

Impact of future price increase on ordering policies for deteriorating items under quadratic demand

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ABSTRACT

When a supplier announces a price increase at a certain time in the future, for each retailer it is important to choose whether to purchase supplementary stock to take benefit of the current lower price or procure at a new price. This article focuses on the possible effects of price increase on a retailer's replenishment strategy for constant deterioration of items. Here, quadratic demand is debated; which is appropriate for the products for which demand increases initially and subsequently it starts to decrease with the new version of the substitute. We discuss two scenarios in this study: (I) when the special order time coincides with the retailer's replenishment time and (II) when the special order time falls during the retailer's sales period. We determine an optimal ordering policy for each case by maximizing total cost savings between special and regular orders during the depletion time of the special order quantity. Scenarios are established and illustrated with numerical examples. Through, sensitivity analysis important inventory parameters are classified. Graphical results, in two and three dimensions, are exhibited with supervisory decision.

1. Introduction

Because of the current rises in the prices of oil and raw materials, the prices of commodities have continued to increase at universal level. This is the serious issue for enterprises. In particular, when supervisors make decisions relating to their inventory policy, it is essential for them to consider increases in goods prices. When a supplier announces an impending price increase from effect at a certain time in the future, it is important for each retailer to decide whether to purchase sufficient stock before the price increase, to take advantage of the present lower price. Researchers have taken the declaration of a price increase problem into account and have proposed various analytical models to get more insights into the decisions relating to inventory policy. Naddor (1966) proposed an infinite horizon economic order quantity (EOQ) model where the supplier announces a price increase. Lev and Soyster (1979) established a finite horizon inventory model and determined optimal ordering policies based on known information

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about an imminent price increase. Later, Goyal (1979) studied Lev and Soyster's (1979) model and proposed an alternative method for determining the optimal policy. Taylor and Bradley (1985) extended Naddor's (1966) model and found the optimal ordering strategies for situations where the price increase does not coincide with the end of an EOQ cycle. Lev and Weiss (1990) subsequently developed a structure of optimal policies and procedures for computing the optimal policy. Goyal *et al.* (1991) offered a review of a study on inventory policies under one-time-only incentives. Tersine (1996) suggested an economic production quantity (EPQ) model under an announced price increase. Ghosh (2003) and Huang and Kulkarni (2003) presented an infinite-horizon deterministic inventory model under an announced price increase. In contrast to single price change models, a small number of continuous price change models exist within inventory management literature. Erel (1992) and Khouja and Park (2003) considered EOQ models with continuous price changes (price increases or reductions). Recently, Tripathi and Tomar (2015) studied optimal ordering policy for deteriorating items with time-dependent demand in response to temporary price discount linked to order quantity. They considered constant rate of deterioration.

The above inventory models account for the impact of price changes and focus on the determination of the optimal special order quantity for the retailer. A weakness with most of them is that they neglect the deterioration of goods, which is a common phenomenon. It is obvious that certain products, such as medicine, volatile liquids, fruits, and vegetables, will deteriorate when kept in storage for a long period. For such products, losses due to deterioration cannot be ignored when determining the optimal order policy. Inventory problems relating to deteriorating items have been studied extensively in previous research. Ghare and Schrader (1963) first established an EOQ model for an exponentially decaying item for which there is constant demand. Later, Covert and Philip (1973) extended Ghare and Schrader's (1963) model and developed an EOQ model for a variable deterioration rate, by assuming a two-parameter Weibull distribution. Philip (1974) then obtained an inventory model with a three-parameter Weibull distribution deterioration rate. Goyal and Giri (2001) provided an excellent and detailed review of the literature on deteriorating inventory since the early 1990s. Moon *et al.* (2005) formulated a model to incorporate two extreme physical characteristics of stored items into inventory model ameliorating (value or utility increase with time) and deteriorating. Bakker *et al.* (2012) have undertaken an up-to-date review of the advances made in the field of inventory control of perishable items (deteriorating inventory) since 2001. Recently, Tayal *et al.* (2014) has investigated an inventory model for deteriorating items with seasonal products and an option of an alternative market. Freshly, Singh *et al.* (2015) an economic order quantity model for deteriorating products having stock dependent demand with trade credit period and preservation technology. There is also a large body of literature on deteriorating inventory. These cover issues such as the type of demand (*viz.*, Begum *et al.* (2012), Deng, *et al.* (2007), Khanra *et al.* (2010), Mishra and Shah (2008); Skouri and Konstantaras (2009); accounting for the time-value of money (*viz.*, Wee and Law (1999, 2001); allowing shortages and backordering *viz.* Shah (1998), Sharma (2006), Yang (2011); considering multiple items *viz.* Sharma (2007a, 2007b, 2009b); the EPQ model *viz.* Min *et al.* (2012), Sharma (2008a, 2008b, 2009c); and the two-warehouse problem *viz.* Pakkala and Achary (1992), Sarma (1987), Yang, (2006).

Most of the above research reveals that retailers are inclined to adopt a special order, with the special order quantity involved being unlimited, when suppliers announce a price increase. In practice, to avoid the retailer hoarding goods for later sale at a higher selling price, the supplier is willing to offer a limited quantity at the current price prior to the price increase. Therefore, the number of goods the retailer can order is also limited. Thus, in order to discuss the above economic topics, this study examines the possible effects of a price increase on a retailer's replenishment policies. The contribution of this article, relative to previous studies, is that we explore inventory decisions and the three issues of the traditional EOQ model simultaneously. These comprise of the following: (1) the retailer expects the price increase at a certain time in the future (as announced by the supplier) and decides whether to place a special order; (2) the goods deteriorate at a constant rate; and (3) the demand is quadratic in nature. Additionally, because the time for placing the special order may or may not coincide with the replenishment time, we consider

two cases: (I) when the special order time matches with the retailer's replenishment time and (II) when the special order time occurs during the retailer's sales period. In Case I, the retailer's optimal order policy is to decide whether to place a larger order which is always larger than regular EOQ. In Case II, the retailer's optimal order policy is to decide whether to place an additional order which is not necessary larger than regular EOQ. The purpose of this study is to determine the retailer's optimal order policies in response to a price increase by maximising the total cost saving between special and regular orders during the depletion time of the special order quantity. We provide two numerical examples to illustrate the theories in practical use, and we conduct a sensitivity analysis of the optimal solution by examining the main parameters.

2. Notations and assumptions

We shall use following notations and assumptions to build up the mathematical model of the problem under consideration.

2.1 Notations

A	Ordering cost per order
C	Purchase cost per unit
i	Inventory holding cost (excluding interest charges) per unit per unit time
θ	Constant deterioration rate, $0 \leq \theta < 1$ and is a constant
k	Price increase per unit
$R(t)$	Quadratic demand
T	Length of replenishment cycle time before price increase
$I(t)$	Inventory level at any instant of time t before the price increase, $0 \leq t \leq T$
Q	Order quantity (units/order) before the price increase
Q_i	Order quantity (units/order) after the price increase
T_i	Length of replenishment cycle time after price increase
Q_s	Special order quantity before the price increase (decision variable)
T_s	Depletion time for the special order quantity Q_s (decision variable)
q	Residual inventory level when the special order is placed
t_q	The length of time until the special order is placed during the retailer's regular replenishment period
T_q	Depletion time for the inventory quantity $Q_s + q$
$I_s(t)$	Inventory level at any instant of time t when the special order is adopted $0 \leq t \leq T_s$
$I_q(t)$	Inventory level at any instant of time t during the time interval $[0, T_q]$
$TC(T)$	Total cost per unit time during the replenishment period T
$TC(T_i)$	Total cost per unit time during the replenishment period T_i
$g_i(T_s)$	Total cost saving between the special order and regular order during the special cycle time for $i, i = 1, 2$

2.2 Assumptions

1. The system under review deals with single item
2. The demand rate is $R(t) = a(1 + bt - ct^2)$, where $a > 0$ denotes the scale demand, b and c denotes linear and quadratic demand rates. The functional form of demand rate suggests that demand increases linearly and decreases quadratically. This demand is justified for electronic gadgets, fashion goods, medicines during epidemics.

3. To reflect the increasing price of raw materials, the supplier announces that the unit price of an item will increase by a given amount k , at a certain future date.
4. The retailer has only one opportunity to replenish its stock at the present price before the price increases
5. The replenishment rate is infinite and the lead-time is zero.
6. Shortages are not allowed
7. There is no replacement or repair of deteriorated units during the period under consideration.

3. Mathematical Model

This article focuses on the possible effects of price increase on a retailer's replenishment strategy, in which, Depletion of the inventory occurs due to the combined effects of demand and physical deterioration. Thus, the change in inventory level before the price increase is described by following differential equation:

$$\frac{dI(t)}{dt} = -R(t) - \theta(t)I(t), \quad 0 < t < T \quad (1)$$

given the boundary condition $I(T) = 0$, the solution of Eq. (1) is represented by

$$I(t) = \frac{a(\theta^2 - \theta b - 2c + \theta^2 T b + 2\theta T c - c T^2 \theta^2) e^{-\theta t + \theta T}}{\theta^3} - \frac{a(\theta^2 - \theta b - 2c + \theta^2 t b + 2\theta t c - c t^2 \theta^2)}{\theta^3}, 0 \leq t \leq T \quad (2)$$

Hence, the order quantity is given by

$$Q = I(0) = \frac{a(\theta^2 - \theta b - 2c + \theta^2 T b + 2\theta T c - c T^2 \theta^2) e^{\theta T}}{\theta^3} - \frac{a(\theta^2 - \theta b - 2c)}{\theta^3} \quad (3)$$

Earlier to the price increase, the purchasing cost C follows the fixed economic order strategy with a unit purchasing cost, C , the total cost during the replenishment period T being the sum of the ordering cost, purchasing cost and holding cost, i.e.,

$$A + CQ + Ci \int_0^T I(t) dt \quad (4)$$

Therefore, the total cost per unit time is

$$TC(T) = \frac{1}{T} \left(A + CQ + Ci \int_0^T I(t) dt \right) \quad (5)$$

It can easily be shown that $TC(T)$ is a convex function of T . Thus, there is a unique value for T (say T_0) that minimises $TC(T)$. The value of T_0 can be obtained by solving the equation $\frac{dTC(T)}{dT} = 0$.

Once the optimal length of replenishment cycle time, T_0 , is obtained, the optimal order quantity, Q_0 , is obtained as follows:

$$Q_0 = \frac{a(\theta^2 - \theta b - 2c + \theta^2 T_0 b + 2\theta T_0 c - c T_0^2 \theta^2) e^{\theta T_0}}{\theta^3} - \frac{a(\theta^2 - \theta b - 2c)}{\theta^3} \quad (6)$$

Next, when unit purchasing cost increases from C to $(C+k)$, the total cost per unit time becomes

$$TC(T_i) = \frac{1}{T_i} \left(A + (C+k)Q_i + (C+k) i \int_0^{T_i} I(t) dt \right), \tag{7}$$

where, $Q_i = \frac{a(\theta^2 - \theta b - 2c + \theta^2 T_i b + 2\theta T_i c - c T_i^2 \theta^2) e^{\theta T_i}}{\theta^3} - \frac{a(\theta^2 - \theta b - 2c)}{\theta^3}$

Similarly, there is a unique value for T_i (say T_{i0}) that minimises $TC(T_i)$. The value of T_{i0} can be obtained by solving the equation $\frac{dTC(T_i)}{dT_i} = 0$. The corresponding optimal order quantity, Q_{i0} , is

$$Q_{i0} = \frac{a(\theta^2 - \theta b - 2c + \theta^2 T_{i0} b + 2\theta T_{i0} c - c T_{i0}^2 \theta^2) e^{\theta T_{i0}}}{\theta^3} - \frac{a(\theta^2 - \theta b - 2c)}{\theta^3} \tag{8}$$

Consequently, when a supplier announces a price increase that is effective from a particular future period, the retailer may place a special order to take benefit of the current lower price, C , before the price increases. On the other hand, the retailer may ignore this notice and place a regular order. The purpose of this study is to determine the optimal special order quantity by maximising the total cost saving between special and regular orders during the depletion time of the special order quantity. As specified previously, two specific situations arise, which we discuss in this article: (I) when the special order time matches with the retailer’s replenishment time and (II) when the special order time take place during the retailer’s sales period. Next, we will formulate the resultant total relevant inventory cost saving function for these two cases.

3.1. Case I: the special order time matches with the retailer’s replenishment time

In this case, if the retailer decides to adopt a special order and orders Q_s units, then the inventory level at time t is

$$I_s(t) = \frac{a(\theta^2 - \theta b - 2c + \theta^2 T_s b + 2\theta T_s c - c T_s^2 \theta^2) e^{-(\theta + \theta T_s) t}}{\theta^3} - \frac{a(\theta^2 - \theta b - 2c + \theta^2 t b + 2\theta t c - c t^2 \theta^2)}{\theta^3}, 0 \leq t \leq T_s \tag{9}$$

The special order quantity at the original unit purchasing price, C , is

$$Q_s = I_s(0) = \frac{a(\theta^2 - \theta b - 2c + \theta^2 T_s b + 2\theta T_s c - c T_s^2 \theta^2) e^{\theta T_s}}{\theta^3} - \frac{a(\theta^2 - \theta b - 2c)}{\theta^3} \tag{10}$$

The total cost of the special order during the time interval $[0, T_s]$ (denoted by $TC S_1(T_s)$) consist of ordering cost, purchasing cost and holding cost, and is represented by

$$TC S_1(T_s) = A + C Q_s + C i \int_0^{T_s} I(t) dt \tag{11}$$

If the retailer places its regular order, then the total cost of a regular order during the time interval $[0, T_s]$ will be divided into two periods (see fig.(1)). In the first period, the retailer orders Q_0 units at the unit purchasing price C . The corresponding total cost is similar to Eq. (4), and is described by

$$A + C Q_0 + C i \int_0^{T_0} I(t) dt \tag{12}$$

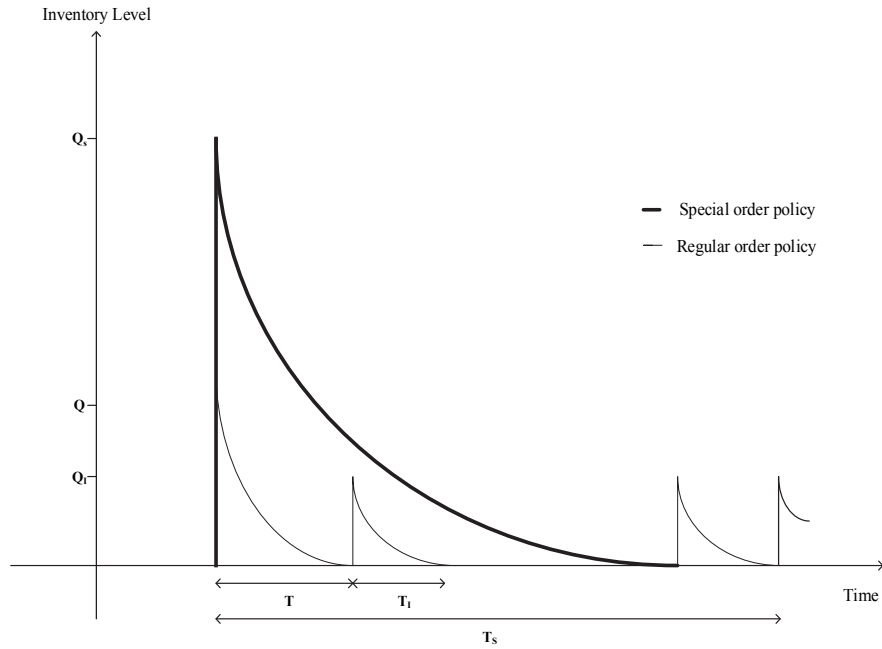


Fig. 1. Special vs. regular order policies when the special order time coincides with the retailer's replenishment time

As to the rest period, the retailer follows regular EOQ policy for the unit purchasing price $(C+k)$. Hence, the total cost during the rest period is given by

$$\frac{T_s - T_0}{T_{I_0}} \left(A + (C+k)Q_{I_0} + (C+k)i \int_0^{T_{I_0}} I(t) dt \right) \quad (13)$$

Subsequently, the total cost of a regular order during the time interval $[0, T_s]$ (denoted by $TCN_1(T_s)$) is

$$TCN_1(T_s) = A + CQ_0 + Ci \int_0^{T_0} I(t) dt + \frac{T_s - T_0}{T_{I_0}} \left(A + (C+k)Q_{I_0} + (C+k)i \int_0^{T_{I_0}} I(t) dt \right) \quad (14)$$

Comparing Eq. (11) with Eq.(14), the total cost saving when the special order coincides with the retailer's replenishment time (i.e., Case I) can be expressed as follows:

$$g_1(T_s) = TCN_1(T_s) - TCS_1(T_s) \quad (15)$$

Simply it can be shown that $g_1(T_s)$ is a concave function of T_s . Hence, there is a unique value for T_s (say T_{s1}) that maximises $g_1(T_s)$. The value of T_{s1} can be obtained by solving the equation $\frac{dg_1(T_s)}{dT_s} = 0$.

3.2. Case II: the special order time take place during the retailer's sales period

Sometimes, the time of the price increase arises during the retailer's sales period. In this state, if the retailer decides to place a special order of quantity Q_s at the present price C , the inventory level will

increase instantaneously from q to $Q_s + q$ when the special order quantity is supplied (see Fig.(2)). Conversely, if the retailer pays no attention to the notification about the price increase, that retailer will not place any orders until the next replenishment. We will formulate the total cost functions for the special and regular order policies and then compare the two.

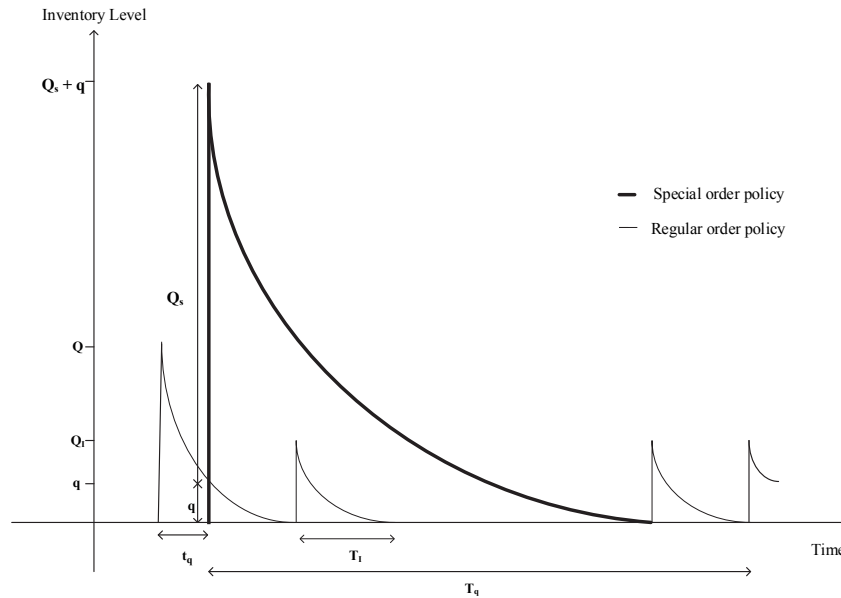


Fig. 2. Special vs. regular order policies when the special order time occurs during the retailer’s sales period

When a special order is placed, the total cost during the time interval $[0, T_q]$ consists of the ordering cost, purchasing cost and holding cost, which is expressed as follows.

As the special order quantity attains, the maximum inventory is given by

$$\begin{aligned}
 Q_s + q &= \frac{a(\theta^2 - \theta b - 2c + \theta^2 T_s b + 2\theta T_s c - c T_s^2 \theta^2) e^{\theta T_s}}{\theta^3} - \frac{a(\theta^2 - \theta b - 2c)}{\theta^3} \\
 &+ \frac{a(\theta^2 - \theta b - 2c + \theta^2 (T_0 - t_q) b + 2\theta (T_0 - t_q) c - c (T_0 - t_q)^2 \theta^2) e^{\theta (T_0 - t_q)}}{\theta^3} - \frac{a(\theta^2 - \theta b - 2c)}{\theta^3}
 \end{aligned} \tag{16}$$

Moreover, the inventory level at time t during the time interval $[0, T_q]$ can be obtained by

$$I_q(t) = \frac{a(\theta^2 - \theta b - 2c + \theta^2 T_q b + 2\theta T_q c - c T_q^2 \theta^2) e^{(-\theta t + \theta T_q)}}{\theta^3} - \frac{a(\theta^2 - \theta b - 2c + \theta^2 t b + 2\theta t c - c t^2 \theta^2)}{\theta^3}, 0 \leq t \leq T_q \tag{17}$$

Now, because of $I_q(0) = Q_s + q$, from Eq. (16) and Eq. (17), we have

$$\begin{aligned}
 &\frac{a(\theta^2 - \theta b - 2c + \theta^2 T_s b + 2\theta T_s c - c T_s^2 \theta^2) e^{\theta T_s}}{\theta^3} - \frac{a(\theta^2 - \theta b - 2c)}{\theta^3} \\
 &+ \frac{a(\theta^2 - \theta b - 2c + \theta^2 (T_0 - t_q) b + 2\theta (T_0 - t_q) c - c (T_0 - t_q)^2 \theta^2) e^{\theta (T_0 - t_q)}}{\theta^3} - \frac{a(\theta^2 - \theta b - 2c)}{\theta^3} \\
 &= \frac{a(\theta^2 - \theta b - 2c + \theta^2 T_q b + 2\theta T_q c - c T_q^2 \theta^2) e^{(-\theta t + \theta T_q)}}{\theta^3} - \frac{a(\theta^2 - \theta b - 2c)}{\theta^3}
 \end{aligned} \tag{18}$$

Now, avoiding higher order terms of θ , we can find T_q as follows

$$Tq = -\frac{1}{\theta^3} \begin{pmatrix} -e^{\theta T_s} \theta^2 + e^{\theta T_s} \theta b + 2e^{\theta T_s} c - e^{\theta T_s} \theta^2 T_s b \\ -2e^{\theta T_s} \theta T_s c + e^{\theta T_s} \theta^2 T_s^2 c + 2\theta^2 - 2\theta b \\ -4c - e^{\theta(T-t_q)} \theta^2 + e^{\theta(T-t_q)} \theta b + 2e^{\theta(T-t_q)} c \\ -e^{\theta(T-t_q)} \theta^2 T b + e^{\theta(T-t_q)} \theta^2 b t_q - 2e^{\theta(T-t_q)} \theta T c \\ + 2e^{\theta(T-t_q)} \theta c t_q + e^{\theta(T-t_q)} c T^2 \theta^2 - 2e^{\theta(T-t_q)} \theta^2 c T t_q + e^{\theta(T-t_q)} \theta^2 c t_q^2 \end{pmatrix} \quad (19)$$

Thus, total cost of the special order during the time interval $[0, T_q]$ (denoted by $TCS_2(T_s)$) can be described as follows:

$$TCS_2(T_s) = A + CQ_s + Ci \int_0^{T_q} I_q(t) dt \quad (20)$$

In contrast, if the retailer ignores announcement of the price increase and places its regular order, the total cost during the time interval $[0, T_q]$ will also be divided into two phases. In the first period, the retailer only has the cost during the depletion time of residual q , $T_0 - t_q$. We use the average cost analysis approach, which gives us the following:

$$\frac{T_0 - t_q}{T_0} \left(A + CQ_0 + Ci \int_0^{T_0} I(t) dt \right) \quad (21)$$

Afterward, the retailer places the regular order with the unit purchase cost $(C + k)$ during the rest period. To obtain the total cost in this period, we use the average cost analysis method, which is given by

$$\frac{T_q - (T_0 - t_q)}{T_{I_0}} \left(A + (C + k)Q_{I_0} + (C + k)i \int_0^{T_{I_0}} I(t) dt \right) \quad (22)$$

As a result, when the retailer ignores the notification and places its regular order during the time interval $[0, T_q]$, the total cost (denoted by $TCN_2(T_s)$) is

$$TCN_2(T_s) = \frac{T_0 - t_q}{T_0} \left(A + CQ_0 + Ci \int_0^{T_0} I(t) dt \right) + \frac{T_q - (T_0 - t_q)}{T_{I_0}} \left(A + (C + k)Q_{I_0} + (C + k)i \int_0^{T_{I_0}} I(t) dt \right) \quad (23)$$

Hence, the total cost saving when the special order time occurs during the retailer's sales period can be formulated as follows:

$$g_2(T_s) = TCN_2(T_s) - TCS_2(T_s) \quad (24)$$

Simply it can be shown that $g_2(T_s)$ is a concave function of T_s . Hence, there is a unique value for T_s (say T_{s_2}) that maximises $g_2(T_s)$. The value of T_{s_2} can be obtained by solving the equation

$$\frac{dg_2(T_s)}{dT_s} = 0.$$

Remark: Note that it is worth placing a special order only when the total cost saving is positive in the above two cases. Otherwise, the retailer will ignore the opportunity to place the special order.

4. Numerical Examples

The developed mathematical model is illustrated in the following numerical examples. The retailer’s total cost per unit time is optimized using Maple XVI is given in Table 1.

Table 1

Numerical data

Parameters	Examples	
	1	2
<i>a</i>	100	100
<i>b</i>	0.2	0.2
<i>c</i>	0.3	0.3
<i>A</i>	50	50
θ	0.2	0.2
<i>i</i>	0.2	0.2
<i>C</i>	20	20
<i>k</i>	4	4

Optimal Solution		
T_s (in years)	0.698	2.189
Q_s (units)	76.59	192.97
$g_i(T_s) \ i = 1,2$ (in \$)	1763.38	846.67

The concavity of the gain functions are shown in Fig. 3 and 4. (Appendix A)

5. Sensitivity analysis

Now, for data of Example 1, we study the effects of various inventory parameters on total cost savings $g_1(T_s)$, decision variables order quantity Q_s and time period T_s by changing them as -20%, -10%, 10% and 20%.

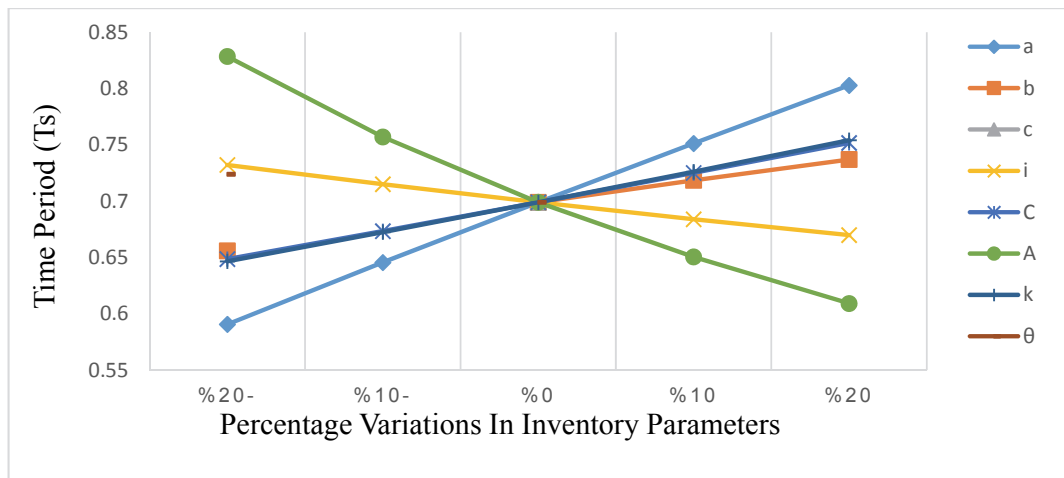


Fig. 5 Variations in time period T_s w. r. t. inventory parameters

From Fig. 5, it is noticed that unit purchase price (C) and unit price increase (k) has huge positive impact on time period. Whereas, scaled demand (a) and linear rate of change of demand (b) increases time period slowly. If holding cost (i) increases then clearly time period decreases. Ordering cost (A) has large negative effect on time period.

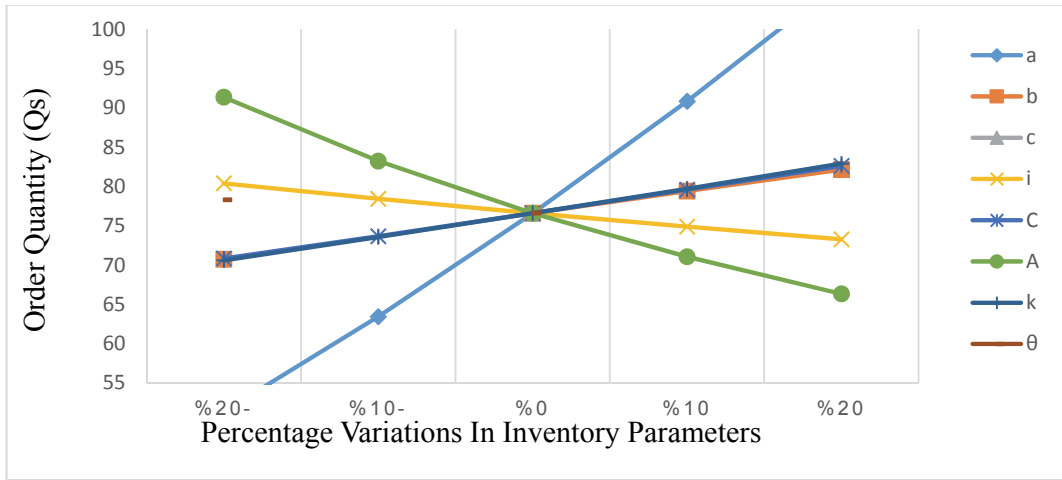


Fig. 6. Variations in order quantity Q_s w. r. t. inventory parameters

From Fig. 6, it is observed that scaled demand (a) has huge positive impact on order quantity. Whereas, unit purchase price (C), unit price increase (k) and linear rate of change of demand (b) increases order quantity slowly. On the other hand, if holding cost (i) increases then clearly order quantity decreases. Ordering cost (A) has big negative effect on order quantity.

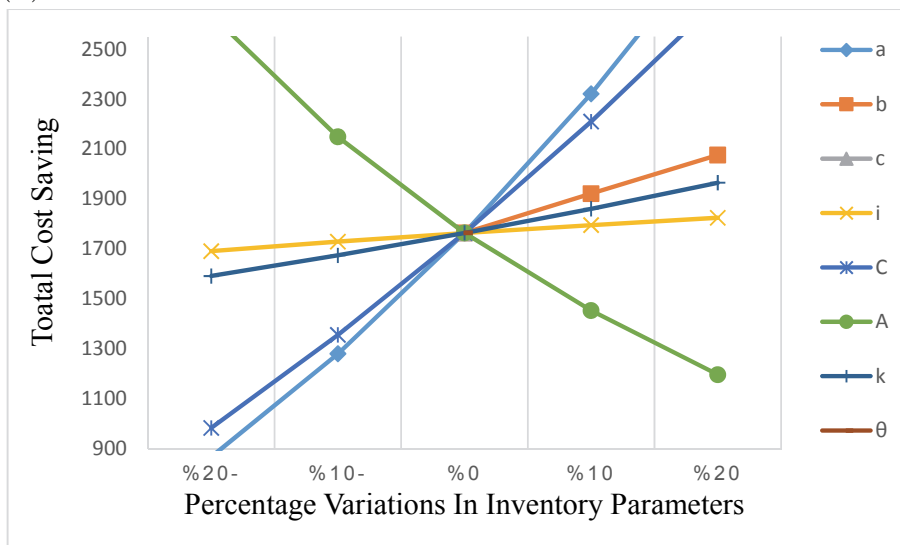


Fig. 7. Variations in total cost saving $g_1(T_s)$ w. r. t. inventory parameters

From Fig. 7, it is observed that scaled demand (a) and unit purchase price (C) has huge positive impact on total cost saving. Holding cost (i), linear rate of change of demand (b) and unit price increase (k) increases total cost saving gradually. Whereas, ordering cost (A) has large negative effect on total cost

saving. Now, for Example 2, we study the effects of various inventory parameters on total cost savings $g_2(T_s)$, decision variables order quantity Q_s and time period T_s by changing them as -20%, -10%, 10% and 20%.

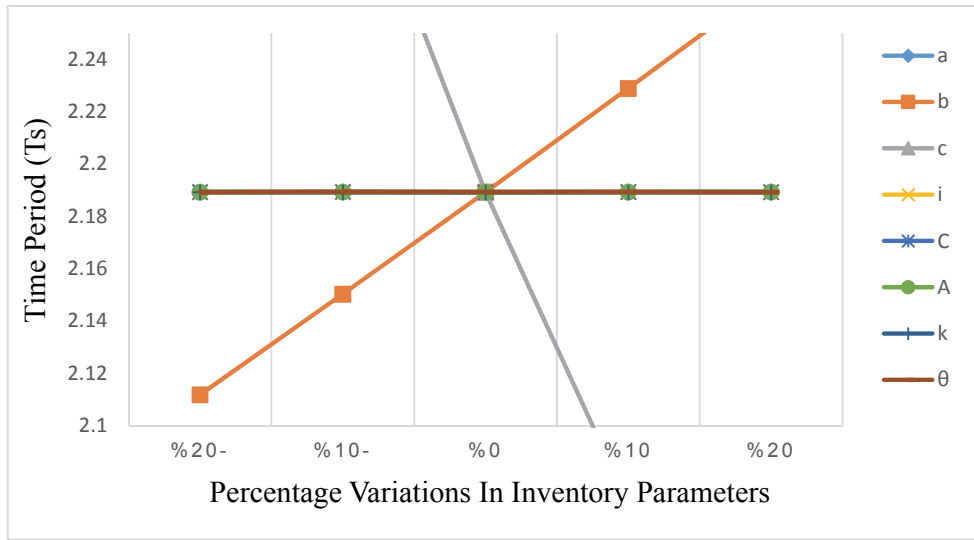


Fig. 8. Variations in time period T_s w. r. t. inventory parameters

From Fig. 8, it is observed that unit price increase (k) and linear rate of change of demand (b) has great positive impact on time period, whereas, quadratic rate of change of demand(c) has huge negative effect on time period.

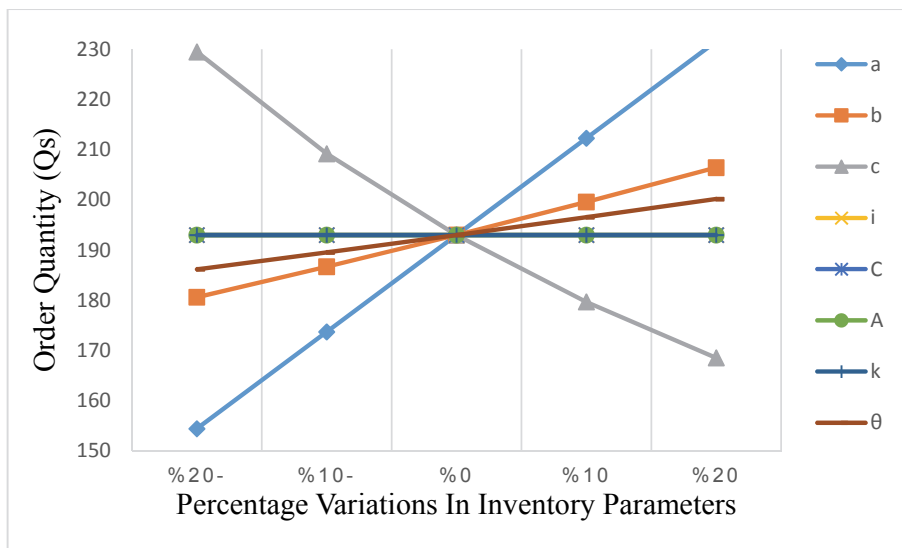


Fig. 9. Variations in order quantity Q_s w. r. t. inventory parameters

From Fig. 9, it is observed that scaled demand (a) has huge positive impact on order quantity. Linear rate of change of demand (b) and deterioration rate (θ) increases order quantity slowly. However, unit

price increase (k) decreases order quantity gently. Quadratic rate of change of demand(c) has enormous negative effect on order quantity.

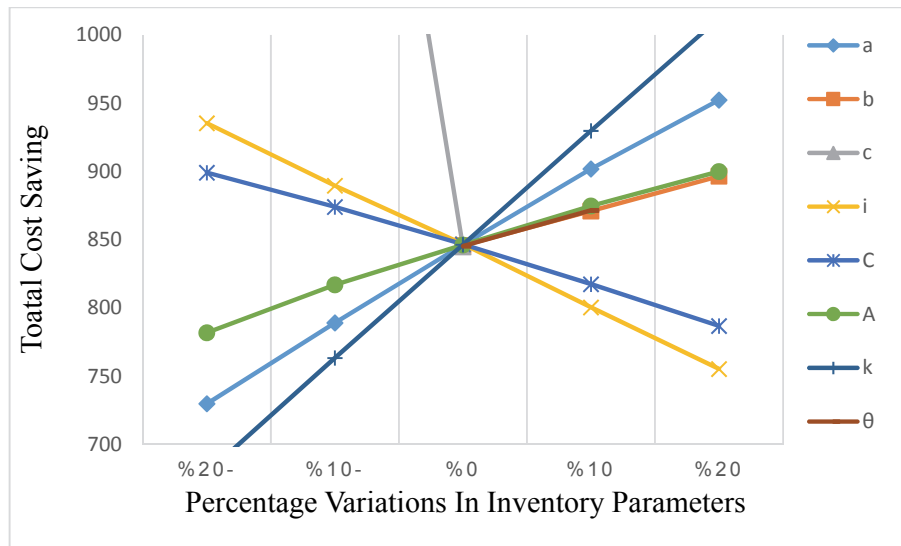


Fig. 10. Variations in total cost saving $g_2(T_s)$ w. r. t. inventory parameters

From Fig. 10, it is observed that scaled demand (a) unit price increase (k) has huge positive impact on total cost saving. Ordering cost(A) increases total cost saving slowly. On the other unit purchase price (C) decreases total cost saving gradually. Holding cost(i) has great negative effect on total cost saving.

5. Conclusion

The article analyzes the effect of a price increase announcement by a supplier on retailers' decision policies when demand is decreasing quadratically and items in retailers' inventory deteriorate at a constant rate. The total cost savings between special and regular ordering is maximized. Results are validated with numerical examples. It is advocated that placing an order at a lower price benefits the retailer. Also, increase in deterioration rate and holding cost decreases total cost savings. When retailers' regular replenishment time coincides with that of the special order time, savings can be increased by having higher residual inventory.

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Appendix A: Concavity of gain functions

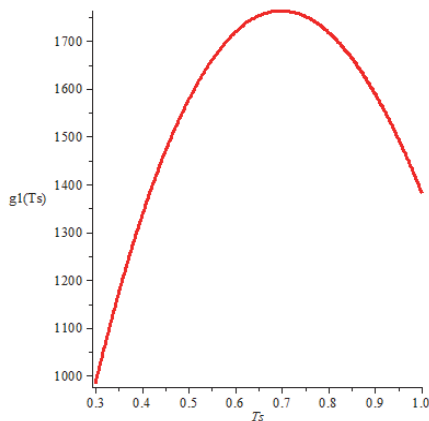


Fig. 3. Concavity of total cost saving $g_1(T_s)$

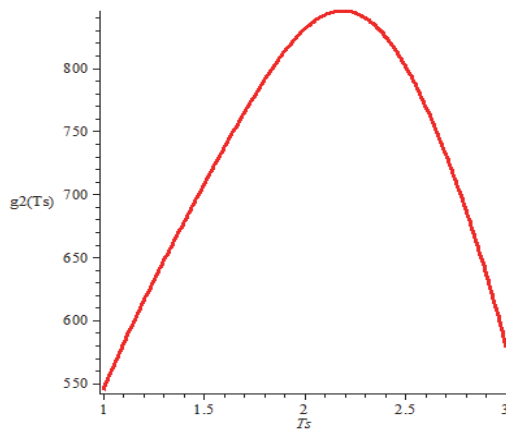


Fig. 4. Concavity of total cost saving $g_2(T_s)$