Journal of Project Management 8 (2023) 199–212

Contents lists available at GrowingScience

Journal of Project Management

homepage: www.GrowingScience.com

A robust solution for optimizing facility location and network design with diverse link capacities

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C H R O N I C L E

Article history:
Received: December 1, 2022
Received in revised format: December 29, 2022
Accepted: January 17, 2023
Available online:
January 20, 2023

Keywords:
Stochastic optimization
Robust optimization
Facility location
Network design
Simulated annealing algorithm

A B S T R A C T

In this paper, the authors proffer a novel mathematical model for the simultaneous optimization of facility location and network design in the presence of uncertainty, with the aim of minimizing operational and transportation costs. The proposed model constitutes a departure from conventional methods in its consideration of probable events in the real world and the incorporation of uncertainty assumptions into the mathematical framework. An algorithm based on simulated annealing is then advanced for the solution of the problem, and the performance of the algorithm is evaluated through comparison with exact methods for problems of modest size, as well as with a basic simulated annealing algorithm for larger problems. The results of these comparisons demonstrate the superiority of the proposed meta-heuristic algorithm. Finally, the robust approach is compared with four other approaches in the presence of uncertainty, with a thorough analysis of the results obtained from each of the methods conducted in a suite of sample problems.

1. Introduction

During the early 1980s, SCM (Supply Chain Management) was introduced to address the competitive landscape among companies (Oliver and Webber, 1982). Over time, corporations began to recognize the importance of incorporating their operations into essential supply chain (SC) processes rather than managing them individually, leading to the further evolution of SCM (La Londe, 1997). The facility location problems have attracted extensive attention in the literature of operations research. In general, the term "location" refers to modeling, formulation and solving problems that can be defined as the optimum location of facilities in a limited area. The related studies demonstrate that network location models have been widely used by individuals and the government. In the facility location problems, the structure of the network is predetermined and the connection between nodes are predefined while determining the optimum location of a facility and designing the main network simultaneously. The real-world problems are analyzed with the assumption that input parameters are fixed while, in practice, data are stochastic; therefore, these assumptions generate solutions which are not optimal and even are infeasible. On the other hand, while making a strategic decision, since the result and consequences are continued for a long time and parameter determination with uncertainty is not possible, considering uncertainty in parameters can result in generating better solutions. The design of a Supply Chain Network (SCN) faces inherent uncertainty in parameters such as costs, demand, and supply. Major disruptions like natural disasters, economic crises or intentional acts can severely impact the performance of the SCN. To optimize the performance of SCN under uncertainty, robust optimization models are employed, which model discrete or continuous uncertain parameters either using the scenario approach or interval-uncertainty modeling (Govindan & Fattahi, 2017). In SCM, risk management has gained considerable attention, but there's a lack of consensus on the definition of supply chain risk. The concept of risk is generally vague and comprehended based on the fear of losing value. However, Heckmann et al. (2015) defined it as the potential loss in terms of objectives due to uncertain

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doi: 10.5267/j.jpm.2023.2.001

variations in SC features caused by triggering events. Based on the classification by (Tang, 2006), SC risks can be categorized as operational and disruption risks, where operational risks stem from intrinsic uncertainties and disruption risks from events like natural disasters and intentional acts (Behdani, 2013; Snyder et al., 2016). These disruption risks can affect the functionality of SC elements either partially or completely for an uncertain duration (Govindan et al., 2017).

Lack of comprehension of historical and current events hinders the capacity to anticipate future developments in supply chain operations, leading to prolonged processes, increased expenses, and inefficiencies. The objective of supply chain network design under uncertainty is to achieve a configuration so that it can perform well under any possible realization of uncertain parameters. The objective of designing a supply chain network in an uncertain environment is to establish a framework that can operate efficiently under any scenario of unpredictable variables. Thus, the innovation of the proposed model is to consider uncertainty for combined facility location and network design problems with multi-type capacitated links in order to cope with the situations taking place suddenly. The result contributes to a significant reduction in costs.

In this paper, a new mathematical model for facility location problems with network design under uncertainty is developed. Demands, capacities, and so on can be changed during a time period; therefore, analyzing and developing the facility location-network design model is one of the fields in which few studies have been conducted. In this paper, this problem is taken into consideration. The developed model can be applied to telecommunication, emergency events, regional planning, pipeline network, energy management, and similar cases. According to our knowledge, it is the first time that the problem is studied under uncertainty. Further, to examine the problem, it is analyzed with different methods. In section (2), the literature survey of location-network design problems is proposed. In section (3), the two-stage mathematical model and the robust model with their assumptions and notations are proposed. In section (4), the proposed algorithm based on simulated annealing is presented. Section (5) evaluates the performance of the proposed algorithm and finally, section (6) contains the conclusion of this study.

2. Literature review

According to Mulvey et al. (1995), the robust optimization (RO) framework consists of two types of robustness - solution robustness and model robustness. This means that the RO problem's solution is "nearly" optimal and "nearly" feasible, respectively, under all possible uncertain parameters. The level of "nearly" depends on the modeler's perspective. When a decision maker is uncertain about the probability distributions of parameters, the expected value criterion cannot be used, making RO a suitable option by defining different robustness measures for the optimization problem. RO problems can handle continuous or discrete uncertain parameters (Govindan et al., 2017). System disorders occur when one or a few of the facilities become out of control during situations such as unfavorable weather conditions, labor force's actions, sabotage, or changes in ownership. When a disruption occurs, customers are forced to use facilities farther away which is cost-inducing. Therefore, it can be effective in improving system performance with regards to the disruptions in network design and deployment of facilities (Shishebori & Babadi, 2015). Snyder and Daskin (2005) were the first to propose an implicit formulation of random p-median problems, where candidate sites could experience stochastic disruptions with equal probability (Snyder & Ülker, 2005; Snyder, 2003). In recent years, due to the large investment in facility location-network design, there has been an increased focus on disruptions and their impact on network design and establishment (Snyder & Daskin, 2007; Qi & Shen, 2007; Qi et al., 2010). Matisziw et al. (2010) studied the effects of disruptions on network services and proposed a multi-objective optimization approach for recovery during network reconstruction after a disaster. (Berman et al., 2010) hypothesized that facilities are not always reliable and that disturbances may occur and studied the impact of the probability of disruptions on optimal facility location, considering that customers may have incomplete information about facilities provided by the government. (O'Hanley & Church, 2011) developed a covered prevention model for facility location, aimed at finding a robust arrangement that would remain efficient even in the event of facility loss. (Peng et al., 2011) investigated the impact of considering reliability in logistics network design with disruptions in facilities and showed that using reliable network design is possible with a slight increase in total space and allocation cost.

Liberatore et al. (2012) developed a three-level mathematical model for the optimization of enrichment programs in the medium capacity distribution systems with limited resources for protection against disturbances, which includes large areas. Jabbarzadeh et al. (2012) have studied the designing of a chain supply which can possibly have partial or complete disturbances in those distribution centers. Shishebori et al. (2013) and Shishebori and Jabalameli (2013) considered the problem of the reliability of facility location-network design with regards to disturbances in the system. In another study, Shishebori et al. (2014) suggested an integrated mathematical formula that not only considers the costs of facility location, establishments, the costs of road construction, and transportation costs, but also the limits of the maximum permissible system disorder cost as well as investment in facility location and the establishing of transportation routes. (Sheppard, 1974) was one of the first people that had proposed a scenario-based approach to facility location. He proposed the choice of the location for the facility to minimize the expected cost. Santoso et al. (2005) presented a stochastic programming model and algorithm to address the real-scale problem of facility location-network design. Tsiakis et al. (2001) considered the uncertainty of demand in a multi-product, multi-category supply chain. Aharon et al. (2009) tackled the multi-periodic stock control problem with a focus on selecting facility sites, capacities, transportation routes, and flows to minimize expected costs. Gülpınar et al. (2013) addressed a stochastic facility location problem where demand is fulfilled by a single product from multiple facilities. Beraldi et al. (2004) studied the emergency medical services design problem under stochastic demand. The topic of home care, with definite or random patient demand, has also been explored, yielding policy analysis or cardinality-constrained models (Jabbarzadeh et al., 2014; Lanzarone & Matta, 2014; Lanzarone & Matta, 2012; Carello & Lanzarone, 2014). Mustapha Anwar Brahami (2022) recently discussed on designing a sustainable supply-chain-network that minimizes cost and environmental impact. A multi-objective model is proposed and solved using a modified Non-dominated Sorting Genetic Algorithm II. The approach considers environmental protection constraints and integrates facility location and transport network design decisions. Numerical experiments and sensitivity analysis are used to evaluate the performance of the proposed approach. Hu and Hu (2015) presented a model for creating a hub-and-spoke transportation network with the aim of reducing operational costs. This network leverages economies of scale and is designed to handle uncertain origin-destination flows between nodes, which can pose challenges in determining node capacities. The researchers devised a stochastic programming approach, combined with expectation theory, to calculate the capacities of the spoke nodes, and then established a stochastic mixed-integer linear programming model to design the network structure and hub capacities. They also created a classification system for nodes based on transportation flows, resource usage, and branch and trunk convective equilibrium. To evaluate node contributions, a multi-attribute utility evaluation function was established, and the researchers examined existing network operational and adjustment strategies based on the results.

Zare and Lotfi (2015) presented a dynamic and probable mixed-integer linear program for managing the forward and reverse supply chains that considered the crucial aspects of efficiency and nimbleness, as well as the capacity for dynamic center openings. The resolution of the dynamic model was found to be more practical, yet the calculation of total costs was undervalued. Zarindast et al. (2018) investigated the challenge of selecting suppliers, transportation methods, and replenishing inventory in the face of uncertain changes in currency exchange rates and discounts. A bi-objective mathematical model was suggested, utilizing a robust and probable programming approach to reduce the overall cost and the number of late deliveries. The results were compared to those produced by a model using chance-constrained programming, demonstrating the benefits of the integrated method over a step-by-step approach in cases of currency fluctuations. Moslemipour et al. (2018) introduced a new hybrid algorithm that combines simulated annealing with a population of initial solutions produced by combining ant colony, clonal selection, and robust layout design methods. This approach can be applied to solve dynamic facility layout problems in both deterministic and stochastic environments, where product demands are treated as normally distributed random variables with a changing probability density function. The hybrid algorithm uses a quadratic assignment-based mathematical model to design a robust layout for the stochastic dynamic layout problem. The efficacy of the proposed algorithm was demonstrated through the resolution of a large number of randomly generated test problems and problems from the literature, resulting in exceptional performance in terms of solution quality and computational efficiency. Farooquie et al. (2017) introduced a gray approach to address uncertainties and enhance performance in the automotive supply chain sector. The complex nature of this industry, with a multitude of suppliers and attendant uncertainties, can lead to disruptions with a significant impact on overall performance. The gray-based method models the relationship between uncertainty and performance, allowing supply chain managers to focus on mitigating uncertainties with the greatest negative impact. Rad et al. (2015) presented a multi-period, multi-product production planning model in a random situation. To tackle the variability of demand, process and setup time, the authors adopted a simulation optimization technique. Rad et al. (2015) proposed a multi-product, multi-period production planning model that accounted for three uncertain parameters, demand rate, process time, and setup time. To determine the optimal response, they used a simulation optimization technique, which calculated the system response rate and approached it through the simulated annealing algorithm. The results were demonstrated through a numerical example.

3. Problem definition

The proposed mathematical model (Rahmaniani & Ghaderi, 2013) assumes that there are multiple paths linking two nodes in the network, each with a distinct capacity, and that each node can have only one facility. The network is customer-to-server and all links are direct, single-directional, and budget-limited.

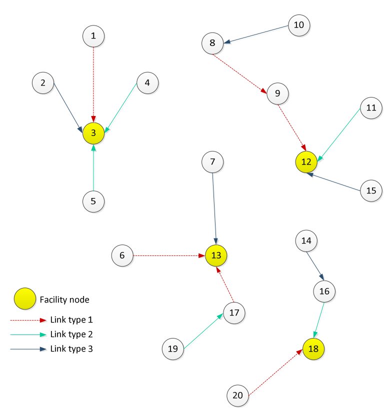

Fig. 1. The topology of the constructed network

Fig. 1 depicts a 20-node network, where 4 nodes have facilities, and the rest are serviced via these main nodes with directed paths. For example, nodes 6, 7, 17, and 19 are supplied by facility node 13 through different path types.

3.1 The two-stage stochastic mathematical model

The mathematical model takes into account real-world factors such as customer demand and transportation costs, which can only be affected in the long-term and can lead to better decision-making when considered uncertain. To handle these uncertainties, the model employs a two-stage approach, utilizing two-stage stochastic programming. In this method, decision variables are split into two categories: first-stage variables, related to facility location and route choice, and second-stage variables, encompassing facility allocation and customer route construction. The proposed model incorporates uncertainty into its design, making it more reflective of actual world conditions, and its two-step approach serves to refine decision-making.

Sets

N	Set of network nodes, $i, j \in \{1,2,\dots,n\}$ and $k \in \{1,2,\dots,n\}$ is the customer set
L	Set of candidate links $i, j, t \in \acute{L}$
T	Set of different types of links $t \in \{1,2,\dots,\acute{T}\}$
S	Considered scenarios set $s \in \{1,2,\dots,\acute{S}\}$

Parameters

D_k^s	Customer *k*'s demand in the scenario ***S***
f_i	Fixed cost for facility establishment in node *i*
c_{ij}^t	Fixed cost for construction of link *(i, j)* of type *t*
tr_{ij}^{ts}	Transportation cost of a link *(i, j)* of a type *t* in a scenario ***S***
d_{ij}	The length of *(i, j)* a link
V_{ij}^t	The capacity of a link *(i, j)* of type *t*
B	Maximum budget available for facility construction
dsr_{ij}^{ts}	1 if the link *(i, j)* of a type *t* in a scenario ***S*** is functioning and 0 otherwise
dsf_i^s	1 if a facility in a node *i* in a scenario ***S***, is not functioning and 0 otherwise

Decision variable

z_i	1 if a facility is constructed in node *i* and 0 otherwise
x_{ij}^t	1 if the link *(i, j)*of type *t* is constructed and 0 otherwise
y_{ij}^{kts}	The fraction of the customer *k*'s demand traveling on a link *(i, j)* of type *t*
w_i^{ks}	The fraction of the customer *k*'s demand served by a facility *i*

The proposed mathematical model is formulated as follow:

$$\min \sum_{s\in S} P^s \sum_{t\in T}\left(\sum_{k}\sum_{(i,j)\in L} D_k^s tr_{ij}^{ts} y_{ij}^{kts}\right) + \sum_{t\in T}\sum_{(i,j)\in L} c_{ij}^t x_{ij}^t \tag{1}$$

subject to

$$z_i + \sum_{t\in T}\sum_{j\in N} x_{ij}^t \le 1\,; \qquad \forall i \in N \tag{2}$$

$$\sum_{t\in T} x_{ij}^t + \sum_{t\in T} x_{ji}^t \le 1; \qquad (i,j)\in L \tag{3}$$

$$\sum_{t\in T}\sum_{j\in N} y_{ij}^{kts} = w_i^{ks} + \sum_{t\in T}\sum_{j\in N} y_{ji}^{kts};\ \ \forall i,k \in N : i \ne k, s\in S \tag{4}$$

$$z_k - dsf_i^s + \sum_{i \in N: i \neq k} w_i^{ks} = 1; \qquad \forall k \in N,\ s \in S \tag{5}$$

$$z_i - dsf_i^s + \sum_{t \in T} \sum_{j \in N} y_{ji}^{its} = 1; \qquad \forall i \in N, s \in S \tag{6}$$

$$w_i^{ks} \leq z_i - dsf_i^s; \qquad \forall i, k \in N, s \in S \tag{7}$$

$$y_{ij}^{kts} \leq x_{ij}^t; \qquad \forall i, j, k \in N, (i, j) \in L, t \in T, s \in S \tag{8}$$

$$\sum_k D_k^s\, y_{ij}^{kts} \leq V_{ij}^t x_{ij}^t; \qquad \forall (i, j) \in L, t \in T, s \in S \tag{9}$$

$$\sum_{i \in N} f_i z_i \leq B; \tag{10}$$

$$y_{ij}^{kts} \leq dsr_{ij}^{ts}; \qquad \forall i, j \in N, (i, j) \in L, t \in T, s \in S \tag{11}$$

$$x_{ij}^t \in \{0,1\}; \qquad \forall (i, j) \in L, t \in T \tag{12}$$

$$z_i \in \{0,1\}; \qquad \forall i \in N, \tag{13}$$

$$y_{ij}^{kts} \geq 0; \qquad \forall (i, j) \in L;\ \forall k \in N; \forall t \in T; s \in S \tag{14}$$

$$w_i^{ks} \geq 0; \qquad \forall i, k \in N : k \neq i\text{،}\ s \in S \tag{15}$$

The model aims to minimize transportation costs and path construction for all scenarios. The constraints of the model ensure that demands of customers are met, that facilities are utilized, and that paths are constructed with limited budget. Constraints (2-5) regulate the construction of paths and demand supply, while constraints (6-11) regulate the functionality of facilities, routing, and route capacity. The decision variables are specified in constraints (12-15). The model considers transportation costs and uncertainties in customer demand and transportation costs to make decisions that increase facility availability.

3.2 Robust model

Robust optimization was one of the most popular topics in the fields of optimization and management science during the late 1990s, which deals with uncertain parameters. In robust optimization, uncertain parameters are described by discrete scenarios (Baohua and Shiwei, 2009). The aim of this optimization method is to obtain an optimal solution, and in this case, almost all uncertain parameters remain feasible. (Mulvey et al., 1995) introduced a robust optimization model which contains two types of robustness: "solution robustness" (the answer in almost all scenarios is the optimal) and "model robustness" (answer in almost all scenarios is feasible). In continuation, the framework for robust optimization will be mentioned in brief. First, some symbols associated with the model will be introduced. x is a vector of variables of the design and y is a vector of control variables. A , B and C are matrixes of parameters, while b and e are parameter vectors. A and b are deterministic and B , C and e , are uncertain. Consider a limited set of scenarios $\Omega = \{1, 2, ..., S\}$ or modeling the uncertain parameters, in which each situation $s \in \Omega$ is a subset of $\{B_s, C_s, e_s\}$ and the probability of the scenario is considered to be $p_s \left(\sum_s p_s = 1\right)$. The control variable y , when placed in a scenario, is shown as y_s for scenario s . Because of the uncertainty of the parameters, the model may be infeasible for some scenarios. δ_s shows the infeasibility of the model under the scenario $\boldsymbol{S}$. If the model is feasible, δ_s equals zero. Otherwise, δ_s is a positive value determined according to constraint (18). A robust optimization model is formulated as follows:

$$\min \sigma(x, y_1, y_2, ..., y_s) + wp(\delta_1, \delta_2, ..., \delta_s) \tag{16}$$

subject to

$$Ax = b \tag{17}$$

$$\forall s \in \Omega \quad B_s x + C_s y_s + \delta_s = e_s \tag{18}$$

$$\forall s \in \Omega \qquad \delta_s \geq 0 \quad y_s \geq 0;\ x \geq 0; \tag{19}$$

There are two parts of the objective function: the first part of the answer is related to the robustness of the solution and shows the desire for lesser costs and risk aversion. The second part is the model's robustness which shows the penalty for answers that did not meet the demand required for a scenario or violated other physical constraints such as capacity. Using the weight *w*, the exchanges between the robust solutions of the first row *δ(0)* is measured and the robust model which is calculated by the penalty *p(0)*, and can be turned into a model by using the multi-criteria decision-making process. ξ is used for the introduction of *f(x,y)* which is a function for the benefit or the cost, $\xi_s = f(x,y_s)$ is used for a scenario ***S***. A high variance for $\xi_s = f(x,y_s)$ shows that the answer is a high-risk decision. Mulvey et al. (1995) proposed a second form variance with for considering the concept of risk and provided a robust answer. To deal with the computational complexity due to its non-linearity, Yu and Li (2000) proposed an absolute value of the two locations, presented as follows:

$$\sigma(0) = \sum_{s\in\Omega} p_s \xi_s + \lambda \sum_{s\in\Omega} p_s \left| \xi_s - \sum_{s'\in\Omega} p_{s'} \xi_{s'} \right| \tag{20}$$

λ, is used to demonstrate the weight of solutions' variance in which, when λ is increased, in all scenarios, the solution is less sensitive while inputs are changed. To minimize the objective function (21), they proposed an effective method. The framework of the method proposed by Yu and Li is designed in constraint (21).

$$\min \sum_{s\in\Omega} p_s \xi_s + \lambda \sum_{s\in\Omega} p_s \left[\xi_s - \sum_{s'\in\Omega} p_{s'} \xi_{s'} + 2\theta_s \right] \tag{21}$$

$$\xi_s - \sum_{s\in\Omega} p_s \xi_s + \theta_s \geq 0; \qquad s \in \Omega \tag{22}$$

$$\theta_s \geq 0; \qquad\qquad s \in \Omega \tag{23}$$

It can be inferred that when the value of ξ_s is greater than the value of $\sum_{s\in\Omega} p_s \xi_s$, then $\theta_s = 0$ and when the value of $\sum_{s\in\Omega} p_s \xi_s$ is greater than the value of ξ_s , then $\theta_s = \sum_{s\in\Omega} p_s \xi_s - \xi_s$.

Regarding the mentioned explanations, the expected value of the objective function of the problem is as follows:

$$E(A) = \sum_{s\in S} P^s \sum_{t\in T} \left(\sum_k \sum_{(i,j)\in L} D_k^s tr_{ij}^{ts} y_{ij}^{kts} \right) + \sum_{(i,j)\in L} c_{ij}^t x_{ij}^t \tag{24}$$

Regarding the calculated expected value, the objective function of the two-stage model is as formulated below:

$$\min \sum_{s\in S} P^s \left[\sum_{t\in T} \left(\sum_k \sum_{(i,j)\in L} D_k^s tr_{ij}^{ts} y_{ij}^{kts} \right) - E(A) + 2\theta_s \right] + E(A) + \sum_{(i,j)\in L} c_{ij}^t x_{ij}^{ts} \tag{25}$$

subject to

$$\sum_{t\in T} \left(\sum_k \sum_{(i,j)\in L} D_k^s tr_{ij}^{ts} y_{ij}^{kts} \right) - E(A) + \theta_s \geq 0 \tag{26}$$

$$\theta_s \geq 0 \tag{27}$$

The objective function minimizes three parts: the variance of total costs, the expected value of traveling costs on routes, and the cost of route construction. The results of a sample with 15 nodes were analyzed with the base model, two-stage model, and robust model. If two scenarios occur where the link between nodes 11 and 12 is destroyed, the base model would not be able to serve the demand of node 11, but the two-stage and robust models consider scenarios and have alternative links to serve all nodes.

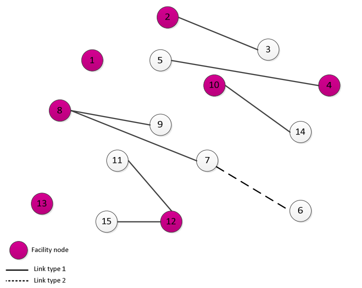

Fig. 2. The result of the base model

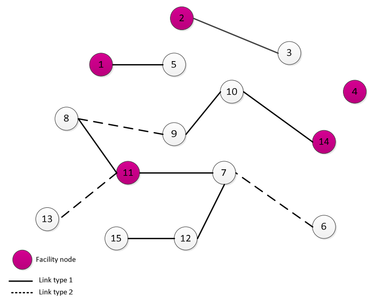

Fig. 3. The result of two stage stochastic model

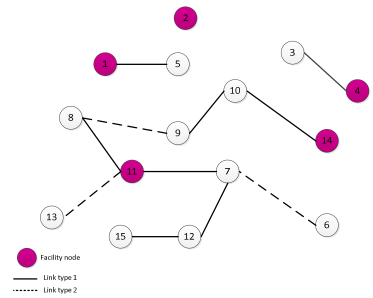

Fig. 4. The result of robust model

4. Solution approach

Regarding the fact that the problem of the basic article is NP-Hard (Rahmaniani and Ghaderi, 2013) and the proposed model in this paper is its expansion; therefore, the proposed model is also NP-Hard. As a result, solving large-sized sample problems is not possible in a reasonable time. Thus, to solve the problem, a meta-heuristic method was used. Examining several initial examples shows that if the variables related to established and non-established facilities (z_i) is determined, the time needed for solving the problem shows a significant decrease; thus, a combined method is used. In this method, selected nodes for establishing facilities are identified by a meta-heuristic algorithm, and the rest of the methods is used in relation to the exact method of solving the problem. In continuation, the proposed meta-heuristic algorithm is explained.

5. The proposed algorithm

In this part, the simulated annealing algorithm, which is used to solve the problem, is explained. The Pseudo code of Simulated Annealing is presented in Fig. 5.

Generate the random vector of $x \in \{0,1\}$
If a solution x is infeasible regarding the budget limit, one of the elements with the value of 1 is randomly selected and converted to 0.This process will continue until the vector x is feasible regarding the budget limit.
Set the current position equal to x .
Calculate the value of $f(x)$.
Set $T = T_0$.
Until the stop criterion of the algorithm is not met:
Select several random neighbors for x and after calculating the value of the objective function, assign the best value to y
Calculate the value of $\Delta = f(y) - f(x)$
If $\Delta < 0$, assign y to a current position otherwise assign y to current position with the probability of $e^{-\Delta/T}$
If the maximum-iteration stop criterion is not met, start from the beginning of the loop otherwise update T and start from the beginning of the loop.
End of the loop.

Fig. 5. Pseudo code of Simulated Annealing

The proposed parameters of the algorithm include the number of iterations, the initial temperature and the number of neighborhood search, which is calculated through trial and error. The initial temperature is taken to be equal to the objective function value of the initial solution, and then in each iteration, this temperature is decreased by a factor of 0.98. The number of neighborhood search is measured as 30.

5.1 Solution structure

In order to display the answers from a string with the length equal to N, where N is the total number of nodes used in the network, Fig. 6 is illustrated. Each cell in the string represents a node. Values inside the cell are either 0 or 1, which represent the probability of establishing facilities in the desired node. Fig. 6 shows a sample solution string in a problem with 9 nodes.

1	**1**	**0**	**0**	**1**	**0**	**1**	**0**	**0**
1	2	3	4	5	6	7	8	9

Fig. 6. Solution string used in simulated annealing algorithm

In the example given in Fig. 6, facilities are created at nodes 1, 2, 5, and 7. The example for solving the meta-heuristic solution methods is to determine the location of these facilities which is determined in each iteration of the simulated annealing algorithm. Then, by keeping the facility nodes at a fixed location, the rest of the problem is calculated by the solver of CPLEX. It is necessary to mention that in the cases where the solution is infeasible, regarding the budget limitation, a cell will be randomly changed from a value of 1 to a value of 0, and this continues until the answer becomes feasible. To explain better the example provided in Fig. 7, the painting is drawn from 3 main points. In Fig. 7, numbers in red are each customer's demand, green indicates the cost of a manufacturing facility in each location. The numbers on the candidate routes show the distance between the two points.

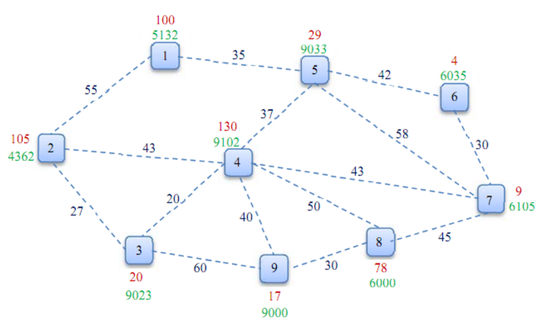

Fig. 7. Hypothetical network of nodes

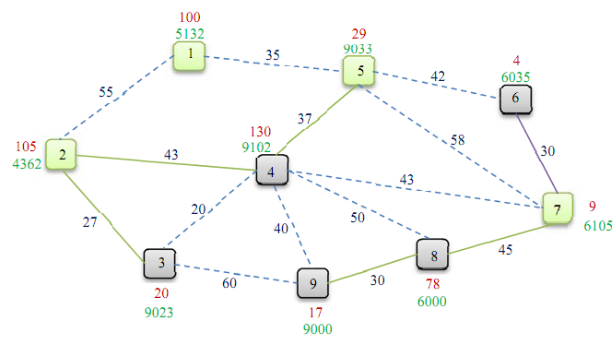

Fig. 8. Solution illustration

It is assumed that there is a probability of establishing three types of road. After determining the location of facilities, with fixing the relevant variables, the model can be solved so that the objective function can be calculated. With regards to the solution considered in Fig. 6, the accurate method of solving this problem is shown in Fig. 8. The green dots are locations for creating facilities, the green paths are type I roads areas and the purple paths are to indicate type III roads. (Type II roads have not been used in the final answer). All the locations that are considered as facility location meet other nodes' needs as well as their own. So, the facility at node 2 services nodes 4 and 3, location 5 to point 4, and point 7 to points 8 and 9. Facility in point 1 satisfy the needs of this area.

6. Results

To evaluate the performance of the meta-heuristic algorithm presented, in this section, large and small-sized problems have been designed. In small-sized problems, the result of the meta-heuristic algorithm is compared to the exact method. In large-sized problems, the effect of increase in the number of neighborhood parameters on the quality of the solutions is evaluated.

6.1 Sample tests generation

Following assumptions are used for the production of sample problems. For each path, three different qualities are taken into consideration (T = 3), in terms of travel costs for Type 1 using Eq. (28) and Eq. (29) are produced.

$$C_{ij}^{1} = ud_{ij}\left(1+r\right) \qquad u \in \left[2,10\right], r \in \left[-0.2, 0.2\right] \tag{28}$$

In which, r and u are random variables.

The transportation cost of each unit on path type 1 is calculated by means of the following formulation.

$$t_{ij}^{1} = d_{ij}(1+r) \tag{29}$$

In order to calculate the magnitude of the space production costs and travel expenses for the Road Type II, a random number in range [1.1,1.2] was produced and was multiplied by the Type I's travel costs. Also, to produce the construction and travel costs of Type II, another random number between [1.15,1.3] was chosen and multiplied with Type I and II (Rahmaniani & Ghaderi, 2013). Traveling waves of type I trip was calculated using Eq. (28) and Eq. (29). Each of the five scenarios was considered for each issue. Uncertainty scenarios classified as follows: excellent, good, average, bad and very bad, and for each scenario, a random occurrence chance was calculated for. In each scenario, the amount of customer demand, transportation cost and the probability of road failure were considered. It also increased the intensity free of changes in the first and fifth scenarios,

6.2 Solving small-sized problems

As the effect of evaluation in increasing the number of local searches after the high-quality service, five problems were produced on a small-sized, and the results were compared with the results of the exact solution. Results are presented in Table 1. The first column in the table shows the number of network nodes. The next column is related to the optimal solution found via exact methods. The next two columns are related to the basic simulated meta-heuristic algorithm and show the objective function and degree of error. The two final columns are related to the simulated annealing algorithms. The degree of error of our suggested algorithms was gained from Eq. (30).

$$G\% = \frac{best - answer}{best}.100 \tag{30}$$

In this equation, the "best" and "answer" are respectively the best response ever found by the methods, and the intended meaning.

Table 1
Result from solving small-size problems with GAMS, SA, and ISA

Test problem	CPLEX	SA		ISA	
N	Opt Sol	answer	G%	answer	G%
5	7010	7010	0	7010	0
7	8132	8132	0	8132	0
9	9258	9298	0.4	9258	0
13	8718	8848	1.5	8848	1.5
15	10322	10759	4.23	10529	2.01
17	12038	12729	5.40	12420	3.10
20	15168	16153	6.50	15736	3.75
22	17488	18135	3.60	18004	2.90
26	20636	22487	8.97	21155	2.51
28	23525	26085	10.88	24666	4.85
Avg	13229	13964	4.15	13576	2.06

The maximum Gap of the basic and proposed meta-heuristic algorithms in small-sized problems are in turn 10.88% and 4.85%. The average value of Gaps for both algorithms is 4.15% and 2.06 %, respectively. In Fig. 9, the quality of the generated solutions is demonstrated.

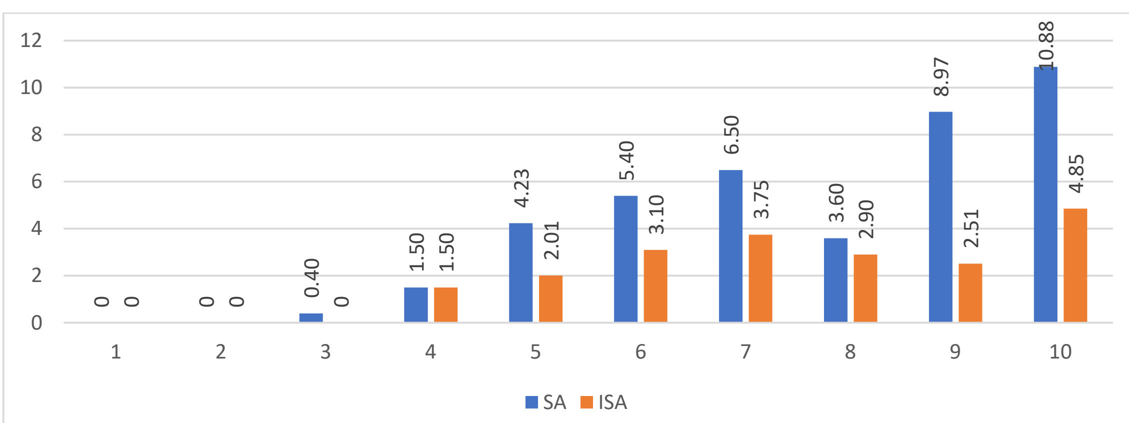

Fig. 9. The quality of solutions in small-sized problem

6.3 Solving large-sized problems

In order to evaluate the performance of the meta-heuristic algorithm and the impact of increasing local search on the quality of the generated solutions, in this section 10, sample problems were solved. In Table 2, results from solving large-sized problems by means of SA and improved SA is illustrated. The times are in second. The second column is used to show the best solution. The third column is assigned to the required time for solving the problems and column 6 shows the error of SA in comparison with ISA. As it can be inferred from Table 2, with increasing the number of local searches, the quality of the solutions is averagely improved almost 9.26%. The maximum improvement is equal to 22.01% which belongs to a sample test with 75 nodes. Regarding the processing time, with increasing the number of neighbors, the required time for solving problems has increased by 32 seconds.

Table 2
Results from solving large-sized problems with SA and ISA

Test problem	ISA		SA		
N	answer	Time(s)	answer	Time(s)	G%
30	31835	1095	33706	1023	5.8
35	42598	1397	42812	1342	0.5
40	46728	1502	48019	1449	2.7
45	54878	1738	54903	1694	0.04
50	61074	2017	62313	1978	2.02
55	68403	2299	73738	2294	7.8
60	75243	2667	87508	2685	16.3
65	85777	3147	102504	3106	19.5
70	96928	3620	112340	3612	15.9
75	115345	4307	140732	4291	22.01
average	67881	2379	75857	2347	9.26

The effect of robust two-stage stochastic programming on the quality of solutions (managerial insight)

Robust SCND has gained less attention in comparison with fuzzy and stochastic programs. However, it must be noted that in many real-world applications, enough historical data are not present to estimate parameters' distributions, but a robust optimization is a suitable tool for handling such a situation (Govindan et al., 2017). Robust SCND is a less popular optimization method compared to fuzzy and stochastic programs, but it is a useful tool for handling situations where there is not enough historical data to estimate parameter distributions (Govindan et al., 2017). On account of being more useful to managers, this section examines the impact of various approaches for dealing with uncertainties on the expenses. For this purpose, four approaches are used to solve sample problems. These approaches include the average value approach (Expected value), stochastic two-stage programming approach (Here & Now), waiting and observation approach (Wait & See) and robust approach. It should be noted that the small-sized problems are solved with exact methods and large-sized problems are solved by using an improved algorithm. In the expected value approach, the values of the parameters in the possible state (average) in the first stage of the model and decisions in this level are made on the basis of the calculated values. Tables 3, 4, 5 and 6 show the data related to solving each sample problem with regards to the mentioned approaches. L is the number of nodes in each sample.

Table 3
Results from Expected Value method

Expected value								
N	L	S1	S2	S3	S4	S5	Avg(EEV)	Variance
1	5	4592	5610	7010	10945	13176	8266.6	10683057.44
2	9	6525.6	8045.2	16558	18629	22763	15704.16	107060069.2
3	13	6349.6	7756.2	9208	16054.5	48067	17487.06	244892088.9
4	17	8296.8	10096.2	12038	47044.2	62702	28035.44	506090333.3
5	22	12084.8	14659.2	17488	53293.2	75798.8	34664.8	648829226.4
6	30	22756.8	27689.2	33038	77754	91950	50637.6	811145506.8
7	35	29924	35641	43860	88859	135836	66824	1621906431
8	40	31780	39766	46740	120770	165640	80939.2	2808432555
9	45	37020.2	48399	56878	99600.1	171089	82597.26	2405630867
10	50	43259.2	50597	63193	168775	205670	106298.84	4542540528
Avg							49145.496	1370721066

In the Here & Now approach, which is the stochastic two-step approach, the model involves all scenarios in making decisions during the first step.

Table 4
Results from Here & Now method

Here & Now								
N	L	S1	S2	S3	S4	S5	Avg(HN)	Variance
1	5	5080	5616	6360	9485	11376	7583.4	5926855.84
2	9	8377.6	10495.2	12878	15567	18535.2	13170.6	12947579.01
3	13	10081.6	12907.2	15828	19414	23395.2	16325.2	22087611.65
4	17	18540.8	22693.2	27342	32622.2	38383.2	27916.28	49442416.95
5	22	21928.8	26791.2	32252	38430.2	45151.2	32910.68	67755802.81
6	30	32664	37940.2	44324	56167.8	67828	47784.8	161907286.7
7	35	43225	50743	62258	76456	93125	65161.4	321411309.8
8	40	49612	60223	75776	91999	115809.4	78683.88	550006143.8
9	45	41498	65320	77418	95392	120031.4	79931.88	708795315.3
10	50	64726	82192	100888	125170	153001.2	105195.4	973972320.3
Avg							47466.36	287425264.2

In the Robust approach, the decisions made in the first step are all taken according to the robust model.

Table 5
Results from Robust method

Robust								
N	L	S1	S2	S3	S4	S5	Avg(RO)	Variance
1	5	5080	5616	6360	9485	11376	7583.4	5926855.84
2	9	8617.6	10843.2	13368	16225	19387.2	13688.2	14564298.53
3	13	10269.6	13129.2	16088	19716	23743.2	16589.2	22623347.97
4	17	18728.8	22915.2	27602	32924.2	38731.2	28180.28	50242527.83
5	22	22116.8	27013.2	32512	38732.2	45499.2	33174.68	68691902.49
6	30	32284.8	39599.3	49807	58376	70127	50038.82	179457218.1
7	35	43465	54094	67324	76738	93275	66979.2	301153628.6
8	40	49076	61953	76432	92161	115902	79104.8	545511043.8
9	45	54906	66123	79850	97531	121155	83913	549362921.2
10	50	64901	82431	111247	125189	153045	107362.6	968864584.6
Avg							48661.418	270639832.9

In the Wait & See approach, the ideal scenario for the problem is imagined, and the assumption is that before making a decision the main scenario has happened and is known thus the model is solved for each of the scenarios.

Table 6
Results from Wait & See method

Wait & See								
N	L	S1	S2	S3	S4	S5	Avg(WS)	Variance
1	5	4096	5028	7010	9485	11376	7399	7370139.2
2	9	2409.6	7611.6	9258	11971	14736	9197.24	17392321.93
3	13	4156.8	7291.2	8718	13775	18516	10491.4	25737923.78
4	17	6312.8	9661.2	12038	14274.2	22704	12998.04	30516571.14
5	22	10232.8	14440.2	17488	20794	27132	18017.4	32892412.42
6	30	14109.6	22204.2	32038	40303	52824	32295.76	183633393.8
7	35	21218.4	35104.2	42860	56566	92352	49620.12	587465534.4
8	40	24211.9	35953.4	46790	77650	121398	61200.66	1221512031
9	45	29732	40628	55892	75691	130023	66393.2	1250672970
10	50	30999	48329	61994	95587	15823	50546.4	750487082.2
Avg							31815.92	410768038

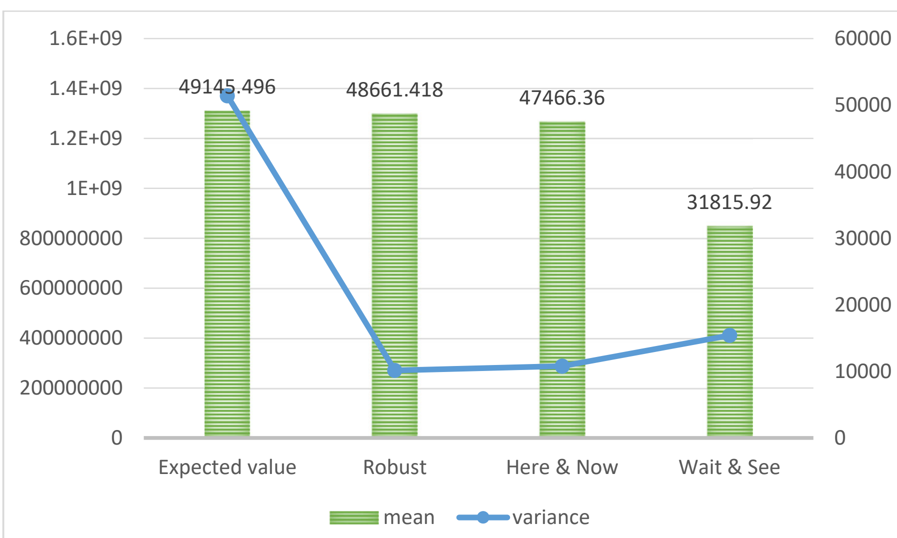

Fig. 10. The of mean and variance obtained from different scenarios in different methods

As can be seen in Fig 10, the maximum value of variance belongs to Expected Value approach and the minimum value of variance has resulted from Robust approach. Regarding the average value of cost, the minimum value has observed in the Wait & See approach. To have a more precise evaluation, EVPI, VSS and PR criteria have been utilized. The mentioned value is calculated by means of the following formulations.

$$EVPI = z_{HN} - z_{ws} \tag{31}$$

$$VSS = z_{EEV} - z_{HN} \tag{32}$$

$$PR = z_{AR} - z_{HN} \tag{33}$$

In Table 7, the values of Expected Value of Perfect Information (EVPI), Value of Stochastic Solution (VSS) and Price of Robustness (PR) for each problem are demonstrated.

Table 7
The value of criteria

N	L	EVPI	VSS	PR
1	5	184.4	683.2	0
2	9	3973.36	5733.56	517.6
3	13	5833.8	1161.86	264
4	17	14918.24	119.32	264
5	22	14893.28	1754.12	264
6	30	15468.84	1873	1910.6
7	35	14972.6	1391.28	424.64
8	40	19136.28	2051.44	326.24
9	45	19389.28	206.42	1225.1
10	50	27126.8	1156.32	504
Avg		13589.69	1613.052	570.018

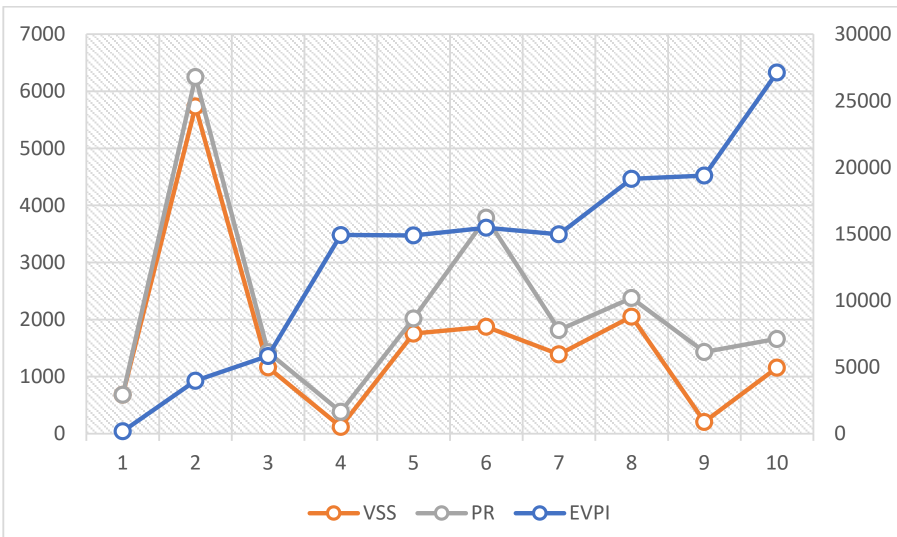

Fig. 11. The value of EVPI, VSS, and PR in different problems

As can be seen in Fig 11 and Table 7, using stochastic two-stage programming decreased the costs for 1613 units in comparison with the expected value. However, using Robust approach added 570 units to the cost. The Wait & See approach decreased 13589 units of cost in average. This shows the importance of forecasting the value of data in the proposed model.

7. Conclusion

Recent attention has been directed towards Supply Chain Network Design (SCND) with disruptions, with various optimization strategies proposed to address the issue (Govindan et al., 2017). Snyder et al. (2016) compiled a review of the management science and operations research models for mitigating disruptions in SCND. In this research, a novel mathematical model for optimizing both the facility location and network design under uncertain conditions is proposed. The model takes into account realistic assumptions regarding the uncertainty, which are of utmost importance for decision makers and managers. The study investigates the uncertainty in data related to the facility location-network design problem, which is caused by fluctuations in demand, changes in travel costs, and road or route damage. To assess the uncertainty, five methods were used including Expected Value, Here & Now stochastic two-step programming approach, Wait & See waiting and observation approach, and the robust approach. The analysis of the cost improvement from each method revealed that the two-stage stochastic programming approach resulted in an average reduction of 1,613 units in comparison to the mean, while the robust approach led to an average increase of 570 units. The robust approach showed a reduction in variance which makes the model more compelling for further study. The wait and see approach resulted in a reduction of cost by an average of 13,589 units, highlighting the importance of forecasting data in the proposed network model. The research also introduces a meta-heuristic approach based on simulated annealing algorithms to solve large-scale problems, with results showing the efficient performance of the algorithm. However, the research also acknowledges that obtaining correct estimates for uncertain parameters can be a challenge and thus, further studies could be carried out based on real-life case studies in a supply chain network.

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