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A sustainable inventory model for growing items considering carbon emissions, product expiry, and profit-sharing policy

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C H R O N I C L E

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A B S T R A C T

In this article, a multi-echelon supply chain for growing and deteriorating items, where the grower has a lot of live newborn items (growing) is discussed. The grower transfers the matured inventory to the processor in each shipment. The processor begins to process the stock as a ready-sale product in the market. The processor also delivers the processed inventory to the retailer in each shipment in the non-processing period of his cycle length. Then the processor offers trade credit to the retailer and makes the retailer agree to share a portion of his profit with him. The product's life cycle when in the hand of the retailer is certain and it expires after some time t. Carbon emission during processing is considered while packing and preserving the livestock for sale. Depending on these assumptions, there are six possibilities to discuss profit values. Sensitivity analysis was also brought to verify the optimal determined values. The profit-sharing sharing method's outcome benefits the processor and the retailer more.

1. Introduction

Trade credit is a company (B2B) agreement by which users can buy items without paying in cash out the advance, and then pay the provider at a later point. Typically, businesses that use trade credit may allow clients to pay in a few days, with the transaction documented by an invoice. Trade credit is a sort of 0% financing that increases a company's assets while postponing payment for a predetermined amount of products or services to a later date and requires no tax to be repaid in connection to the payback period. In general, offering trade credit to a buyer always gives benefits to a company's cash flow. The duration a credit is granted is defined by the company granting the advance and is negotiated upon by both companies granting the favor and the firm receiving it. Trade credit can also be used to help businesses finance short-term expansion. Trade credit, which is a sort of borrowing with no interest, is frequently used to boost sales. In the end, trade credit is a type of commercial lending that is extremely beneficial to firms. It is an equity mortgage that allows a buyer to purchase products with repayment at a future stage at no additional cost. This results in enhanced free cash flow and the reduction of traditional finance costs. Profit sharing is a method in which staff is paid a percentage of the company's net earnings based on a predetermined written formula. Such benefits, which may vary depending on salary or compensation, are separate from and in addition to ordinary wages. Profit sharing is a sort of pre-tax employee contribution plan in which employees receive a portion of a company's profits. The profit-sharing payments are determined by: 1. Profitability of a company 2.Regular salary and bonuses for employees 3. The amount is determined by the company. A profit-sharing plan (PSP) pays employees a percentage of the company's earnings over a certain period of time (e.g., a year). In most cases, a worker gets a proportion or cash of the company's profits in cash or stock holdings. Many companies provide revenue sharing as a lifetime pension to their employees. If an employer does not earn a profit during the time frame (e.g., year), they are not required to contribute that year. Since these two are playing the most important role in revenue earning and also concentrate on customer welfare we planned to include this in our model to make it more user-friendly for the growing

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items inventory model. By considering these possibilities, the model has been developed to overcome these situations in real life.

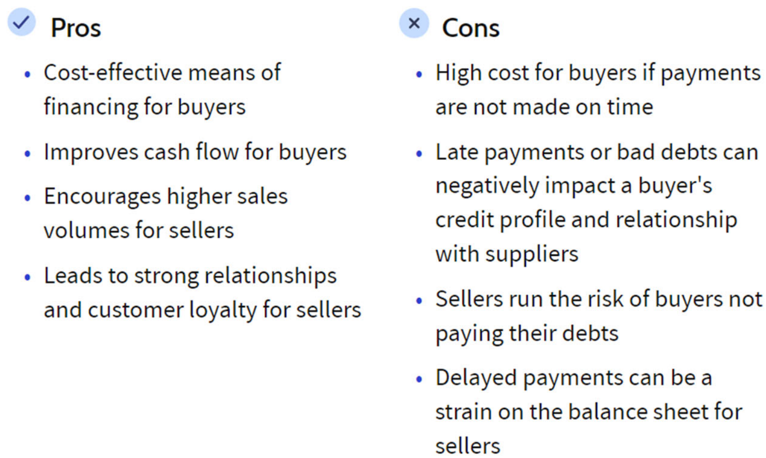

Fig. 1. Trade credit Effects

In this study, we created an inventory model for growing items for a grower-processor-retailer three-tiered supply chain, taking into account the selling price-dependent demand of the livestock and the aforementioned issues of trade credit and profit sharing. In addition to this, we have added the feature that carbon emission is emitted during the packing and preserving process of the product as a real-life assumption. Profit sharing and credit period offers are considered for the retailer and processor to make maximum profit in this sale for both. The paper is organized as a Literature review in section 2, and model formulation followed by the literature review, notations, and assumptions were given to make our model clear to understand throughout this work, solution procedure is obtained to verify the results of the developed mathematical model, numerical examples are given and optimal profit values are obtained.

2. Literature Review

2.1 Growing items inventory supply chain

In general, economic order(production) quantity (EOQ/EPQ) models for manufacturing products have typically been provided. Various EOQ/EPQ models have been proposed in the literature, each incorporating certain significant properties of a specific category of item. Rezaei (2014) suggests a new class of inventory models, specifically one for expanding inventory items. Growing inventory items include poultry and cattle. Hidayat's (2020) proposed scheme modifies three presumptions of the classical EOQ (i.e) purchased objects do not proliferate, infinite capacity, and an unlimited budget. Inventory models for growing items that take into account quality aspects, allowable shortages with complete backorder, and holding costs during both the growth and consumption periods, a model of nonlinear programming is developed in Alfares and Afzal (2021). Several inventory items, such as livestock, are living entities and can thus grow during the replenishment cycle. To establish the ideal inventory strategy that minimizes overall inventory cost in both owned and rented facilities, a mathematical model is developed by Sebatjane(2019). Sebatjane's (2019) offers an inventory system in which the ordered items, such as cattle, might expand during the inventory replenishment cycle. Furthermore, it is expected that some of the things are of worse quality than desired. It is also assumed that live newborn goods are ordered and nourished until they reach a customer-specified weight before being butchered. Before all slaughtered things are sold, they are screened to separate the high-quality items from the low-quality ones. Moon et al. (2005) included these two opposing physical features of stored things into an inventory model, and created models for improving/degrading items with time-varying demand patterns across a finite planning horizon while accounting for the effects of inflation and the time value of money. Gharaei and Almehdawe (2020, 2021) introduced Economic Growing Quantity (EGQ), a new group of inventory models focused on growing commodities in agricultural businesses like fishing, livestock, and cattle. For a growing item, an EGQ inventory model takes into account the probability density functions of lifespan and mortality. It also takes into account the growing activities of live and dead objects. Abbasi et al. (2022) developed a theory of growing economic order quantity was developed in this work, which was presented using a fictitious numerical example. It was supposed that a corporation acquires day-old chicks, feeds and raises them until they reach the required weight, and then sells them after quality control. Nobil and Taleizadeh (2019)'s study described an economic order quantity (EOQ) model for growing objects. In this supply chain, a buyer orders commodities such as animals and poultry, which develop and achieve their optimal weight over time. Shortages are not permitted, and order numbers must be integer values. A two-tiered sustainable supply chain model with a supplier–retailer scenario is studied in this study of Choudhury and Mahata (2021). The supplier's primary responsibility is to breed newborn animals in accordance with a biological growth plan. Carbon emissions are calculated based on the transportation of killed items to the store. The goal of Sebatjane(2019)'s is to provide a model of coordinated inventory control for growing goods in a supply chain that includes farming, processing, and retail operations. The purpose of Malekitabar et al. (2019)'s research is to investigate the growth phase in the supplier and then in the farmer's sites in order to maximize the profit of the supplier as the leader and the farmer as the follower in a Stackelberg game. An effective system for inventory control in a three-tiered supply chain for growing commodities is developed by Sebatjane (2021), including farming, processing, and retail echelons. Customer demand is thought to be reliant on inventory level and expiration date at the retailer end of the chain. It is explored the usefulness of a profit-enhancing mechanism that modifies the standard zero-ending supply chain at the retailing end of the chain. Sebatjane(2020b) described a model for inventory management in a perishable food industry supply chain that starts with farm work that raises live inventory items and concludes with the demand for processed inventory. A processing step connects the agricultural and consumption (retail) stages, during which living inventory is converted into a consumable form. An integrated inventory model is developed by Sebatjane(2022) with the goal of optimizing the performance of the entire food supply chain. The processing echelon's goal is to convert live growing materials into processed food products. Once processed, the goods are vulnerable to deterioration at both the processing and retail levels.

2.1.1 EOQ/EGQ models with trade credit offers

Mittal and Sharma (2021) proposed a model for a specific type of inventory, namely increasing goods. Poultry and animals are two real-world examples of growing objects. By planning a broad scientific model that might be used for a variety of growing objects, followed by a particular numerical model focusing on a certain type of chicken. Teng's (2012) paper, they enhanced the stable demand model to include a time-dependent linear non-decreasing demand function. The demand function of a product grows with time during the development stage of its life cycle (particularly for high-tech products). Mahata et al.'s study shows that the retailer's ideal payment term and restocking time exist and are distinct. The aim of the study is to develop an inventory policy for deteriorating items so that demand for these products is not dependent on stock levels with trade credit and preservation technology investment in Singh et al. (2016)'s work. Lou and Wang's (2013) article, looks at how trade credit affects demand but has a detrimental influence on collecting the lender's existing debt. We first offer an economic order quantity model from the purchaser's perspective in order to determine the seller's optimal trade credit and order quantity at the same time. The recently written model tacitly presumed that the supplier would provide a delay time to the retailer, but the retailer would not provide a trade credit period to his/her consumer. In this note, we suppose that the store additionally uses trade credits to stimulate customer demand in terms of developing their replenishment model. Cárdenas-Barrón et al. (2020) research considers an economic order quantity (EOQ) stock model with nonlinear dependent demand and nonlinear holding cost. It is designed from the retailer's perspective, with the supplier providing a trade credit period. The standard concept of zero-ending inventory level is relaxed in this work. Dye and Yang (2015)'s worked at sustainability initiatives in the context of combined trade credit and inventory management, where demand is determined by the duration of the credit period provided by the merchant to its clients. The effects of the credit term and environmental rules on the inventory model are quantified. Sarkar's (2015) research has two objectives. The first step is to evaluate the trade-credit policies of suppliers and retailers for fixed-lifespan products and time-varying degradation. We assume that suppliers provide full trade credit to retailers, but retailers only provide trade agreement credit to their customers. Taleizadeh(2020) presents an inventory model with heterogeneous inventory ordering policies. The ordering policies consider a hybrid payment strategy with numerous prepayment and partial trade credit schemes tied to order quantity. Giri et al. (2018) , worked on the manufacturer-retailer supply chain model for inventory products with trade credit offers and profit sharing strategy, this model certainly benefits the manufacturer and retailer in a good manner. Since it benefits in a useful manner, we tried to implement this strategy in the three-echelon supply chain model for growing and deteriorating items in the work of Sebatjane(2022), with selling price-dependent demand and trade credit offers also with profit-sharing by the retailer to the processor is developed.

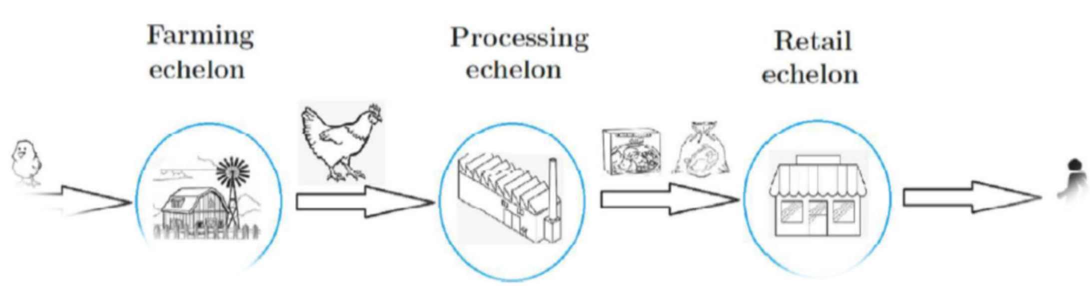

Fig. 2. Representation of the Supply chain

2.2 Contribution

In this model, we have worked on a multi-echelon supply chain for growing and deteriorating items. A farmer(grower)-processor-retailer with a single growing item is considered. Here, the demand rate of the processor and retailer is price-dependent. In the processing period, the product emits carbon-di-oxide during the whole process since we have considered the emission cost for the processor's inventory cycle. The processor also offers a trade credit period to the retailer, and the retailer also agrees to pay a fixed portion of his profit to the processor. It is a win-win situation for both the processor and the retailer in this model. The retailer obtains the packed processed product from the processor since every processed product of livestock must have a limiting period of consumption since we have considered the expiration rate in the retailer's inventory level. The optimal values are obtained and the joint total profit cost of the three-echelon system is obtained and results are verified.

3. Notations

Notations	*Description*
	Grower's Notations
g	Number of newborns, the grower receives in his lot
G_0	Lot size of the Grower
$G(t)$	Growth function of the product received by the grower
p_g	Unit price of the grower
T_g	Time period of the grower'sinventory cycle
G_1	The product's maximum possible weight
p_v	purchase price of the grower'slot per unit from the farmer
β	Constant increase in weight ratio of the growing item
γ	Growth rate of the product over time
$I_g(t)$	Inventory level of the grower over the time period $0 \leq t \leq T_g$
$N_f(t)$	Dietary feeding function of the growing item
β_1	Feeding ratio of the growing inventory over time
a	Constant demand for the growing items in the processor's lot
b	Ratio of change in demand pattern because of the sales price
α_1	*live stock survived during growing period*
w_1	*maturity weight attained*
	Processor's Notations
P_r	Processing rate of the processor in time period T_{v_1}.
T_p	Time period of the processor's echelon
L_p	Processor's weight of the inventory at the initial time period of the cycle
p_p	Unit price of the processor's lot
T_{v_1}	Processing time of the received inventory
T_{v_2}	Non − processing time of the retailer
θ_p	Deterioration rate of the Processor over the time T_p
$I_{v_1}(t_1)$	Inventoryleveloftheprocessorintheprocessingtime$0 \leq t_1 \leq T_{v_1}$
$I_{v_2}(t_2)$	Inventory level of the processor in the processing time $0 \leq t_2 \leq T_{v_2}$
δ	Profit ratio shared by the retailer to the processor
	Retailer'sNotations
$I_r(t)$	Inventory level of the retailer in $[0, T_r]$
L_r	Lot size of the retailer
p_r	Unit price of the retailer's lot
$\theta_r(t)$	Deterioration rate of the retailer
c_d	Deterioration cost of the growing items in the inventory
i_m	Rate of interest paid to the processor, if the total cost is paid beyond the fixedtime
i_e	Interest earned from the source(bank)
i_c	Interest Charged from the retailer by the source(bank)
i_v	Opportunity lost sales cost by for the processor

4. Assumptions

1. A three-echelon growing supply chain model for deteriorating items is considered. (grower-processor-retailer)

2. The demand of the retailer is linearly decreasing function with selling price p_r, $D_r = a - bp_r$, where $a > 0$, $b > 0$.

3. Shortages are not allowed to occur in the retailer's inventory, since the processor's inventory level is always greater than the retailer's demand rate and all the replenishments of the cycle are made instantaneously.

4. The processor provides a trade credit term M to the retailer in exchange for the retailer sharing his profit from the company's sales during the credit period M. If the retailer settles just after the credit period M, the processor must pay the complete interest on the appraised value until the payment time R at a rate of interest i_m.

5. If the payback time is longer than the credit period and shorter than the total time used, the retailer pays the processor the agreed profit sharing up to the credit period.

6. If the retailer pays the processor just before the credit period, the profit is shared just for that period. Due to the retailer's failure to pay on trade credit during this period, the manufacturer lacks the opportunity to gain. To offset the opportunity cost of a loan offer, the processor may request that the merchant contribute a portion of his revenue during the credit period.

7. Throughout the activity, the retailer puts his profits to an interest-giving organization (bank) with a rate of interest i_e that is obtainable without any conditions to the retailer.

5. Model Formulation

Throughout the time T_g, the grower feeds the live newborn items up to the maturity stage w_1(reaches the maximum possible weight), then ships the grown items L_g to the processor in each shipment; after some time, the lot in the processor's echelon depletes due to demand D_p and deterioration θ_p and reaches zero in time T_{v_2}. After the processing period (T_{v_1}) ends the processor ships the lot L_p, to the retailer in the non-processing time T_{v_2}, in each shipment, the products in the processor's lot L_p also depletes in the ratio θ_r after received by the retailer and reaches to zero due to demand D_r and deterioration. The process of the three supply chain players is explained in the upcoming sections.

5.1 Grower's Echelon

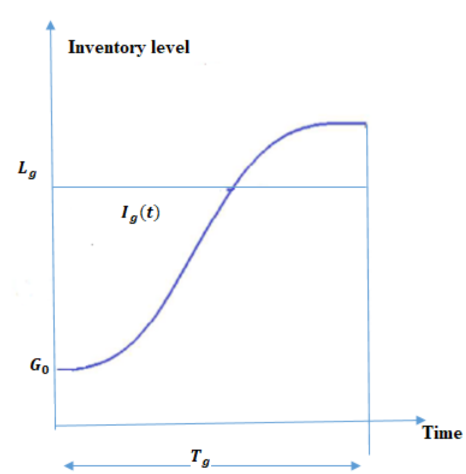

Fig. 3. Grower's Inventory as in (Sebatjane & Adetunji, 2020)

At the start of each cycle ($t = 0$), the grower accepts orders for g live newborn products. The things are reared by the farmer over the duration of the cultivation phase, and at the end of the growth period T_g, the farmer ships a complete lot of usable inventory to the slaughterer. Since g neonatal things are obtained as a current farming cycle begins, and each item's weight can be modulated by,

$$G(t) = G_1(1 + \beta e^{-\gamma t})^{-1} \tag{1}$$

The mass of the grower's inventory over the interval $[0, T_g]$ is given by,

$$I_g(t) = gG(t) = gG_1(1 + \beta e^{\gamma t})^{-1} \quad 0 \le t \le T_g \tag{2}$$

The grower's original lot size at the start of each cycle is weighted at $(t = 0)$, is

$$G_0 = I_g(0) = gG_1(1 + \beta)^{-1}$$

$$\therefore g = \frac{G_0(1 + \beta)}{G_1} \tag{3}$$

We get,

$$I_g(t) = \frac{G_0(1 + \beta)}{(1 + \beta e^{-\gamma t})} \quad 0 \le t \le T_g \tag{4}$$

The sales revenue of grower is given as, $SR_g = p_g\alpha 1 g w_1$. The grower's total profit is the sum of the setup and feeding cost of the live newborns, that is given by Sebatjane (2022).

$$\begin{aligned} TPC_g &= SR_g - \frac{PC_g}{T} - SC_g - FC_g \\ &= p_g\alpha_1 g w_1 - \frac{p_v w_0 g}{T_g} - \frac{K_g}{T_g} - \frac{c_g}{T_g}\left[\frac{P_r T_p(1 + \beta e^{-\gamma T_g})^{-1}}{G_1(1 + \beta)e^{-\alpha_1 T_g}}\right]\left(\frac{e^{\beta_1 T_g} - 1}{\beta_1}\right) \end{aligned} \tag{5}$$

5.2 Processor's Echelon

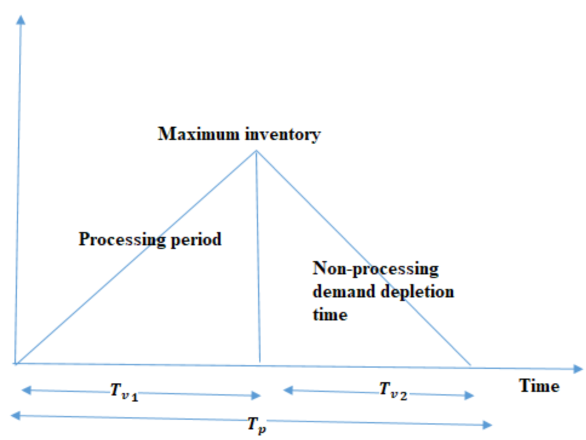

Fig. 4. Processor's Inventory Cycle

Every T_p time units, the processor receives an order for L_p weight units of inventory. The processor delivers N evenly proportioned shipments of processed inventory to the retailer in every T_r time units based on the lot size of L_p weight units. It can be separated into two pieces based on the processor's cycle time T_p, namely the preparation and non-preparation portions T_{v_1}, T_{v_2} .

$$I_p(t) = \int_0^{T_{v_1}} I_{v_1}(t_1)dt_1 + \int_0^{T_{v_2}} I_{v_2}(t_2)dt_2 \tag{6}$$

The prepared inventory is generated at rate P_r , and deteriorates at constant rate θ_p . The processor inventory depletes owing to demand and deterioration, and it accumulates due to processing. As a result, the inventory level is shown here over the time window $[0, T_p]$ is, (where $D_p = a - bp_p$)

$$\frac{dI_{v_1}(t_1)}{dt_1} = (P_r - D_p) - \theta_p I_{v_1}(t_1), \qquad 0 \le t_1 \le T_{v_1} \tag{7}$$

Similarly, after the processing time of the cycle, the prepared inventory degrades due to supply and deterioration; even so, there is no gathering of the treated inventory in time $[0, T_{v_2}]$.

$$\frac{dI_{v_2}(t_2)}{dt_2} = -D_p - \theta_p I_{v_2}(t_2), \qquad 0 \le t_2 \le T_{v_2} \tag{8}$$

The boundary conditions $I_{v_1}(0) = I_{v_2}(T_{v_2}) = 0$, is applied to find the inventory level of the processor's cycle,

$$I_{v_1}(t_1) = \frac{P_r - D_p}{\theta_p}\left(1 - e^{-\theta_p t_1}\right), \quad 0 \le t_1 \le T_{v_1} \tag{9}$$

$$I_{v_2}(t_2) = \frac{D_p}{\theta_p}\left(e^{-\theta_p (T_{v_2} - t_2)} - 1\right), \quad 0 \le t_2 \le T_{v_2} \tag{10}$$

The boundary conditions $I_{v_1}(T_{v_1}) = I_{v_2}(0)$, is used to find out the time period of processing,

$$\approx T_{v_1} = \frac{P_r}{(P_r - D_p)} T_{v_2}\left(1 + \frac{1}{2}\theta_p T_{v_2}\right) \tag{11}$$

The processing amount is identical to the quantity received by the processor from the grower.

Carbon emission during processing Livestock:

Livestock processing emission is a frequent fallacy that chickens don't affect climate change since, unlike cows, they don't release methane during the digestion process. However, greenhouse gases (GHGs), such as CO2 from fossil fuels and nitrous oxide from fertilizer applications, are still released in order to produce chicken feed. Additionally, nitrous oxide, that's even more powerful than bio-gas and has 298 times the overall heating capability of CO2 over 100 years, is released by chicken manure. Only 50% of the emissions associated with chicken production occur prior to slaughter. Typically, chicken flesh is processed into a range of products, such as boneless, skinless meat and chicken nuggets; each of these stages requires a substantial amount of energy and water, which considerably increases the GHG carbon output of chicken products as in Goodman(1999). A European Union directive mandates that all animals be stunned before being slaughtered in order to guarantee that they are not feeling any pain during the process. Electrical, mechanical, or gas-based stunning techniques are only a few ways to make an animal unconscious. There is one exception: animals slaughtered in accordance with religious rites at slaughterhouses. Concerns over the requirements for animal care in the processing of pigs and poultry have grown. This method of stunning has grown in popularity, especially in Europe, because it can reduce the risk of injury during the stunning process because the animals do not need to be restrained beforehand. It can also improve the quality of the meat and is generally regarded as one of the most humane stunning techniques. There are usually limits on the transportation of animals, including transporting them to slaughterhouses, when livestock culls are implemented to stop and stop the spread of illness as given in the data of the article by Edinburgh Sensors, 2018.

By considering all these issues, we have included the carbon emission cost during processing process, since these emission cost may increase the total cost of the processor, which affects the profit of the processor and also it may change the total profit of the integrated system of the supply chain, since we have to introduce some carbon emission regulation policy to reduce the emission such as carbon tax, cap and trade policy, carbon cap and offset. In our model we have worked on carbon tax policy as in Rout et al. (2020).

1. When the processor's lot size is smaller than demand during T_p $(D_p T_p)$, the quantity of degrading inventory throughout this processor's cycle and carbon emission on degradation is defined. When the processor's degradation cost of c_d per weight unit is taken into account as,

$$DC_{pr} = \frac{c_d + c_d'}{T_p}\left(L_p - D_pT_p\right) = \frac{c_d + c_d'}{T_p}\left(P_rT_{v_1} - D_pT_p\right)$$

where c_d', is the emission parameter for degradation cost.

2. The processor's holding cost for the processed inventory per unit time,

$$HC_{pr} = \frac{h_p + h_p'}{T_p}\int_0^{T_{v_1}} I_{v_1}(t_1)dt_1 + \int_0^{T_{v_2}} I_{v_2}(t_2)dt_2 = \frac{h_p + h_p'}{T_p}\left[\frac{(P_r - D_p)T_{v_1}^2}{2}\left(1 - \frac{\theta_pT_{v_1}}{3}\right) + \frac{D_pT_{v_2}^2}{2}\left(1 + \frac{\theta_pT_{v_2}}{3}\right)\right]$$

h_d'- carbon emission cost for holding the inventory.

3. The carbon emission due to processing is, where e_p, emission cost for the processing period per unit item,

$$CE_p = e_pP_rT_{v_1}$$

4. The processor's opportunity cost due to trade credit offer is,

$$OLC_p = \frac{i_pp_p}{T_p}\int_0^R Ddt = \frac{i_pp_pD_pR}{T_p}$$

Case IA: When $R < M$

Here, the profit sharing ratio of the processor is same for the upcoming case IC also.

The Profit shared is $PF_{p_{11}} = PF_{p_{13}} = \delta(p_r - p_p)D_rM$

Then the total profit of the processor in this case is given as

$$TP_{pIA}(T, p_p, R) = (p_r - p_p)D_p - \frac{K_{pr}}{T_p} - \frac{p_g\alpha_1gw_1}{T_p} - e_pP_rT_{v_1} - \frac{c_d + c_d'}{T_p}\left(P_rT_{v_1} - D_pT_p\right) + i_m(R - M)p_pD_p + \frac{\delta(p_r - p_p)D_pM}{T_p} - \frac{h_p + h_p'\left[\frac{(P_r - D_p)T_{v_1}^2}{2}\left(1 - \frac{\theta_pT_{v_1}}{3}\right) + \frac{D_pT_{v_2}^2}{2}\left(1 + \frac{\theta_pT_{v_2}}{3}\right)\right] - \frac{i_pp_pD_pR}{T_p}}{T_p} \quad (12)$$

Case IB: When $R < M$

Here, the profit sharing ratio of the processor is same for the upcoming case II C also.

The Profit shared is $PF_{p_{12}} = PF_{p_{23}} = \delta(p_r - p_p)D_rR$

$$TP_{p_{IB}}(T, p_p, R) = (p_r - p_p)D_p - \frac{K_{pr}}{T_p} - \frac{p_g\alpha_1gw_1}{T_p} - e_pP_rT_{v_1} - \frac{c_d + c_d'}{T_p}\left(P_rT_{v_1} - D_pT_p\right) + \frac{\delta(p_r - p_p)D_pR}{T_p} - \frac{h_p + h_p'}{T_p}\left[\frac{(P_r - D_p)T_{v_1}^2}{2}\left(1 - \frac{\theta_pT_{v_1}}{3}\right) + \frac{D_pT_{v_2}^2}{2}\left(1 + \frac{\theta_rT_{v_2}}{3}\right)\right] - \frac{i_pp_pD_pR}{T_p} \quad (13)$$

Case IC: When $M < T$

$$TP_{p_{IC}}(T_p, p_p, R) = (p_r - p_p)D_p - \frac{K_{pr}}{T_p} - \frac{p_g\alpha_1gw_1}{T_p} - e_pP_rT_{v_1} - \frac{c_d + c_d'}{T_p}\left(P_rT_{v_1} - D_pT_p\right) + i_m(R - M)p_pD_p - \frac{h_p + h_p'}{T_p}\left[\frac{(P_r - D_p)T_{v_1}^2}{2}\left(1 - \frac{\theta_pT_{v_1}}{3}\right) + \frac{D_pT_{v_2}^2}{2}\left(1 + \frac{\theta_pT_{v_2}}{3}\right)\right] - \frac{i_pp_pD_pR}{T_p} + \frac{\delta(p_r - p_p)D_pM}{T_p} \quad (14)$$

Case II A: When $R > T$

Here, the profit sharing ratio of the processor is same for the upcoming case II B also.

The Profit shared is $PF_{p_{21}} = PF_{p_{22}} = \delta(p_r - p_p)D_rT_r$

$$TP_{p_{IIA}}(T_p, p_p, R) = (p_r - p_p)D_p - \frac{K_{pr}}{T_p} - \frac{p_g \alpha_1 g w_1}{T_p} - e_p P_r T_{v_1} - \frac{c_d + c_d'}{T_p}\left(P_r T_{v_1} - D_p T_p\right) + \delta(p_r - p_p)D_p - \frac{h_p + h_p'}{T_p}\left[\frac{(P_r - D_p)T_{v_1}^2}{2}\left(1 - \frac{\theta_p T_{v_1}}{3}\right) + \frac{D_p T_{v_2}^2}{2}\left(1 + \frac{\theta_p T_{v_2}}{3}\right)\right] - i_p p_p D_p R \quad (15)$$

Case II B: When $M > T$

$$TP_{p_{IIB}}(T_p, p_p, R) = (p_r - p_p)D_p - \frac{K_{pr}}{T_p} - \frac{p_g \alpha_1 g w_1}{T_p} - e_p P_r T_{v_1} - \frac{c_d + c_d'}{T_p}\left(P_r T_{v_1} - D_p T_p\right) + \delta(p_r - p_p)D_p - \frac{h_p + h_p'}{T_p}\left[\frac{(P_r - D_p)T_{v_1}^2}{2}\left(1 - \frac{\theta_p T_{v_1}}{3}\right) + \frac{D_p T_{v_2}^2}{2}\left(1 + \frac{\theta_p T_{v_2}}{3}\right)\right] - i_p p_p D_p R + i_m(R - M)p_p D_p \quad (16)$$

Case II C: When $R < T_p$

$$TP_{p_{IIC}}(T, p_p, R) = (p_r - p_p)D_p - \frac{K_{pr}}{T_p} - \frac{p_g \alpha_1 g w_1}{T_p} - e_p P_r T_{v_1} - \frac{c_d + c_d'}{T_p}\left(P_r T_{v_1} - DT_p\right) + \frac{\delta(p_r - p_p)D_p R}{T_p} - \frac{h_p + h_p'}{T_p}\left[\frac{(P_r - D_p)T_{v_1}^2}{2}\left(1 - \frac{\theta_p T_{v_1}}{3}\right) + \frac{D_p T_{v_2}^2}{2}\left(1 + \frac{\theta_p T_{v_2}}{3}\right)\right] - \frac{i_p p_p D_p R}{T_p} \quad (17)$$

5.3 Retailer's Echelon

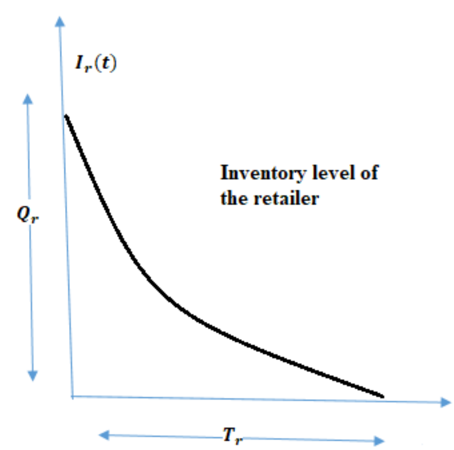

Fig. 5. Retailer's Inventory Cycle

In the retailer's cycle, L_r weight units of treated stock time T_r units are delivered to the store. The treated inventory deteriorates at $\theta_r(t)$, which is time dependent. All the livestock the retailer have is definitely going to expire (i.e) not worth for human consumption. therefore, the expiration rate of the product is u, where, $0 \le \theta_r \le 1$ as in Madhud(2021).

$$\theta_r(t) = \frac{1}{1 + u - t}, \quad 0 \le u \le T_r \le u$$

As a result, the processed inventory is decreased during the replenishment cycle due to both customer needs and deterioration. Then the processed inventory throughout the time interval $[0, T_r]$ becomes, (where $D_r = (a - b * p_r)$).

$$\frac{dI_r(t)}{dt} = -D_r - \theta_r(t)I_r(t) \quad 0 \le t \le T_r \text{ where } \theta_r(t) = \frac{1}{1+u-t}, \quad 0 \le t \le u \quad (18)$$

The retailer's demand (inventory) at any time t, can be solved using the given boundary condition, $I_r(T_r) = 0$, and the inventory level of the retailer is given as,

$$I_r(t) = D_r(1+u-t)\, ln\left(\frac{1+u-t}{1+u-T}\right) \quad 0 \le t \le T_r \tag{19}$$

The prior order size received by the store at the opening of each loop is,

$$L_r = I_r(0) = D_r(1+u)\, ln\left(\frac{1+u}{1+u-T}\right)$$

1. During the retailer's cycle, of duration T_r, the quantity of deteriorating inventory is described as the order quantity (L_r), less than the demand during T_r. Taking the firm's degradation cost of c_d per weight unit into account, the firm's deterioration cost per unit time is,

$$DC_r = \frac{c_d(1+u)\, ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r(1+u)\, ln\left(\frac{1+u}{1+u-T}\right) - D_r T_r\right]$$

2. The holding cost of the retailer's inventory after received from the processor is given as,

$$HC_r = \frac{h_r}{T_r}\int_0^{T_r} I_r(t)dt = \frac{h_r D_r}{T_r}\left(\frac{(1+u)^2}{2} ln\left(\frac{1+u}{1+u-T}\right) - \frac{(1+u)}{2}T_r + \frac{1}{4}T_r^2\right)$$

5.1 Profit sharing and Interest earned & paid by the Retailer

Case I: $M \le T_r$

Case IA: When $M \le R \le T_r$

The sales revenue of the retailer in this case is given as $SR_1 = p_r D_r M$.

According to the agreement, the retailer gives the ratio of profit to the processor,

$$PF_r = \delta(p_r - p_p)D_r M$$

to the manufacturer from his sales revenue.

In case, if $R > M$, then the retailer pays the interest amount on purchase cost at the rate i_p, then

The total interest earned by the retailer $IE_{R1} = i_e p_r \int_0^R D_r t dt = \frac{i_e p_r D_r R^2}{2}$

According to this in time R, the retailer has to deposit the purchase price to the processor along with his share in profit and interest. It may lead to other two situations in the inventory cycle, they are,

Case I A1

If the total paid profit share is more than the revenue of the retailer at that time, since the amount has been adjusted from the source with an interest rate i_c.

In the end the interest paid by the retailer in the cycle is,

$$IP_{R_{I.1}} = i_c(T_r - R)\left[(p_p D_r T_r - p_r D_r R) + \delta(p_r - p_p)D_r M + i_m(R-M)p_p D_r T_r - \frac{i_e p_r D_r R^2}{2}\right] \tag{20}$$

The retailer's entire profit in this case is,

$$TP_{R_{1.1}} = \begin{pmatrix} p_r D_r M - \frac{K_r}{T_r} - \frac{p_p \alpha_1 g w_1}{T_r} - \frac{c_d (1+u)\, ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r (1+u)\, ln\left(\frac{1+u}{1+u-T}\right) - D_r T_r\right] \\ -\frac{\delta(p_r - p_p) D_r M}{T_r} - i_m (R-M) p_p D + \frac{i_e p_r D}{2T_r}(T_r - R)^2 - i_c (T_r - R) \\ \left[(p_p D_r T_r - p_r D_r R) + \delta(p_r - p_p) D_r M + i_m (R-M) p_p D_r T_r - \frac{i_e p_r D_r R^2}{2}\right] \\ + \frac{h_r D_r}{T_r}\left(\frac{(1+u)^2}{2} ln\left(\frac{1+u}{1+u-T}\right) - \frac{(1+u)}{2} T_r + \frac{1}{4} T_r^2\right) \end{pmatrix} \quad (21)$$

Case I.A2 What if the retailer earns more profit than to be paid(shared)

In this case, the retailer keeps the additional amount and at the end of the cycle, the interest earned by the retailer is,$IE_{R_{1.2.1}}$ is

$$= i_e (T_r - R)\left[(p_r D_r R - p_p D_r T_r) - \delta(p_r - p_p) D_r M - i_m (R-M) p_p D_r T_r + \frac{i_e p_r D_r R^2}{2}\right] \quad (22)$$

In the time $(T_r - R)$, the retailer has his revenue to the same source and at the end of the cycle, the interest earned is,

$$IE_{R_{1.2.2}} = i_e p_r \int_0^{T_r - R} D_r t dt = \frac{i_e p_r D_r (T_r - R)^2}{2} \quad (23)$$

$$TP_{R_{1.2}} = \begin{pmatrix} p_r D_r M - \frac{K_r}{T_r} - \frac{c_d (1+u)\, ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r (1+u)\, ln\left(\frac{1+u}{1+u-T}\right) - D_r T_r\right] \\ -\frac{\delta(p_r - p_p) D_r M}{T_r} - i_m (R-M) w D + \frac{i_e p_r D_r}{2T_r}(T_r - R)^2 - \frac{p_p \alpha_1 g w_1}{T_r} + i_e (T_r - R) \\ \left[(p_r D_r R - p_p D_r T) - \delta(p_r - p_p) D_r M - i_m (R-M) p_p D_r T + \frac{i_e p_r D_r R^2}{2}\right] \\ + \frac{h_r D_r}{T_r}\left(\frac{(1+u)^2}{2} ln\left(\frac{1+u}{1+u-T}\right) - \frac{(1+u)}{2} T_r + \frac{1}{4} T_r^2\right) \end{pmatrix} \quad (24)$$

Case IB: When $R \leq M \leq T_r$

The fraction of the profit, the processor gets, when this chance flows, $PF_{r2} = \delta(p_r - p_r) D_r R$.

Here as the payment is made in advance, then there is no need to pay any interest .

$$\text{Interest earned upto the payment time } IE_{pt} = \frac{i_e p_r D_r R^2}{2} \quad (25)$$

$$\text{I. E after the payment and until the end } IE_{pe} = \frac{i_e p_r D_r (T_r - R)^2}{2} \quad (26)$$

The I. C by the processor is

$$IC_{p1} = i_c \left[(p_p T_r - p_r R) D + \delta(p_r - p_p) D_r R - \frac{i_e p_r D_r R^2}{2}\right] \quad (27)$$

Similarly, the interest earned in this case is,

$$IE_1 = i_e \left[(p_r R - p_p T_r) D_r - \delta(p_r - p_p) D_r R + \frac{i_e p_r D_r R^2}{2}\right](T_r - R) \quad (28)$$

Then the total profit of the earned by the retailer from the above possibilities is,

$$TP_{RIB_1}(T,p_r,=p_rD_rM-\frac{K_r}{T_r}-\frac{c_d(1+u)\,ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r(1+u)\,ln\left(\frac{1+u}{1+u-T}\right)-D_rT_r\right]-\frac{p_p\alpha_1gw_1}{T_r} -\frac{h_rD_r}{T_r}\left(\frac{(1+u)^2}{2}ln\left(\frac{1+u}{1+u-T}\right)-\frac{(1+u)}{2}T_r+\frac{1}{4}T_r^2\right) \tag{29}$$

$$-\frac{\delta(p_r-p_p)D_rR}{T_r}+\frac{i_ep_rD_r}{2T}(T_r-R)^2-\frac{i_c\left[(p_pT-p_rR)D_r+\delta(p_r-p_p)D_rR-\frac{i_ep_rD_rR^2}{2}\right]}{T_r}$$

$$TP_{RIB_2}(T,p_r,R)=p_rD_rM-\frac{K_r}{T_r}-\frac{p_p\alpha_1gw_1}{T_r}-\frac{h_rD_r}{T_r}\left(\frac{(1+u)^2}{2}ln\left(\frac{1+u}{1+u-T}\right)-\frac{(1+u)}{2}T_r+\frac{1}{4}T_r^2\right) \tag{30}$$

$$-\frac{\delta(p_r-p_p)D_rR}{T_r}+\frac{i_e\left[(p_rR-p_pT)D-\delta(p_r-p_p)D_rR+\frac{i_ep_rD_rR^2}{2}\right](T_r-R)}{T_r}-\frac{c_d(1+u)\,ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r(1+u)\,ln\left(\frac{1+u}{1+u-T}\right)-D_rT_r\right]$$

Case IC: When $M\le T_r\le R$

The retailer pays at the end of the cycle, the sales revenue of the retailer is, $p_rD_rT_r$

∴ The profit sharing in this case for the retailer is $\delta(p_r-p_p)D_rM$.

$$IP_r=i_m(R-M)p_pD_rT_r \tag{31}$$

Here there is no discussion taking any bank loan or from any other source. Then, the interest earned by the retailer in this case is, $i_ep_r\int_0^T D_rtdt=\frac{i_ep_rD_rT_r^2}{2}$.

Interest earned by the retailer is,

$$IE_{R3}=\frac{i_ep_rD_rT_r^2}{2}+\left(p_rD_rT_r+\frac{i_ep_rD_rT_r^2}{2}\right)(R-T_r)i_e \tag{32}$$

Hence, the total profit for the retailer at the end of the cycle is,

$$TP_{R_{IC}}(T,p_r,R)=(p_r-p_p)D_r-\frac{K_r}{T_r}-\frac{h_rD_r}{T_r}\left(\frac{(1+u)^2}{2}ln\left(\frac{1+u}{1+u-T}\right)-\frac{(1+u)}{2}T_r+\frac{1}{4}T_r^2\right) -\frac{c_d(1+u)\,ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r(1+u)\,ln\left(\frac{1+u}{1+u-T}\right)-D_rT_r\right]+\frac{i_ep_rD_rT_r}{2} -\frac{p_p\alpha_1gw_1}{T_r}-\frac{\delta(p_r-p_p)D_rM}{T_r}-i_m(R-M)p_p+\left(p_rD_r+\frac{i_ep_rD_rT_r}{2}\right)(R-T_r)i_eD_r \tag{33}$$

Case II When $T_r\le M$

Similar to the above case I, we consider the other 3 various options of the total profit of the retailer when $T_r\le M$.

Case II A

The profit share for the processor in this case is,

$PF_{IIA}=\delta(p_r-p_p)D_rT_r$

In this case, there is no interest to be paid, because the retailer pays before the trade credit period.

$$\text{Interest earned by the retailer upto time } T=\frac{i_ep_rD_rT_r^2}{2} \tag{34}$$

The interest is same as the previous case $i_e(R-T_r)\left(p_rD_rT_r+\frac{i_ep_rD_rT_r^2}{2}\right)$ (35)

The retailer deposits his remaining amount in the bank, for $(M-R)$ period, and the interest earned with this is,

$$i_e\int_0^{M-R}\left[(p_r-p_p)D_rT_r-\delta(p_r-p_p)D_rT_r+\frac{i_ed_rD_rT_r^2}{2}+\left(p_rD_rT_r+\frac{i_ep_rD_rT_r^2}{2}\right)\right]i_e(R-T_r)dt \quad (36)$$

Hence, the entire profit earned by the retailer in this case is,

$$\boxed{TP_{rIIA}(T_r,p_r,R)}$$

$$=\left(\begin{array}{l}(p_r-p_p)D-\frac{K_r}{T_r}-\frac{c_d(1+u)\,ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r(1+u)\,ln\left(\frac{1+u}{1+u-T}\right)-D_rT_r\right]\\ -\delta(p_r-p_p)D_r+\frac{i_ep_rD_rT_r}{2}-\frac{p_p\alpha_1gw_1}{T_r}+i_e(M-R)\\ \left(\left[(p_r-p_p)D_rT_r-\delta(p_r-p_p)D_rT_r+\frac{i_ed_rD_rT_r^2}{2}+\left(p_rD_rT_r+\frac{i_ep_rD_rT_r^2}{2}\right)\right]i_e(R-T_r)\right)\\ -\frac{h_rD_r}{T_r}\left(\frac{(1+u)^2}{2}ln\left(\frac{1+u}{1+u-T}\right)-\frac{(1+u)}{2}T_r+\frac{1}{4}T_r^2\right)\end{array}\right) \quad (37)$$

Case IIB When $T_r\le M\le R$

Here, the profit sharing to the processor is same as the sub case.

The interest earned upto the time

$$(R-T_r)\ is\ i_e\left[\frac{p_rD_rT_r(R-T_r)}{2}+\frac{i_ep_rD_rT_r^2}{2}(R-T_r)\right] \quad (38)$$

The interest earned by the processor after the credit period is

$$i_m(R-M)p_pD_rT_r$$

The interest paid in this time period is null. The total profit for this time period of the retailer is given as,

$$TP_{R_{IIB}}=(p_r-w)D_r\frac{K_r}{T_r}-\frac{p_p\alpha_1gw_1}{T_r}-\delta(p_r-p_p)D_r-i_m(R-M)p_pD_r+\frac{i_ep_rD_rT_r}{2}$$
$$+i_e\left[\frac{p_rD_rT_r(R-T_r)}{2}+\frac{i_ep_rD_rT_r^2}{2}(R-T_r)\right]-\frac{h_rD_r}{T_r}\left(\frac{(1+u)^2}{2}ln\left(\frac{1+u}{1+u-T}\right)-\frac{(1+u)}{2}T_r+\frac{1}{4}T_r^2\right)-\frac{c_d(1+u)\,ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r(1+u)\,ln\left(\frac{1+u}{1+u-T}\right)-D_rT_r\right] \quad (39)$$

Case II C When $R\le T_r\le M$

The profit share by the retailer is $\delta(p_r-p_p)D_rR$

The interest paid to the source(bank) is

$$i_c(T_r-R)\left[(p_pT_r-p_rR)D_r+\delta(p_r-p_p)D_rR-\frac{i_ep_rD_rR}{2}\right] \quad (40)$$

The interest earned by the retailer in time (T_r-R) is

$$i_e(T_r - R)\left[(p_rR - p_pT_r)D_r - \delta(p_r - p_p)D_rR + \frac{i_ep_rD_rR^2}{2}\right] \tag{41}$$

The interest earned upto the trade credit period is

$$i_e(M - T_r)\big[(p_r - p_p)D_rT_r - \delta(p_r - p_p)D_rR - i_c(T_r - R)$$
$$\left((p_pT_r - p_rR) + \delta(p_r - p_p)D_rR - p_rD_rR - \frac{i_ep_rD_rR^2}{2}\right) + \frac{i_ep_rD_r}{2}(T_r - R)^2\big] \tag{42}$$

The total profit per unit of the retailer in this case is,

$$\begin{aligned} TP_{R_{IIC_1}} = {} & (p_r - p_p)D_r - \frac{K_r}{T_r} - \frac{c_d(1+u)\,ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r(1+u)\,ln\left(\frac{1+u}{1+u-T}\right) - D_rT_r\right] \\ & - \frac{\delta(p_r - p_p)D_rR}{T_r} + \frac{i_ep_rDR^2}{2T} + \frac{i_ep_rD}{2T}(T-R)^2 - \frac{p_p\alpha_1 gw_1}{T_r} \\ & - \frac{i_c(T-R)\left[(p_pT - p_rR)D + \delta(p - p_p)DR - \frac{i_ep_rDR^2}{2}\right]}{T} \\ & + \frac{i_e(M - T_r)}{T_r} \\ & \times \left[\begin{array}{l}((p_r - p_p)D_rT_r - \delta(p_r - p_p)D_rR - i_c(T_r - R) \\ \left((p_pT - p_rR)D_r + \delta(p_r - p_p)D_rR - \frac{i_ep_rD_rR^2}{2}\right) + i_c\frac{p_rD_r}{2}(T_r - R)^2\Big)\end{array}\right] \\ & - \frac{h_rD_r}{T_r}\left(\frac{(1+u)^2}{2}ln\left(\frac{1+u}{1+u-T}\right) - \frac{(1+u)}{2}T_r + \frac{1}{4}T_r^2\right) \end{aligned} \tag{43}$$

$$TP_{R_{IIC_2}} = \left(\begin{array}{l} (p_r - p_p)D_r - \frac{K_r}{T_r} - \frac{c_d(1+u)\,ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r(1+u)\,ln\left(\frac{1+u}{1+u-T}\right) - D_rT_r\right] - \frac{\delta(p_r - p_p)D_rR}{T_r} \\ + \frac{i_ep_rD_rR^2}{2T_r} + \frac{i_ep_rD_r}{2T_r}(T_r - R)^2 - \frac{i_e(T_r - R)\left[(p_rR - p_pT_r)D_r + \delta(p_r - p_p)D_rR - \frac{i_ep_rD_rR^2}{2}\right]}{T_r} \\ + \frac{i_e(M - T_r)}{T_r}\left[\begin{array}{l}((p_r - w)D_rT_r - \delta(p_r - p_p)D_rR - i_e(T_r - R) \\ \left((p_rR - p_pT_r)D_r + \delta(p_r - p_p)D_rR - \frac{i_ep_rD_rR^2}{2}\right) + i_e\frac{p_rD_r}{2}(T_r - R)^2\Big)\end{array}\right] - \frac{p_p\alpha_1 gw_1}{T_r} \\ - \frac{h_rD_r}{T_r}\left(\frac{(1+u)^2}{2}ln\left(\frac{1+u}{1+u-T}\right) - \frac{(1+u)}{2}T_r + \frac{1}{4}T_r^2\right) \end{array}\right) \tag{44}$$

6. Integrated Supply Chain

Considering all of the instances covered in the preceding subcategories, the efficient supply chain chain's profit margin per unit time is provided by,

$$TPG(T, p, R) = TP_g(T_g, p_g, R) + TP_{pij}(T_p, p_p, R) + TP_{rij}(T_r, p_r, R)$$

$$where\ i = I, II, j = A, B, C$$

$$TPG(T,p,R)=\begin{cases} TPG_1(T,p,R) & if, M\le R\le T\\ TPG_2(T,p,R) & if, M\le R\le T\\ TPG_3(T,p,R) & if, R\le M\le T\\ TPG_4(T,p,R) & if, R\le M\le T\\ TPG_5(T,p,R) & if, M\le T\le R\\ TPG_6(T,p,R) & if, T\le R\le M\\ TPG_7(T,p,R) & if, T\le M\le R\\ TPG_8(T,p,R) & if, R\le T\le M\\ TPG_9(T,p,R) & if, R\le T\le M \end{cases} \tag{45}$$

In the total profit function, the decision variables p, T, represents the corresponding price of the retailer and processor, p_r, p_p, and the time period T as T_r, T_p, T_g.

$$\begin{aligned} TPG_1(T,p,R) &= p_g\alpha_1 g w_1 - \frac{p_g w_0 g}{T_g} - \frac{K_g}{T_g} - \frac{c_g}{T_g}\left[\frac{P_r T_p(1+\beta e^{-\gamma T_g})^{-1}}{G_1(1+\beta)e^{-\alpha_1 T_g}}\right]\left(\frac{e^{\beta_1 T_g}-1}{\beta_1}\right) - i_m(R-M)p_p D \\ &+ \frac{i_e p_r D}{2T_r}(T_r-R)^2 + (p_r-p_p)D_p - \frac{K_{pr}}{T_p} - e_p P_r T_{v_1} - \frac{c_d+c_d'}{T_p}\left(P_r T_{v_1} - D_p T_p\right) + i_m(R \\ &- M)p_p D_p - -\frac{i_p p_p D_p R}{T_p} \\ &+ \frac{\delta(p_r-p_p)D_p M}{T_p}\frac{h_p+h_p'}{T_p}\left[\frac{(P_r-D_p)T_{v_1}^2}{2}\left(1-\frac{\theta_p T_{v_1}}{3}\right) + \frac{D_p T_{v_2}^2}{2}\left(1+\frac{\theta_p T_{v_2}}{3}\right)\right] \\ &- \frac{p_p\alpha_1 g w_1}{T_r} - \frac{\delta(p_r-p_p)D_r M}{T_r} + p_r D_r M - \frac{K_r}{T_r} \\ &- \frac{c_d(1+u)\ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r(1+u)\ln\left(\frac{1+u}{1+u-T}\right) - D_r T_r\right] - \frac{p_g\alpha_1 g w_1}{T_p} - i_c(T_r \\ &- R)\left[\left(p_p D_r T_r - p_r D_r R\right) + \delta\left(p_r-p_p\right)D_r M + i_m(R-M)p_p D_r T_r \right. \\ &- \left.\frac{i_e p_r D_r R^2}{2}\right] - \frac{h_r D_r}{T_r}\left(\frac{(1+u)^2}{2}\ln\left(\frac{1+u}{1+u-T}\right) - \frac{(1+u)}{2}T_r + \frac{1}{4}T_r^2\right) \end{aligned} \tag{46}$$

$$\begin{aligned} TPG_2(T,p,R) &= p_g\alpha_1 g w_1 - \frac{p_g w_0 g}{T_g} - \frac{K_g}{T_g} - \frac{c_g}{T_g}\left[\frac{P_r T_p(1+\beta e^{-\gamma T_g})^{-1}}{G_1(1+\beta)e^{-\alpha_1 T_g}}\right]\left(\frac{e^{\beta_1 T_g}-1}{\beta_1}\right) - \frac{p_p\alpha_1 g w_1}{T_r} \\ &+(p_r-p_p)D_p - \frac{K_{pr}}{T_p} - e_p P_r T_{v_1} - \frac{c_d+c_d'}{T_p}\left(P_r T_{v_1} - D_p T_p\right) + i_m(R-M)p_p D_p - \\ &\frac{h_p+h_p'}{T_p}\left[\frac{(P_r-D_p)T_{v_1}^2}{2}\left(1-\frac{\theta_p T_{v_1}}{3}\right) + \frac{D_p T_{v_2}^2}{2}\left(1+\frac{\theta_p T_{v_2}}{3}\right)\right] - \frac{i_p p_p D_p R}{T_p} + \frac{\delta(p_r-p_p)D_p M}{T_p} \\ &+(p_r-w)D_r\frac{K_r}{T_r} - \frac{p_g\alpha_1 g w_1}{T_p} - \frac{c_d(1+u)\ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r(1+u)\ln\left(\frac{1+u}{1+u-T}\right) - D_r T_r\right] \\ &-\delta(p_r-p_p)D_r - i_m(R-M)p_p D_r + \frac{i_e p_r D_r T_r}{2} + i_e\left[\frac{p_r D_r T_r(R-T_r)}{2} + \frac{i_e p_r D_r T_r^2}{2}(R - \right. \\ &\left. T_r)\right] - \frac{h_r D_r}{T_r}\left(\frac{(1+u)^2}{2}\ln\left(\frac{1+u}{1+u-T}\right) - \frac{(1+u)}{2}T_r + \frac{1}{4}T_r^2\right) \end{aligned} \tag{47}$$

$$TPG_3(T,p,R) = p_g\alpha_1 g w_1 - \frac{p_g w_0 g}{T_g} - \frac{K_g}{T_g} - \frac{c_g}{T_g}\left[\frac{P_r T_p (1+\beta e^{-\gamma T_g})^{-1}}{G_1(1+\beta)e^{-\alpha_1 T_g}}\right]\left(\frac{e^{\beta_1 T_g}-1}{\beta_1}\right) - \frac{p_p\alpha_1 g w_1}{T_r} + p_r D_r M - \frac{K_r}{T_r} - \frac{h_r D_r}{T_r}\left(\frac{(1+u)^2}{2} ln\left(\frac{1+u}{1+u-T}\right) - \frac{(1+u)}{2}T_r + \frac{1}{4}T_r^2\right) - \frac{\delta(p_r-p_p)D_r R}{T_r} + \frac{i_e p_r D_r}{2T}(T_r-R)^2 - \frac{i_c\left[(p_p T - p_r R)D_r + \delta(p_r-p_p)D_r R - \frac{i_e p_r D_r R^2}{2}\right]}{T_r} + (p_r-p_p)D_p - \frac{K_{pr}}{T_p} - e_p P_r T_{v_1} - \frac{c_d+c_d'}{T_p}\left(P_r T_{v_1} - D_p T_p\right) + \frac{\delta(p_r-p_p)D_p R}{T_p} - \frac{h_p+h_p'}{T_p}\left[\frac{(P_r-D_p)T_{v_1}^2}{2}\left(1-\frac{\theta_p T_{v_1}}{3}\right) + \frac{D_p T_{v_2}^2}{2}\left(1+\frac{\theta_r T_{v_2}}{3}\right)\right] - \frac{i_p p_p D_p R}{T_p} - \frac{p_g\alpha_1 g w_1}{T_p} - \frac{c_d(1+u)\, ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r(1+u)\, ln\left(\frac{1+u}{1+u-T}\right) - D_r T_r\right] \quad (48)$$

$$TPG_4(T,p,R) = p_g\alpha_1 g w_1 - \frac{p_g w_0 g}{T_g} - \frac{K_g}{T_g} - \frac{c_g}{T_g}\left[\frac{P_r T_p\left(1+\beta e^{-\gamma T_g}\right)^{-1}}{G_1(1+\beta)e^{-\alpha_1 T_g}}\right]\left(\frac{e^{\beta_1 T_g}-1}{\beta_1}\right) - \frac{p_p\alpha_1 g w_1}{T_r} + (p_r-p_p)D_p - \frac{K_{pr}}{T_p} - e_p P_r T_{v_1} - \frac{c_d+c_d'}{T_p}\left(P_r T_{v_1} - D_p T_p\right) + \frac{\delta(p_r-p_p)D_p R}{T_p} - \frac{h_p+h_p'}{T_p}\left[\frac{(P_r-D_p)T_{v_1}^2}{2}\left(1-\frac{\theta_p T_{v_1}}{3}\right) + \frac{D_p T_{v_2}^2}{2}\left(1+\frac{\theta_r T_{v_2}}{3}\right)\right] - \frac{i_p p_p D_p R}{T_p} - \frac{p_g\alpha_1 g w_1}{T_p} + p_r D_r M - \frac{K_r}{T_r} - \frac{c_d(1+u)\, ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r(1+u)\, ln\left(\frac{1+u}{1+u-T}\right) - D_r T_r\right] - \frac{\delta(p_r-p_p)D_r R}{T_r} + \frac{i_e\left[(p_r R - p_p T)D - \delta(p_r-p_p)D_r R + \frac{i_e p_r D_r R^2}{2}\right](T_r-R)}{T_r} - \frac{h_r D_r}{T_r}\left(\frac{(1+u)^2}{2} ln\left(\frac{1+u}{1+u-T}\right) - \frac{(1+u)}{2}T_r + \frac{1}{4}T_r^2\right) \quad (49)$$

$$TPG_5(T,p,R) = p_g\alpha_1 g w_1 - \frac{p_g w_0 g}{T_g} - \frac{K_g}{T_g} - \frac{c_g}{T_g}\left[\frac{P_r T_p (1+\beta e^{-\gamma T_g})^{-1}}{G_1(1+\beta)e^{-\alpha_1 T_g}}\right]\left(\frac{e^{\beta_1 T_g}-1}{\beta_1}\right) - \frac{p_p\alpha_1 g w_1}{T_r} - \frac{p_g\alpha_1 g w_1}{T_p} - \frac{h_r D_r}{T_r}\left(\frac{(1+u)^2}{2} ln\left(\frac{1+u}{1+u-T}\right) - \frac{(1+u)}{2}T_r + \frac{1}{4}T_r^2\right) + (p_r-p_p)D_p - \frac{K_{pr}}{T_p} - e_p P_r T_{v_1} - \frac{c_d+c_d'}{T_p}\left(P_r T_{v_1} - D_p T_p\right) + i_m(R-M)p_p D_p - \frac{h_p+h_p'}{T_p}\left[\frac{(P_r-D_p)T_{v_1}^2}{2}\left(1-\frac{\theta_p T_{v_1}}{3}\right) + \frac{D_p T_{v_2}^2}{2}\left(1+\frac{\theta_p T_{v_2}}{3}\right)\right] - \frac{i_p p_p D_p R}{T_p} + \frac{\delta(p_r-p_p)D_p M}{T_p} + (p_r-p_p)D_r - \frac{K_r}{T_r} - \frac{c_d(1+u)\, ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r(1+u)\, ln\left(\frac{1+u}{1+u-T}\right) - D_r T_r\right] - \frac{\delta(p_r-p_p)D_r M}{T_r} - i_m(R-M)p_p + \frac{i_e p_r D_r T_r}{2} + \left(p_r D_r + \frac{i_e p_r D_r T_r}{2}\right)(R-T_r)i_e D_r \quad (50)$$

$$TPG_6(T,p,R) = p_g\alpha_1 g w_1 - \frac{p_g w_0 g}{T_g} - \frac{K_g}{T_g} - \frac{c_g}{T_g}\left[\frac{P_r T_p(1+\beta e^{-\gamma T_g})^{-1}}{G_1(1+\beta)e^{-\alpha_1 T_g}}\right]\left(\frac{e^{\beta_1 T_g}-1}{\beta_1}\right) - \frac{p_p\alpha_1 g w_1}{T_r} + (p_r - p_p)D_p - \frac{K_{pr}}{T_p} - e_p P_r T_{v_1} - \frac{c_d + c_d'}{T_p}\left(P_r T_{v_1} - D_p T_p\right) + \delta(p_r - p_p)D_p + \frac{i_e p_r D_r T_r}{2} - \frac{h_r D_r}{T_r}\left(\frac{(1+u)^2}{2}ln\left(\frac{1+u}{1+u-T}\right) - \frac{(1+u)}{2}T_r + \frac{1}{4}T_r^2\right) - i_p p_p D_p R - \frac{p_g\alpha_1 g w_1}{T_p} - \frac{h_p + h_p'}{T_p}\left[\frac{(P_r - D_p)T_{v_1}^2}{2}\left(1 - \frac{\theta_p T_{v_1}}{3}\right) + \frac{D_p T_{v_2}^2}{2}\left(1 + \frac{\theta_p T_{v_2}}{3}\right)\right] + (p_r - p_p)D - \frac{K_r}{T_r} - \delta(p_r - p_p)D_r - \frac{c_d(1+u)\,ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r(1+u)\,ln\left(\frac{1+u}{1+u-T}\right) - D_r T_r\right] + i_e(M-R)\left(\left[(p_r - p_p)D_r T_r - \delta(p_r - p_p)D_r T_r + \frac{i_e d_r D_r T_r^2}{2} + \left(p_r D_r T_r + \frac{i_e p_r D_r T_r^2}{2}\right)\right] i_e(R - T_r)\right) \tag{51}$$

$$TPG_7(T,p,R) = p_g\alpha_1 g w_1 - \frac{p_g w_0 g}{T_g} - \frac{K_g}{T_g} - \frac{c_g}{T_g}\left[\frac{P_r T_p(1+\beta e^{-\gamma T_g})^{-1}}{G_1(1+\beta)e^{-\alpha_1 T_g}}\right]\left(\frac{e^{\beta_1 T_g}-1}{\beta_1}\right) - \frac{p_p\alpha_1 g w_1}{T_r} - \frac{p_g\alpha_1 g w_1}{T_p} + (p_r - p_p)D_p - \frac{K_{pr}}{T_p} - e_p P_r T_{v_1} - \frac{c_d + c_d'}{T_p}\left(P_r T_{v_1} - D_p T_p\right) + \delta(p_r - p_p)D_p - \frac{h_p + h_p'}{T_p}\left[\frac{(P_r - D_p)T_{v_1}^2}{2}\left(1 - \frac{\theta_p T_{v_1}}{3}\right) + \frac{D_p T_{v_2}^2}{2}\left(1 + \frac{\theta_p T_{v_2}}{3}\right)\right] - i_p p_p D_p R + i_m(R-M)p_p D_p - \frac{h_r D_r}{T_r}\left(\frac{(1+u)^2}{2}ln\left(\frac{1+u}{1+u-T}\right) - \frac{(1+u)}{2}T_r + \frac{1}{4}T_r^2\right) + (p_r - w)D_r\frac{K_r}{T_r} - \frac{c_d(1+u)\,ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r(1+u)\,ln\left(\frac{1+u}{1+u-T}\right) - D_r T_r\right] + \frac{i_e p_r D_r T_r}{2} - \delta(p_r - p_p)D_r - i_m(R-M)p_p D_r + i_e\left[\frac{p_r D_r T_r(R - T_r)}{2} + \frac{i_e p_r D_r T_r^2}{2}(R - T_r)\right] \tag{52}$$

$$TPG_8(T,p,R) = p_g\alpha_1 g w_1 - \frac{p_g w_0 g}{T_g} - \frac{K_g}{T_g} - \frac{c_g}{T_g}\left[\frac{P_r T_p\left(1+\beta e^{-\gamma T_g}\right)^{-1}}{G_1(1+\beta)e^{-\alpha_1 T_g}}\right]\left(\frac{e^{\beta_1 T_g}-1}{\beta_1}\right) - \frac{p_p\alpha_1 g w_1}{T_r} - \frac{\delta(p_r - p_p)D_r R}{T_r} + (p_r - p_p)D_p - \frac{K_{pr}}{T_p} - e_p P_r T_{v_1} - \frac{c_d + c_d'}{T_p}\left(P_r T_{v_1} - D T_p\right) + \frac{\delta(p_r - p_p)D_p R}{T_p} - \frac{h_r D_r}{T_r}\left(\frac{(1+u)^2}{2}ln\left(\frac{1+u}{1+u-T}\right) - \frac{(1+u)}{2}T_r + \frac{1}{4}T_r^2\right) - \frac{i_c(T-R)\left[(p_p T - p_r R)D + \delta(p - p_p)DR - \frac{i_e p_r D R^2}{2}\right]}{T} - \frac{h_p + h_p'}{T_p}\left[\frac{(P_r - D_p)T_{v_1}^2}{2}\left(1 - \frac{\theta_p T_{v_1}}{3}\right) + \frac{D_p T_{v_2}^2}{2}\left(1 + \frac{\theta_p T_{v_2}}{3}\right)\right] - \frac{i_p p_p D_p R}{T_p} - \frac{p_g\alpha_1 g w_1}{T_p} + (p_r - p_p)D_r - \frac{K_r}{T_r} - \frac{c_d(1+u)\,ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r(1+u)\,ln\left(\frac{1+u}{1+u-T}\right) - D_r T_r\right] + \frac{i_e p_r D R^2}{2T} + \frac{i_e p_r D}{2T}(T-R)^2 + \frac{i_e(M - T_r)}{T_r} \times \left[\begin{array}{l}\left((p_r - p_p)D_r T_r - \delta(p_r - p_p)D_r R - i_c(T_r - R)\right. \\ \left.\left((p_p T - p_r R)D_r + \delta(p_r - p_p)D_r R - \frac{i_e p_r D_r R^2}{2}\right) + i_c\frac{p_r D_r}{2}(T_r - R)^2\right)\end{array}\right] \tag{53}$$

$$TPG_9(T,p,R) = p_g\alpha_1 g w_1 - \frac{p_g w_0 g}{T_g} - \frac{K_g}{T_g} - \frac{c_g}{T_g}\left[\frac{P_r T_p(1+\beta e^{-\gamma T_g})^{-1}}{G_1(1+\beta)e^{-\alpha_1 T_g}}\right]\left(\frac{e^{\beta_1 T_g}-1}{\beta_1}\right) - \frac{p_p\alpha_1 g w_1}{T_r} - \frac{p_g\alpha_1 g w_1}{T_p} - \frac{K_r}{T_r} + (p_r - p_p)D_p - \frac{K_{pr}}{T_p} - e_p P_r T_{v_1} - \frac{c_d + c_d'}{T_p}\left(P_r T_{v_1} - DT_p\right) - \frac{i_p p_p D_p R}{T_p} - \frac{\delta(p_r - p_p)D_r R}{T_r}$$

$$+\begin{pmatrix} -\frac{h_r D_r}{T_r}\left(\frac{(1+u)^2}{2} ln\left(\frac{1+u}{1+u-T}\right) - \frac{(1+u)}{2}T_r + \frac{1}{4}T_r^2\right) \\ +\frac{i_e p_r D_r R^2}{2T_r} + \frac{i_e p_r D_r}{2T_r}(T_r - R)^2 - \frac{i_e(T_r-R)\left[(p_r R - p_p T_r)D_r + \delta(p_r - p_p)D_r R - \frac{i_e p_r D_r R^2}{2}\right]}{T_r} \\ +\frac{i_e(M-T_r)}{T_r}\left[\begin{matrix}((p_r - w)D_r T_r - \delta(p_r - p_p)D_r R - i_e(T_r - R) \\ \left((p_r R - p_p T_r)D_r + \delta(p_r - p_p)D_r R - \frac{i_e p_r D_r R^2}{2}\right) + i_e\frac{p_r D_r}{2}(T_r - R)^2\end{matrix}\right)\right] \end{pmatrix} \quad (54)$$

$$+\frac{\delta(p_r - p_p)D_p R}{T_p} + (p_r - p_p)D_r - \frac{c_d(1+u)\, ln\left(\frac{1+u}{1+u-T}\right)}{T}\left[D_r(1+u)\, ln\left(\frac{1+u}{1+u-T}\right) - D_r T_r\right] - \frac{h_p + h_p'}{T_p}\left[\frac{(P_r - D_p)T_{v_1}^2}{2}\left(1 - \frac{\theta_p T_{v_1}}{3}\right) + \frac{D_p T_{v_2}^2}{2}\left(1 + \frac{\theta_p T_{v_2}}{3}\right)\right]$$

7. Solution Methodology

By applying the same solution procedure followed in Mittal and Sharma (2021)'s work, we have found the optimal values of the profit function and the concavity of the profit function also satisfied. Due to the complexity in finding the derivatives with respect to all the decision variables and it seems hard to derive it analytically, we have adapted the solution methodology followed in Giri et al. (2018)'s work, which maximize the profit values of the Integrated Supply Chain. We implemented the algorithm developed in Thangam(2009)'s model, to find the optimal values in all the profit functions mentioned above. We have to find the first derivative of the respective profit functions, $TPG_{i_i}(T,p,R)$, where $i = 1,\ldots,9$, to find the optimal values of the decision variable, and then to prove the concavity of the profit, we have to substitute the values of the decision variables in the second derivative of the respective profit function. To clarify that the values of the second derivative of the profit fucntion should satisfy the concavity conditions, as given in Thangam (2009). Then the final stage is that obtained optimal values which maximizes the profit for both grower, retailer and processor respectively.

Example. (Basic Numerical Data) $P_r = 3500; K_r = 2500; K_p = 2500; K_g = 3000; h_r = 1; p_r = 50; h_p = 0.5; p_p = 30; p_f = 15; \alpha_1 = 0.9; g = 179; C_d = 2; c_g = 1; p_v = 10; \alpha = 51; \beta = 5; M = 0.1; i_v = 0.1; i_c = 0.18; \gamma = 0.12; \delta = 0.4; \beta_1 = 25; i_e = 0.14; i_m = 0.15; w_0 = 8.5; w_1 = 30; T_g = 0.36; \theta_r = (1-\alpha_1); T_{v_2} = Tv1 - Tp; R = 0.45; a = 200; b = 0.15; T_r = 0.39; h_p' = 0.1;\ c_d' = 0.6; e_p = 1;$.

These are the various parameters used in the numerical example to verify the developed model. The optimal values are obtained and the results are verified.

8. Results for various cases

8.1 For Processor

Example . For **Case 1A**, $R < M$, the numerical data given below varies with other cases, so $R = 0.2918,\ M = 0.3$ Then, $T_p^* = 0.378$, The total profit obtained by the processor in this case is, **Rs.69,092.**

Example . For **Case 1B**, $R < M$, the credit period $R = 0.2899$, the the time period, $T_p^* = 0.376$, then the total profit obtained in this case is, **Rs. 68,952.**

Example . For **Case 1C**, $M < T_p$, the numerical data given below varies with other cases, so $R = 0.2918,\ M = 0.3$ Then, $T_p^* = 0.378$, The total profit obtained by the processor in this case is, **Rs.68,946.**

Example . For **Case 2A**, $R > T_p$, the numerical data given below varies with other cases, so $R = 0.4,\ M = 0.299$ Then, $T_p^* = 0.398$, The total profit obtained by the processor in this case is, **Rs.66,621.**

Example . For **Case 2B**, $M > T_p$, the numerical data given below varies with other cases, so $R = 0.4$, $M = 0.3$ Then, $T_p^* = 0.378$, The total profit obtained by the processor in this case is, **Rs.65,999.**

Example . For **Case 2C**, $R < T_p$, the numerical data given below varies with other cases, so $R = 0.25$, $M = 0.299$ Then, $T_p^* = 0.48$, The total profit obtained by the processor in this case is, **Rs.69,796.**

8.2 For Retailer

Example . For **Case 1A**, $M \leq R < T_r$, the numerical data given below varies with other cases, so $R = 0.35$, $M = 0.2$ Then, $T_r^* = 0.39$, The total profit obtained by the processor in this case is, For **1A1:Rs.65,569.** and **For 1AII: Rs. 65,511**

Example . For **Case 1B**, $R \leq M \leq T_r$, the credit period $R = 0.2$, the the time period, $T_r^* = 0.41$, $M = 0.4$ then the total profit obtained in this case is, **For 1BI:Rs. 66,438.** and for **1BII: Rs. 66,153.**

Example . For **Case 1C**, $M < T_r \leq R$, the numerical data given below varies with other cases, so $R = 0.5$, $M = 0.3$ Then, $T_r^* = 0.36$, The total profit obtained by the processor in this case is, **Rs.39,545.**

Example . For **Case 2A**, $T_r \leq M$, the numerical data given below varies with other cases, so $R = 0.3$, $M = 0.4$ Then, $T_p^* = 0.39$, The total profit obtained by the processor in this case is, **Rs.65,306.**

Example . For **Case 2B**, the numerical data given below varies with other cases, so $R = 0.45$, $M = 0.4$ Then, $T_r^* = 0.39$, The total profit obtained by the processor in this case is, **Rs.65,137.**

Example . For **Case 2C**, $R < T_r \leq M$, the numerical data given below varies with other cases, so $R = 0.3$, $M = 0.4$ Then, $T_r^* = 0.36$, The total profit obtained by the processor in this case is, **Rs.69,796.**

9. Numerical Analysis

Optimal results on Profit for different subcases

Cases	R	T	Grower	Processor	Retailer	Total Profit in Rs.
IA1	0.3209	0.3925	42,006	69,092	65,569	**1,76,667**
IA2	0.1045	0.3467	42,006	69,468	65,511	**1,76,985**
IB1	0.1016	0.4014	42,006	68,952	66,438	**1,77,396**
IB2	0.2102	0.3997	42,006	69,059	66,153	**1,77,218**
IC	0.4204	0.4015	42,006	68,946	39,545	**1,50,497**
IIA	0.4213	0.3990	42,006	66,621	65,306	**1,73,933**
IIB	0.4509	0.3989	42,006	65,999	65,137	**1,73,142**
IIC1	0.3017	0.3578	42,006	69,786	64,653	**1,76,445**
IIC2	0.2897	0.3486	42,006	68,987	63,546	**1,74,449**

10. Discussion

Here we compared some of the results obtained by changing the parameters of demand and credit period. By changing the constant demand a, value the lot size of the system varies and the profit oscillates accordingly, the constant demand pattern may change due to the seasonal demand and change in price of the product, the change in price may cause the delay in the depletion of the demand and it can increase the time length of the inventory system. If we change the rate of b, it directly affects the lot size and profit, for higher the b value the demand is low compared to the other values of b, it may decrease the net profit of the system. And if the credit period increases, the payment time and the cycle length will increase and it decreases the value of the profit of the entire system.

From the obtained optimal values, it is stated clearly that, the case IB1, has maximum profit and also the credit period offered is less than the time period of the entire system, but the processor gains profit less than the retailer. In general, the higher the selling price gives more profit to the retailer, than the processor and the entire profit value may hike, it seems a bit less to the processor.

In case A1, the retailer earns more profit than case A2,but the time period seems bit higher than A1, and also the whole system's profit hikes more in A2.

In case B1, the credit period offered is lower than the time period of the cycle and the profit of the processor and retailer is comparatively a bit low to the case B2, but the period offered is low, and in case B2, time period T is bit lower than in B1, the profit of the retailer and processor seems a bit improved compared to case B1.

In case C1, the credit period offered is higher than the time period, it improves the total profit of the processor and retailer also and it effects in whole system's profit.

In case IIA, the retailer earns the profit comparably lower than the other cases obtained above, but the processor earns profit more than the other cases, undoubtedly it is one of the highest profit of the system compared to other cases obtained.

In case IIB, the credit period offered is a bit more higher, than it has been offered in other cases, it also lowered the profit values of the processor and retailer and also it is the second lowest profit value of the system.

Similar to the case IIB, the time period of the system is lower compared to all other values of the cases given, the R value is a bit low, because of the lower time period, the profit values of the retailer and processor oscillates and reduces the system's profit.

The credit period offered here is less than the time period of the system but the time period is the least one and the profit of the system also oscillates but doesn't affects much to care.

10. Conclusion

A multi-echelon supply chain for growing items with product expiry is developed in this study. The grower begins his cycle by maturing immature items to a specific stage and selling them to the processor after they reach the maximum growing stage; the processor then slaughters the matured livestock, preserves it, and packs it into a specific form; during the packing process, it emits carbon, which costs the processor in inventory costs and he has to pay the tax for carbon emissions; after the packing process, he sends the product to the retailer and he offers There are six possibilities for these assumptions, which are discussed in the developed model, solution procedure is obtained, and numerical analysis and sensitivity analysis are given to verify the sustainability of the model. Further, this model can be developed by considering stochastic demand patterns, advertisement patterns, a carbon cap, and trade regulations be added. Finally, the model can be developed by making assumptions about green products and government subsidies.

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