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Management Science Letters

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A new improved estimator for the population mean using twofold auxiliary information under simple random sampling

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C H R O N I C L E

Article history:
Received: March 10, 2023
Received in revised format: March 25 2023
Accepted: May 4, 2023
Available online: May 4, 2023

Keywords:
Simulation
Simple random sampling
Auxiliary variable
Bias
Mean square error
PRE
Simulation

A B S T R A C T

In this manuscript, the mean of the study and the auxiliary variable, as well as the rank of the auxiliary variable, were needed to develop a new, improved ratio-in-regression type estimator for population mean. Up to the first order of approximation, expressions for the bias and mean square error of the existing and proposed estimators are computed. The effectiveness and stability of our new, enhanced estimator are evaluated using simulation and two actual data sets. The suggested estimator's superior performance to all other considered estimators is shown both conceptually and numerically. The mean square error is the lowest, and PREs out-performs other known estimators by a factor of more than one hundred. Overall, we draw the conclusion that the suggested new improved estimator outperforms all its predecessors.

1. Introduction

It is common knowledge in survey sampling that effective use of the auxiliary variable can increase estimators' efficiency, both during the designing and estimate stages. The estimation of unknown population parameters, such as mean, median, mode, percentage, variance, etc., is one goal of sample surveys. It is preferable to use simple random sampling when the population under consideration is homogeneous (SRS). When the study variable (Y) is linked to the auxiliary variable, standard estimators, such as ratio, products, and regression type of estimators, are frequently employed to estimate population parameters (X). The rank of the auxiliary variable is linked to the research variable whenever there is a positive correlation between the two variables. In different sampling schemes, Hussain et al. (2020), Ahmad et al. (2022), and Irfan et al. (2022) proposed certain estimators employing dual auxiliary variables. There are several significant works that discuss the population mean under simple random sampling using the auxiliary data, such as Kadilar and Cingi (2006), Singh et al. (2012), Shabbir et al. (2014), Grover and Kaur (2014), Singh and Khalid (2015), Muneer et al. (2017), Zaman (2020), Kumar et al. (2021), Singh et al. (2021), Riyaz et al. (2022), Rather et al. (2022), Bulut and Zaman (2022) and Adichwal et al. (2022). By eliminating the edge of connection between the study variable and the auxiliary variable, the dual usage of the auxiliary variable may improve the accuracy of estimators. We build a new, superior estimate for the finite population mean utilising dual auxiliary variables under simple random sampling in this article since dual use of the auxiliary variable for population mean has very rarely been addressed in the literature.

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doi: 10.5267/j.msl.2023.5.001

The article's remaining sections have been organised as follows. We go over a few notations and symbols for the population mean under simple random sampling in Section 2. We evaluated a few current estimators in Section 3. In Section 4, a new, improved estimator for simple random sampling is proposed. In Section 5, theoretical comparisons are provided. In Section 6, a summary statistic is provided. Section 7 provides a simulation. Section 8 of the text is devoted to discussion. Section 9 of the essay discusses the conclusion.

2. Notations and symbols

Let a finite population $\Delta=\{\Delta_1,\ \Delta_2,\dots,\Delta_N\}$ consist of N distinct units, and a sample of size n is drawn from Δ by using simple random sampling without replacement (SRSWOR). Let y_i, x_i and r_{xi} be the values of the study variable (y), auxiliary variable (x), and the rank of the auxiliary variable (r_x) for the i^{th} unit respectively. Let $s_y^2=\sum_{i=1}^{n}(y_i-\bar{y})^2/(n-1)$, $s_x^2=\sum_{i=1}^{n}(x-\bar{x})^2/(n-1)$, $s_{r_x}^2=\sum_{i=1}^{n}(r_{xi}-\bar{r}_x)^2/(n-1)$, represents the sample variance that corresponds to the population variances, i.e. $S_y^2=\sum_{i=1}^{N}(Y_i-\bar{Y})^2/(N-1)$, $S_x^2=\sum_{i=1}^{N}(X_i-\bar{X})^2/(N-1)$, $S_{r_x}^2=\sum_{i=1}^{N}(R_{xi}-\bar{R}_x)^2/(N-1)$, respectively. Also $\bar{y}$, $\bar{x}$ and $\bar{r}_x$ be the sample means corresponding to the population mean $\bar{Y}$, $\bar{X}$ and $\bar{R}_x$ respectively.
To obtain the bias and MSE of the existing and proposed estimators, are given by:
$\varepsilon_0=\frac{\bar{y}-\bar{Y}}{\bar{Y}}$, $\varepsilon_1=\frac{\bar{x}-\bar{X}}{\bar{X}}$, $\varepsilon_2=\frac{\bar{r}_x-\bar{R}_x}{\bar{R}_x}$, such that $E(\varepsilon_i)=0$, for i = (0,1,2),
$E(\varepsilon_0^2)=\lambda C_y^2=\Psi_{200}$, $E(\varepsilon_1^2)=\lambda C_x^2=\Psi_{020}$, $E(\varepsilon_2^2)=\lambda C_{rx}^2=\Psi_{002}$,
$E(\varepsilon_0\varepsilon_1)=\lambda\rho_{yx}C_yC_x=\Psi_{110}$, $E(\varepsilon_0\varepsilon_2)=\lambda\rho_{y\,r_x}C_yC_{r_x}=\Psi_{101}$, $E(\varepsilon_1\varepsilon_2)=\lambda\rho_{x\,r_x}C_xC_{r_x}=\Psi_{011}$,
$C_y=\frac{S_y}{\bar{Y}}$, $C_x=\frac{S_x}{\bar{X}}$, $C_{r_x}=\frac{S_{rx}}{\bar{R}_x}$, $\lambda=\left(\frac{1}{n}-\frac{1}{N}\right)$.

3. Literature review

In this section, we go over a number of simple random sampling-related estimators that are available in the literature.The traditional mean estimator $\hat{\bar{Y}}$ is given by:

$$\mathrm{Var}(\hat{\bar{Y}})=\bar{Y}^2\Psi_{200} \tag{1}$$

(i) Cochran (1940) suggested the ratio estimator $\hat{\bar{Y}}_R$, is given by:

$$\hat{\bar{Y}}_R=\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) \tag{2}$$

The bias and MSE of $\hat{\bar{Y}}_R$, are given as:

$$\mathrm{Bias}(\hat{\bar{Y}}_R)=\bar{Y}(\Psi_{020}-\Psi_{110}),$$

and

$$\mathrm{MSE}\left(\hat{\bar{Y}}_R\right)\cong\bar{Y}^2(\Psi_{200}+\Psi_{020}-2\Psi_{110}) \tag{3}$$

(ii) Murthy (1964) suggested the usual product estimator:

$$\hat{\bar{Y}}_P=\bar{y}\left(\frac{\bar{x}}{\bar{X}}\right) \tag{4}$$

The bias and MSE of $\hat{\bar{Y}}_P$, is given by:

$$\mathrm{Bias}(\hat{\bar{Y}}_P)=\bar{Y}\Psi_{110},$$

and

$$\mathrm{MSE}\left(\hat{\bar{Y}}_P\right)\cong\bar{Y}^2(\Psi_{200}+\Psi_{020}+2\Psi_{110}) \tag{5}$$

(iii) Bahl and Tuteja (1991) suggested the following estimators:

$$\hat{\bar{Y}}_{BT,R}=\bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right), \tag{6}$$
$$\hat{\bar{Y}}_{BT,P}=\bar{y}\exp\left(\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right). \tag{7}$$

The biases and MSEs of $\hat{\hat{Y}}_{BT,R}$, $\hat{\hat{Y}}_{BT,P}$, are given by:

$\text{Bias}\left(\hat{\hat{Y}}_{BT,R}\right) = \bar{Y}\left(\frac{3}{8}\,\Psi_{020} - \frac{1}{2}\Psi_{110}\right)$

and

$$\text{MSE}\left(\hat{\hat{Y}}_{BT,R}\right) = \frac{\bar{Y}^2}{4}\left(4\Psi_{200} + \Psi_{020} - 4\Psi_{110}\right).$$
$$\text{Bias}\left(\hat{\hat{Y}}_{BT,P}\right) = \bar{Y}\left(\frac{1}{2}\,\Psi_{110} - \frac{1}{8}\Psi_{020}\right), \tag{8}$$

and

$$\text{MSE}\left(\hat{\hat{Y}}_{BT,P}\right) = \frac{\bar{Y}^2}{4}\left(4\Psi_{200} + \Psi_{020} + 4\Psi_{110}\right). \tag{9}$$

(iv) The difference estimator $\hat{\hat{Y}}_{dif}$, given as:

$$\hat{\hat{Y}}_{dif} = \bar{y} + \text{d}\,(\bar{X} - \bar{x}), \tag{10}$$

where d is an appropriate chosen constant. The minimum variance of $\hat{\hat{Y}}_{dif}$ at the optimum value $d_{opt} = \frac{\bar{Y}\Psi_{110}}{\bar{X}\Psi_{020}}$, is given as:

$$\text{Var}(\hat{\hat{Y}}_{dif})_{min} = \frac{\bar{Y}^2\,(\Psi_{200}\Psi_{020} - \Psi_{110})}{\Psi_{020}}, \tag{11}$$

(v) Rao (1991) suggested the following estimator:

$$\hat{\hat{Y}}_{R,D} = Q_1\,\bar{y} + Q_2\,(\bar{X} - \bar{x}), \tag{12}$$

The properties of $\hat{\hat{Y}}_{R,D}$, given as:

$\text{Bias}\left(\hat{\hat{Y}}_{R,D}\right) = \bar{Y}\,(Q_1 - 1),$

and

$\text{MSE}\left(\hat{\hat{Y}}_{R,D}\right) = \bar{Y}^2 - 2\,Q_1\bar{Y}^2 + Q_1^2\bar{Y}^2 + Q_1^2\bar{Y}^2\Psi_{200} - 2Q_1Q_2\,\bar{Y}\bar{X}\,\Psi_{110} + Q_2^2\bar{Y}^2\Psi_{020}$.

The optimum values of Q_1, Q_2 are given :

$$Q_{1opt} = \frac{\Psi_{020}}{(\Psi_{020}\Psi_{200} - \Psi_{110}^2 + \Psi_{020})},$$
$$Q_{2opt} = \frac{\bar{Y}\Psi_{110}}{\bar{X}(\Psi_{200}\Psi_{020} - \Psi_{110}^2 + \Psi_{020})},$$

The MSE of $\hat{\hat{Y}}_{R,D}$ at Q_1 and Q_2:

$$\text{MSE}\left(\hat{\hat{Y}}_{R,D}\right)_{min} = \frac{\bar{Y}^2(\Psi_{200}\Psi_{020} - \Psi_{110}^2)}{(\Psi_{200}\Psi_{020} - \Psi_{110}^2 + \Psi_{020})} \tag{13}$$

(vi) The suggested estimator of Singh et al. (2009):

$$\hat{\hat{Y}}_S = \bar{y}\,\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right), \tag{14}$$

The bias and MSE of $\hat{\hat{Y}}_{Singh}$, is given by:

$\text{Bias}\left(\hat{\hat{Y}}_S\right) = \bar{Y}\left(\frac{3}{8}\,\Psi_{020} - \frac{1}{2}\Psi_{110}\right),$

and

$$\mathrm{MSE}\left(\hat{\hat{Y}}_S\right) \cong \frac{\bar{Y}^2}{4}\left(4\Psi_{200} + \Psi_{020} - 4\Psi_{110}\right). \tag{15}$$

(vii) The suggested estimator of Grover and Kaur (2011), is given by:

$$\hat{\hat{Y}}_{Gk} = \{Z_1\bar{y} + Z_2(\bar{X} - \bar{x})\}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right), \tag{16}$$

The properties of $\hat{\hat{Y}}_{Gk}$, is given by:

$$\mathrm{Bias}\left(\hat{\hat{Y}}_{Gk}\right) = \bar{Y}(Z_2 - 1) + \frac{3}{8}Z_1\bar{Y} + \frac{1}{2}Z_2\bar{X}\,\Psi_{020} - \frac{1}{2}\bar{Y}\,\Psi_{110},$$

and

$$\mathrm{MSE}\left(\hat{\hat{Y}}_{Gk}\right)_{min} \cong Z_2^2\bar{X}^2\Psi_{020} + Z_1^2\bar{Y}^2\Psi_{200} + 2Z_1Z_2\bar{Y}\bar{X}\,\Psi_{020} - 2Z_1Z_2\bar{Y}\bar{X}\Psi_{110} + \bar{Y}^2 - 2Z_1\bar{Y}^2 + Z_1^2\bar{Y}^2 + Z_1\bar{Y}^2\Psi_{110} - Z_2\bar{Y}\bar{X}\Psi_{020} - 2Z_1^2\bar{Y}^2\Psi_{110} - \frac{3}{4}Z_1\bar{Y}^2\Psi_{020} + Z_1^2\bar{Y}^2\Psi_{020}. \tag{17}$$

The optimum values of Z_1 and Z_2 are given as:

$$Z_{1(opt)} = \frac{\Psi_{020}(\Psi_{020}-8)}{8(-\Psi_{200}\Psi_{020}-\Psi_{110}^2-\Psi_{020})},$$
$$Z_{2(opt)} = \frac{\bar{Y}(\Psi_{020}^2-\Psi_{020}\Psi_{020}+4\Psi_{200}\Psi_{020}-4\Psi_{110}^2-4\Psi_{020}+8\Psi_{110})}{8\bar{X}(\Psi_{200}\Psi_{020}-\Psi_{110}^2+\Psi_{020})},$$

The minimal MSE of $\hat{\hat{Y}}_{Gk}$, are given by:

$$\hat{\hat{Y}}_{GK\,min} = \frac{\bar{Y}^2}{64}\left(64 - 16\Psi_{020} - \frac{\Psi_{020}(-8+\Psi_{020})^2}{\Psi_{020(1+\Psi_{200})-\Psi_{110}^2}}\right) \tag{18}$$

(viii) Ahmad et al. (2022) suggested the following estimator, is given by:

$$\hat{\hat{Y}}_s = \left\{W_1\bar{Y} + W_2\left(\frac{\bar{X}-\bar{x}}{\bar{X}}\right) + W_3\left(\frac{\bar{R}_x-\bar{r}_x}{\bar{R}_x}\right)\right\}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right), \tag{19}$$

when (a=1) and (b=0).

The bias and MSE of $\hat{\hat{Y}}_s$, are given by:

$\mathrm{Bias}\,(\hat{\hat{Y}}_s) = \bar{Y}(W_1\text{-}1) + \frac{3}{8}W_1\bar{Y}\Psi_{020} + \frac{1}{2}W_2\,\Psi_{020} - \frac{1}{2}W_1\bar{Y}\Psi_{110} + \frac{1}{2}W_2\,\Psi_{011}$, and

$$\mathrm{MSE}\left(\hat{\hat{Y}}_s\right) \cong \bar{Y}^2(W_1\text{-}1) + W_1^2\bar{Y}^2\Psi_{200} + W_2^2\Psi_{020} + W_3^2\Psi_{002} + W_1^2\bar{Y}^2\Psi_{020} - W_2\bar{Y}\Psi_{020} + 2W_1W_2\bar{Y}\Psi_{020} - \frac{3}{4}W_1\bar{Y}^2\Psi_{020} + W_1\bar{Y}^2\Psi_{110} - 2\bar{Y}^2\Psi_{110} - 2W_1W_2\bar{Y}\,\Psi_{110} - 2W_1W_3\bar{Y}\Psi_{101} - W_3\bar{Y}\Psi_{011} + 2W_1W_3\bar{Y}\Psi_{011} - 2W_1W_3\Psi_{011} \tag{20}$$

The optimum values of W_1, W_2 and W_3, achieved by diminishing Eq. (20), are given below:

$$W_{1(opt)} = \frac{8-\Psi_{020}}{8\{1+\Psi_{200}(1-R^2{}_{yx_1x_2})\}},$$

$$W_{2(opt)} = \frac{\bar{Y}\left[\Psi_{020}^{3/2}(R_{x_1x_2}-1)+\Psi_{200}^{1/2}(-8+\Psi_{020})(R_{yx_1}-R_{x_1x_2}R_{yx_1})+4\Psi_{020}^{1/2}(R_{x_1x_2}^2-1)\{-1+\Psi_{200}(1-R_{yx_1}^2)\}\right]}{8\Psi_{020}^{1/2}(R_{x_1x_2}^2-1)\{-1+\Psi_{200}(1-R_{yx_1}^2)\}},$$
$$W_{3(opt)} = \frac{\bar{Y}\,\Psi_{200}^{1/2}(8-\Psi_{020})(R_{yx_1}-R_{x_1x_2}R_{yx_1})}{8\,\Psi_{020}^{1/2}(R_{x_1x_2}^2-1)\{-1+\Psi_{200}(1-R_{yx_1x_2}^2)\}}.$$

The minimum mean square error of $\hat{\hat{Y}}_s$, at W_1, W_2, and W_3, are given as:

$$\mathrm{MSE}\left(\hat{\hat{Y}}_s\right)_{min} \cong \frac{\bar{Y}^2\{64\Psi_{200}(1-R_{yx_1x_2}^2)-\Psi_{020}^2-16\Psi_{020}\,\Psi_{200}(1-R_{yx_1x_2}^2)\}}{64\{1+\Psi_{200}(1-R_{yx_1x_2}^2)\}}, \tag{21}$$

where

$$R^2_{yx_1x_2} = \left(\frac{\Psi^2_{110}\Psi_{002}+\Psi^2_{101}\Psi_{020}-2\Psi_{101}\Psi_{110}\Psi_{011}}{\Psi_{200}(\Psi_{020}\Psi_{002}-\Psi^2_{011})}\right).$$

4. Proposed estimator

When used properly, the auxiliary variable can improve the design and estimate stages of estimators' precision. When there is a high correlation between the study variable and the auxiliary variable, the study variable's rank is also related to it. We presented a ratio-in-regression type exponential estimator for calculating the population means based on simple random sampling, drawing inspiration from Ahmad et al. (2022). Dual use of auxiliary variables has not been explored frequently in the literature on survey sampling, which is why we are driven to do so. Our enhanced ratio-in-regression type estimator under simple random sampling has the main benefit of being more adaptable and effective than the current estimators.

$$\hat{\bar{Y}}_{ss} = \text{Ꞇ}_{11}\hat{\bar{Y}} + \text{Ꞇ}_{12}\left(\bar{X} - \hat{\bar{X}}\right)\exp\left(\frac{\bar{X}-\hat{\bar{X}}}{\bar{X}+\hat{\bar{X}}}\right) + \text{Ꞇ}_{13}\left(\bar{R}_x - \hat{\bar{R}}_x\right)\exp\left(\frac{\bar{R}_x-\hat{\bar{R}}_x}{\bar{R}_x+\hat{\bar{R}}_x}\right), \tag{22}$$

where Ꞇ_{11}, Ꞇ_{12} and Ꞇ_{13} are unknown constants.

Solving $\hat{\bar{Y}}_{ss}$ given in Eq. (22),

$$\hat{\bar{Y}}_s = \text{Ꞇ}_{11}\bar{Y}(1+\varepsilon_0) - \text{Ꞇ}_{12}\bar{X}\varepsilon_1\left(1-\frac{1}{2}\varepsilon_1+\frac{3}{8}\varepsilon_1^2\right) - \text{Ꞇ}_{13}\bar{R}_x\varepsilon_2\left(1-\frac{1}{2}\varepsilon_2+\frac{3}{8}\varepsilon_2^2\right)$$

$$\hat{\bar{Y}}_{ss} - \bar{Y} = (\text{Ꞇ}_{11}-1)\bar{Y} + \text{Ꞇ}_{11}\bar{Y}\varepsilon_0 - \text{Ꞇ}_{12}\bar{X}\left(\varepsilon_1-\frac{1}{2}\varepsilon_1^2\right) - \text{Ꞇ}_{13}\bar{R}_x\left(\varepsilon_2-\frac{1}{2}\varepsilon_2^2\right) \tag{i}$$

$$\text{Bias}\left(\hat{\bar{Y}}_{ss}\right) = (\text{Ꞇ}_{11}-1)\bar{Y} + \frac{1}{2}\bar{X}\text{Ꞇ}_{12}\Psi_{020} + \frac{1}{2}\text{Ꞇ}_{13}\bar{R}_x\Psi_{002}$$

Simplify Eq. (i), we have

$$\text{MSE}\left(\hat{\bar{Y}}_{ss}\right) = (\text{Ꞇ}_{11}-1)^2\bar{Y}^2 + \text{Ꞇ}_{11}^2\bar{Y}^2\varepsilon_0^2 + \text{Ꞇ}_{12}^2\bar{X}^2\varepsilon_1^2 + \text{Ꞇ}_{13}^2\bar{R}x^2\varepsilon_2^2 + 2\,\text{Ꞇ}_{11}(\text{Ꞇ}_{11}-1)\bar{Y}^2\varepsilon_0 - 2(\text{Ꞇ}_{11}-1)\text{Ꞇ}_{12}\bar{Y}\bar{X}\left(\varepsilon_1-\frac{1}{2}\varepsilon_1^2\right) - 2(\text{Ꞇ}_{11}-1)\text{Ꞇ}_{13}\bar{Y}\bar{R}_x\left(\varepsilon_2-\frac{1}{2}\varepsilon_2^2\right) - 2\text{Ꞇ}_{11}\text{Ꞇ}_{12}\bar{Y}\bar{X}(\varepsilon_0\varepsilon_1) - 2\text{Ꞇ}_{11}\text{Ꞇ}_{13}\bar{Y}\bar{R}_x(\varepsilon_0\varepsilon_2) + 2\text{Ꞇ}_{12}\text{Ꞇ}_{13}\bar{Y}\bar{R}_x(\varepsilon_1\varepsilon_2)$$

$$= (\text{Ꞇ}_{11}-1)^2\bar{Y}^2 + \text{Ꞇ}_{11}^2\bar{Y}^2\Psi_{200} + \text{Ꞇ}_{12}^2\bar{X}^2\Psi_{020} + \text{Ꞇ}_{13}^2\bar{R}_{x^2}\Psi_{002} + 2(\text{Ꞇ}_{11}\text{Ꞇ}_{12}-\text{Ꞇ}_{13})\bar{Y}\bar{X}\frac{\Psi_{020}}{2} + 2(\text{Ꞇ}_{11}\text{Ꞇ}_{13}-\text{Ꞇ}_{13})\bar{Y}\bar{R}_x\frac{\Psi_{002}}{2} - 2\text{Ꞇ}_{11}\text{Ꞇ}_{12}\bar{Y}\bar{X}\Psi_{110} - 2\text{Ꞇ}_{11}\text{Ꞇ}_{13}\bar{Y}\bar{R}_x\Psi_{101} + 2\text{Ꞇ}_{12}\text{Ꞇ}_{13}\bar{X}\bar{R}_x\Psi_{011}$$

$$\text{MSE}\left(\hat{\bar{Y}}_{ss}\right) = (\text{Ꞇ}_{11}-1)^2\bar{Y}^2 + \text{Ꞇ}_{11}^2\bar{Y}^2\Psi_{200} + \text{Ꞇ}_{12}^2\bar{X}^2\Psi_{020} + \text{Ꞇ}_{13}^2\bar{R}_{x^2}\Psi_{002} - 2\text{Ꞇ}_{12}\bar{Y}\bar{X}\frac{\Psi_{020}}{2} - 2\text{Ꞇ}_{13}\bar{Y}\bar{R}_x\frac{\Psi_{002}}{2} + 2\text{Ꞇ}_{11}\text{Ꞇ}_{12}\bar{Y}\bar{X}\left(\frac{\Psi_{020}}{2}-\Psi_{110}\right) + 2\text{Ꞇ}_{11}\text{Ꞇ}_{13}\bar{Y}\bar{R}_x\left(\frac{\Psi_{002}}{2}-\upsilon_{101}\right) + 2\text{Ꞇ}_{12}\text{Ꞇ}_{13}\bar{X}\bar{R}_{x^2}\Psi_{011} \tag{23}$$

The optimum values of Ꞇ_{11}, Ꞇ_{12} and Ꞇ_{13} are given by:

$$\text{Ꞇ}_{11(opt)} = \frac{\left(2\lambda\frac{\Psi_{200}^{1/2}}{\lambda^{1/2}}\right)(\text{ʃ}_{11}+\text{Ƥ}_{11}) + \lambda\left(T+\frac{\Psi_{020}}{\lambda}\right) - 4}{4\Psi_{200}(R^2_{yx}-1) + 4\Psi_{200}^{1/2}(\text{ʃ}_{11}+\text{Ʀ}_{11}) + \lambda\left(T+\frac{\Psi_{020}}{\lambda}\right) - 4},$$

$$\text{Ꞇ}_{12(opt)} = \frac{\frac{\Psi_{200}^{1/2}}{\lambda^{1/2}}\bar{Y}\left[2\Psi_{200}^{1/2}\lambda^{1/2}\left\{\frac{\Psi_{002}^{1/2}}{\lambda^{1/2}}\text{ʃ}_{11} + \frac{\Psi_{020}^{1/2}}{\lambda^{1/2}}\left(\frac{\Psi^2_{101}}{\Psi_{200}\Psi_{002}}\right)\right\}\Psi_{002}^{1/2}\Psi_{020}^{1/2}\text{ʈ}_{11} + 4\text{ʄ}_{11}\right]}{\left[\frac{\Psi_{020}^{1/2}}{\lambda^{1/2}}\bar{X}\left(1-\frac{\Psi^2_{101}}{\Psi_{200}\Psi_{002}}\right)\left\{4\Psi_{200}(R^2_{yx}-1) + \lambda\Psi_{200}^{1/2}(\text{ʃ}_{11}+\text{Ƥ}_{11}) + \lambda\left(T+\frac{\Psi_{020}}{\lambda}\right) - 4\right\}\right]},$$

$$\text{Ꞇ}_{13(opt)} = \frac{\frac{\Psi_{200}^{1/2}}{\lambda^{1/2}}\bar{Y}\left[2\Psi_{200}^{1/2}\lambda^{1/2}\left\{\frac{\Psi_{002}^{1/2}}{\lambda^{1/2}}\text{ʃ}_{11} + \frac{\Psi_{020}^{1/2}}{\lambda^{1/2}}\left(\frac{\Psi^2_{101}}{\Psi_{200}\Psi_{002}}\right)\right\} + 4\text{ʄ}_{11}\right]}{\left[\frac{\Psi_{020}^{1/2}}{\lambda^{1/2}}\bar{X}\left(1-\frac{\Psi^2_{101}}{\Psi_{200}\Psi_{002}}\right)\left\{4\Psi_{200}(R^2_{yx}-1) + \lambda\Psi_{200}^{1/2}(\text{ʃ}_{11}+\text{Ƥ}_{11}) + \lambda\left(T+\frac{\Psi_{020}}{\lambda}\right) - 4\right\}\right]}.$$

where

$$\text{ʈ}_{11} = \frac{\Psi_{110}}{\sqrt{\Psi_{200}}\sqrt{\Psi_{020}}} - \frac{\Psi_{101}}{\sqrt{\Psi_{200}}\sqrt{\Psi_{002}}},\ R^2_{yx} = \left(\frac{\Psi^2_{110}\Psi_{002}+\Psi^2_{101}\Psi_{020}-2\Psi_{101}\Psi_{110}\Psi_{011}}{\Psi_{200}(\Psi_{020}\Psi_{002}-\Psi^2_{110})}\right),$$

$$\text{T} = \frac{\left\{\frac{\Psi_{002}^{1/2}}{\lambda^{1/2}} - \frac{\Psi_{020}^{1/2}}{\lambda^{1/2}}\left(\frac{\Psi_{011}}{\sqrt{\Psi_{020}}\sqrt{\Psi_{002}}}\right)\right\}^2}{1 - \frac{\Psi_{0110}^2}{\Psi_{020}\Psi_{002}}},\ \text{ʄ}_{11} = \frac{\frac{\Psi_{020}^{1/2}}{\lambda^{1/2}}\left(\frac{\Psi_{101}}{\sqrt{\Psi_{200}}\sqrt{\Psi_{002}}} - \frac{\Psi_{011}}{\sqrt{\Psi_{020}}\sqrt{\Psi_{002}}} - \frac{\Psi_{110}}{\sqrt{\Psi_{200}}\sqrt{\Psi_{020}}}\right)}{1 - \frac{\Psi_{0110}^2}{\Psi_{020}\Psi_{002}}},$$

$$\text{Ɽ}_{11} = \frac{\left\{\frac{\Psi_{101}}{\sqrt{\Psi_{200}}\sqrt{\Psi_{002}}}\left(\frac{\Psi_{200}^{1/2}}{\lambda^{1/2}}\right) - \frac{\Psi_{110}}{\sqrt{\Psi_{020}}\sqrt{\Psi_{002}}}\left(\frac{\Psi_{002}^{1/2}}{\lambda^{1/2}}\right)\right\}^2}{1 - \frac{\Psi_{011}^2}{\Psi_{020}\Psi_{002}}},\ \text{Ᵽ}_{11} = \frac{\frac{\Psi_{002}^{1/2}}{\lambda^{1/2}}\left(\frac{\Psi_{110}}{\sqrt{\Psi_{200}}\sqrt{\Psi_{002}}} - \frac{\Psi_{010}}{\sqrt{\Psi_{020}}\sqrt{\Psi_{002}}} - \frac{\Psi_{101}}{\sqrt{\Psi_{200}}\sqrt{\Psi_{002}}}\right)}{1 - \frac{\Psi_{011}^2}{\Psi_{020}\Psi_{002}}}.$$

Putting the optimum values of Ʈ_{11}, Ʈ_{12}, and Ʈ_{13} in Eq. (23), we get the minimal MSE of $\hat{\hat{Y}}_{ss}$, given by:

$$MSE\left(\hat{\hat{Y}}_{ss}\right)_{min} = \frac{\bar{Y}^2\Psi_{200}\{T(\lambda) - \text{Ᵽ}_{11}(\lambda) + \Psi_{020} + 4(R_{yx}^2 - 1)\}}{\left\{\lambda\left(4\frac{\Psi_{200}}{\lambda}(R_{yx}^2 - 1) + 4\frac{\Psi_{020}^{1/2}}{\lambda^{1/2}}(\text{ʄ}_{11} + \text{Ᵽ}_{11}) + T + \frac{\Psi_{020}}{\lambda} - 4\right)\right\}}. \tag{24}$$

5. Theoretical comparison

In this Section, we performed a theoretical comparison of the adopted and proposed estimator:

(i) Taking Eq. (1) and Eq. (24),

$$MSE\left(\hat{\hat{Y}}_{ss}\right)_{min} < \text{Var}(\hat{\hat{Y}}) \text{ if}$$

$$\text{Var}(\hat{\hat{Y}}) - MSE\left(\hat{\hat{Y}}_{ss}\right)_{min} > 0$$

$$\frac{\bar{Y}^2\Psi_{200}\left[\frac{(\Upsilon_{11})}{\sqrt{\lambda\left(\frac{1}{2}\right)}}[\Upsilon_{12}]\right]}{\lambda\left[\frac{\Upsilon_{13}}{\sqrt{\lambda\left(\frac{1}{2}\right)}}\right]} > 0$$

where

$\Upsilon_{11} = (T-4)\lambda + R_{yx}^2\Psi_{200} + \Psi_{020} - 4\Psi_{200}\sqrt{\lambda\left(\frac{1}{2}\right)} + 4\sqrt{\Psi_{020}}\lambda\,(\text{ʄ}_{11} + \text{Ᵽ}_{11})$,

$\Upsilon_{12} = T(\lambda) - \text{Ᵽ}_{11}\lambda + \Psi_{020} + 4R_{yx}^2 - 4$,

$\Upsilon_{13} = \left((T-4)\lambda + 4R_{yx}^2\Psi_{200} + \Psi_{020} - 4\Psi_{200}\right)\sqrt{\lambda\left(\frac{1}{2}\right)} + 4\sqrt{\Psi_{020}}\lambda\,(\text{ʄ}_{11} + \text{Ᵽ}_{11})$

(ii) Taking Eq. (3) and Eq. (24),

$$MSE\left(\hat{\hat{Y}}_{ss}\right)_{min} < \text{MSE}\left(\hat{\hat{Y}}_R\right) \text{ if}$$

$$\text{MSE}\left(\hat{\hat{Y}}_R\right) - MSE\left(\hat{\hat{Y}}_{ss}\right)_{min} > 0$$

$$\frac{\bar{Y}^2\left[\frac{(-\Psi_{020} + 2\Psi_{110} - \Psi_{200})\lambda(\Upsilon_{11})}{\sqrt{\lambda\left(\frac{1}{2}\right)}} + \Psi_{200}[\Upsilon_{12}]\right]}{\lambda\left[\frac{\Upsilon_{13}}{\sqrt{\lambda\left(\frac{1}{2}\right)}}\right]} > 0$$

(iii) Taking Eq. (5) and Eq. (24),

$$MSE\left(\hat{\hat{Y}}_{ss}\right)_{min} < \text{MSE}\left(\hat{\hat{Y}}_P\right) \text{ if}$$

$$\text{MSE}\left(\hat{\hat{Y}}_P\right) - MSE\left(\hat{\hat{Y}}_{ss}\right)_{min} > 0.$$

$$\frac{\bar{Y}^2\left[\frac{(-\Psi_{020}+2\Psi_{110}+\Psi_{200})\lambda(\Upsilon_{13})}{\sqrt{\lambda\left(\frac{1}{2}\right)}}+\Psi_{200}[\Upsilon_{12}]\right]}{\lambda\left[\frac{\Upsilon_{14}}{\sqrt{\lambda\left(\frac{1}{2}\right)}}\right]} > 0.$$

where

$$\Upsilon_{14} = \left((T-4)\lambda + 4R_{yx}^2\Psi_{200} + \Psi_{020} + 4\Psi_{200}\right)\sqrt{\lambda\left(\frac{1}{2}\right)} + 4\sqrt{\Psi_{020}}\lambda\,(\mathfrak{f}_{11} + \mathfrak{p}_{11})$$

(iv) Taking Eq. (8) and Eq. (24),

$$MSE\left(\hat{\hat{Y}}_{ss}\right)_{min} < \text{MSE}\left(\hat{\hat{Y}}_{BT,R}\right) \text{if}$$

$$\text{MSE}\left(\hat{\hat{Y}}_{BT,R}\right) - MSE\left(\hat{\hat{Y}}_{ss}\right)_{min} > 0$$

$$\frac{\bar{Y}^2\left[\frac{\left(-\frac{1}{4}\Psi_{020}+\Psi_{110}-\Psi_{200}\right)\lambda(\Upsilon_{13})}{\sqrt{\lambda\left(\frac{1}{2}\right)}}-\Psi_{200}[\Upsilon_{12}]\right]}{\lambda\left[\frac{\Upsilon_{13}}{\sqrt{\lambda\left(\frac{1}{2}\right)\lambda}}\right]} > 0$$

(v) Taking Eq. (9) and Eq. (24),

$$MSE\left(\hat{\hat{Y}}_{ss}\right)_{min} < \text{MSE}\left(\hat{\hat{Y}}_{BT,P}\right) \text{if}$$

$$\text{MSE}\left(\hat{\hat{Y}}_{BT,P}\right) - MSE\left(\hat{\hat{Y}}_{ss}\right)_{min} > 0$$

$$\frac{\bar{Y}^2\left[\frac{\left(-\frac{1}{4}\Psi_{020}+\Psi_{110}+\Psi_{200}\right)\lambda(\Upsilon_{13})}{\sqrt{\lambda\left(\frac{1}{2}\right)}}+\Psi_{200}[\Upsilon_{12}]\right]}{\lambda\left[\frac{\Upsilon_{13}}{\sqrt{\lambda\left(\frac{1}{2}\right)\lambda}}\right]} > 0$$

(vi) Taking Eq. (11) and Eq. (24),

$$MSE\left(\hat{\hat{Y}}_{ss}\right)_{min} < \text{Var}(\hat{\hat{Y}}_{dif})_{min} \text{ if}$$

$$\text{Var}(\hat{\hat{Y}}_{dif})_{min} - MSE\left(\hat{\hat{Y}}_{ss}\right)_{min} > 0$$

$$\frac{\bar{Y}^2\left[\frac{(-\Psi_{020}\Psi_{110}+\Psi_{110})\lambda(\Upsilon_{13})}{\sqrt{\lambda\left(\frac{1}{2}\right)}}+\Psi_{200}[\Upsilon_{12}]\right]}{\left[\frac{\Upsilon_{13}}{\sqrt{\lambda\left(\frac{1}{2}\right)}}\right]} > 0$$

(vii) Taking Eq. (13) and Eq. (24),

$$MSE\left(\hat{\hat{Y}}_{ss}\right)_{min} < \text{MSE}\left(\hat{\hat{Y}}_{R,D}\right)_{min} \text{ if}$$

$$\text{MSE}\left(\hat{\hat{Y}}_{R,D}\right)_{min} - MSE\left(\hat{\hat{Y}}_{ss}\right)_{min} > 0$$

$$\frac{\bar{X}^2\left[\frac{(\Psi_{020}\Psi_{200}-\Psi_{110}^2)\lambda(\mathrm{Y}_{13})}{\sqrt{\lambda\left(\frac{1}{2}\right)}}-(\Psi_{200}+1)\Psi_{020}-\Psi_{110}^2[\mathrm{Y}_{12}]\right]}{((\Psi_{200}+1)\Psi_{020}-\Psi_{110}^2)\left[\frac{\mathrm{Y}_{13}}{\sqrt{\lambda\left(\frac{1}{2}\right)}}\right]}>0$$

(viii) Taking Eq. (15) and Eq. (24),

$$MSE\left(\hat{\hat{\bar{Y}}}_{ss}\right)_{min}<\mathrm{MSE}\left(\hat{\hat{\bar{Y}}}_{Singh}\right)\ \text{if}$$

$$\mathrm{MSE}\left(\hat{\hat{\bar{Y}}}_{Singh}\right)-MSE\left(\hat{\hat{\bar{Y}}}_{ss}\right)_{min}>0.$$

$$\frac{\bar{Y}^2\left[\frac{\left(-\frac{1}{4}\Psi_{020}+\Psi_{110}-\Psi_{200}\right)\lambda(\mathrm{Y}_{13})}{\sqrt{\lambda\left(\frac{1}{2}\right)\lambda}}+\Psi_{200}[\mathrm{Y}_{12}]\right]}{\lambda\left[\frac{\mathrm{Y}_{13}}{\sqrt{\lambda\left(\frac{1}{2}\right)\lambda}}\right]}>0$$

(ix) Taking Eq. (18) and Eq. (24),

$$MSE\left(\hat{\hat{\bar{Y}}}_{ss}\right)_{min}<\mathrm{MSE}\left(\hat{\hat{\bar{Y}}}_{GK\,min}\right)\ \text{if}$$

$$\mathrm{MSE}\left(\hat{\hat{\bar{Y}}}_{GK\,min}\right)-MSE\left(\hat{\hat{\bar{Y}}}_{ss}\right)_{min}>0$$

$$\frac{\frac{1}{64}\left[\frac{(-X^2\,(16\Psi_{020}-64)(-\Psi_{110}^2+\Psi_{200}+1)\lambda(\mathrm{Y}_{13})}{\sqrt{\lambda\left(\frac{1}{2}\right)}}-\Psi_{200}[\mathrm{Y}_{12}]\right]}{[\Psi_{020}(-\Psi_{110}^2+\Psi_{200}+1)]\,\lambda\left[\frac{\mathrm{Y}_{13}}{\sqrt{\lambda\left(\frac{1}{2}\right)}}\right]}>0$$

(x) Taking Eq. (21) and Eq. (24),

$$MSE\left(\hat{\hat{\bar{Y}}}_{ss}\right)_{min}<\mathrm{MSE}\left(\hat{\hat{\bar{Y}}}_{Haq}\right)_{min}\ \text{if}$$

$$\mathrm{MSE}\left(\hat{\hat{\bar{Y}}}_{Haq}\right)_{min}-MSE\left(\hat{\hat{\bar{Y}}}_{ss}\right)_{min}>0$$

$$\frac{\bar{Y}^2\left\{64\Psi_{200}\left(1-R_{y.xrx}^2\right)-\Psi_{020}^2-16\Psi_{020}\Psi_{200}\left(1-R_{y.xrx}^2\right)\right\}}{64\left\{1+\Psi_{200}\left(1-R_{y.xrx}^2\right)\right\}}$$

$$-\frac{\bar{Y}^2\Psi_{200}\left\{T(\lambda)-B_{11}(\lambda)+\Psi_{020}+4\left(R_{yx}^2-1\right)\right\}}{\left\{\lambda\left(4\frac{\Psi_{200}}{\lambda}(R_{yx}^2-1)+4\,\frac{\Psi_{020}^{1/2}}{\lambda^{1/2}}(S_{11}+B_{11})+T+\frac{\Psi_{020}}{\lambda}-4\right)\right\}}>0.$$

6. Data Description

In this section, we use two real data sets for numerical comparison of the adopted and proposed estimators in simple random sampling.

We use the following expression to obtain the PREs:

$$\mathrm{PRE}(.)=\frac{Var(\hat{\bar{Y}})}{MSE(\hat{\hat{\bar{Y}}}_{i(min)})}*100,$$

where i = ($\hat{\hat{\bar{Y}}}_R$, $\hat{\hat{\bar{Y}}}_P$, $\hat{\hat{\bar{Y}}}_{BT,R}$, $\hat{\hat{\bar{Y}}}_{BT,P}$, $\hat{\hat{\bar{Y}}}_{dif}$, $\hat{\hat{\bar{Y}}}_{R,D}$, $\hat{\hat{\bar{Y}}}_{Singh}$, $\hat{\hat{\bar{Y}}}_{Gk}$, $\hat{\hat{\bar{Y}}}_{S,}$ $\hat{\hat{\bar{Y}}}_{ss}$).

Population I: (Source: Koyuncu and Kadilar (2009))

Y= Instructors aggregate,

X= Pupil aggregate in both elementary and secondary levels in Turkey, R_x= Rank of X variable

Table 1
Summary statistics for population-I

N=923	S_x^2 = 455071	ρ_{yx} = 0.9543029
N = 180	C_x^2 = 3.47646	ρ_{yrx} = 0.6444158
λ = 0.00447	C_x = 1.8645	ρ_{xrx} = 0.6306615
$\bar{Y}$ = 436.4345	$\bar{R}_x$ = 462	S_{yx} = 15266040
S_y^2 = 562409.3	C_{rx}^2 = 0.3329725	S_{yrx} = 128836.4
C_y^2 = 2.952667	C_{rx} = 0.577037	S_{xrx} = 3586380
C_y =1.71833	β_y = 18.62857	
$\bar{X}$ = 11440.5	β_x = 21.61308	

Table 2
MSE and PREs using population-I

Estimators	MSE	PRE
$\hat{\bar{Y}}$	2515.169	100
$\hat{\bar{Y}}_R$	267.6354	939.7742
$\hat{\bar{Y}}_P$	14223.6	17.68306
$\hat{\bar{Y}}_{BT,R}$	651.0631	386.3172
$\hat{\bar{Y}}_{BT,P}$	5859.952	42.92132
$\hat{\bar{Y}}_{dif}$	224.6194	1119.747
$\hat{\bar{Y}}_{R,D}$	224.3549	1121.067
$\hat{\bar{Y}}_{Singh}$	651.0631	386.3172
$\hat{\bar{Y}}_{GK}$	222.7643	1129.072
$\hat{\bar{Y}}_s$	215.2417	1168.538
$\hat{\bar{Y}}_{ss}$	205.5355	1223.722

Population 2: (Source: Koyuncu and Kadilar (2009))

Y= Instructors aggregate,
X= Classes aggregate in elementary and secondary levels, R_x= Rank of X variable

Table 3
Summary statistics for population-II

N=923	S_x^2 = 195784	ρ_{yx} = 0.9543029
N = 180	C_x^2 = 1.76384	ρ_{yrx} = 0.64907
λ = 0.00447	C_x = 1.3280	ρ_{xrx} = 0.72913
$\bar{Y}$ = 436.4345	$\bar{R}_x$ = 462	S_{yx} = 325003
S_y^2 = 562409.3	C_{rx}^2 = 0.3329725	S_{yrx} = 129767.1
C_y^2 = 2.952667	C_{rx} = 0.577037	S_{xrx} = 86008.5
C_y =1.71833	β_y = 18.62857	
$\bar{X}$ = 333.1647	β_x = 14.3732	

Table 4
MSE and PREs using population-II

Estimators	MSE	PRE
$\hat{\bar{Y}}$	2515.169	100
$\hat{\bar{Y}}_R$	209.696	1199.44
$\hat{\bar{Y}}_P$	12365.7	20.3398
$\hat{\bar{Y}}_{BT,R}$	986.809	254.879
$\hat{\bar{Y}}_{BT,P}$	4794.77	52.4565
$\hat{\bar{Y}}_{dif}$	102.41	2455.97
$\hat{\bar{Y}}_{R,D}$	102.355	2457.29
$\hat{\bar{Y}}_{Singh}$	986.809	254.879
$\hat{\bar{Y}}_{GK}$	101.809	2466.62
$\hat{\bar{Y}}_s$	79.3012	3171.67
$\hat{\bar{Y}}_{ss}$	73.7003	3412.7

7. Simulation study

To compare the effectiveness of the suggested estimators with their existing counterparts when the auxiliary variable and rank of the auxiliary variable are used, we conduct a simulation study in this section. Three populations are taken into account for this reason. Tables 5 provide information about this populations' PRE. Three populations totaling 1,0000 were created from a multivariate normal distribution using various covariance matrices. The correlations between the X and Y variables in each of these populations are different, with Population-I being positively correlated, Population-II being negatively correlated, and Population-III having a positive correlation. Below are the population averages and a covariance matrix:

Population-I

$$\mu_1 = \begin{bmatrix} 5 \\ 5 \end{bmatrix},$$

and

$$\Sigma_1 = \begin{bmatrix} 4 & 9.6 \\ 9.6 & 64 \end{bmatrix}$$

$\rho_{XY} = 0.59985$

Population-II

$$\mu_2 = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

and

$$\Sigma_2 = \begin{bmatrix} 2 & 4 \\ 6 & 10 \end{bmatrix}$$

$\rho_{XY} = 0.89377$

Population-III

$$\mu_3 = \begin{bmatrix} 5 \\ 5 \end{bmatrix},$$

and

$$\Sigma_3 = \begin{bmatrix} 4 & -9.7 \\ -9.7 & 65 \end{bmatrix}$$

$\rho_{XY} = -0.5978$

The Percentage Relative Efficiency (PRE) is calculated as follows:

$$\text{PRE}\left(\hat{\bar{Y}}_{ss}, \hat{\bar{Y}}\right) = \frac{\text{Var}\left(\hat{\bar{Y}}\right)}{\text{MSE}\left(\hat{\bar{Y}}_{ss}\right)_{min}} \times 100,$$

Table 5

PREs of estimators using simulation for populations I-III,

Estimators	Population-I	Population-II	Population-III
$\hat{\bar{Y}}$	100	100	100
$\hat{\bar{Y}}_R$	24.285068	50.254867	18.06951
$\hat{\bar{Y}}_P$	4.625123	10.033594	4.541537
$\hat{\bar{Y}}_{BT,R}$	38.85633	399.3905	34.85600
$\hat{\bar{Y}}_{BT,P}$	13.60701	23.59207	37.81075
$\hat{\bar{Y}}_{dif}$	152.2060	483.0771	145.206
$\hat{\bar{Y}}_{R,D}$	156.2063	488.0772	147.2063
$\hat{\bar{Y}}_{Singh}$	38.85633	399.3905	34.85600
$\hat{\bar{Y}}_{GK}$	156.2063	497.0772	154.2063
$\hat{\bar{Y}}_s$	186.663	530.1053	196.2137
$\hat{\bar{Y}}_{ss}$	240.8563	588.1077	250.6259

8. Discussion

We used two actual data sets to test the effectiveness of our suggested estimator under simple random sampling. Tables 1 and 3 include the summary statistics of these data sets. According to the mathematical findings, which are shown in Tables 2 and 4, the suggested estimator is effective in terms of effectiveness. A similar PRE based on simulation is shown in Table 5. It can be demonstrated that the suggested estimator outperforms all its competitors. The suggested estimator in SRS produces the best results when the variables Y and X have a positive correlation, as shown by the percent relative efficiency. Overall, we can say that the suggested estimator performs better than every other estimate now in use.

9. Conclusion

With the help of an auxiliary variable based on the sample mean and rank of the auxiliary variable, we have developed a new, improved estimator for the population mean under simple random sampling in this article. Using a simulation study and two real data sets, the suggested estimators are contrasted with their current counterparts in order to assess their robustness and generalizability. The first order of approximation is used to derive MSE expressions. The numerical outcome shows that the suggested estimators outperform their existing counterparts. Therefore, for future evaluation, we strongly advise using the suggested estimators.

Data Availability

All the data used for this study can be found inside the manuscript.

Funding

No specific funding was received to conduct this study.

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