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Mathematical modeling for exploring the effects of overtime option, rework, and discontinuous inventory issuing policy on EMQ model

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ABSTRACT

This study employs mathematical modeling to explore the effects of overtime option, rework, and discontinuous end-item issuing policy on the economic manufacturing quantity (EMQ) model. Conventional EMQ model assumed that all products fabricated are of good quality and are issued under continuous policy. In real world, however, nonconforming items are randomly produced, due to diverse unexpected factors in fabrication process. When finished items are to be distributed to outside locations, discontinuous multi-shipment policy is often used rather than continuous rule. In addition, with the intention of increasing short-term capacity or shortening replenishment cycle length to smooth the production planning, adopting overtime option can be an effective strategy. To cope with the aforementioned features in real production systems, this study incorporates overtime option, rework, and multi-shipment policy into the EMQ model and explores their joint effects on optimal lot size and number of shipments, and on other relevant system parameters. Mathematical modeling and Hessian matrix equations enable us to derive the optimal policies to the problem. Through the use of numerical example, the applicability of research result is exhibited and a variety of significant effects of these features on the proposed system are revealed.

1. Introduction

A mathematical modeling is used in this study to explore the effects of overtime option, rework, and discontinuous end-item issuing policy on the economic manufacturing quantity (EMQ) model. Unlike the conventional EMQ model (Taft, 1918) which finds the most economic manufacturing lot size with simple assumptions of perfect product quality and continuous issuing policy; in real world, nonconforming items are sometimes produced, due to diverse unexpected factors in fabrication process (Mak, 1985; Henig & Gerchak, 1990; Boone et al., 2000; Maddah et al., 2010; Chiu et al., 2015a,b; Kim et al., 2015; Koley et al., 2015; Koren & Palčič, 2015; Jindal & Solanki, 2016; Zhang et al., 2016; Dhaiban et al., 2017). Reworking and repairing the nonconforming items can help reduce total manufacturing costs (Yum & McDowell, 1987; Zargar, 1995; Inderfurth, 2006; Taleizadeh et al., 2010; Chiu et al., 2016a,b; Jawla & Singh, 2016; Khanna et al., 2017). Besides, when finished items are to be distributed to outside locations, the discontinuous multi-shipment policy is practically used rather than the continuous policy. Golhar and Sarker (1992) proposed an algorithm to decide optimal lot size for an

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EMQ-based system by considering discontinuous just-in-time frequent shipments. Their results indicated that the cost function of the problem is convex, and if both fabrication uptime and cycle length are an integer multiple of time interval of deliveries, an optimal delivery size can be derived. Furthermore, they explored the economic impact of setup and ordering costs reduction on the problem.

Sarker and Khan (2001) examined a fabrication-inventory system with a periodic delivery policy. Their objective was to derive optimal batch size that minimizes total system costs. A raw materials procurement policy is used to meet its requirements in lot for in-house fabrication needs. End items are distributed at fixed time interval to meet product demands. Accordingly, a cost function for the proposed fabrication-inventory-shipment integrated system was constructed and analyzed. Finally, they proposed a solution procedure to determine the optimal raw materials ordering policy as well as optimal fabrication batch size. Kuhn and Liske (2011) studied a supply and production planning problem which combines the economic lot-sizing and vehicle routing decisions. They established a mathematical model with an exact solution process for a given delivery policy. Through computation of an extensive range of test examples, they obtained analytical results regarding important insight information to the problem that can help determine the most economic solutions. Extra papers relating to diverse characteristics of discontinuous issuing policies in supply chain systems can also be found elsewhere (Hahm & Yano, 1992; Hill, 1995; Cetinkaya & Lee, 2000; Abdul-Jalbar et al., 2005; Grunder et al., 2013; Balaji et al., 2015; Cao et al., 2015; Lemma et al., 2015; Aboumasoudi et al., 2016; Chiu et al., 2016c; Qu & Ji, 2016; Setiawan, 2016).

Furthermore, with the intention of increasing short-term capacity or shortening replenishment cycle length or leveling work/machine loadings to smooth the production planning, adopting overtime option can be an effective strategy. Dixon et al. (1983) examined a manufacturing system with known time-varying demands where no stock-out situation was allowed. The objective was to decide the lot size and refilling timing that minimizes the expected system cost. Flexibility in production includes both regular and overtime with permitted time-varying capacities. They presented a heuristic algorithm and tested on a number of large sets of problems to show the outstanding performance of their proposed heuristic to the problems. Yura (1994) considered a production scheduling problem with the goals of simultaneously meeting workers' preferences in working times as well as meeting due-date constraints. Note that when workloads are heavy, it is not possible to find a solution to satisfy all workers' preferences. Therefore, the scheduling problem aims to minimize total necessary overtime to satisfy workers' preferences. Linear goal programming problems were built and formulated to solve the problem. Numerical example was provided to show how solution process works and what kind of analytical results to the problem can be obtained.

Robinson and Sahin (2001) incorporated overtime option and extra fixed charges for cleanup or inspection into conventional economic production quantity (EPQ) model. Two separate production problems each with mathematical modeling and optimization process were examined by considering separate manufacturing restrictions, setup cost, and availability of overtime capacity. A few experimental sets of problems were used to confirm applicability and excellent performance of the proposed algorithms. Zobolas et al. (2008) intended to improve the master production schedule (MPS) in make-to-order manufacturing systems featuring excessive demand. An algorithm for solving the proposed MPS problem was proposed, which incorporates various constraints such as the production orders guided by tardiness, earliness and overtime penalties. The resource utilization was determined by an intermediate tool - rough cut capacity planning (RCCP) with extra features of positive lead time, overtime option, earliness and tardiness. The proposed model was then solved by a Genetic algorithm. Various benchmarking problems against industrial real data were tested to show that improvements can be gained over the results from conventional RCCP solution process.

Extra papers that explored different characteristics of manufacturing systems with overtime options can also be referred elsewhere (Axsater, 1981; Conley, 1985; Holm & Kiander, 1993; Schank, 2005; Campbell, 2012; Bazdar et al., 2015; Yilmaz et al., 2016; Renna, 2017). Nevertheless, little attention has been paid for the exploration of joint effects of overtime, rework, and discontinuous issuing policy on EMQ model and this paper aims at filling the gap.

2. Problem, assumption and mathematical modeling

This study uses mathematical modeling to inspect the effects of overtime option, rework, and discontinuous issuing policy on EMQ model. Description of the problem with mathematical modeling is presented below. First of all, since the overtime option can increase the capacity or expedite manufacturing output rate, we assume α_1 denotes the adjusted percentage of fabrication rate due to adoption of a particular overtime plan, and the resulting fabrication rate P_A is as follows:

$$P_A = (1+\alpha_1)P, \tag{1}$$

where P denotes the standard fabrication rate per year. Consequently, manufacturing setup cost K_A and unit cost C_A are higher than standard setup cost K and unit cost C. Their relationships are assumed as follows:

$$K_A = (1+\alpha_2)K, \tag{2}$$

$$C_A = (1+\alpha_3)C, \tag{3}$$

where α_2 and α_3 are the respective cost increase proportions. The annual product demand rate is λ. The manufacturing process is imperfect, assuming an x percentage of defective items may randomly be fabricated, at a rate of d_A.

$$d_A = xP_A. \tag{4}$$

Maximal on-hand inventories of finished products H_1 (Fig. 1) in the end of uptime t_{1A} are

$$H_1 = (P_A - d_A)t_{1A}. \tag{5}$$

All defective items are assumed to be repairable through a rework process in each cycle (see Fig. 1 and Fig. 2), at a rate of P_{1A}. The relationship between P_{1A} and standard reworking rate P_1 is as follows,

$$P_{1A} = (1+\alpha_1)P_1. \tag{6}$$

Maximal on-hand inventories of finished products H at the end of rework time t_{2A} are

$$H = H_1 + P_{1A}t_{2A}. \tag{7}$$

At the end of rework process, when the entire production lot is completed, n fixed quantity installments of finished products are delivered to buyer during t_{3A} (Fig. 1). The time interval t_{nA} between two consecutive distributions is as follows:

$$t_{nA} = \frac{t_{3A}}{n} \tag{8}$$

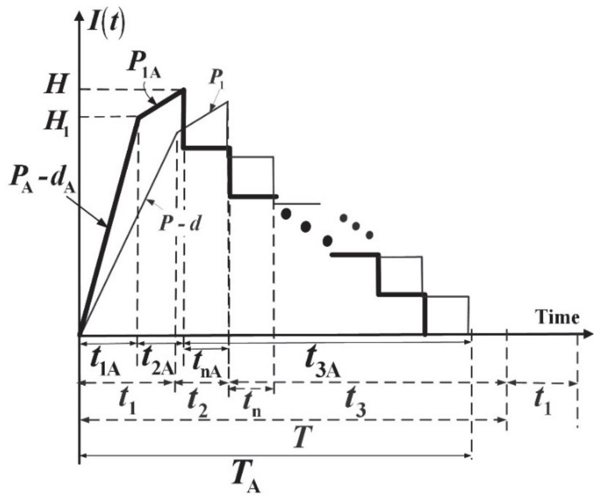

Fig. 1. Inventory level of finished products in any given cycle of the proposed EMQ-based system (in bold lines) compared to that in the classic EMQ model (in thin lines)

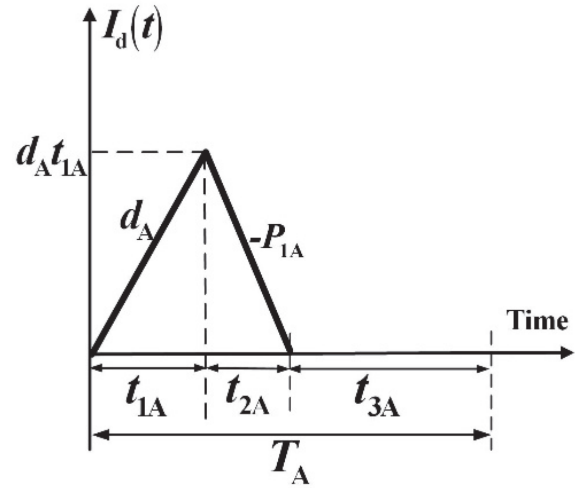

Fig. 2. Inventory level of defective products in the proposed EMQ-based system

Inventory status of finished items during t_{3A} of the proposed EMQ-based system is depicted in Fig. 3.

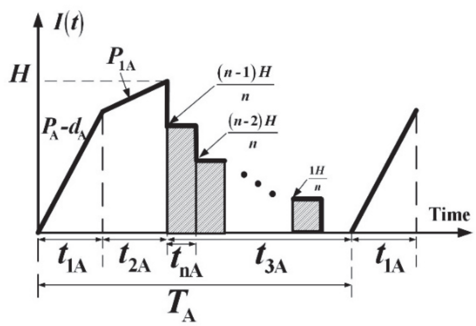

Fig. 3. Inventory level of finished products in t_{3A} of the proposed EMQ-based system

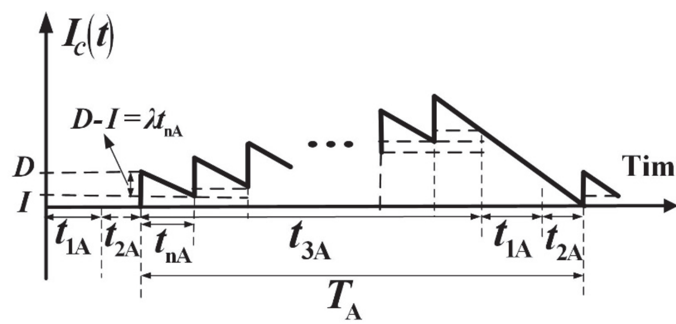

Fig. 4. Status of on-hand stock at the buyer's end

Fig. 4 illustrates the status of on-hand stocks at the buyer's end. Fixed quantity D per shipment and the leftover stocks I (after demand during t_{nA} is satisfied) are as follows:

$$D = \frac{H}{n}, \tag{9}$$

$$I = D - \lambda t_{nA}. \tag{10}$$

Other notations employed in this study are defined as follows:

Q = manufacturing lot size – one of the decision variables,
T_A = manufacturing cycle time,
n = number of shipments per cycle – the other decision variable,
C_{RA}= unit reworking cost, $C_{RA} = (1 + \alpha_3)C_R$
h = unit holding cost per year,
h_1 = unit holding cost of reworked item per year,
K_1 = fixed cost per shipment,
C_T = transportation cost per product,
h_2 = unit holding cost per year at the buyer's end,

$TC(Q, n)$ = total cost per cycle of the proposed EMQ-based system,
$E[T_A]$ = the expected cycle time of the proposed EMQ-based system,
$E[TCU(Q, n)]$ = the expected total system cost per unit time of the proposed EMQ-based system,
$I(t)$ = level of on-hand inventories at time t,
$I_d(t)$= level of on-hand defective items at time t,
$I_c(t)$= level of on-hand inventories at buyer end at time t,
t_1 = uptime of the classic EMQ model,
d = production rate of defective items in the classic EMQ model,
t_2 = rework time of the classic EMQ model,
t_3 = delivery time of the classic EMQ model,
T = cycle time of the classic EMQ model,
C_R = unit reworking cost of the classic EMQ model,
t_n = interval of time between two consecutive shipments in the classic EMQ model.

The following manufacturing lot-size, uptime, reworking time, delivery time, and manufacturing cycle time can be obtained accordingly (see Fig. 1):

$$Q = P_A t_{1A} \tag{11}$$

$$t_{1A} = \frac{Q}{P_A} = \frac{Q}{(1+\alpha_1)P} = \frac{H_1}{P_A - d_A} = \frac{H_1}{\left[(1+\alpha_1)P\right](1-x)} \tag{12}$$

$$t_{2A} = \frac{xQ}{P_{1A}} = \frac{xQ}{(1+\alpha_1)P_1} \tag{13}$$

$$t_{3A} = T_A - t_{1A} - t_{2A} = \frac{Q}{\lambda} - \frac{Q}{(1+\alpha_1)P} - \frac{xQ}{(1+\alpha_1)P_1} \tag{14}$$

$$T_A = t_{1A} + t_{2A} + t_{3A} = \frac{Q}{\lambda} \tag{15}$$

Total defective items produced at the end of uptime are (Fig. 2):

$$d_A t_{1A} = xQ \tag{16}$$

During the delivery time), total inventories (Fig. 3) are

$$\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right) H t_{3A} = \left(\frac{1}{n^2}\right)\left[\frac{n(n-1)}{2}\right] H t_{3A} = \left(\frac{n-1}{2n}\right) H t_{3A} \tag{17}$$

Observing Fig. 4 and from the relevant definitions of system parameters (Eqs. (8-10)), we can obtain total inventories at buyer's end in a replenishment cycle as follows (Chiu et al., 2015):

$$n\left(D - \frac{\lambda t_{nA}}{2}\right) t_{nA} + \frac{n(n-1)}{2} I t_{nA} + \frac{nI}{2}\left(t_{1A} + t_{2A}\right) = \frac{1}{2}\left[\frac{H t_{3A}}{n} + T_A\left(H - \lambda t_{3A}\right)\right]. \tag{18}$$

Total cost per cycle of the proposed EMQ-based system $TC(Q, n)$ comprises manufacturing setup cost, variable production cost, rework cost, fixed and variable transportation costs, inventory holding costs for perfect quality products in setup, rework, and delivery time, holding cost for nonconforming items in rework time, and stock holding cost at the customer side. Hence, $TC(Q, n)$ is as follows:

$$TC(Q,\ n) = K_{\mathrm{A}} + C_{\mathrm{A}}Q + C_{\mathrm{RA}}\left(xQ\right) + nK_1 + C_TQ + h\left[\frac{H_1 + d_{\mathrm{A}}t_{1\mathrm{A}}}{2}\left(t_{1\mathrm{A}}\right) + \frac{H_1 + H}{2}\left(t_{2\mathrm{A}}\right)\right]$$
$$+ h\left(\frac{n-1}{2n}\right)Ht_{3\mathrm{A}} + h_1\frac{P_{1\mathrm{A}}t_{2\mathrm{A}}}{2}\left(t_{2\mathrm{A}}\right) + \frac{h_2}{2}\left[\frac{Ht_{3A}}{n} + T_A\left(H - \lambda t_{3A}\right)\right]. \tag{19}$$

Replacing K_{A}, C_{A}, C_{RA}, d_{A}, P_{A}, and $P_{1\mathrm{A}}$ with their corresponding standard variables (K, C, C_{R}, d, P, and P_1), $TC(Q, n)$ is computed as follows,

$$TC(Q,\ n) = \left(1+\alpha_2\right)K + \left(1+\alpha_3\right)CQ + \left(1+\alpha_3\right)C_{\mathrm{R}}xQ + nK_1 + C_TQ$$
$$+ h\left[\frac{H_1 + x\left[\left(1+\alpha_1\right)P\right]t_{1\mathrm{A}}}{2}\left(t_{1\mathrm{A}}\right) + \frac{H_1 + H}{2}\left(t_{2\mathrm{A}}\right)\right] + h\left(\frac{n-1}{2n}\right)Ht_{3\mathrm{A}}$$
$$+ h_1\frac{\left[\left(1+\alpha_1\right)P_1\right]t_{2\mathrm{A}}}{2}\left(t_{2\mathrm{A}}\right) + \frac{h_2}{2}\left[\frac{Ht_{3A}}{n} + T_A\left(H - \lambda t_{3A}\right)\right]. \tag{20}$$

By substituting Eqs. (5-7) and Eq. (13) to Eq. (15) in $TCU(Q, n)$, and also by applying the expected values of x to deal with its randomness, and with extra derivations, $\mathrm{E}[TCU(Q, n)]$ can be obtained as follows:

$$E\left[TCU\left(Q,\ n\right)\right] = \frac{E\left[TC\left(Q,\ n\right)\right]}{E\left[T_{\mathrm{A}}\right]} = \lambda\left[(1+\alpha_3)C + (1+\alpha_3)C_RE[x]\right] + \frac{\left[(1+\alpha_2)K\right]\lambda}{Q}$$
$$+ \frac{nK_1\lambda}{Q} + C_T\lambda + \frac{hQ}{2} + \frac{\lambda Q\left(h_1 - h\right)E[x]^2}{2(1+\alpha_1)P_1} + \frac{hQ\lambda E[x]}{2(1+\alpha_1)P_1}$$
$$+ \left(h_2 - h\right)Q\left(\frac{1}{2n}\right)\left\{1 - \frac{\lambda}{(1+\alpha_1)}\left[\frac{1}{P} + \frac{E[x]}{P_1}\right]\right\} + \frac{h_2Q}{2}\frac{\lambda}{(1+\alpha_1)}\left[\frac{1}{P} + \frac{E[x]}{P_1}\right]. \tag{21}$$

3. Determining manufacturing lot size and shipments per cycle

First, we verify the following Hessian matrix equations (Rardin, 1998) and make certain that $E[TCU(Q, n)]$ is convex (see Appendix for details):

$$\begin{bmatrix} Q & n \end{bmatrix} \cdot \begin{pmatrix} \dfrac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q^2} & \dfrac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q \partial n} \\ \dfrac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q \partial n} & \dfrac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial n^2} \end{pmatrix} \cdot \begin{bmatrix} Q \\ n \end{bmatrix} = \frac{2\left[\left(1+\alpha_2\right)K\right]\lambda}{Q} > 0. \tag{22}$$

Then, by setting the first and the second partial derivatives of $E[TCU(Q, n)]$ equal to 0 and solving the following linear system (from Eqs. (A-1) and (A-3)):

$$\frac{\partial E\left[TCU\left(Q,n\right)\right]}{\partial Q} = -\frac{\left[(1+\alpha_2)K\right]\lambda}{Q^2} - \frac{nK_1\lambda}{Q^2} + \frac{h}{2} + \frac{\lambda\left(h_1 - h\right)E[x]^2}{2(1+\alpha_1)P_1} + \frac{h\lambda E[x]}{2(1+\alpha_1)P_1}$$
$$+ \left(h_2 - h\right)\left(\frac{1}{2n}\right)\left\{1 - \frac{\lambda}{(1+\alpha_1)}\left[\frac{1}{P} + \frac{E[x]}{P_1}\right]\right\} + \frac{h_2}{2}\frac{\lambda}{(1+\alpha_1)}\left[\frac{1}{P} + \frac{E[x]}{P_1}\right] = 0, \tag{23}$$

$$\frac{\partial E\left[TCU\left(Q,n\right)\right]}{\partial n} = \frac{K_1\lambda}{Q} - \left(h_2 - h\right)Q\left(\frac{1}{2n^2}\right)\left\{1 - \frac{\lambda}{(1+\alpha_1)}\left[\frac{1}{P} + \frac{E[x]}{P_1}\right]\right\} = 0, \tag{24}$$

and with extra derivatives of the aforementioned linear system, we may find the optimal manufacturing lot size and the number of shipments per cycle as follows:

$$Q^* = \sqrt{\frac{2\lambda\left[(1+\alpha_2)K + nK_1\right]}{h + \frac{h\lambda E[x]}{(1+\alpha_1)P_1} + \frac{\lambda\left[h_1 - h\right]E[x]^2}{(1+\alpha_1)P_1} + \frac{h_2\lambda}{(1+\alpha_1)}\left[\frac{1}{P} + \frac{E[x]}{P_1}\right] + \frac{(h_2 - h)}{n}\left\{1 - \frac{\lambda}{(1+\alpha_1)}\left[\frac{1}{P} + \frac{E[x]}{P_1}\right]\right\}}}, \tag{25}$$

and

$$n^* = \sqrt{\frac{(1+\alpha_2)K\left(h_2 - h\right)\delta}{K_1\left\{h + \frac{h\lambda E[x]}{(1+\alpha_1)P_1} + \frac{\lambda\left(h_1 - h\right)E[x]^2}{(1+\alpha_1)P_1} + \frac{h_2\lambda}{(1+\alpha_1)}\left[\frac{1}{P} + \frac{E[x]}{P_1}\right]\right\}}}, \tag{26}$$

where $\delta = \left\{1 - \frac{\lambda}{(1+\alpha_1)}\left[\frac{1}{P} + \frac{E[x]}{P_1}\right]\right\}$.

4. Numerical example with sensitivity analyses

A numerical example with sensitivity analyses is provided below to demonstrate the applicability of the proposed method. Assuming that the following variables are in an economic manufacturing quantity model and by considering overtime option, rework, and discontinuous product issuing policy:

P = 20,000 items per year,
α_1 = 0.5,
P_A = 30,000 items per year (since $P_A = (1 + \alpha_1)P$),
K = \$5,000,
α_2 = 0.1 (assuming $\alpha_2 = 0.2(\alpha_1)$),
K_A = \$5,500 (since $K_A = (1 + \alpha_2)K$),
C = \$100 per item,
α_3 = 0.25 (assuming $\alpha_3 = 0.5(\alpha_1)$),
C_A = \$125 (since $C_A = (1 + \alpha_3)C$),
h = \$30,
λ = 4,000 items per year,
x = [0, 0.2], which follows the uniform distribution,
C_R = \$60,
h_1 = \$40,
K_1 = \$800 per shipment,
C_T = \$0.5 per product,
h_2 = \$80.

4.1 Optimality and rework issues

Applying Eqs. (25-26), and Eq. (21), the following optimal replenishment lot size, shipments per cycle, and the expected system cost can be found: Q^* = 1,025, n^* = 3, and $E[TCU(Q^*, n^*)]$ = \$593,652. Further analyses illustrate the behavior of $E[TCU(Q, n)]$ with respective to Q (Fig. 5), and effect of nonconforming rate on different system cost components of $E[TCU(Q, n)]$ in the proposed EMQ-based system (Fig. 6).

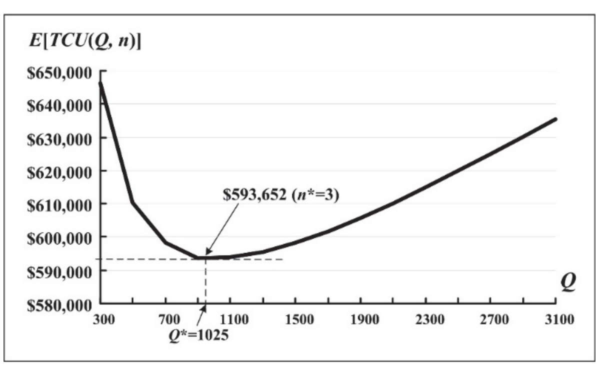

Fig. 5. Behavior of $E[TCU(Q, n)]$ with respective to Q in the proposed EMQ-based system

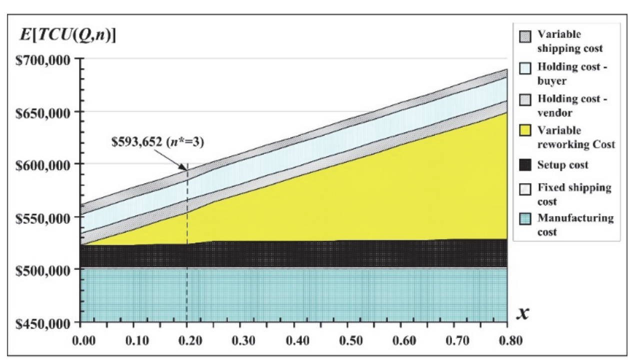

Fig. 6. Effect of nonconforming rate on different system cost components of $E[TCU(Q, n)]$

From Fig. 6, it is noted that as nonconforming rate x goes up, variable reworking cost increases significantly, and certainly, so does the expected total system cost $E[TCU(Q, n)]$. The joint effects of replenishment lot size Q and number of shipments n per cycle on the expected total system costs $E[TCU(Q, n)]$ are depicted in Fig. 7. It can be seen that the optimal operating policy (Q^* = 1,025, n^* = 3) is reconfirmed.

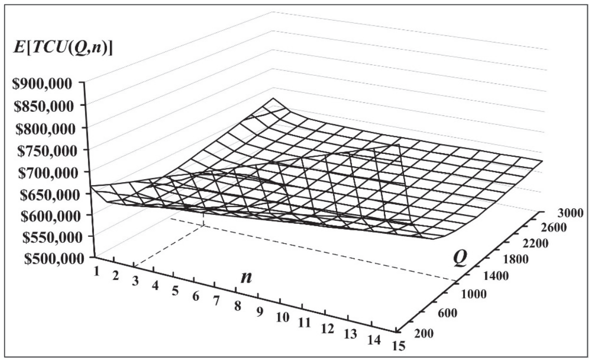

Fig. 7. Joint effects of replenishment lot size and shipments per cycle on the expected total system costs $E[TCU(Q, n)]$

4.2 Joint effects from overtime relating factors and rework

With further investigation on the effects of variations in α_i (i.e., those parameters associated with overtime option) on the proposed system, certain important information is revealed (as shown in Appendix B: Table B-1 and Table B-2). Joint effects of variations in adjusted factor α_1 and random nonconforming rate x on the expected total system cost are shown in Fig. 8. It is noted that as α_1 increases (i.e., more overtime are put into operation), expected total system cost $E[TCU(Q, n)]$ goes up notably; as nonconforming rate x goes higher, $E[TCU(Q, n)]$ rises accordingly. From analytical results shown in Table B-1, the impact of overtime output adjusted rate α_1 on the optimal replenishment lot size Q^* and shipments per cycle n^* is exposed as illustrated in Fig. 9.

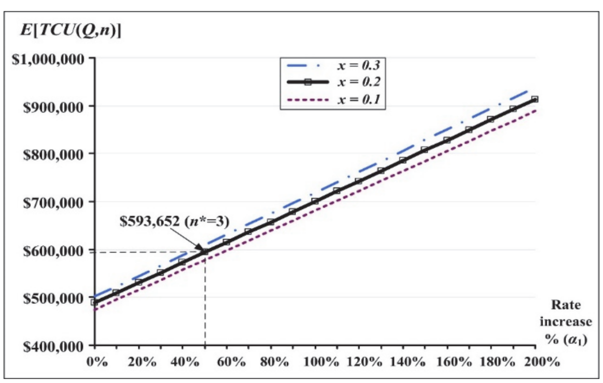

Fig. 8. Joint effects of adjusted factor α_1 and nonconforming rate x on $E[TCU(Q, n)]$

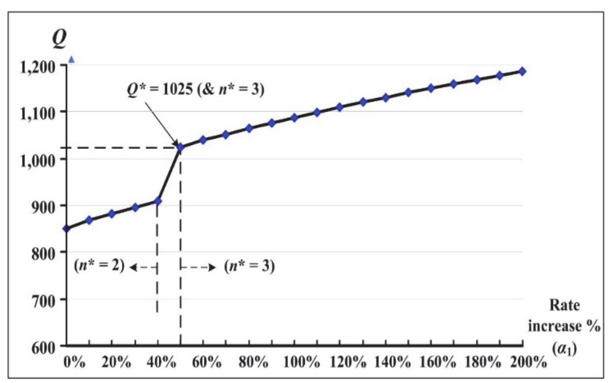

Fig. 9. Impact of variations in α_1 on the optimal replenishment lot size and shipments per cycle

It indicates that as α_1 increases (i.e., more overtime are put into operation), Q^* goes up and n^* increases from 2 to 3 at α_1 = 50% and up. The analytical results (from Table B-1, Appendix B) also point out the effect of overtime unit cost increase rate α_3 on the expected total system cost $E[TCU(Q, n)]$ (Fig. 10). For instance, at α_3 = 25% (i.e., corresponding α_1 = 50%), $E[TCU(Q, n)]$ increases 21.64% as compared to that in the same system operation without overtime option. As α_3 goes higher, the expected total system cost increases significantly.

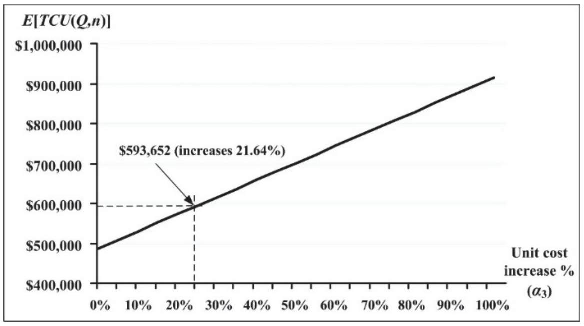

Fig. 10. Effect of overtime unit cost increase rate α_3 on the expected total system cost

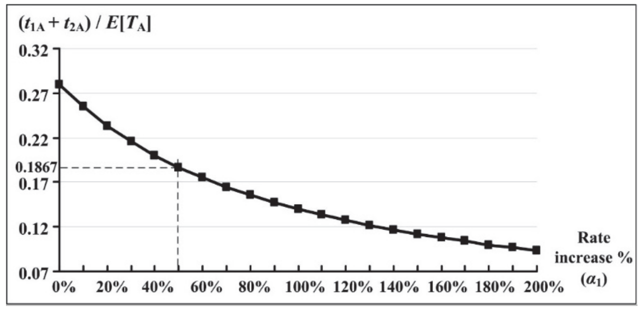

Fig. 11. The impact of variations in overtime output adjusted rate α_1 on machine utilization

Further, from the analytical results shown in Table B-2, the impact of variations in overtime output adjusted rate α_1 on machine utilization is revealed (Fig. 11). It can be seen that at α_1 = 50%, machine utilization (i.e., the ratio of (the sum of uptime t_{1A} and rework time t_{2A}) over the expected cycle length $E[T_A]$) drops 18.67%; and as α_1 goes up, machine utilization keep on declining accordingly.

5. Conclusions

An EMQ-based model incorporating an overtime option, rework, and a discontinuous issuing policy for end items has been developed in this study. Such a decision-support model not only provides the optimal replenishment lot size and number of shipments per cycle (see Figs. 5 and 7), but also allows production managers to identify the individual and combined effects of various important system factors on the system's decision variables, expected total cost per unit time, and machine utilization. These critical factors include random defective rate (see Fig. 8), output-adjusted rate due to overtime (see Figs. 9 and 11), and unit cost-increase rate due to overtime (see Fig. 10). Without such an in-depth study, a variety of information related to managerial decision making remains ignored.

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Appendix A

From Eq. (21), derivations of Hessian matrix equations (Rardin, 1998) are presented as follows:

$$\frac{\partial E\left[TCU\left(Q,n\right)\right]}{\partial Q} = -\frac{\left[(1+\alpha_2)K\right]\lambda}{Q^2} - \frac{nK_1\lambda}{Q^2} + \frac{h}{2} + \frac{\lambda\left(h_1-h\right)E[x]^2}{2(1+\alpha_1)P_1} + \frac{h\lambda E[x]}{2(1+\alpha_1)P_1} + \left(h_2-h\right)\left(\frac{1}{2n}\right)\left\{1-\frac{\lambda}{(1+\alpha_1)}\left[\frac{1}{P}+\frac{E[x]}{P_1}\right]\right\} + \frac{h_2}{2}\frac{\lambda}{(1+\alpha_1)}\left[\frac{1}{P}+\frac{E[x]}{P_1}\right], \tag{A-1}$$

$$\frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q^2} = \frac{2[(1+\alpha_2)K]\lambda}{Q^3} + \frac{2nK_1\lambda}{Q^3}, \tag{A-2}$$

$$\frac{\partial E\left[TCU\left(Q,n\right)\right]}{\partial n} = \frac{K_1\lambda}{Q} - \left(h_2-h\right)Q\left(\frac{1}{2n^2}\right)\left\{1-\frac{\lambda}{(1+\alpha_1)}\left[\frac{1}{P}+\frac{E[x]}{P_1}\right]\right\}, \tag{A-3}$$

$$\frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial n^2} = \left(h_2-h\right)Q\left(\frac{1}{n^3}\right)\left\{1-\frac{\lambda}{(1+\alpha_1)}\left[\frac{1}{P}+\frac{E[x]}{P_1}\right]\right\}, \tag{A-4}$$

$$\frac{\partial E\left[TCU\left(Q,n\right)\right]}{\partial Q\partial n} = -\frac{K_1\lambda}{Q^2} - \left(h_2-h\right)\left(\frac{1}{2n^2}\right)\left\{1-\frac{\lambda}{(1+\alpha_1)}\left[\frac{1}{P}+\frac{E[x]}{P_1}\right]\right\}. \tag{A-5}$$

With extra derivations, the resulting Hessian matrix equations are as follows:

$$\begin{bmatrix} Q & n \end{bmatrix} \cdot \begin{pmatrix} \frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q^2} & \frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q\partial n} \\ \frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q\partial n} & \frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial n^2} \end{pmatrix} \cdot \begin{bmatrix} Q \\ n \end{bmatrix} = \frac{2\left[\left(1+\alpha_2\right)K\right]\lambda}{Q} > 0 \tag{A-6}$$

Since α_2, K, λ, and Q are all positive, thus, Eq. (A-6) is positive. So, *E[TCU(Q, n)]* is strictly convex function for all Q and n different from zero.

Appendix B

Table B-1

Joint effects of variations in adjusted factors α_1 and α_3 on the optimal replenishment lot size and shipments, the expected total system cost, and system cost increase %

α_1 %	α_3 %	Q^*	n^*	$E[TCU(Q)]$	% of cost increases
0%	0%	865	2	$488,041	-
10%	5%	878	2	$509,004	4.30%
20%	10%	890	2	$530,067	8.61%
30%	15%	901	2	$551,206	12.94%
40%	20%	911	2	$572,405	17.29%
50%	**25%**	**1,025**	**3**	**$593,652**	**21.64%**
60%	30%	1,036	3	$614,805	25.97%
70%	35%	1,047	3	$636,000	30.32%
80%	40%	1,057	3	$657,229	34.67%
90%	45%	1,067	3	$678,488	39.02%
100%	50%	1,076	3	$699,771	43.38%
110%	55%	1,085	3	$721,075	47.75%
120%	60%	1,094	3	$742,396	52.12%
130%	65%	1,103	3	$763,734	56.49%
140%	70%	1,112	3	$785,084	60.86%
150%	75%	1,120	3	$806,447	65.24%
160%	80%	1,128	3	$827,819	69.62%
170%	85%	1,136	3	$849,201	74.00%
180%	90%	1,144	3	$870,590	78.38%
190%	95%	1,152	3	$891,986	82.77%
200%	100%	1,160	3	$913,389	87.15%

Table B-2

Effect of variations in adjusted factor α_1 on fabrication uptime, reworking time, expected cycle length, machine utilization and its decrease %

α_1 %	Uptime (t_{1A})	Reworking time (t_{2A})	$E[T_A]$	Machine utilization $(t_{1A}+t_{2A})/E[T_A]$	% utilization decreases
0%	0.0426	0.0170	0.2128	0.2800	-
10%	0.0394	0.0158	0.2168	0.2545	-9.09%
20%	0.0368	0.0147	0.2205	0.2333	-16.67%
30%	0.0345	0.0138	0.2240	0.2154	-23.08%
40%	0.0325	0.0130	0.2273	0.2000	-28.57%
50%	**0.0342**	**0.0137**	**0.2563**	**0.1867**	**-33.33%**
60%	0.0325	0.0130	0.2597	0.1750	-37.50%
70%	0.0309	0.0124	0.2630	0.1647	-41.18%
80%	0.0296	0.0118	0.2661	0.1556	-44.44%
90%	0.0283	0.0113	0.2691	0.1474	-47.37%
100%	0.0272	0.0109	0.2720	0.1400	-50.00%
110%	0.0262	0.0105	0.2748	0.1333	-52.38%
120%	0.0252	0.0101	0.2775	0.1273	-54.55%
130%	0.0244	0.0097	0.2801	0.1217	-56.52%
140%	0.0236	0.0094	0.2826	0.1167	-58.33%
150%	0.0228	0.0091	0.2851	0.1120	-60.00%
160%	0.0221	0.0088	0.2875	0.1077	-61.54%
170%	0.0215	0.0086	0.2899	0.1037	-62.96%
180%	0.0209	0.0083	0.2922	0.1000	-64.29%
190%	0.0203	0.0081	0.2944	0.0966	-65.52%
200%	0.0198	0.0079	0.2967	0.0933	-66.67%