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Adaptive optimal control of the production inventory system in supply chain management with completely unknown dynamics

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A B S T R A C T

This paper describes the adaptive optimal control of the inventory production system with Weibull-distributed deterioration items. First of all, the dynamic model of the system is presented with all possible disturbances and uncertainties. Then, it is controlled using an adaptive and optimal controller. In this method, by having numerical data from the output of the system without using its dynamic equations, an LQR controller is estimated for it. This is important and practical because in physical systems under significant disturbances and fundamental uncertainties, the dynamic equations of the system will not have the former reliability; And it is possible to change the equations of motion by adding any non-linearity so that the conventional controllers will suffer an error. Finally, it is shown that due to the nature of the system and existing uncertainties, the used method has a clear advantage over other optimal control methods and their application in optimizing the inventory production system in the supply chain.

1. Introduction

Optimal control theory is a useful tool for solving dynamic inventory and production problems. The production system includes the manufacturing plant and the finished goods in the warehouse to store those products that are made but not sold immediately. Excess inventory is sold during periods of high demand. The advantages of having products in inventory are: first to meet demand, second by using a warehouse to store surplus production. The company must assess the high costs of production and find the quantity that should be produced to keep the total cost to a minimum. The main objective of the paper is to minimize the difference between the actual production flow rate and the required production flow rate (Zaher & Zaki, 2014). As it is known, controlling a supply chain system despite all the uncertainty and disturbances on it is very vital; according to the system conditions and requirements, it is selected or designed from among all available controllers. However, in order to compensate for disturbances and uncertainties and increase the stability of the system, an adaptive and optimal control system is introduced that can provide an estimate of the optimal control benefits using the system data at any moment. This is important because the dynamic equations of the system in a state that faces major disturbances or fundamental uncertainties can be different from the state that is in a steady state. Therefore, conventional control systems will not be able to handle strong disturbances. Emamverdi et al. (2011) presented the optimal control of the production inventory system with perishable items in which the rate of deterioration follows the Weibull distribution. They set the optimal production rate to minimize total production and inventory costs. Foul et al. (2009) presented a production inventory system consisting of two stores. The model is presented as an optimal control problem with two state variables, the inventory levels in the first store

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and the same in the second store. This paper also considered three control variables, production, remanufacturing, and disposal rates. By using Pontryagin's minimum principle, optimal control of the reverse logistics model of a manufacturing inventory system found. Varbie et al. (2009) presented a model in which a novel policy iteration technique is used to solve the continuous-time LQR problem online without using knowledge about the detailed dynamics of the system. Chaudhary et al. (2013) considered market segmentation as a vital element of marketing in industrialized countries. They used a market segmentation approach in a single-item inventory system with deteriorating items and optimal control using Pontryagin's maximum principle. Adida and Perakis (2007) investigated a continuous-time optimal control model for a dynamic inventory and pricing system problem without stock orders. They presented a continuous time solution approach using Pontryagin's principle for bounded problems. They showed the role of capacity and the dynamic nature of demand in the model. Yang and Wee (2006) defined deterioration as obsolete decay, damage, spoilage, evaporation, theft, and loss of ultimate value or loss of existence of a product that affects. In terms of decreasing utility from the original version, Singh and Kumar (2011) presented a method based on a genetic algorithm to improve inventory performance in supply chain management using MATLAB software. The algorithm considered in this section is the work introduced by Emamverdi et al. (2011. In this research, a computational method for extracting the optimal benefits of the adaptive controller for a system with uncertain dynamics has been implemented in a supply chain system. In this method, by having numerical data from the output of the system and without using its dynamic equations, an LQR controller is estimated for it. This is important and practical because in physical systems under major disturbances and fundamental uncertainties, the dynamic equations of the system will not have the former reliability; And it is possible to change the equations of motion by adding any non-linearity so that the conventional controllers will suffer an error. Some important features of the introduced method are as follows:

1- Using the dynamic programming technique in the estimation-comparative method
2- Iterative solution of Riccati's algebraic equation (which is a definition of LQR optimal control)
3- Use of online system input and output information
4- No need to use a mathematical model for system dynamics
5- The used algorithm is completely fast and online and can be used in surveillance and security systems with the requirement of high response speed.

This optimal controller extracts the optimal control input without the need to know the mathematical model of the system and only by using online measurements of the system states.

2. Problem Statement

As stated by Boukas et al. (2000), a production system consisting of one machine and producing one type of item is considered. The dynamics of the stock level can be described by the following differential equation:

$$\dot{x}(t) = Ax(t) + Bv(t - \tau(t)) + B_1 w(t) \tag{1}$$

where x(t) represents the number of produced parts in the stock level at time t, $\tau(t)$ *is* the processing time, A, B, and B_1 are known constant matrices, *v(t)* is the production rate of the production system, and *w(t)* is an energy bounded disturbance from L_2 [0, ∞). Now, by implementing an adaptive and optimal controller, the system can be returned to its original and stable state. For this purpose, the procedure is similar to Jiang and Jiang (2012). In this case, by considering a completely unknown system in terms of dynamics (but with linearization capability), a method is implemented that can extract the optimal control output. If we consider a linearized system (Eq. 1) in the presence of disturbance as follows:

$$\dot{x} = Ax(t) + Bv(t - \tau(t)) + B_1 w(t) = Ax + B(u + \tilde{u}) \tag{2}$$

where $\tilde{u}$ is the disturbance to the system. First of all, it is possible to remove the disturbance from the system and rewrite the system equation in the standard form without disturbance. Now, the optimal control output will be as follows:

$$u = -Kx \tag{3}$$

which optimizes the following cost function:

$$\int_0^\infty (x^T Qx + u^T Ru)dt \tag{4}$$

3. Solution

The matrix of *K* coefficients in Eq. 3 is extracted in the following form Jiang and Jiang (2012):

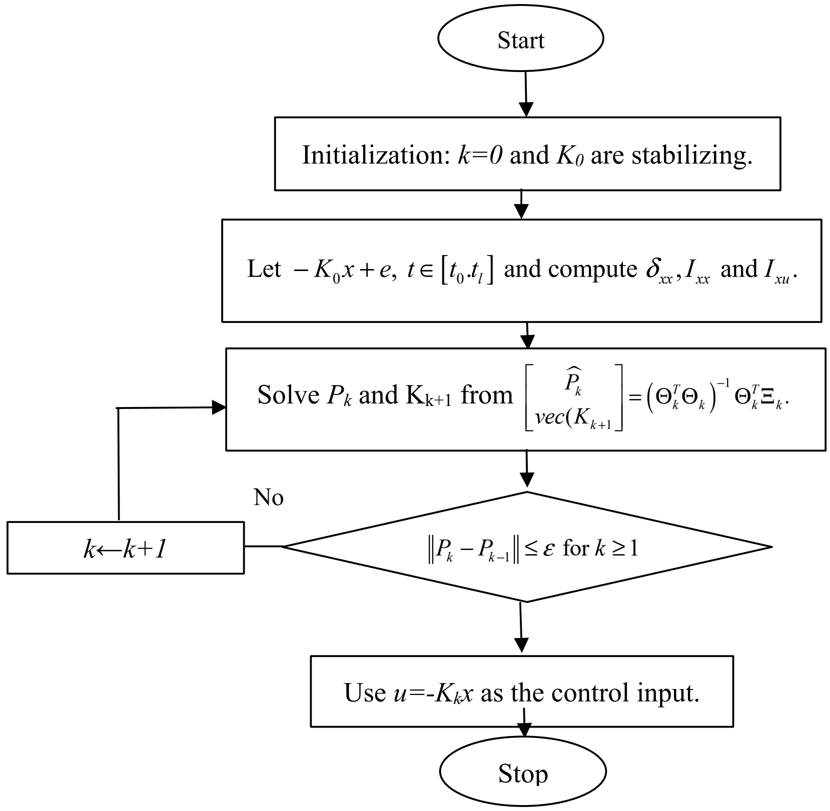

Fig. 1. A view of the implemented optimal adaptive controller (Jiang & Jiang, 2012)

where x_i is the states of the system in the i-th sampling of the signal and we have:

$$\overline{x}=[x_1^2,x_1x_2,...,x_1x_n,x_2^2,x_2x_3,...,x_{n-1}x_n,x_n^2]^T \tag{5}$$

and also:

$$\begin{aligned}\delta_{xx}&=[\overline{x}(t_1)-\overline{x}(t_0),\overline{x}(t_2)-\overline{x}(t_1),...,\overline{x}(t_l)-\overline{x}(t_{l-1})]^T\\ I_{xx}&=\left[\int_{t_0}^{t_1}x\otimes x d\tau,\int_{t_1}^{t_2}x\otimes x d\tau,...,\int_{t_{l-1}}^{t_l}x\otimes x d\tau\right]^T\\ I_{xu}&=\left[\int_{t_0}^{t_1}x\otimes u d\tau,\int_{t_1}^{t_2}x\otimes u d\tau,...,\int_{t_{l-1}}^{t_l}x\otimes u d\tau\right]^T\end{aligned} \tag{6}$$

where ⊗ is the Kronecker multiplier. and finally:

$$\begin{aligned}\Theta_k&=[\delta_{xx},-2I_{xx}(I_n\otimes K_k^TR)-2I_{xu}(I_n\otimes R)]\\ \Xi_k&=-I_{xx}\text{vec}(Q_k)\\ Q_k&=Q+K_k^TRK_k\end{aligned} \tag{7}$$

that the estimation of the optimal control coefficients K_i and the unknown coefficients in the Lyapunov equation P_i is equal to:

$$\begin{bmatrix}\hat{P}_k\\ vec(K_{k+1})\end{bmatrix}=(\Theta_k^T\Theta_k)^{-1}\Theta_k^T\Xi_k \tag{8}$$

As stated, in the above algorithm and the stated method, it is easy to design an optimal controller free of existing disturbances and uncertainties for a linear system (here the supply chain system). This optimal controller extracts the optimal control input without the need to know the mathematical model of the system and only by using online measurements of the system states.

4. Simulation study

In the following, we compare the results by examining an example from Boukas et al. (2000). A system with the following equation is the subject of research:

$$A = \begin{bmatrix} 0.5 & 1 \\ 0.1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix} \tag{9}$$

The weighting matrices are selected to be:

$$Q = I_2, \quad R = 1. \tag{10}$$

Now, by simulating this system and recording the outputs, we apply the following disturbances to the system:

$$w = 100 \sum_{i=1}^{100} \sin(\omega_i) \tag{11}$$

where i=1,2,...100 and each ω_i is randomly selected in the interval $[-500,500]$. In the same conditions, the convergence of the matrix of optimal coefficients is as follows:

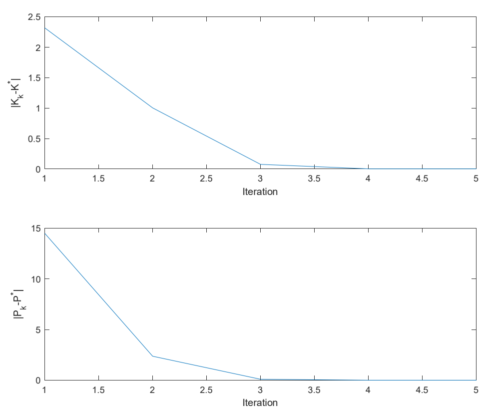

Fig. 2. The results obtained by the method presented in this research

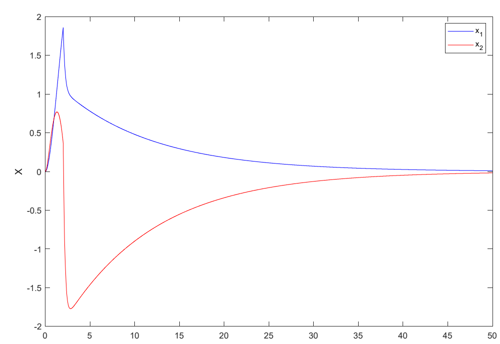

Fig. 3. Outputs of the system by applying the adaptive optimal controller in the Time 2

As is known, Fig. 2 shows the number of numerical repetitions required to reach the desired error in the estimation of K and P matrices. As it is known, for a fixed P, initially, the introduced method provides a good estimate of K. After that, by starting the repetition loops and reducing the error rate from the optimal value, the presented method reaches the convergence in the acceptable error range in a short time. The outputs of the system are given Fig. 2. As shown in Fig. 3, the system states have an initial error and have started moving outside the acceptable range. In Time 2 onwards, by applying the control method introduced to the system, the optimal coefficients matrix was estimated. In the following, this matrix of coefficients is used by the corresponding controller (LQR) and the state vectors of the system lead to close to the desired values. The improvement in system motion from Time 2 onwards is clearly visible. Generally, as shown in Fig. 3, in Time 2 by applying the control law, the system is close to its stable state and the efficiency of the controller can be seen well. For a better Comparison, a system as a four-echelon serial supply chain (SC) realization for a capacity–inventory management model is considered. the general structure of a four-echelon serial SC is integrated by factory (F), distributors (D), wholesalers (W), and retailers (R), as presented in Fig. 4:

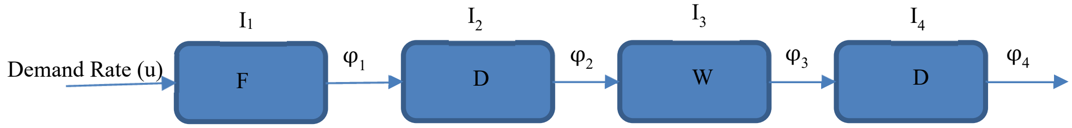

Fig. 4. Schematic of four-echelon serial SC realization for a capacity–inventory management model.

The dynamics equation of the SC as the set of coupled ordinary differential equations are (Taboada et al., 2022):

$$\begin{cases} \dot{I}_1 = -u - \varphi_1 \dfrac{I_1}{C_1} \\ \dot{I}_2 = \varphi_1 \dfrac{I_1}{C_1} - \varphi_2 \dfrac{I_2}{C_2} \\ \dot{I}_3 = \varphi_2 \dfrac{I_2}{C_2} - \varphi_3 \dfrac{I_3}{C_3} \\ \dot{I}_4 = \varphi_3 \dfrac{I_3}{C_3} - \varphi_4 \dfrac{I_4}{C_4} \end{cases} \tag{12}$$

which can be rewritten as follows:

$$\dot{I} = AI + Bu, \quad \begin{cases} I = \begin{bmatrix} I_1 & I_2 & I_3 & I_4 \end{bmatrix}^T, \quad B = \begin{bmatrix} -1 & 0 & 0 & 0 \end{bmatrix}^T \\ A = \begin{bmatrix} -\dfrac{\varphi_1}{C_1} & 0 & 0 & 0 \\ \dfrac{\varphi_1}{C_1} & -\dfrac{\varphi_2}{C_2} & 0 & 0 \\ 0 & \dfrac{\varphi_2}{C_2} & -\dfrac{\varphi_3}{C_3} & 0 \\ 0 & 0 & \dfrac{\varphi_3}{C_3} & -\dfrac{\varphi_4}{C_4} \end{bmatrix}. \end{cases} \tag{13}$$

where C_i is the i[th] echelon capacity of the system, I_i is the i[th] inventory level and the φ_i is the i[th] production rate. The weighting matrices are selected to be:

$$Q = [0], \quad R = 1. \tag{14}$$

The outputs of the system are as follows:

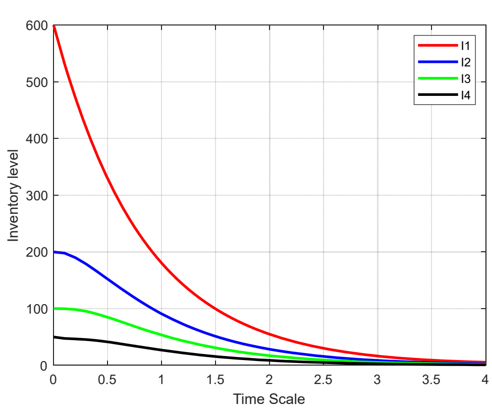

Fig. 5. Outputs of the system by applying the adaptive optimal controller

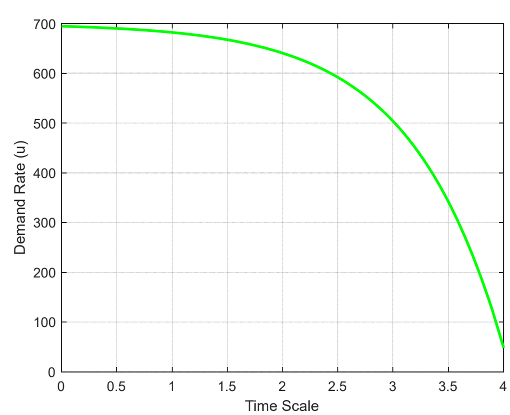

Fig. 6. Input of the system by applying the adaptive optimal controller

Fig. 5 presents the inventory level for factories and distributors for a time horizon of 4 arbitrary units. It can be seen that factory inventory starts from a higher inventory level compared to distributors. Also in this figure, an analysis of wholesaler and retailer inventory levels is presented considering a higher inventory level for wholesalers than retailers. Fig. 5 presents the demand rate graph for a time horizon of 4 arbitrary units. considering time evolves to the final time horizon, the demand on the serial supply chain tends to be zero.

5. Conclusion

In this paper, describes the adaptive optimal control of the inventory production system with Weibull-distributed deterioration items. In this method, by having numerical data from the output of the system without using its dynamic equations, an LQR controller is estimated for it. by applying the control method introduced to the system, the optimal coefficients matrix was estimated. In the following, this matrix of coefficients is used by the corresponding controller (LQR) and the state vectors of the system lead to close to the desired values. By applying the control law, the system is close to its stable state and the efficiency of the controller has been seen. Finally, it is shown that due to the nature of the system and existing uncertainties, the used method has a clear advantage over other optimal control methods and their application in optimizing the inventory production system in the supply chain.

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