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An empirical study on measuring the relative efficiency using DEA method: A case study of bank industry

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A B S T R A C T

Data Envelopment Analysis (DEA) has been widely used as an effective tool for measuring the relative efficiency of similar units by considering various input/output parameters. This paper examines DEA models for the estimation and improvement of organizational inputs and outputs in order to enhance management and decision making processes. We propose an empirical DEA analysis on banking sector by considering several financial and non-financial inputs and outputs. The relative efficiencies of various branches of banks are analyzed in different scenarios. The preliminary results indicate that there are some non-financial items that could significantly change the overall performance of a unit along with other financial items.

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1. Introduction

Measuring the relative efficiency is one of the main concerns on many financial institutions such as insurance companies, banks, etc. DEA has been one of the most effective tools for calculating the relative efficiency of similar units. The advantage of using DEA is that one may use the non-financial factors along with the financial figures to have a fair comparison of different units. DEA has become a popular method among practitioners due to simple implementation and interpretation. During the past few years, there has been tremendous interest on using DEA models for measuring the relative efficiency of banks in the world (Haslem et al., 1999; Mercan et al., 2003). Yang et al. (2010) proposed an integrated bank performance assessment and management planning using hybrid minimax reference point – DEA approach.

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Staub et al. (2010) studied different factors affecting the relative efficiency of Brazilian banks such as cost and technical efficiencies for a time period between the years of 2000 to 2007. They reported that Brazilian banks suffered from low levels of efficiency compared with European or North American banks. They also reported that state-owned banks were significantly more cost efficient than other foreign banks. However, they did not find any evidence to claim that the differences in economic efficiency were due to the type of activity and bank size. Avkiran (2010) studied the relationship between the supper-efficiency estimations and some major key financial ratios for some Chinese banking sector. The method provides some opportunity to detect the inefficient units where there is a low correlation between the supper-efficiency and good financial ratios. Lin et al. (2009) performed different DEA models for 117 branches of a certain bank in Taiwan and reported an overall technical efficiency of 54.8 percent for all banks. The results of their survey also showed that most branches were relatively inefficient. Thoraneenitiyan and Avkiran (2009) surveyed the implementation of an integrated DEA and SFA to measure the impact of restructuring and country-specific factors on the efficiency of post-crisis east Asian banking systems from 1997 to 2001. They reported that banking system inefficiencies were mainly attributed to country-specific conditions, such as high interest rates, concentrated markets and economic development. DEA was also used for banking decisions. For instance, Che et al. (2010) used a combination of Fuzzy analytical hierarchy procedure (AHP) and DEA as a decision making facility for making bank loan decisions.

This paper is organized as follows. We first present the problem statement of DEA method in section 2. Section 3 presents an in-depth discussion of different DEA models for input and output estimation together with efficiency improvement and mathematical calculation methods. We present the implementation of the DEA approach for banking sector in section 4. Finally, concluding remarks are given in the last section to summarize the contribution of the paper.

2. Data Envelopment Analysis

The constant return to scale DEA (CCR) was first proposed by Charnes, et al. (1978, 1994) as a mathematical tool for measuring the relative efficiency of decision making units (DMU). One may easily understand how a given DMU works whenever a production function is available. However, in many cases obtaining an analytical form for this function is not possible. Therefore, we form a set of production feasibility which constituts of some principles such as fixed-scale efficiency, convexity and feasibility as follows,

$$T_C = \left\{ (X,Y) \middle| X \geq \sum_{j=1}^{n} \lambda_j X_j , Y \leq \sum_{j=1}^{n} \lambda_j Y_j , \lambda_j \geq 0, j = 1, \cdots n \right\}, \tag{1}$$

where X and Y represent the input and output vectors, respectively. The CCR production feasibility set border defines the relative efficiency in which any off-border DMU is regarded as inefficient. The CCR model can be determined in two forms of either input or output oriented. The input CCR aims to decrease the maximum input level with a ratio of θ so that, at least, the same output is produced, i.e.:

$$\begin{aligned} &\min \quad \theta \\ &\text{subject to} \\ &\theta X_p - \sum_{j=1}^{n} \lambda_j X_{ij} \geq 0, \\ &\sum_{j=1}^{n} \lambda_j Y_{rj} \geq Y_{rp}, \\ &\lambda_j \geq 0, \qquad j = 1, \cdots, n. \end{aligned} \tag{2}$$

Model (2) is called envelopment form of input CCR where θ is the relative efficiency of the DMU and it is an easy assignment to show that the optimal value of θ, θ^*, is always between zero and one.

In an input oriented DEA model, once the efficiency of a DMU unit, DMU_p, drops in case of inefficiency, one may directs it towards the border to make it efficient. In the case of the output oriented DEA model, the primary aim is to maximize the output level, φ , by using the same amount of input. The model can be formulated as follows,

$$\begin{aligned} &\min \quad \varphi \\ &\text{subject to} \\ &\sum_{j=1}^{n} \lambda_j X_{ij} \le X_{ip}, \\ &\sum_{j=1}^{n} \lambda_j Y_j \ge \varphi Y_{ip}, \\ &\lambda_j \ge 0, \quad j = 1, \cdots, n. \end{aligned} \tag{3}$$

3. DEA Models for Estimating and Improving Inputs and Outputs

3.1 Output estimation

Consider n different DMUs as $\{DMU_j : j=1,...,n\}$ using m inputs to generate s outputs. Let y_{ri} and x_{ij} be the rth output, $r = (1, \cdots, s)$ and the ith input, $i = (1, \cdots m)$ of the jth DMU, $j = (1, \cdots n)$, respectively. Let φ^* be the efficiency level of the DMU_p where it has a value of one or higher, i.e. the measured unit is either efficient or inefficient. Suppose that we increase the inputs of DMU_p from x_p to $\alpha_{ip} = x_{ip} + \Delta x_{ip}$ where $\Delta x_p \ge 0$ and $\Delta x_p \ne 0$ and we want to know how much output DMU_p would be generated. That is we want to estimate the output vector $y_{rp(new)} = (y_{1p(new)}, y_{2p(new)}, ...y_{sp(new)})$, where we present them as $\beta_{rp} = (\beta_{1p}, \beta_{2p}, ...\beta_{sp})$, for the sake of the simplicity. We also consider two conditions for our problem statement. First, it is assumed that as the inputs increase, φ^* remains unchanged and second, it is assumed that as the inputs increase the efficiency will also increase. If efficiency increase is not the target and the efficiency of DMU_p remains at φ^*, the outputs of the measured unit can be calculated by solving the following,

$$\begin{aligned} &\max \quad \beta_p = (\beta_{1p}, \cdots, \beta_{sp}) \\ &\text{subject to} \\ &\sum_{j=1}^{n} \lambda_j X_{ij} \le \alpha_{ip} \\ &\sum_{j=1}^{n} \lambda_j Y_{rj} \ge \varphi_p^* \beta_p \\ &\beta_p \ge Y_p \\ &\lambda_j \ge 0 \quad j = 1...n. \end{aligned} \tag{4}$$

Model (4) is a multi-purpose problem to solve where we may assign relative weights (w_p) to each output (y_{ip}) using a multiple criteria decision making methods such as AHP. Let

$\beta_{rp} = (\beta_{1p}, \beta_{2p}, ...\beta_{sp}) = \sum_{r=1}^{s} w_r \beta_{rp}$. Therefore we have,

$$\begin{aligned} &\max \quad \beta_p = (\beta_{1p}, \cdots, \beta_{sp}) = \sum_{r=1}^{s} w_r \beta_{rp} \\ &\text{subject to} \\ &\sum_{j=1}^{n} \lambda_j X_{ij} \le \alpha_{ip} \end{aligned}$$

$$\sum_{j=1}^{n}\lambda_j Y_{rj} \geq \varphi_p^* \beta_p \tag{5}$$
$$\beta_p \geq Y_p$$
$$\lambda_j \geq 0 \qquad j = 1...n.$$

Let Δx_p be the increase on the inputs of unit *p* and η be the percentage of the increase on φ^*. In order to obtain the output for unit *p* we may replace φ^* with $(1-\frac{\eta}{100})\varphi^*$ in (5) which yields,

$$\max \quad \beta_p = (\beta_{1p}, \cdots, \beta_{sp}) = \sum_{r=1}^{s} w_r \beta_{rp}$$

subject to

$$\sum_{j=1}^{n}\lambda_j X_{ij} \leq \alpha_{ip},$$
$$\sum_{j=1}^{n}\lambda_j Y_{rj} \geq \left[(1-\eta/100)\varphi_p^*\right]\beta_p, \tag{6}$$
$$\beta_p \geq Y_p,$$
$$\lambda_j \geq 0 \qquad j = 1...n.$$

3.2 Input estimation

Let θ^* be the optimal efficiency value of the DMU measured by model (2) and we intend to increase the production of DMU$_p$ by $\Delta y_p \geq 0$, that is $y_{rp(new)} = \beta_{rp} = y_{rp} + \Delta y_{rp}$. Assuming a constant efficiency of the measured DMU we can estimate the inputs of the unit *p* with similar approach given in the previous section. Let $x_{ip(new)} = (x_{1p(new)}, x_{2p(new)}, ...x_{mp(new)}) = \alpha_{ip} = (\alpha_{1p}, \alpha_{2p}, ...\alpha_{mp})$ and to simplify the solution of the multi-purpose function, one may rewrite the target function as $\alpha_{ip} = (\alpha_{1p}, \alpha_{2p}, ...\alpha_{mp}) = \sum_{i=1}^{m} w_i \alpha_{ip}$ and solve the following model,

$$\min \quad \alpha_{ip} = (\alpha_{1p}, \alpha_{2p}, ...\alpha_{mp}) = \sum_{i=1}^{m} w_i \alpha_{ip}$$

subject to

$$\sum_{j=1}^{n}\lambda_j X_{ij} \leq \theta^* \alpha_{ip} \qquad i = 1...m$$
$$\sum_{j=1}^{n}\lambda_j Y_{rj} \geq \beta_{rp} \qquad r = 1...s \tag{7}$$
$$\alpha_{ip} \geq x_{ip}$$
$$\lambda_j \geq 0 \qquad j = 1...n.$$

Let δ be the percentage increase in efficiency of θ^* resulted when the outputs are increased. Let θ^* is replaced with $(1+\frac{\delta}{100})\theta^*$. Therefore we have,

$$\min \quad \alpha_{ip} = (\alpha_{1p}, \alpha_{2p}, ...\alpha_{mp}) = \sum_{i=1}^{m} w_i \alpha_{ip}$$

subject to

$$\sum_{j=1}^{n}\lambda_j X_{ij} \leq (1+\delta/100)\theta^* \alpha_{ip} \qquad i = 1...m \tag{8}$$
$$\sum_{j=1}^{n}\lambda_j Y_{rj} \geq \beta_{rp} \qquad r = 1...s$$

$\alpha_{ip} \geq x_{ip}$

$\lambda_j \geq 0 \qquad j = 1...n.$

However, if the amount of efficiency increase is not specified and the measured organization requires such increase as a precondition for increase in the outputs, then the input estimation of model (7) will be changed to model (8) where $\theta \geq \theta^*$ is an additional condition.

4. Analysis and Results

In this section we present the details of our DEA implementation for an Iranian banking system. The data for the input and the output are collected for the fiscal year of 2006. The study uses four inputs and six outputs shown in Fig. 1.

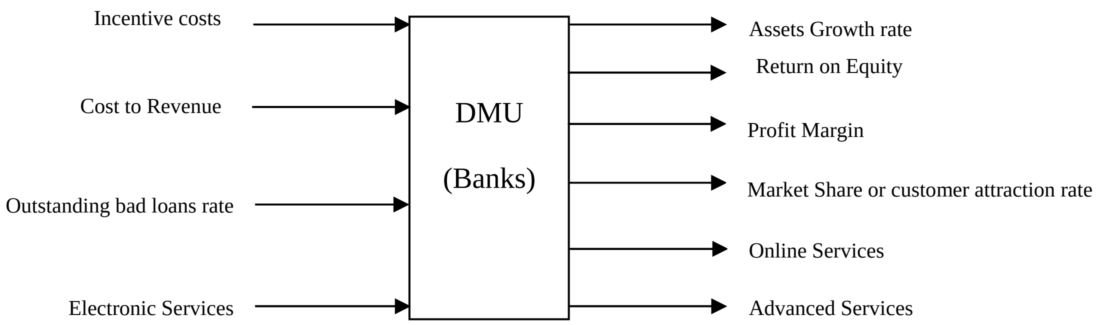

Fig. 1. The input and the output of DEA model

The input data for all ten units are summarized in Table 1 where the first column represents the number of electronic services in terms of the number, the second column shows the percentage of the incentive costs, the thirst column depicts the ratio of cost to income in terms of the percentage and the last column provides the information of the rate of the outstanding bad loans.

Table 1
The necessary input data

DMU	Electronic services	Incentive costs	Cost/Income	Outstanding bad loans rate
1	1305	23.03	52.84	2.68
2	1906	18.72	42.77	9.5
3	1758	18.50	60	15
4	1500	5.30	60.2	8.5
5	745	17	57.90	7.3
6	517	3	96	14
7	957	16	72	7.8
8	1310	21	83	10
9	982	17	48	9.5
10	793	13	51	6.5

The output data are also tabulated in Table 2 where the first column shows the online services in terms of the number, the second column represents the number of advanced services, the third column provides the growth rate of assets, the fourth column specifies the rate of growth margin and the last column shows the growth in market share to absorb new customers.

Table 2
The output information

DMU	Online services	Advanced services	Assets growth rate	Investment return	Profit margin	Customer attraction rate
1	1376	74	17.42	4.81	1.48	22.91
2	1560	57	12.98	7.16	2.62	25.8
3	1842	8	47.59	7	8	29
4	1315	37	18.9	1.4	2.7	34.50
5	787	34	20.13	1.23	3	21.8
6	409	10	10.28	1.02	4	13
7	650	18	12.10	1.4	2.8	18
8	1120	36	18	3.5	3.8	21
9	980	28	15	2.5	5.3	19
10	784	37	23.5	1.6	1.9	21

Applying the DEA models explained in the previous section yields the relative efficiencies of different units in two different cases. Table 3 summarizes the results of the efficiency estimation in two cases of input and output based.

Table 3
Efficiency measurement results

DMU	1	2	3	4	5	6	7	8	9	10
Output oriented efficiency	1	1	1	1.182	1	1	1.52	1.20	1	1
Input oriented efficiency	1	1	1	0.845	1	1	0.65	0.83	1	1

As we can observe from Table 4, there are three inefficient units of 4, 7 and 8 when we solve the system based on the output oriented system and the rest of them are located on efficient frontier which means they are efficient. Since we use only four inputs we can change their values to find out how to make the inefficient unit efficient. We have performed this analysis in different cases and the following is the summary of our experience.

Table 4
The relationships between the number of electronic services and asset growth

Electronic services	1400	1450	1500	1550	1600	1650	1700	1750	1800
Asset growth rate	0.189	0.189	0.237	0.249	0.26	0.27	0.284	0.296	0.308

We have studied the relationship between the online services and the growth rate on assets. Table 4 summarizes the results of our survey. As we can observe from Table 4, when the number of electronic services is limited to 1400, we could expect approximately 19 percent of growth on banks' assets. However, when we increase the number of services from 1400 to 1900 we could expect an over 30 percent growth on our assets. In other word, an increase on electronic services could significantly contribute to asset growth. We have also studied the relationship between the electronic service, as an input element, and online service as output factor. Table 5 summarizes the details of our survey between these two items. As we can see, an increase to the number of electronic services could also contribute to the number of online services. In other word, a 400 increase on the number of electronic services could result to an increase of about 268 online services.

Table 5
The relationships between the number of electronic services and the number of online services

Electronic services	1400	1450	1500	1550	1600	1650	1700	1750	1800
Online Services	1315	1315	1358	1401	1445	1487	1530	1530	1573

Another interesting issue is to study the relationship between the efficiency and the growth on assets. Table 6 shows the details of our study on these two factors for DMU 4.

Table 6
The relationships between the efficiency and the asset growth rate

Efficiency (θ)	1.182	1.175	1.15	1.125	1.1	1.075	1.05	1.025	1.00
Asset growth rate	0.237	0.24	0.253	0.266	0.28	0.295	0.31	0.326	0.343

As we can observe from Table 6 when unit 4 is completely inefficient with $\theta = 1.182$ the average growth on total assets is limited to 0.237. However, we can expect an over 10 percent increase on asset growth when unit 4 becomes efficient. There is also a similar pattern on the number of online services and the efficiency of this unit which is shown on Table 7.

Table 7
The relationships between the efficiency and the asset growth rate

Efficiency (θ)	1.182	1.175	1.15	1.125	1.1	1.075	1.05	1.025	1.00
Online services	1315	1323	1353	1384	1416	1450	1485	1523	1562

One of the most important observations is the strong relationship between the efficiency and the number of online services. As we can learn from Table 7, when $\theta = 1.182$ for unit 4, the number of online services is limited to only 1315. However, an increase of only 247 online services makes the unit efficient.

Table 8
The effects of simultaneous changes on input/output parameters for unit number four

	Asset Growth Rate	Investment Return	Profit Margin	Customer Attraction rate	Online Services	Advanced Services
Φ = 1.182 Electronic Service = 1500 Outstanding bad loan rate = 0.085	0.237	0.014	0.027	0.345	1315	37
Φ = 1.15 Electronic Service = 1600 Outstanding bad loan rate = 0.1	0.318	0.014	0.027	0.345	1453	37
Φ = 1.1 Electronic Service = 1650 Outstanding bad loan rate = 0.12	0.387	0.014	0.027	0.382	1582	37
Φ = 1.08 Electronic Service = 1700 Outstanding bad loan rate = 0.13	0.419	0.014	0.027	0.409	1667	37

The other interesting issue is to study the effects of the simultaneous changes on different input and output parameters on the efficiency ratio. We have made different changes on two input parameters as well as six output factors and the results are summarized in Table 8.

As we can observe from Table 8, it is possible to increase the efficiency of unit four even when the input parameters are increased. This could be done by hiking the number of online services as well as trying to have a better customer attraction rate.

5. Conclusion

The present study has attempted to examine different DEA models for the estimation and improvement of organizational inputs and outputs in order to enhance management and decision making processes. The results of this survey show that financial figures such as profit margin and return on equity are not the only signals of performance measurement and there are other non-financial characteristics that could affect the efficiency of a bank such as the number of online services, electronic services, etc. The study also emphasizes that there is a strong relationship between some non-financial and financial items such as financial asset growth and electronic services.

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