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Huff-type competitive facility location model with foresight in a discrete space

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A B S T R A C T

Consider a chain as leader that wants to open p new facilities in a linear market, like metro. In this market, there is a competitor, called follower. The leader and the follower have established some facilities in advance. When the leader opens p new facilities, its competitor, follower, reacts the leader's action and opens r new facilities. The optimal locations for leader and follower are chosen among predefined potential locations. Demand is considered as demand points and is assumed inelastic. Considering huff model, demand points are probabilistically absorbed by all facilities. The leader's objective is maximization of its market share after opening follower's new facilities. For solving leader problem, first the follower's problem is solved for all leader's potential locations and the best location for leader is obtained and then, a heuristic model is proposed for leader problem when the leader and the follower want to open one new facility. Computational results show that the proposed method is efficient for large scale problems.

1. Introduction

The aim of location models is to find optimal location for one or more new facilities among existing ones. Location theory is so widespread and entails too many different issues. The vast part of this theory embraces "Location" in monopoly condition in which a chain, a company, a vendor or a manufacturer captures the market, dominantly. In this case, a set of facilities that is going to be located, supply their own products or services and the ownership of all facilities belongs to one person or one company that has no competitor in the market. The review of this kind of researches is carried out by Drezner (1995).

Practically, this assumption is not true in real situations and most of the times, there are more than one company that exist in the market and compete with one another in offering their own products and services. Facility location in this state is more realistic than the aforementioned one in monopoly condition. Competitors for opening their new facilities should consider several factors like customer

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behavior and competitors' reactions. This kind of facility location is called *competitive facility location*.

Researches on competitive facility location models are originated by Hotelling (1929). He considered the competitive facility location under the conditions that customers are uniformly distributed on a line segment, and each of competitors can locate her/his own facility at any locations in this space. All customers use the closest facility in Hotelling model. Huff (1946) defined the attractive function of facility for customers by considering not only the distance but also the quality of facility. In fact, Huff formulated a model for capturing market share assuming that the probability that a customer patronizes a facility has a direct relationship with quality and opposite relationship with the function of distance. Competitive facility location is categorized differently in consideration with various factors. From the standpoint of competitors' reactions, it can be divided into three categories of static competition, competition with foresight and dynamic competition. In this paper, the second category, i.e. competition with foresight is considered. In this type of competition, a chain that has tendency towards opening one or more new facilities knows this fact that after the facility location is determined by the chain, prior or upcoming competitors response immediately and locate similar facilities.

Competitive facility location can be studied based on different aspects such as patronizing behavior, dispensability and indispensability of offered products and services (demand elasticity or inelasticity), customer criteria for selecting facilities, the number of new facilities which are going to be located, etc. As mentioned before, in this paper, competition with foresight is considered and a chain as leader locates new facilities in a linear market. In this case, the leader's competitor is called follower and it is assumed that the leader and the follower have some facilities before locating new ones. The follower reacts to the leader's new facilities location and opens new facilities once leader discloses her decision. The objective of both leader and follower is to maximize their own market share.

In the competitive facility location literature, this kind of problems is known as Stackelberg problems. This type of problem was introduced by Hakimi (1983) for the first time. He used the word "medianoid" for follower problem when follower wants to maximize his market share, profit and etc. Also the word "centroid" for leader problem is used when maximization of her market share, profit or other objective functions is equivalent to minmax of follower’s objective function. In fact, a $(r|x_p) - medianoid$ problem is the one in which the follower is tracing to locate r new facilities in order to maximize his objective function while the leader did the same action before i.e. leader is locating p facilities at x_P points. In a $(r|p) - centroid$ problem, the leader finds the optimal location of new facility of P, when the follower will respond to leader’s action with locating r new facilities and maximization of her objective function is equivalent to minmax of follower’s one. In general, when the demand is inelastic, the leader’s problem will be a $(r|p) - centroid$ problem.

There are also other articles about this kind of problems have been published. Some of these articles have analyzed this problem on a network (Hakimi, 1996, 1990; Serra & revelle, 1994; García & Pelegrín, 1997). A review referring this kind of problems up to the year 1996 is seen in Eiselt and Laporte (1997). Some other papers have solved leader-follower problem for continuous and discrete spaces. An exact algorithm has been used for solving leader-follower problems in a single facility case by Drezner (1982). Macias and Perez (1995) used effective algorithms for solving follower problems with rectilinear distances. Bhadury et al. (2003) solved a $(r|p) - centroid$ problem using alternating heuristic. Drezner and Drezner (1998, 2002) also suggested heuristics for Huff type problems. The leader-follower problem was also proposed to determine the location and design of new facilities in a way that uses branch and bound method (Sáiz et al., 2009) and heuristics (Redondo et al., 2010) for solving the problems. In this paper, huff type leader and follower problem in discrete linear space is considered.

This paper is organized as follows. In section 2, the suggested model is proposed and its solution process is presented in section 3. In section 4, the efficiency of this solution process becomes clear by mingling some examples. Finally, in section 5, obtained results and future researches are offered.

2. Proposed model

In this section, the proposed mathematical model is presented. There are two competitors in the linear market (for example in a metro) with some facilities in this space. First competitor, leader, intends to open p new facilities in p of the predefined potential locations but on the other hand she knows that its competitor, follower, surely responds to its counterpart actions by opening r new facilities. Generally, there are m facilities in the space prior to opening new facilities and the first t facilities belong to the leader and the next $m-t$ facilities correspond to the follower. After opening new facilities by leader and follower, the number of facilities will be $m+p+r$.

The demand in this model is supposed as the demand point (n demand points exist in this space). The existing facilities are located in m of n demand points and the rest of $n-m$ points can be considered as potential locations for new leader's facilities. Since the follower cannot open his new facilities in location of leader's new facilities, the number of potential locations for follower is equal to $n-m-p$. Considering gravity model, demand points are absorbed by the facilities, probabilistically. The amount of a given facility attraction for a customer depends on the distance between the customer and the facility. It also depends on the other characteristics of the facility which determine its quality. Considering Huff model, the amount of attraction has a direct relationship with the quality of the facility and a opposite relationship with the function of the distance between facilities and demand points. It is also supposed that the quality levels of all facilities such as new and existing facilities are predefined. The leader and the follower objective in locating new facilities is to maximize their own market share. First, leader finds optimal locations of p new facilities and knows that the follower will locate r new facilities right after her emprise. It is deemed that the follower is rational and is following to find optimal locations for opening his own facilities.

The following notations are used for formulating the model:

m: The number of existing facilities

p: The number of leader's new facilities

r: The number of follower's new facilities

n: The number of demand points

n_{pot}^{L}: The number of potential locations for leader ($n_{pot}^{L}=n-m$)

n_{pot}^{F}: The number of potential locations for follower ($n_{pot}^{F}=n-m-p$)

i: Index of existing facility; $i=1,2,\dots,t$ Leader's existing facilities and $i=t+1,t+2,\dots,m$ Follower's existing facilities

j: Index of demand points $j=1,2,\dots,n$

k: Index of leader's new facilities $k=1,2,\dots,p$

h: Index of follower's new facilities $h=1,2,\dots,r$

s_L: Index of potential locations for leader $s_L=1,2,\dots,n_{pot}^{L}$

s_F: Index of potential locations for follower $s_F = 1,2, \dots, n_{pot}^F$

z_i: The location of i^{th} existing facility

y_j: The location of j^{th} demand point

p_s^L: The location of s_L^{th} potential location for leader

p_s^F: The location of s_F^{th} potential location for follower

b_j: Buying power of j^{th} demand point (The population or total wealth represented by demand point j)

x_{kL}: The location of k^{th} leader's new facility

x_{hF}: The location of h^{th} follower's new facility

d_{ij}: The distance between i^{th} existing facility and j^{th} demand point

$d_{x_{kL}j}$: The distance between k^{th} leader's new facility and j^{th} demand point

$d_{x_{hF}j}$: The distance between h^{th} follower's new facility and j^{th} demand point

q_{ij}: Quality of i^{th} existing facility for j^{th} demand point

q_{Lj}: Quality of leader's new facilities for j^{th} demand point

q_{Fj}: Quality of follower's new facility for j^{th} demand point

A_{ij}: The amount of i^{th} existing facility attraction for j^{th} demand point

A_{Lj}: The amount of leader's new facility attraction for j^{th} demand point

A_{Fj}: The amount of follower's new facility attraction for j^{th} demand point

$XP_{s_L k}$: A binary variable which is equal to 1 if leader opens her k^{th} new facility in s_L^{th} potential location, 0 otherwise

$XP_{s_F h}$: A binary variable that is equal to 1 if leader opens his h^{th} new facility in s_F^{th} potential location, 0 otherwise

As mentioned before, in conformity with Huff rule, there is a direct relationship between the customer's attraction and the quality. Also there is a reverse relationship between the customer's attraction and the function of distance. Therefore, the amount of i^{th} facility attraction for j^{th} customer is as follows,

$$A_{ij} = {q_{ij}}\big/{(1 + d_{ij}^2)} \tag{1}$$

The amounts of leader's (A_{Lj}) and follower's (A_{Fj}) new facilities attraction for j^{th} demand point are also calculated as follows,

$$A_{Lj} = {q_{Lj}}\big/{(1 + d_{x_{kL}j}^2)}\,, \tag{2}$$

$$A_{Fj} = {q_{Fj}}\Big/{(1 + d^2_{x_{hF}j})}. \tag{3}$$

Market share is calculated based on the buying power of all customers multiplying the probability of customer attraction. Let x_{kL} be the best location of the k^{th} leader's new facility and x_{hF} be the best location of the h^{th} follower's new facility. Therefore the leader's market share is as follows,

$$M_L = \sum_{j=1}^{n} b_j \frac{\sum_{i=1}^{t} \frac{q_{ij}}{(1+d^2_{ij})} + \sum_{k=1}^{p} \sum_{s_L=1}^{n^L_{pot}} \frac{q_{Lj}}{(1+d^2_{x_{kL}j})} XP_{s_L k}}{\sum_{i=1}^{t} \frac{q_{ij}}{(1+d^2_{ij})} + \sum_{k=1}^{p} \sum_{s_L=1}^{n^L_{pot}} \frac{q_{Lj}}{(1+d^2_{x_{kL}j})} XP_{s_L k} + \sum_{i=t+1}^{m} \frac{q_{ij}}{(1+d^2_{ij})} + \sum_{h=1}^{r} \sum_{s_F=1}^{n^F_{pot}} \frac{q_{Fj}}{(1+d^2_{x_{hF}j})} XP_{s_F h}} \tag{4}$$

or

$$M_L = \sum_{j=1}^{n} b_j \frac{\sum_{i=1}^{t} \frac{q_{ij}}{1+(z_i-y_j)^2} + \sum_{k=1}^{p} \sum_{s_L=1}^{n^L_{pot}} \frac{q_{Lj}}{1+(p^L_s-y_j)^2} XP_{s_L k}}{\sum_{i=1}^{t} \frac{q_{ij}}{1+(z_i-y_j)^2} + \sum_{k=1}^{p} \sum_{s_L=1}^{n^L_{pot}} \frac{q_{Lj}}{1+(p^L_s-y_j)^2} XP_{s_L k} + \sum_{i=t+1}^{m} \frac{q_{ij}}{1+(z_i-y_j)^2} + \sum_{h=1}^{r} \sum_{s_F=1}^{n^F_{pot}} \frac{q_{Fj}}{1+(p^F_s-y_j)^2} XP_{s_F h}} \tag{5}$$

The follower's market share is also as follows,

$$M_F = \sum_{j=1}^{n} b_j \frac{\sum_{i=t+1}^{m} \frac{q_{ij}}{(1+d^2_{ij})} + \sum_{h=1}^{r} \sum_{s_F=1}^{n^F_{pot}} \frac{q_{Fj}}{(1+d^2_{x_{hF}j})} XP_{s_F h}}{\sum_{i=1}^{t} \frac{q_{ij}}{(1+d^2_{ij})} + \sum_{k=1}^{p} \sum_{s_L=1}^{n^L_{pot}} \frac{q_{Lj}}{(1+d^2_{x_{kL}j})} XP_{s_L k} + \sum_{i=t+1}^{m} \frac{q_{ij}}{(1+d^2_{ij})} + \sum_{h=1}^{r} \sum_{s_F=1}^{n^F_{pot}} \frac{q_{Fj}}{(1+d^2_{x_{hF}j})} XP_{s_F h}} \tag{6}$$

or

$$M_F = \sum_{j=1}^{n} b_j \frac{\sum_{i=t+1}^{m} \frac{q_{ij}}{1+(z_i-y_j)^2} + \sum_{h=1}^{r} \sum_{s_F=1}^{n^F_{pot}} \frac{q_{Fj}}{1+(p^F_s-y_j)^2} XP_{s_F h}}{\sum_{i=1}^{t} \frac{q_{ij}}{1+(z_i-y_j)^2} + \sum_{k=1}^{p} \sum_{s_L=1}^{n^L_{pot}} \frac{q_{Lj}}{1+(p^L_s-y_j)^2} XP_{s_L k} + \sum_{i=t+1}^{m} \frac{q_{ij}}{1+(z_i-y_j)^2} + \sum_{h=1}^{r} \sum_{s_F=1}^{n^F_{pot}} \frac{q_{Fj}}{1+(p^F_s-y_j)^2} XP_{s_F h}} \tag{7}$$

The follower with the knowledge of optimal location of leader's new facilities is beginning to locate his own facilities. Given x_{kL} , Problem $F(P(x_{kL}))$ of the follower is the $(r|x_{kL}) - medianoid$ problem:

$$\text{Max } M_F = \sum_{j=1}^{n} b_j \frac{\sum_{i=t+1}^{m} \frac{q_{ij}}{(1+d^2_{ij})} + \sum_{h=1}^{r} \sum_{s_F=1}^{n^F_{pot}} \frac{q_{Fj}}{(1+d^2_{x_{hF}j})} XP_{s_F h}}{\sum_{i=1}^{t} \frac{q_{ij}}{(1+d^2_{ij})} + \sum_{k=1}^{p} \sum_{s_L=1}^{n^L_{pot}} \frac{q_{Lj}}{(1+d^2_{x_{kL}j})} XP_{s_L k} + \sum_{i=t+1}^{m} \frac{q_{ij}}{(1+d^2_{ij})} + \sum_{h=1}^{r} \sum_{s_F=1}^{n^F_{pot}} \frac{q_{Fj}}{(1+d^2_{x_{hF}j})} XP_{s_F h}} \tag{8}$$

subject to

$$\sum_{s_F=1}^{n^F_{pot}} XP_{s_F h} = 1 \qquad h = 1,2,\dots,r \tag{9}$$

$$\sum_{h=1}^{r} XP_{s_F h} \leq 1 \qquad s_F = 1,2,\dots,n_{pot}^{F} \tag{10}$$

$$\sum_{h=1}^{r} \sum_{S_F=1}^{n_{pot}^{F}} XP_{s_F h} = r \tag{11}$$

$$XP_{s_F h} \in \{0,1\} \tag{12}$$

The leader has set up its new facilities in x_{kL} ,objective function (8) maximizes follower's market share and constraint (9) states that each new facility locates in only one of the potential locations and constraint (10) shows that each potential location can be the host of, at most, one of the new facilities and finally constraint (11) ensures that the number of potential locations occupied by the new facilities exactly is equal to the number of follower's new facilities. Let $x_{hF}^{*}(x_{kL})$ be the optimal solution of $F(P(x_{kL})$ problem. Problem LP (Leader Problem) for the leader is the $(r|p) - centroid$ problem as follows:

$$\max \quad M_L = \sum_{j=1}^{n} b_j \frac{\sum_{i=1}^{t} \frac{q_{ij}}{(1+d_{ij}^2)} + \sum_{k=1}^{p} \sum_{s_L=1}^{n_{pot}^{L}} \frac{q_{Lj}}{(1+d_{x_{kL}j}^2)} XP_{s_L k}}{\sum_{i=1}^{t} \frac{q_{ij}}{(1+d_{ij}^2)} + \sum_{k=1}^{p} \sum_{s_L=1}^{n_{pot}^{L}} \frac{q_{Lj}}{(1+d_{x_{kL}j}^2)} XP_{s_L k} + \sum_{i=t+1}^{m} \frac{q_{ij}}{(1+d_{ij}^2)} + \sum_{h=1}^{r} \sum_{s_F=1}^{n_{pot}^{F}} \frac{q_{Fj}}{(1+d_{x_{hF}^{*}j}^2)} XP_{s_F F}} \tag{13}$$

subject to

$$\sum_{s_L=1}^{n_{pot}^{L}} XP_{s_L k} = 1 \qquad k = 1,2,\dots,p \tag{14}$$

$$\sum_{k=1}^{p} XP_{s_L k} \leq 1 \qquad s_L = 1,2,\dots,n_{pot}^{L} \tag{15}$$

$$\sum_{k=1}^{p} \sum_{s_L=1}^{n_{pot}^{L}} XP_{s_L k} = p \tag{16}$$

$$XP_{s_L k} \in \{0,1\} \tag{17}$$

The leader's objective function in Eq. (13) and the constraints (14-16) are described as the follower's objective function and constraints, respectively.

3. Solution approach

In this section we explain the solution procedure for the proposed method. As we already explained, the leader knows that after she locates her new facilities, the follower who is assumed to be a rational person will surely open his own new facilities at the optimal locations. Therefore, the follower's problem in order to maximize his market share can be solved by considering arbitrary locations for the leader facilities. Next, we find the optimal locations for the follower facilities in order to maximize follower's market share. The follower problem is a mixed integer nonlinear programming problem whose solution can be determined for small and medium scale problems but we may end up having a local optimum solution for large-scale problems.

In small scales, the follower problem is solved for all leader's potential locations (which equals $n - m$ points) in order to obtain an exact solution for the leader problem. The exact solution of the follower problem which exists in the other optimal points (the follower can't open his new facilities in the place the leader has located her facilities before) is achieved for each of these points. Since the leader problem in this paper is a $(r|p) - centroid$ problem, the maximum value of the leader's market share is in the locations where the maximum value of follower's market share is minimized. Therefore, given the optimal solutions for the follower for all leader potential points, the best location for the leader's new facility is the one with the minmax of follower's objective function.

This method is time-consuming for large-scale problems and it may lose its efficiency in the leader-follower problems in spite of offering the exact solutions. In the following, the proposed method for large-scale problems when the leader and the follower want to open a new facility is proposed. Before we go further, we review some the necessary issues associated with the proposed method.

There are different points which could be eliminated from the leader's possible optimal point set and reduce the set of the potential points which reduces the burden of the computations. There are four rules in the following which are used in reducing probable optimal point set:

1. Demand is assumed to be inelastic in this paper and leader does not intend to open her new facilities in the proximity of her existing facilities so that they would not lose their market.
2. There are many real-world cases where the optimal solution of the leader problem is located approximately in the middle of two existing facilities or at the beginning points and terminal points of the market.
3. There are also cases where the leader chooses the optimal location of her new facility in the proximity of the follower's existing facilities to cannibalize his market share.
4. The points with higher buying power are more prone to obtain optimal solution for the leader's problem.

In summary, among all potential alternatives, we choose the ones which are located between leader's two existing facilities and they are also in the proximity of the follower facilities or the points with relatively higher buying power. The following summarizes the necessary steps of the proposed method.

Step 1: Prepare set *A* which incorporates the appropriate alternatives based on the above four rules

Step 2: Solve the follower problem for each elements of the set *A* and obtain the optimal points for the follower problem and the values of the follower's objective functions, If the solution of the follower problem for all points of set *A* becomes similar, go to step 4 and if more than one solution is get go to step 3

Step 3: Among different solutions for follower problem, keep the point with the minimum objective function value and eliminate the others

Step 4: Substitute the leader problem with the follower problem for the point which is obtained

The leader problem is solved with the known solution of the follower problem. The obtained solution of this problem is the leader's optimal solution. The obtained solution of this problem is the leader's optimal solution.

4. Numerical examples

In this section, some instances are used to evaluate the results of proposed model and solution approach. Seven examples are offered in this section. The first example is the $(2|2) - centroid$ small-sized problem. The other 6 examples are the $(1|1) - centroid$ problems. The second one is a large-scale and we used the proposed method to examine the performance of the proposed method for this type of problems. Five instances with medium size are considered and solved by general approach and proposed method to compare the results of proposed solution approach with optimal solutions. The first small size problem and the large-scale problem are described with details data in the following section.

Example 1: There are 15 demand points and 5 existing facilities in the market. Three of these facilities belong to the leader and the others are considered as follower's facilities. The leader wants to open two new facilities and knows that the follower will open two new facilities after her action. As mentioned in section 2, facilities are located in demand points, so there are 10 potential points for leader's new facilities and 9 for follower's new facilities. Each demand point has a different buying power from the others. The buying power is randomly selected for different demand points in a range of 1 to 10. Quality values are also determined randomly in a range of 1 to 5 for new and existing leader and follower facilities. The locations of demand points and the leader-follower existing facilities are stated in the following and are depicted in Fig. 1.

y_j=0, 1, 2…, 14

z_i=1, 7, 11, 4, 13;   i=1, 2,3 for leader and i=4,5 for follower

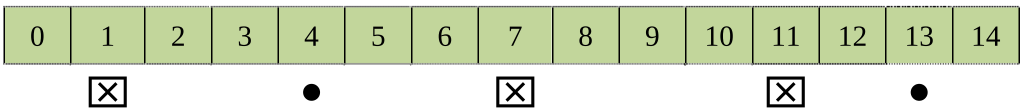

Fig. 1. The location of leader's and follower's existing facilities (☒ for leader and ● for follower)

The buying power of demand points and the potential locations are stated respectively as follows:

b_j = 8,8,8,4,2,9,3,8,5,7,2,2,5,10,7

p_s^L = 0,2,3,5,6,8,9,10,12,14

First, this problem is solved by general approach and then we use the proposed method to solve it to compare the results.

For the implementation of the general approach, first, it is assumed that the leader opens her new facilities in $x_{1L} = 0$ and $x_{2L} = 2$. Then the follower problem is solved. The optimal solution of follower problem is 40.55 at the points of $x_{1F} = 3$ and $x_{2F} = 8$. The follower problem is solved for all potential points, similarly and the results are shown in Table 1.

Table 1
The result of solving follower's problems for all leader's potential locations

x_{kL}	0,2	0,3	0,5	0,6	0,8	0,9	0,10	0,12	0,14	2,3	2,5	2,6
x_{hF}^*	3,8	2,8	2,8	2,8	2,9	2,8	2,8	2,8	2,8	0,8	0,8	0,8
M_F^*	40.55	40.35	39	39.21	38.59	39.18	39.8	38.19	37.69	39.84	38.15	38.16
x_{kL}	2,8	2,9	2,10	2,12	2,14	3,5	3,6	3,8	3,9	3,10	3,12	3,14
x_{hF}^*	0,9	0,8	0,8	0,8	0,8	0,8	0,8	0,9	0,8	0,8	0,8	0,8
M_F^*	37.21	37.77	38.34	36.69	36.17	38.28	38.16	36.96	37.43	37.96	36.27	35.73
x_{kL}	5,6	5,8	5,9	5,10	5,12	5,14	6,8	6,9	6,10	6,12	6,14	8,9
x_{hF}^*	0,8	0,9	0,8	0,8	0,8	0,8	0,9	0,8	0,8	0,8	0,8	0,6
M_F^*	38.22	36.52	36.69	37.07	35.27	34.68	37.24	37.23	37.53	35.67	35.05	37.44
x_{kL}	8,10	8,12	8,14	9,10	9,12	9,14	10,12	10,14	12,14			
x_{hF}^*	0,9	0,9	0,9	0,8	0,8	0,8	0,8	0,8	0,8			
M_F^*	37.52	35.42	34.78	38.7	36.57	35.77	37.65	36.81	37.05			

Fig. 2 depicts the optimal solutions of follower's objective functions for all leader's potential points.

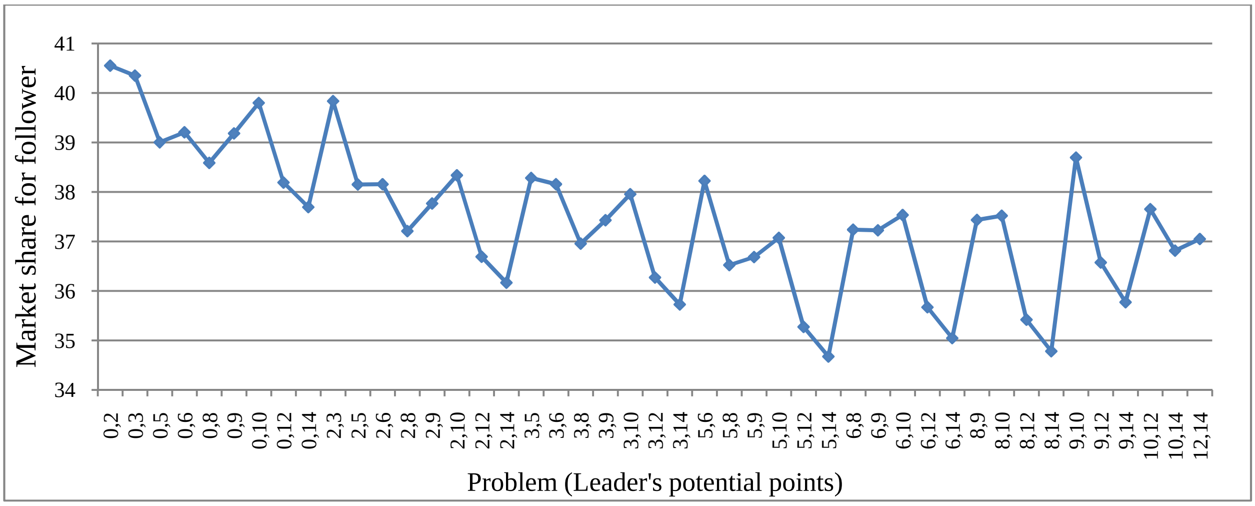

Fig. 2. The graph of maximum follower's market share for all leader's potential locations

As we can observe from Table 1 and Fig. 2, the minimum value of maximized objective functions in each case is associated with the points of $x_{1L} = 5$ *and* $x_{2L} = 14$ which is the optimal solution for the leader problem because these points maximizes leader's objective function. Therefore, the optimal solutions of the problem is $x_{kL}^* = 5{,}14$ and $x_{hF}^* = 0{,}8$. Fig. 3 illustrates the optimal locations both for leader and follower facilities.

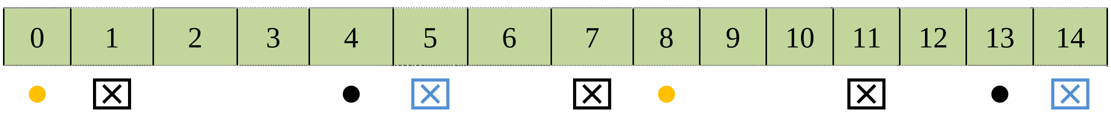

Fig. 3. The location of leader's and follower's existing and new facilities

Example 2: like the prior example, the problem data are as follows:

$n = 100, m = 20, t = 10, p = r = 1$

$y_j = 0,1,2, \dots ,99$

$z_i = 7,18,25,36,43,50,63,72,86,95,2,15,21,30,42,55,66,70,80,99;$ i=1,2,…,10 for leader and i=11,12,…,20 for follower. Table 2 and Table 3 demonstrate the information of b_j and p_s^L.

Table 2
The buying power of demand points (b_j) for example 2

	8	8	8	4	2	9	3	8	5	7	2	2	5	10	7	3	2	10	9	4
	7	8	2	3	6	5	7	6	9	2	10	9	7	7	5	8	7	4	7	8
b_j	8	5	3	5	4	6	4	5	8	2	2	5	5	7	3	4	7	4	7	7
	7	6	3	9	6	3	5	8	7	8	1	3	8	8	3	4	7	2	8	3
	6	10	10	7	2	8	7	4	2	2	2	3	1	4	4	2	7	3	7	4

Table 3
The potential locations (p_s^L) for example 2

	0	1	3	4	5	6	8	9	10	11	12	13	14	16	17	19	20	22	23	24
p_s^L	26	27	28	29	31	32	33	34	35	37	38	39	40	41	44	45	46	47	48	49
	51	52	53	54	56	57	58	59	60	61	62	64	65	67	68	69	71	73	74	75
	76	77	78	79	81	82	83	84	85	87	88	89	90	91	92	93	94	96	97	98

The following figure has depicted the above example.

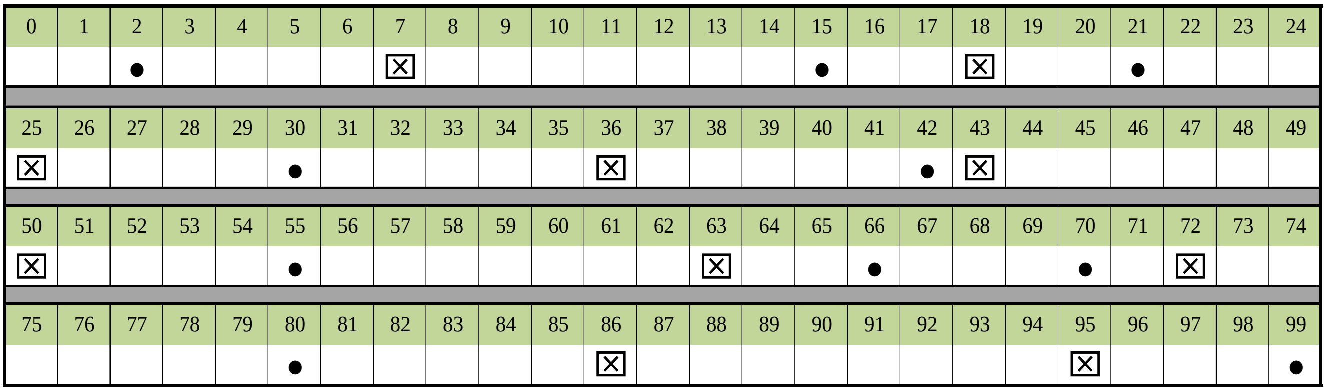

Fig. 4. The location of existing and new facilities of leader and follower

As Fig. 4 shows the set *A* is selected as follows:

A= {1,13,14,22,31,40,41,56,67,68,69,81,82,98}

The main reason that set *A* is arranged in this form is because the point 1 is adjacent to the follower's existing facility, far from leader's existing facility and we are supposed to capture the market share of the initial points in the linear market. The points 14, 22, 31, 41, 56, 67, 68, 69, 81 and 98 are located between two leader's existing facilities and they also are adjacent to the follower's existing facility and ultimately, the points 13, 40 and 82 maintain high values of buying power and they are near to the follower's existing facility. The follower problem is solved for all the elements of the set *A* and the results are shown in Table 4.

Table 4
The result of solving follower's problems for all the elements of the set *A*

x_{kL}	1	13	14	22	31	40	41	56	67	68	69	81	82	98
x_{hF}^*	38	38	38	38	39	85	48	38	38	38	38	38	38	38
M_F^*	259.9	256.7	257.1	263.3	255.3	261.3	262.2	255.6	258.8	258.8	260.2	256.9	259.2	264.8

The minimum value among these objective functions is 255.3 and the other optimal solutions are eliminated. The follower's new facility location is $x_{1F}^* = 39$. The leader and the follower problems are substituted with each other for the resulted point and the leader problem is solved with the point of $x_{1F}^* = 39$. The optimal solution of the leader problem for the point of $x_{1F}^* = 39$ is equal to $x_{1L}^* = 31$.

The results of five other examples are shown in Table 5. For each example the buying power and quality values are randomly generated. The first column represents the information of each example and the second column shows the location of leader's and follower's existing facilities. For each example, the follower problem is solved for all leader's potential locations and the optimal locations for follower and the leader exist in the third and the forth column, respectively. The set *A* for each example is in the fifth column and the follower problem is solved for elements of this set and the results are in the sixth column. In the seventh column the location with minimum market share is kept according to step 3 of proposed method and finally the best location for the leader is obtained by substituting the leader and followers problem shown in the last column.

Table 5
The computational results of solving five examples

	z_i	x_{1F}^*	x_{1L}^*	The set A	x_{1F}^* for elements of the set A	Selected x_{1F}^*	x_{1L}^*
n=15 m=5 t=4	{0,4,7,11} {5}	9	13	{6,12,13}	{13,9,9}	9	13
n=25 m=10 t=5	{2,8,13,18,24} {0,6,11,17,22}	7	21	{5,10,20,21}	{7,7,7,7}	7	21
n=25 m=10 t=5	{6,8,16,20,24} {5,11,14,17,23}	2	13	{3,4,12,13,22}	{2,2,2,2}	2	13
n=50 m=15 t=10	{0,7,16,21,26,28, 34,38,44,48} {2,14,27,35,45}	8	13	{1,3,5,12,13,18,30,31 ,36,40,46}	{8,8,39,8,8,8,8,8,8,8,8}	8	13
n=50 m=20 t=10	{0,7,16,21,26,28, 34,38,44,48} {2,5,14,18,23,29, 35,40,45,49}	8	13	{3,4,12,13,19,24,30,3 1,36,41,46}	{8,8,8,8,8,8,8,8,8,8,8}	8	13

x_{1L}^* is obtained by substituting leader with follower problem

5. Conclusions and future research

In this paper, a mixed-integer nonlinear programming model (MINLP) is proposed to formulate competitive location problem in competition with foresight environment (huff type leader and follower problem). Leader and follower have some located facilities from the past and they intend to locate new facilities in discrete linear space to maximize their market share. Demand is considered as a point with inelastic feature. Leader and follower have to locate their new facilities among predefined potential locations which are considered on unallocated demand points. For small-scale problems, we solve randomly generated problems and determined the optimal solution as well as the location of new facilities for leader and follower. For large-scale problems we have proposed a heuristic approach where the leader and the follower intend to open one new facility. Computational experiments are shown the efficiency of the heuristic procedure for randomly generated test problems. As a future work, we could consider elasticity for demand and reformulate the resulted problem. Also, it is desired to develop some efficient meta-heuristic algorithms to solve the proposed problem for large-scale problems.

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