors such as profitability, insolvency, risk management, organizational ownership, liquidity, asset quality, etc using the ratio assessment requires a different approach. To manage the challenges posed by using the ratio analysis in bank efficiency assessment. In recent decades, frontier analysis methods have dominated contemporary studies, since they offer accurate technical information about banks or organization performance when measuring efficiency levels, and they take into account factors related to firm productivity.

Most recent studies that focused on the frontier analysis approach for efficiency assessment in the banking industry and the application of Data Envelopment Analysis(DEA) methodology for performance analysis are, Clara et al. (2024) for Angolan commercial banks, Olohunlana et al. (2023) for Nigerian commercial banks, Li (2020) for Chinese commercial banks, Vilaça et al. (2019) for Portugal banks, Henriques et al. (2018) also analyzed Brazilian banking, Triki et al. (2016) for African banks, Gamachis Garamu (2016) worked on Ethiopian Commercial Banks, Gahé et al. (2016) examined banking sector of Côte d'Ivoire. Other previous studies that applied DEA methods are; Schaffnit et al. (1997) worked on a Canadian bank, and Sherman and Ladino (1995) on a US bank, while Vassiloglou, Giokas (1990, 1991) worked on a Greek bank.

Our study focused on the African banking context because according to empirical evidence, it is a region that is more profitable than other regions in the world and at the same time the only region that has not experienced any banking crisis in recent decades but suffers most economically in any global crisis. Therefore, it is prudent that factors that ensure banking stability and foster performance are keen to empirical research.

The study is mainly motivated by, first, [1]inadequate studies examining bank Technical efficiency assessment in the whole region. Because existing literature reveals that recent studies are mostly country-level focused. For instance, Olohunlana et al. (2023) studied the technical efficiency of Nigerian commercial banks, Gahé et al. (2016) focused on Technical efficiency assessment of the banking sector of Côte d'Ivoire and Gamachis Garamu (2016) technical Efficiency and Productivity of Ethiopian Commercial Banks. Secondly, it is a less developed banking and financial system, has inadequate economic integration, and lacks common banking regulation and supervisory frameworks. These factors contribute to increased effects on bank performance and efficiency, particularly in cross-border banking operations within the sub- region and continent. This study holds significance as Africa envisions to establish a resilient and stable banking and financial system. In the quest to doing so, developing a consolidated financial system, economic integration, and minimizing the spillover effects of global financial system volatility, pandemics, and dynamics of geopolitics is very relevant, for trade harmonization among member states of the continent, especially the upcoming African Continental Free Trade Area (AfCFTA) Agenda. Therefore, determining the bank's technical efficiency will not only enhance the robustness of the banking industry but will facilitate bank resource allocation and fund mobilization in the region. Finally, when there are efficient financial intermediaries, it attracts savings from various sources for an onward allocation into productive activities and sectors that bring benefits to all the investment players and the entire economy (Gulde et al., 2006). The most powerful tool for economic growth is when a banking system efficiently channels its financial resources into productive use (Levine, 1997).

The main purpose of this study is to assess banks' technical efficiency in the African banking industry using Data Envelopment Analysis under the non-parametric approach. Furthermore, we attempt to scrutinize the factors that explain bank performance(output) assessment and to ascertain whether African banks operate effectively and efficiently (intermediation role) through economic (EE) and allocative efficiency(AE) as well. Whether banks are efficiently converting their inputs into better output(loans), thus, in our case, the conversion of available banks' assets and liabilities into loan production for customers. Using the banks' total deposits, net income, and the number of employees) as input, and to produce total loans (output).

To the best of our knowledge, there are no studies in the African banking context on the above-mentioned reasons and the scope that inspired this paper. Our paper presents three novel contributions. Firstly, it introduces a new model to assess a bank's lending capacity through loan production and modified the Banker-Charnes- Cooper,1948 (BCC) and Charnes, Cooper, and Rhodes,1978 (CCR) equation, by incorporating variables such as balance sheet net income, total deposits, and human resources(bank employees). This approach is distinct from previous studies such as Pires et al. (2023) and Gahé et al. (2016). Secondly, our study advances by evaluating the economic and allocative efficiency of banks in Africa, a unique perspective that integrates technical, economic, and allocative efficiency measures simultaneously. Hence, our research distinguishes between pure and scale inefficiencies of Decision-Making Units (DMUs) in the African banking context. Lastly, our findings

[1] Inadequate studies examining Technical Efficiency of African commercial banks on a regional level in recent decades.

offer new insights on how banks could combine certain assets and liabilities to determine efficiency levels to enhance the technical and operational efficiency of banks, suggesting strategies for optimizing resource utilization and improving DMU performance through unit structural adjustments.

The rest of the paper proceeds as follows. In section 2, present the literature review. In section 3, we introduce the methodology. Section 4, data and model, section 5. Results and discussion. Section 6, concludes the paper.

2. Literature review

The application of Data Envelopment Analysis (DEA) models has emerged as a pivotal method for assessing the efficiency of decision-making units (DMUs) such as banks, by examining their input-output relationships. Within this framework, two primary orientation models are frequently employed: the input-oriented Constant Returns to Scale (CRS) model and the Variable Returns to Scale (VRS) model, each offering distinct perspectives on efficiency assessment. The input-oriented CRS model, proposed by the Charnes, Cooper, and Rhodes (CCR) model, aims to evaluate the relative efficiency of DMUs while maintaining input levels constant. On the other hand, [2]the input-oriented VRS model, often referred to as the Banker-Charnes-Cooper (BCC) model, accommodates variable returns to scale by allowing input levels to fluctuate.

A DMU of a firm can be classified as being at a technical efficiency level, if the firm can realize maximum outputs with a minimum set of inputs at any given time (Atkinson and Cornwell, 1994). Assessing each DMU's degree of efficiency could be inferred as an increasing production level with the same input without utilizing additional resources (Farell,1957). In addition, from the profit maximization viewpoint, firms are more inclined to use their resources in an economically efficient manner to maximize production output with little resources and, more importantly, to tailor unit decisions based on cost and benefit analysis, of the technical and allocative efficiency theory (Sullivan et al., 2003). Based on the return to scale concept, the assessment of firms' technical efficiency can be further classified into Pure Technical Efficiency (PTE) and Scale Technical Efficiency (STE). PTE evaluates how production departments or units manage their resources, while STE ensures that production units operate at an optimal scale. However, the resource-based theory highlights the significance of developing internal resources for optimal firm performance (Anifowose et al., 2018; Bayraktaroglu et al., 2019; Isola et al., 2020).

For an extensive investigation into the input-output relationships of Technical Efficiency, two main approaches are often considered. The "Input-oriented" approach assesses the production unit's capacity to achieve certain output levels with minimal input quantities. Essentially, it quantifies the level of inputs that can be proportionally reduced without affecting output quantities (Coelli et al., 2005). Conversely, the "Output-oriented" approach examines the production unit's ability to maximize output with a given quantity of inputs and production technology. It explores how output quantities can be changed without altering input quantities (Kamgna and Dimou, 2008). Simply, technical inefficiency arises either from producing below the technically feasible level with a given input quantity and technology or from utilizing input quantities beyond what is necessary for a given output level. Then, there's a need to consider the type of techniques being applied to the chosen orientation ("input or output") . The two prominent techniques that are commonly employed to measure the technical efficiency of firms, including banks.

Firstly, the parametric approach, introduced by researchers such as (Cobb Douglas and Translog), entails approximating the effective production function using a predefined functional form. This method offers a mathematical equation that outlines the efficient frontier, independent of the dataset. Consequently, it allows for easier specification and enhanced analysis of the various algebraic properties of this function. These techniques can adopt either a deterministic or stochastic stance. Deterministic approaches attribute any deviation from the frontier to inefficiency, while stochastic approaches attribute such deviations to inherent inefficiency. However, they are easily influenced by certain hazards and measurement errors. Due to this, various authors, including Farrell (1957), Timmer (1971), Afriat (1972), and Richmond (1974), have proposed different techniques aimed at approximating the efficient frontier based on deterministic methods. However, both parametric and deterministic approaches have their limitations, particularly their strong sensitivity to extreme observations and the restrictive nature of the functional form assigned to the frontier function. Moreover, the stochastic parametric approach primarily addresses some of the limitations of the deterministic approach by providing insight into the origin of deviations from the efficient frontier. According to Amara and Romain (2000), this method proposes that the error term consists of two independent components: a purely random component distributed on each side of the production frontier (two-sided error term), and a component representing technical efficiency distributed on one side of the frontier (one-sided error term). In both scenarios, utilizing parametric approaches is often not feasible because it necessitates the formulation of a cost or profit function for the firm under review, which may not always be practicable in several types of businesses in today's world. Therefore, the non-parametric approach serves as an alternative to address these challenges. Secondly, the non-parametric approach involves examining a frontier that is not tied to any specific functional form: the isoquant is estimated by the ratio of outputs to inputs of each Decision-Making Unit (DMU). This approach is typically deterministic in nature. The method entails placing all DMUs in a sample and representing each of their performances by a point on a graph. Subsequently, an efficient frontier is drawn. In the case of DEA, this frontier connects all points that envelop the cloud of points from the top. The points situated on this frontier represent efficient units, while those below it are considered "ineffective" or "under- effective" units. Furthermore, the distance of each

[2] VRS Model proposed by Banker-Charnes-Cooper (BCC), accommodates variable returns to scale by allowing input levels to fluctuate

point from the frontier serves as a measure of its technical efficiency level. It's important to note that this efficiency is relative, as it depends on the most efficient units within the sample. Under the non-parametric approach, DEA is considered a prime example, featuring two main models: Constant Return to Scale (CRS) and Variable Return to Scale (VRS). A robust measure of scale efficiency for a firm involves calculating the difference between the technical efficiency ratios obtained through the DEA-CRS and DEA-VRS methods for the same firm, as proposed by Coelli et al. (2005). To obtain this measure, both DEA-CRS and DEA-VRS methods should be estimated using the same programming or statistical software. For instance, if there is a variance in efficiency ratios estimated under the two DEA methods (CRS and VRS) for a specific firm, it indicates that the firm is not operating at an optimal scale. Essentially, scale inefficiency is defined as the disparity between CRS technical inefficiency and VRS technical inefficiency. This concept allows for further study and analysis of a firm's efficiency or inefficiency levels. According to Amara and Romain (2000), the popularity of the DEA method has expanded the analysis of technical efficiency to encompass multi-products and situations with non-constant returns to scale, including within the banking sector.

Reviewing recent studies employing DEA methods in the banking industry reveals significant insights. Clara et al. (2024) utilized the DEA approach to examine the determinants of efficiency in the Angolan banking sector from 2014 to 2019, employing the CCM under an input-oriented framework. Their findings showed the statistical significance of the Solvency ratio, the relationship between liabilities and equity, and return on equity in Angolan banking efficiency. Similarly, Li et al. (2020) analyzed the efficiency of 101 commercial banks in China from 2015 to 2017 using the bootstrapped DEA approach. Additionally, Vilaça et al. (2019) investigated the determinants of Portuguese bank efficiency, focusing on the DEA, Charnes, Cooper, and Rhodes (CCR), and Banker, Charnes, and Cooper (BCC) models from the first half of 2005 to the first half of 2017. They identified size, capital adequacy, seniority, and the country's macroeconomic situation as influential factors. Henriques et al. (2018) examined Brazilian banking efficiency from 2012 to 2016 using DEA, analyzing a dataset of 37 Brazilian banks provided by the Central Bank of Brazil. Their evaluation using the two classic DEA models revealed higher efficiency levels in the BCC model compared to the CCR model. Hence, their findings challenged the notion that larger banks are inherently more efficient.

Furthermore, several studies have specifically focused on specific African banking industry utilizing the DEA method. For instance, Olohunlana et al. (2023) employed the DEA approach to analyze the intellectual capital efficiency of Nigerian listed banks. Their findings revealed that only 8.33% of the sampled Nigerian commercial banks operate at optimum capacity in utilizing their intellectual capital, while 91.67% are inefficient. The study also identified that bank size and directors' shareholdings positively impact intellectual capital efficiency, whereas market and ownership concentration hinder the attainment of optimum intellectual capital efficiency. Similarly, Gamachis Garamu (2016) investigated the Technical Efficiency and Productivity of Ethiopian Commercial Banks using the DEA approach. The study aimed to examine the relative technical efficiency and productivity change of the commercial banks during the study period. Their results indicated that, on average, Ethiopian commercial banks were relatively technically inefficient, with scale inefficiency being the leading source of inefficiency. The study also revealed an average Total Factor Productivity (TFP) change of 0.965 during the study period, with technical efficiency regress being the primary contributor to the loss of TFP. Additionally, Gahé et al. (2016) conducted a study on Technical Efficiency Assessment using data envelopment analysis in the banking sector of Côte d'Ivoire. Their findings unveiled that Ivorian banks do not efficiently allocate loans. Moreover, a classification of banks by ownership and origin revealed that foreign ownership private banks are relatively more efficient than public ownership ones. Further analysis attributed the source of inefficiency to an incompatibility of production scale. Lastly, Triki et al. (2016) highlighted issues related to the regulation and efficiency of African banks, involving 42 countries, utilizing the DEA methodology. The authors emphasized the necessity for regulations to be tailored to the level of risk and the size of banks.

Now the question that emerges is how and what specific approach is required in assessing banks' Technical Efficiency .The predominant and widely accepted technique for evaluating banks' technical efficiency (TE) is the Data Envelopment Analysis (DEA) proposed by Charnes et al. (1978). This mathematical programming model empirically estimates the relationship between production functions and production efficiency. It constructs a frontier based on data from production units, where each unit's efficiency is calculated relative to the frontier. Empirical studies in the banking industry, such as those by Henriques et al. (2018), Gahé et al. (2016), and Gamachis Garamu (2016), have extensively utilized this technique. Previous researchers, particularly in Francophone Africa, including Joumady (2000), Tanimoune (2003), Kamgna and Dimou (2008), Dannon (2009), and Kablan (2009), have also employed this method for assessing bank technical efficiency. DEA methods offer advantages as they do not impose a priori conditions on the functional form of the estimated frontier. They are well-suited for measuring efficiency in firms or banks with complex production processes and services involving multiple inputs and outputs. However, DEA may be ideal for small-sample empirical studies (Ludwin and Guthrie, 1989). Despite its advantages, DEA has some weaknesses. Firstly, a significant number of observations may be identified as efficient, especially when the sum of inputs and outputs is small relative to the number of observations. Secondly, DEA methods may only distinguish between economically viable and technically efficient units, potentially overlooking other important factors. However, these weaknesses can be addressed by imposing a priori constraints on virtual multipliers, as suggested by Farrell et al. (1957) and further developed by Charnes et al. (1978) and Banker et al. (1984). The approaches to assessing bank efficiency with DEA vary, including production, intermediation, and cost approaches. This study focuses on the intermediation approach, which measures bank production in monetary units, considering inputs such as total deposits, net income, human resources, and outputs like loan production capacity. Additionally, a modern approach incorporating elements of information theory in bank activities and risk management has emerged (Freixas et al., 1999). In contrast, the production approach views banks as service providers to customers, with outputs including services provided to savers and borrowers. Inputs may include physical capital and labor. This study adopts the intermediation approach under the non-parametric DEA method for investigation.

3. Methodology

There are various forms of DEA models. The study gives an overview of the mathematical framework of the two main orientation models that are frequently employed in empirical studies under the "input-oriented" thus, Constant Returns to Scale (CRS) model and the Variable Returns to Scale (VRS) model, each offering distinct view on efficiency assessment.

The input-oriented CRS model, proposed by Charnes, Cooper, and Rhodes (CCR), aims to evaluate the relative efficiency of DMUs while maintaining input levels constant. Mathematically, the CCR model gives:

max θ

subject to

$$\sum_{j=1}^{m} \lambda_j \mathcal{Y}_{rj} \leq \theta \mathcal{Y}_{ij}, \quad r = 1,2,\dots,s \tag{1}$$

$$\sum_{j=1}^{m} \lambda_j = 1, \lambda_j \geq 0, \quad j = 1,2,\dots,m \tag{2}$$

In this model, $\mathcal{Y}_{ij}$ represents the outputs of DMU i, $\mathcal{Y}_{rj}$ represents the outputs of efficient DMUj for each output, λ_j are the weights assigned to the efficient DMUs, and θ represents the efficiency score of DMU i.

On the other hand, the input-oriented VRS model, often referred to as the Banker-Charnes- Cooper (BCC) model, accommodates variable returns to scale by allowing input levels to fluctuate. The BCC model can be expressed as:

max θ

subject to

$$\sum_{i=1}^{n} \lambda_i x_{ir} \geq \theta x_{ij}, \quad r = 1,2,\dots,m \tag{3}$$

$$\sum_{i=1}^{n} \lambda_i = 1, \lambda_i \geq 0, \quad j = 1,2,\dots,n \tag{4}$$

Here, x_{ij} represents the inputs of DMU i, x_{ir} represents the inputs of efficient DMU r for each input, $\lambda_i$are the weights assigned to the efficient DMUs, and θ represents the efficiency score of DMU j. Both models offer different advantages and disadvantages. The CRS model provides insights into the efficiency of DMUs under constant input levels, allowing for direct comparisons of efficiency. However, it assumes that returns to scale remain constant, which may not hold all the time in real-world scenarios where input levels could vary tremendously. Conversely, the VRS model adapt to variable returns to scale, offering a more flexible approach to efficiency assessment. This model allows for a deeper understanding of the efficiency features, particularly in situations where input levels are subject to change. However, the VRS model may also introduce additional complexity and computational demands due to its allowance for variable returns to scale. To enhance the clarity of our analysis on efficiency assessment in the African banking industry, we utilized the presentation format suggested by Kablan (2009) and Coelli (2005)**.** The notation format assumes that there are K production factors (Inputs) and Q goods (Outputs) for each bank i (i = 1, 2..., n). Denoted by x_i and $\mathcal{Y}_i$ are the vector of inputs utilized by bank i and the vector of goods produced by the same bank, respectively. Let's consider $K \times N$ as the matrix of inputs X and $Q \times N$ as the matrix of outputs Y. The optimal approach for introducing DEA is through its ratio form. Consequently, we aim to acquire a ratio-based measure for each bank's outputs across all inputs, represented by $\frac{u'\mathcal{Y}_i}{V'x_i}$ where u and v are the vectors of dimensions $Q \times 1$ and $K \times 1$ respectively, the optimal weights are identified by resolving the subsequent mathematical programming problem:

$$max_{u,v}\left(\frac{u'\mathcal{Y}_i}{V'x_i}\right), st \quad \frac{u'\mathcal{Y}_j}{V'x_j} \leq 1, \; j = 1,2, \dots n \tag{5}$$

$$u, v \geq 0$$

This entails determining the values for u and v such that the efficiency measure of the i -th bank is optimized, while ensuring that all efficiency measures remain within the range of zero to one. However, a drawback of this specific ratio formulation is its infinite solution set. In other words, if (u*, v*) is a solution, then (αu*, αv*) is also a solution, and so forth. To address this issue, one can impose the constraint $V'x_i = 1$, which gives:

$max_{u,v} \quad (\mu' \mathcal{Y}_i)$
subject to
$$V'x_j = 1 \tag{6}$$
$$\mu' \mathcal{Y}_{j-} \, V'x_j \leq 0, \; j = 1,2, \dots n \tag{7}$$

The above presents a switch in notation from u and v to μ and v which signifies transformation, this transition is called the "multiplier form "of the linear programming problem. Solving the problem in this form presents some challenges. However, leveraging on the duality in linear programming allows us to derive an equivalent envelopment form of the problem, which can simplify the solution process as ;

$min_{\theta\lambda} \quad \theta$

subject to

$$-\mathcal{Y}_i + \mathrm{Y}\lambda \geq 0 \tag{8}$$

$$\theta x_i - \mathrm{X}\lambda \geq 0 \tag{9}$$

$$\lambda \geq 0 \tag{10}$$

In this envelopment form, θ represents a scalar while λ stands for a vector of constants with dimensions N × 1 . Compared to the multiplier form, this form imposes fewer constraints (K + Q < N + 1) making it the right approach for solving the problem. The resulting θ value serves as the efficiency score for the i-th bank, falling within the range of 0 and 1, thereby satisfying the expression $\theta \in [0, 1]$. An efficiency score of 1 signifies a technically efficient bank is positioned on the frontier. It's worth noting that it is not relevant to assume that the linear programming problem needs to be solved **N** times, once for each bank in our sample. This method assumes constant returns to scale. However, to handle changes in scale economies (variable returns to scale), the convexity constraint $N1'\lambda = 1$ can be incorporated to formulate the following program:

$min_{\theta\lambda} \quad \theta$

subject to

$$N1'\lambda = 1 \tag{11}$$

$$-\mathcal{Y}_i + \mathrm{Y}\lambda \quad \geq 0 \tag{12}$$

$$\theta x_i - \mathrm{X}\lambda \geq 0 \tag{13}$$

$$\lambda \geq 0 \tag{14}$$

where $N1$ represents a vector of ones with dimensions N × 1, this formulation enables the creation of a convex hull composed of intersecting planes that tightly envelope the data points, surpassing the CRS conical hull, and providing technical efficiency scores equal to or greater than those obtained using the CRS model. The VRS specification has been dominant since the 1990s. Building on the work of Berg et al. (1993), the study estimates technical efficiency under both CRS and VRS assumptions.

4. Data and sample

Our data set covers 70 commercial banks from 19 African countries, including Benin, Botswana, Eswatini, Egypt, Ghana, South Africa, Ivory Coast, Kenya, Mauritius, Morocco, Namibia, Niger, Nigeria, Tanzania, Uganda, Sudan, Togo, Tunisia, and Zambia, during the period from 2009 to 2020. Financial data for each bank was collected from the Bloomberg database. Information regarding balance sheet total loans, balance sheet net income, total deposits, and the number of employees was extracted directly from the balance sheet and financial statements of each bank.

4.1. Variables and model specifications

Measuring bank efficiency requires careful selection of input and output variables under Charnes, Cooper and Rhodes (CCR) and Banker, Charnes and Cooper (BCC) input oriented DEA models , which is typically done through either the intermediate or production approaches (Berger & Humphrey, 1997; Sharma et al., 2012; Farrell, 1957). The intermediate approach views banks as intermediaries facilitating asset transfers between surplus and deficit units, while the production approach treats banks as producers with tangible inputs and outputs.

In our study, we adopted the non -parametric and intermediate approach, by selecting three input variables and one output; inputs are, balance sheet net income, total deposits (demand, savings, and fixed deposits), and human resources (total number of employees). Balance sheet total loans are considered outputs due to their significance as direct performance and risk assessment indicators (Rao & Tekeste, 2012) .Specifically the DEA-Window approach for its simplicity and effectiveness in panel time series analysis (Charnes et al., 1985). In this approach, each bank is treated as a Decision Making Unit (DMU) for each year, allowing comparison of its performance over time and against other banks in the same period. Applying DEA on the "input-oriented" under both VRS and CRS assumption, our focus is on maximizing the ratio of loans (output) while considering the constraint of available net income, deposits, and employees (inputs), known as the intermediation approach. Finally, we estimated the following the empirical model:

$$Y_{loan} = f(\mathrm{X}_{deposit} + \mathrm{X2}_{netincome} + \mathrm{X3}_{employees}) \tag{15}$$

where,

Y_{loan} is the volume of bank total loans as the output, and as the function of total deposits ($\mathrm{X}_{deposit}$), total net income ($\mathrm{X2}_{netincome}$) and human resources (number of employees)

($\mathrm{X3}_{employees}$) termed as the inputs.

Data Envelopment Analysis Program (DeaR) version 1.4.1 software is used to measure the technical efficiency of commercial banks in this study.

Table 1
Variables and Sources

List of Variables	Definition	Source
Output :		
Loans (Y)	Banks' balance sheet total loans	Bloomberg database
Inputs :		
Deposits (X)	Banks' balance sheet total deposits	Bloomberg database
Net income (X2)	Banks' balance sheet total net income	Bloomberg database
Employees (X3)	Banks' total number of employees	Bloomberg database

[a]Note: Inputs (deposits, net income, number of employees) and output (total loan)used

5. Results and discussion

Using an input-oriented DEA model, for both Constant Returns to Scale (CRS) and Variable Return to Scale (VRS) assumptions, in time-series panel data to evaluate commercial banks' technical efficiency. Studies systematically classify technical efficiency scores obtained from CRS into two components: pure efficiency and scale efficiency (inefficiency). Pure efficiency or inefficiency represents the proportion of total technical efficiency attributable to fully efficient DMUs under VRS. Scale efficiency denotes the fraction of total technical efficiency explained by the bank's production scale alignment. Any disparity in technical efficiency scores between CRS and VRS models for a specific Decision Making Unit (DMU) indicates scale inefficiency, as outlined by Coelli (2005).

The results obtained from the CRS-DEA analysis indicate that only 47.1% of the Decision Making Units (DMUs), representing banks, fall within the technical efficiency range of less than or equal to 0.1. Additionally, 7.1% of the DMUs exhibit efficiency scores equal to or less than 0.2. Similarly, 7.1%, 4.3%, 7.1%, 1.4%, 4.3%, and 4.3% of DMUs respectively scored within the efficiency ranges of 0.3 to 0.4 and 0.5 to 0.8. However, a notable 17.1% of DMUs are identified as fully efficient. Consequently, a total of 12 out of 70 banks demonstrate a technical efficiency score of one (1) under CRS. In particular, DMUs 1, 4, 10, 14, 18, 20, 23, 27, 49, 61, 63, and 69 show technical efficiency. The average level of technical efficiency under CRS is 33% as reported in Appendix A.

In examining the outcomes from the VRS-DEA model, we observe that only 21.4% of the Decision Making Units (DMUs), were technically efficient within the range of less than or equal to 0.3. Furthermore, 10% of the DMUs show efficiency scores equal to or less than 0.4. Also, 4.3%, 10%, 2.9%, and 10% of DMUs obtain efficiency levels equal to or less than 0.5 and 0.8, respectively, while 20% of DMUs operate within the technical efficiency range of less than or equal to 1. In particular, 21.4% of DMUs are identified as fully efficient (Pure efficiency). Hence, a total of 15 out of 70 banks achieve a technical efficiency score of one (1). Specifically, DMUs 1, 4, 9, 10, 11,14, 18, 20, 23,24, 27, 49, 61, 63, and 69 represent pure technical efficiency(PTE) banks. Thus, an increase in 3 more banks (DMU 9,11 and 24) attained total pure technical efficiency as compared to the CRS-model (TE) . The average level of technical efficiency under VRS is 67% as shown in Appendix A.

Analyzing results from both modeling assumptions, in the Variable Returns to Scale (VRS) scenario, efficiency scores range from a minimum of 0.229 to a maximum of 1, with an average efficiency score of 0.667. In contrast, under the Constant Returns to Scale (CRS) assumption, scores range between the lowest value of 0.002 and the highest of 1, yielding an average total efficiency score of 0.332.

Table 2
VRS and CRS Summary Statistics

MODEL	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
CRS - DEA MODEL	0.002	0.041	0.134	0.332	0.611	1.000
VRS -DEA MODEL	0.219	0.319	0.702	0.667	0.997	1.000
SCALE EFFIC.	1.000	1.531	4.060	13.432	11.990	366.737

[b]Source : DEAR 1.4.1 computation

This disparity between the two models suggests a decrease in efficiency scores when applying the CRS assumption. In short, under the VRS 21.4 % of banks are purely technically efficient (PTE) while under CRS 17.1% are technically efficient. As an example, under the VRS assumption, DMUs 9, 11, and 24 achieved technical efficiency, whereas these same DMUs were technically inefficient with values of 0.026, 0.765, and 0.530, respectively with the CRS.

The computation of scale efficiency scores reveals that, on average, African commercial banks are plagued by a 34% level of inefficiency in scaling. In essence, thus a failure to capitalize on the potential of scale economies to enhance their outputs. African banks struggle more with an issue of production scale mismatch rather than the matter of management inefficiency, particularly in terms of resource allocation for lending activities in an economically efficient manner. The inefficiency predominantly arises from operating below optimal scale rather than from deficiencies in management practices. In that sense, banks' practices should aim at maximizing the conversion of available resources (assets and liabilities) such as net income, deposits, and human resources (employees) into loan assets production.

We observed a varied correlation among the variables (inputs and output). Specifically, the inputs- banks' human resources (number of employees) and net income are positively correlated with banks' balance sheet total loans (output). Conversely, there exists a negative correlation between banks' total deposits and total loans. This suggests that the accumulation of bank deposits does not necessarily translate into an equivalent to loans production capacity extended to customers . This finding aligns with the proposition that commercial banks in Africa often face challenges in providing loans to customers, leading to difficulties in accessing credit. However, we identified a remarkably strong positive relationship of over 82% between human resources (number of employees) and total bank deposits.

We conducted further analysis by computing banks' economic efficiency (EE) and allocative efficiency (AE) as reported in [Appendix, B], utilizing a scoring scale ranging from 0 (lowest) to 1 (highest) for both metrics. Our findings continue to reveal the prevalent inefficiency among African banks within our sample. Regarding EE, efficiency levels obtained ranged from a minimum of 0.042 to a maximum of 1.44, surpassing the maximum score of 1. Similarly, AE presented a range from 0.050 to 1.44. On average, EE score stood at 0.302 (30%), while AE scored at 0.487 (49%). Unsurprisingly, only two Decision-Making Units (DMUs), specifically DMU 18 and DMU 61, attained both economic and allocative efficiency, with DMU 18 operating optimally at a level of 44% under both EE and AE. Our results suggest the banks are operating inefficiently both economically and allocative. Nonetheless, banks have higher allocative efficacy than economics.

Table 3
Economic and Allocative Efficiency Summary Statistics

Description	Min.	1st Qu.	Median	Mean	3rd Qu.	Max .
Economic Efficiency	0.042	0.118	0.191	0.302	0.413	1.442
Allocative Efficiency	0.050	0.198	0.461	0.483	0.722	1.441

[c]Source: DEAR 1.4.1 computation

Comparing our results to other studies, we find an average Technical Efficiency (TE) of 0.67, consistent with the findings of Julius et al. (2024), who reported an average TE of 0.60 in their study of commercial banks within the Southern African Development Community (SADC) region. Assessing allocative efficiency(AE) under the DEA input-oriented Variable Returns to Scale (VRS) model, we find an average AE of 0.49, slightly diverging from Tito et al.(2023) findings with an average AE of 0.38. Our study confirms the significant discrepancies in TE scores between focused country-level analyses and broader regional perspectives. Commercial banks in Africa display TE scores ranging from 55% to 80% under the DEA framework. For example, Pires et al. (2023) found an average TE of 0.719 in Angolan commercial banks studies, while Gahé et al. (2016) reported an average TE of 0.79 for commercial banks in Cote d'Ivoire. Additionally, Gamachis Garamu (2016) observed an average TE of 0.71 in Ethiopian commercial banks, whereas Olohunlana et al. (2023) found a mean TE of 0.58 in their study of the intellectual capital of Nigerian commercial banks. Moreover, while Gahé et al. (2016) found a 0.95 strong correlation between loans and bank deposits, we found a -0.019 correlation between bank loans (output) and total deposits (input). Instead, we identified a significant correlation of 0.82 between total bank deposits and human resources (number of employees). Indicating the important role this new model and variables play in optimizing bank asset and liability utilization and management for operational efficiency.

In short, considering appropriate steps and practices to ensure maximum efficiency score (1) of the banks in the study sample in the African bank sector, an average effort of 0.33 (under VRS) and 0.67 (under CRS) is required. As the efficiency level does not currently reach 1, there is a need to adjust the bank's policies to attain the optimal level of loans. In computing the ratio of inefficiency to total efficiency, it was determined that banks could enhance their total efficiency level to 144% (as obtained by DMU 18 under EE and AE assessment).

Under the assumption of variable returns to scale, their pure technical efficiency could potentially be increased to 33%. Moreover, the scale technical efficiency adjustment could also rise by 34%, and inefficiency by 66%, without altering the transformation rate of deposits, net income, and human resources. This enhancement can be achieved by adopting an optimal level of production. In light of these findings, it is evident that African commercial banks are facing challenges in optimizing the transformation of their available resources into desired production levels. The implications of these inefficiencies extend to the overall economic development and financial stability of the region.

In addressing the inefficiencies identified, our study proposes the adoption of advanced technology such as a centralized national database linked to banking systems to monitor credits (loans) to help in risk profiling and risk management, automate loan processing, and improve operational efficiency. Additionally, banks need to diversify their loan portfolios and explore alternative lending models to effectively utilize their deposit, income, and human resource base.

6. Conclusion

This paper examined the efficiency of 70 commercial banks from 19 African countries with a dataset from 2009 to 2020. Using DEA methodology of the non-parametric approach to measuring banks' technical efficiency through the application of two models, Variable Return Scale (VRS) and Constant Return to Scale (CRS) under intermediation perspective.

Our results reveal that the average technical efficiency score under VRS is 67% and the CRS is 33%. Our findings indicate that most of the banks in our sample were technically inefficient and struggled with an average scale efficiency(inefficiency) score of 34% due to production scale mismatch rather than the matter of management inefficiency, particularly in terms of resource allocation for lending activities in an economic efficient manner, however, assessment using VRS suggests that banks have higher efficacy assessment level than the CRS assumption, thus, more banks attained Pure Technical Efficiency (PTE) than Technical Efficiency(TE) . In the second phase, we conducted further analysis by computing banks' economic efficiency (EE) and allocative efficiency (AE),. On the average score, we find that banks were operating at an EE level of 30% and AE of 49% respectively, with only one bank (DMU 18) operating optimally at a level of 44% under both EE and AE. Also, finds that African banks have higher allocative efficacy than economics.

The study's revelation of a substantial gap in efficiency scores between the VRS-DEA and CRS-DEA models reveals the critical influence of deposits, net income, and human resources on the inefficiency of loan transformation. This disparity emphasizes the need for banks to carefully evaluate their operational processes and resource allocation to achieve a balanced and efficient loan transformation model. Future studies could look at the effects of cost efficiency and other variables such as non-performing loans (NPL) banks' efficiency level in the African banking industry.

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Appendix

A. Data Envelopment Analysis(DEA) Model for Technical and Scale Efficiency Assessment

	VRS		CRS		
DMUs	Technical Efficiency Scores	Ranks	Technical Efficiency Scores	Ranks	Scale Efficiency
1	1	1	1	1	1
2	0.942	25	0.232	29	0.709
3	0.999	16	0.16	33	0.839
4	1	1	1	1	1
5	0.936	26	0.151	34	0.785
6	0.93	28	0.066	48	0.864
7	0.93	28	0.023	60	0.907
8	0.931	27	0.094	39	0.837
9	1	1	0.026	58	0.974
10	1	1	1	1	1
11	1	1	0.765	13	0.235
12	0.994	21	0.403	24	0.592
13	0.995	20	0.438	22	0.557
14	1	1	1	1	1
15	0.994	22	0.639	18	0.355
16	0.994	22	0.725	14	0.269
17	0.997	18	0.688	16	0.308
18	1	1	1	1	1
19	0.998	17	0.133	36	0.865
20	1	1	1	1	1
21	0.996	19	0.209	31	0.787
22	0.702	35	0.002	70	0.7
23	1	1	1	1	1
24	1	1	0.53	19	0.47
25	0.708	33	0.642	17	0.066
26	0.701	36	0.385	25	0.316
27	1	1	1	1	1
28	0.702	34	0.253	28	0.449
29	0.709	32	0.466	21	0.244
30	0.951	24	0.71	15	0.24
31	0.718	31	0.346	27	0.372
32	0.696	38	0.415	23	0.281
33	0.699	37	0.352	26	0.347
34	0.58	40	0.083	42	0.497
35	0.577	41	0.095	38	0.482
36	0.59	39	0.201	32	0.39
37	0.538	42	0.041	52	0.497
38	0.775	30	0.469	20	0.306
39	0.531	43	0.04	53	0.491
40	0.527	44	0.043	51	0.484
41	0.524	45	0.03	55	0.494
42	0.481	47	0.082	43	0.399
43	0.478	48	0.087	41	0.392
44	0.395	49	0.135	35	0.26
45	0.487	46	0.105	37	0.381
46	0.322	52	0.021	61	0.301
47	0.319	53	0.028	56	0.29
48	0.238	66	0.014	62	0.224
49	1	1	1	1	1
50	0.322	51	0.078	45	0.244
51	0.378	50	0.232	30	0.147
52	0.307	54	0.039	54	0.267
53	0.302	55	0.074	46	0.227
54	0.293	60	0.012	65	0.281
55	0.289	63	0.056	49	0.233
56	0.281	65	0.049	50	0.232
57	0.295	57	0.093	40	0.203
58	0.228	68	0.013	64	0.215
59	0.225	69	0.013	63	0.212
60	0.219	70	0.007	68	0.212
61	1	1	1	1	1
62	0.294	58	0.078	44	0.216
63	1	1	1	1	0
64	0.294	59	0.011	67	0.283
65	0.291	61	0.012	66	0.279
66	0.288	64	0.024	59	0.264
67	0.298	56	0.073	47	0.224
68	0.29	62	0.026	57	0.264
69	1	1	1	1	1
70	0.229	67	0.004	69	0.225
Average	0.667		0.332		0.336

Notes: [d]source: Technical Efficiency scores from DeaR version 1.4.1

2. [e]source : Scale efficiency from Authors computation 2024

3. [f]Scale efficiency is the difference between Technical efficiency of VRS and CRS-DEA

B. Summary of Economic Efficiency (EE) and Allocative Efficiency (AE) Scores

DMUs	Economic Efficiency Scores	Ranks	Allocative Efficiency Scores	Ranks
1	0.1769	38	0.1769	56
2	0.0757	62	0.0804	63
3	0.2601	27	0.2603	48
4	0.4138	18	0.4138	41
5	0.1627	42	0.1739	57
6	0.1483	44	0.1595	59
7	0.1777	37	0.1911	53
8	0.142	47	0.1525	60
9	0.2449	29	0.2449	51
10	0.8776	4	0.8776	7
11	0.4568	12	0.4568	37
12	0.056	67	0.0564	68
13	0.0548	68	0.0551	69
14	0.5284	11	0.5284	28
15	0.0598	65	0.0602	66
16	0.0577	66	0.0581	67
17	0.8661	5	0.8687	8
18	1.4411	1	1.4411	1
19	0.1245	51	0.1247	62
20	0.0629	64	0.0629	65
21	0.4115	19	0.4132	42
22	0.3257	25	0.4642	35
23	0.0502	69	0.0502	70
24	0.0736	63	0.0736	64
25	0.2502	28	0.3535	45
26	0.1733	39	0.247	50
27	0.1886	36	0.1886	55
28	0.1332	48	0.1897	54
29	0.1195	52	0.1684	58
30	0.8123	7	0.8544	11
31	0.4108	20	0.5724	24
32	0.434	16	0.6231	22
33	0.5995	8	0.8574	9
34	0.2639	26	0.4547	38
35	0.127	50	0.2201	52
36	0.5543	10	0.9388	4
37	0.4485	14	0.8337	13
38	0.3826	23	0.4938	33
39	0.4342	15	0.8169	16
40	0.4511	13	0.8558	10
41	0.1296	49	0.2473	49
42	0.3948	22	0.8215	15
43	0.3948	21	0.8253	14
44	0.2049	32	0.5187	30
45	0.4318	17	0.8875	6
46	0.1476	45	0.4588	36
47	0.1707	40	0.5358	27
48	0.1049	58	0.4415	40
49	0.8507	6	0.8507	12
50	0.2016	33	0.6254	21
51	0.3491	24	0.9234	5
52	0.1932	35	0.6302	20
53	0.1959	34	0.6493	19
54	0.1602	43	0.5468	26
55	0.2158	31	0.7467	18
56	0.1471	46	0.5239	29
57	0.0417	70	0.1411	61
58	0.1179	53	0.5172	31
59	0.1136	55	0.5038	32
60	0.1066	57	0.4872	34
61	1	2	1	2
62	0.2337	30	0.7945	17
63	0.942	3	0.942	3
64	0.114	54	0.3879	43
65	0.11	56	0.3776	44
66	0.0991	61	0.3444	46
67	0.1007	60	0.338	47
68	0.1704	41	0.5878	23
69	0.5652	9	0.5652	25
70	0.1015	59	0.4428	39
Average	0.302		0.4869	