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# Integrated batch production and maintenance scheduling for multiple items processed on a deteriorating machine to minimize total production and maintenance costs with due date constraint

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A B S T R A C T

This paper discusses an integrated model of batch production and maintenance scheduling on a deteriorating machine producing multiple items to be delivered at a common due date. The model describes the trade-off between total inventory cost and maintenance cost as the increase of production run length. The production run length is a time bucket between two consecutive preventive maintenance activities. The objective function of the model is to minimize total cost consisting of in process and completed part inventory costs, setup cost, preventive and corrective maintenance costs and rework cost. The problem is to determine the optimal production run length and to schedule the batches obtained from determining the production run length in order to minimize total cost.



## 1. Introduction

Delay in delivering order in a manufacturing company to consumer can be caused by several factors, such as no coordination between production and maintenance divisions in the manufacturing system, among others. It may cause in the following conditions. First, the operation of a busy machine has to be stopped as the scheduled maintenance activity should be started. Second, the machine could break down if the scheduled maintenance activity is not conducted.

A real example is well described by the following case. Company X gets some orders of machinery works in large quantities from its partner industries. It processes the orders in batches with constant sizes determined by the production division. Meanwhile, the maintenance division carries out any machine repair only when a failure of the machine occurs (reactive maintenance). Late delivery orders to

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consumers cannot be avoided if machine repair time takes long time because of machine breakdown or the busy machine should be stopped for maintenance. This condition frequently occurs.

It could be drawn some roots of problems, namely, firstly, the maintenance department has not yet applied a preventive maintenance system, although the data of machine failure times, intervals between failures, and the cost of each failure have been well recorded. Secondly, the production department schedules batches with constant sizes, whereas according to Dobson et al. (1987, 1989), Halim and Ohta (1993, 1994) and Yusriski et al. (2015), inconstant batch sizes bring to a better flow time. Thirdly, the data shows that machine failures occur in the busy time of production, so that the failures decrease the productivity of shop floor.

Several literatures on maintenance, such as Barlow and Prochan (1965), Sherwin and Bossche (1993), Ebeling (1997), Rigdon and Basu (2000), and Jiang and Murthy (2008) discuss the theories on reliability, maintainability, and cost optimization for maintenance scheduling activities. It could be noted that the literatures do not take production scheduling into account in their discussions.

Some research that integrate batch production scheduling and maintenance scheduling are Lee and Rosenblatt (1987), Wang and Sheu (2001), Tseng (1996), Ben-Daya and Noman (2006), Lin and Hou (2005), Chelbi et al. (2008), Elferik and Ben-Daya (2010), Fitouhi and Nourelfath (2012) and Suliman and Jawad (2012). The researchers discuss models considering a deteriorating machine, single item, discrete product types, and decision on the number and size of the batches and also an optimal maintenance time as affected by trade off of setup cost, quality cost, restoration cost, inventory holding cost, and reward cost. The researchers did not consider due date in their discussion.

This research deals with integrating model of batch scheduling and maintenance scheduling on a single deteriorating machine that produces a number of parts of multiple items to be delivered on a common due date. The model decision variables are the number and schedule of preventive maintenance, the length of interval between two successive preventive maintenances (production run), number and schedule of batches in each production run. The model objective is to minimize total cost consisting of the holding cost of the work in process, the holding cost of finished parts, setup cost, preventive maintenance cost, corrective maintenance cost, and rework cost.

## 2. Inventory holding cost formulation for in-process batch and completed batch for multiple items processed on single machine

The inventory holding cost concept in this model is developed from Halim and Ohta (1994) having developed in process and completed part inventory holding costs for just in time (JIT) environment. The model objective has considered due date and it has accommodated the condition that all parts do not need to arrive at the shop at time zero simultaneously but at the times when the production process is stated.

Suppose an order with $p$ types of items. Let $q_1, q_2, ..., q_p$ as quantity of each type of the items. All parts will be processed on a single machine. Each part requires only a process to complete the operation with processing time $t_1, t_2, ..., t_p$ respectively. All parts will be delivered on a common due date $d$. The parts are processed in batches. A machine that used in the processing parts is a deteriorating machine with increasing failure rate Weibull. Let $L_{[i_{kj}]}$ stand for a batch scheduled in $i^{th}$ position in the $k^{th}$ production run with part type-$j$, $Q_{[i_{kj}]}$ for quantity of batch $L_{[i_{kj}]}$, $s$ for setup time required before any batch to be processed, $c_{1j}$ for unit inventory holding cost of finished part type-$j$ per unit part per time unit and $c_{2j}$ for unit inventory holding cost of the work in process part type-$j$ per unit part per time unit, where $j$ = 1, 2, …, $p$.

### *2.1 Inventory Holding Cost in a Production Run*

Zahedi et al. (2014) discussed integrated batch production and maintenance scheduling for single item processed on a single machine. For a multiple items case, parts with different types could be processed in the same production run. Accordingly, the inventory holding cost formulation will follows the formulation for single item processed on single machine with $g$ production runs with no preventive maintenance (PM) interval and the number of production runs becomes a number of the type of items with set $g = p$ (See Fig. 1).

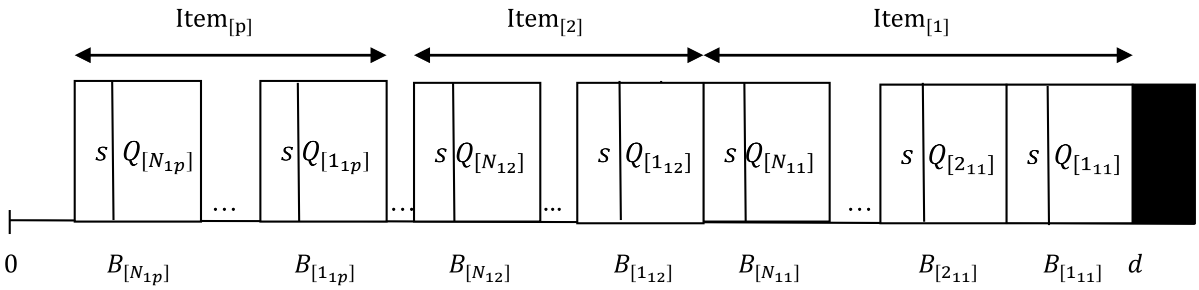


**Fig. 1.** Batches position in multiple items single machine with single production run in total actual flow time criteria

Assuming that each production run is considered as an item with no PM intervals and the number of runs becomes a number of the type of items, then the total inventory holding cost for single production run for multiple items single machine can be formulated as follows:

$$\begin{aligned} ToIC_{[1,p]} &= c_{11} \textstyle\sum_{i_{11}=1}^{N_{11}-1} \left\{ \sum_{l_{11}=1}^{i_{11}} \left( t_1 Q_{[l_{11}]} + s \right) \right\} Q_{[(i+1)_{11}]} \\ &+ \frac{c_{11}+c_{21}}{2} t_1 \textstyle\sum_{i_{11}=1}^{N_{11}} Q_{[i_{11}]}^2 + \frac{c_{21}-c_{11}}{2} t_1 \sum_{i_{11}=1}^{N_{11}} Q_{[i_{11}]} \\ &+ \textstyle\sum_{j=2}^{p} [\, c_{1j} \sum_{i_{1j}=1}^{N_{1j}-1} \left\{ \sum_{l_{1j}=1}^{i_{1j}} \left( t_j Q_{[l_{1j}]} + s \right) \right\} Q_{[(i+1)_{1j}]} \\ &+ \frac{c_{1j}+c_{2j}}{2} t_j \textstyle\sum_{i_{1j}=1}^{N_{1j}} Q_{[i_{1j}]}^2 + \frac{c_{2j}-c_{1j}}{2} t_j \sum_{i_{1j}=1}^{N_{1j}} Q_{[i_{1j}]} \\ &+ c_{1j} \textstyle\sum_{i_{1j}=1}^{N_{1j}} Q_{[i_{1j}]} \left( \sum_{l_{1j}=1}^{i_{1j}-1} (t_j Q_{[l_{1j}]} + s) + \sum_{m_{1(j-1)}=1}^{N_{1(j-1)}} (t_{(j-1)} Q_{[m_{1(j-1)}]} + s) \right) ] \end{aligned} \tag{1}$$

*2.2 Inventory Holding Cost in Two Production Runs*

Let $q$ parts be divided into $N_{1j}$ batches in first production run and $N_{2j}$ batches in the second production run, where the sizes of each batch are $Q_{[ikj]}$ ($i$ = 1, 2, ..., $N_k$, $k$ = 1, 2 and $j$ = 1, 2, …, $p$). If the planning horizon consists of two preventive maintenance intervals, the condition can be shown in Fig. 2.

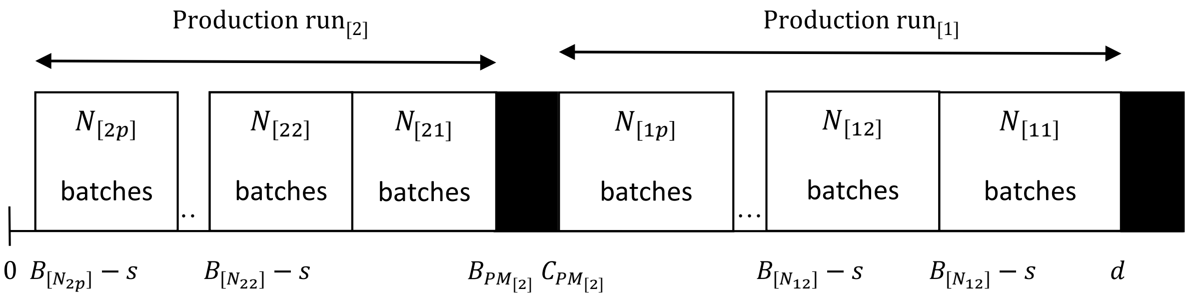


**Fig. 2.** Batches position in multiple items single machine with two production runs in total actual flow time criteria

In the same way with single production run, the total inventory holding cost for the first production run $ToIC_{[1]}$ is the same as $ToIC_{[1,p]}$ (Eq. (1)) and the total inventory holding cost for the second production run can be formulated as follows:

$$\begin{aligned} ToIC_{[2]} &= c_{11} \textstyle\sum_{i_{21}=1}^{N_{21}-1} \left\{ \sum_{l_{21}=1}^{i_{21}} \left( t_1 Q_{[l_{21}]} + s \right) \right\} Q_{[(i+1)_{21}]} \\ &+ \frac{c_{11}+c_{21}}{2} t_1 \textstyle\sum_{i_{21}=1}^{N_{21}} Q_{[i_{21}]}^2 + \frac{c_{21}-c_{11}}{2} t_1 \sum_{i_{21}=1}^{N_{21}} Q_{[i_{21}]} \end{aligned}$$

$$+\sum_{j=2}^{p}[\, c_{1j}\sum_{i_{2j}=1}^{N_{2j}-1}\left\{\sum_{l_{2j}=1}^{i_{2j}}\left(t_jQ_{[l_{2j}]}+s\right)\right\}Q_{[(i+1)_{2j}]}$$
$$+\frac{c_{1j}+c_{2j}}{2}t_j\sum_{i_{2j}=1}^{N_{2j}}Q_{[i_{2j}]}^2+\frac{c_{2j}-c_{1j}}{2}t_j\sum_{i_{2j}=1}^{N_{2j}}Q_{[i_{2j}]}$$
$$+c_{1j}\sum_{i_{2j}=1}^{N_{2j}}Q_{[i_{2j}]}\,[\sum_{l_{2j}=1}^{i_{2j}-1}(t_jQ_{[l_{2j}]}+s)+\sum_{m_{2(j-1)}=1}^{N_{2(j-1)}}(t_{(j-1)}Q_{[m_{2(j-1)}]}+s)$$
$$+t_{PM}+\sum_{i_{1j}=1}^{N_{1j}}\left\{\left(t_jQ_{[i_{1j}]}+s\right)\right\}] \tag{2}$$

Then $ToIC_{[2,p]}=ToIC_{[1]}+ToIC_{[2]}$ is

$$ToIC_{[2,p]}=\sum_{k=1}^{2}[c_{11}\sum_{i_{k1}=1}^{N_{k1}-1}\left\{\sum_{l_{k1}=1}^{i_{k1}}\left(t_1Q_{[l_{k1}]}+s\right)\right\}Q_{[(i+1)_{k1}]}$$
$$+\frac{c_{11}+c_{21}}{2}t_1\sum_{i_{k1}=1}^{N_{k1}}Q_{[i_{k1}]}^2+\frac{c_{21}-c_{11}}{2}t_1\sum_{i_{k1}=1}^{N_{k1}}Q_{[i_{k1}]}$$
$$+\sum_{j=2}^{p}[\, c_{1j}\sum_{i_{kj}=1}^{N_{kj}-1}\left\{\sum_{l_{kj}=1}^{i_{kj}}\left(t_jQ_{[l_{kj}]}+s\right)\right\}Q_{[(i+1)_{kj}]}$$
$$+\frac{c_{1j}+c_{2j}}{2}t_j\sum_{i_{kj}=1}^{N_{kj}}Q_{[i_{kj}]}^2+\frac{c_{2j}-c_{1j}}{2}t_j\sum_{i_{kj}=1}^{N_{kj}}Q_{[i_{kj}]}$$
$$+c_{1j}\sum_{i_{kj}=1}^{N_{kj}}Q_{[i_{kj}]}\,[\sum_{l_{kj}=1}^{i_{kj}-1}(t_jQ_{[l_{kj}]}+s)+\sum_{m_{k(j-1)}=1}^{N_{k(j-1)}}(t_{(j-1)}Q_{[m_{k(j-1)}]}+s)$$
$$+(k-1)t_{PM}+\sum_{n=1}^{k-1}\sum_{i_{nj}=1}^{N_{nj}}\left\{\left(t_jQ_{[i_{nj}]}+s\right)\right\}]] \tag{3}$$

### *2.3 Inventory Holding Cost in Three Production Runs*

If the planning horizon consists of three preventive maintenance intervals and three production runs, the condition can be shown in Fig. 3. Let $q$ parts be divided into $N_{1j}$ batches in the first production run, $N_{2j}$ batches in the second production run, and $N_{3j}$ batches in the third production run, where the sizes of each batch are $Q_{[ikj]}$ ($i$ = 1, 2, ..., $N_k$, $k$ = 1, 2, 3 and $j$ = 1, 2, …, $p$).

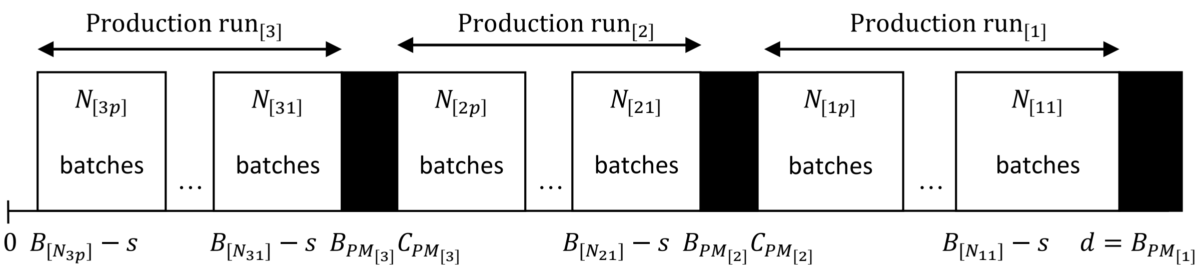


**Fig. 3.** Batches position in multiple items single machine with three production runs in total actual flow time criteria

In the same way with two production runs, the total inventory holding cost for the first production run $ToIC_{[1]}$ is the same as $ToIC_{[1,p]}$ (Eq. (1)) and the total inventory holding cost for the second production run is the same as Eq. (2). The total inventory holding cost for the third production run can be formulated as follows:

$$ToIC_{[3]}=c_{11}\sum_{i_{31}=1}^{N_{31}-1}\left\{\sum_{l_{31}=1}^{i_{31}}\left(t_1Q_{[l_{31}]}+s\right)\right\}Q_{[(i+1)_{31}]}$$
$$+\frac{c_{11}+c_{21}}{2}t_1\sum_{i_{31}=1}^{N_{31}}Q_{[i_{31}]}^2+\frac{c_{21}-c_{11}}{2}t_1\sum_{i_{31}=1}^{N_{31}}Q_{[i_{31}]}$$
$$+\sum_{i_{3j}=1}^{N_{3j}-1}\left\{\sum_{l_{3j}=1}^{i_{3j}}\left(t_jQ_{[l_{3j}]}+s\right)\right\}Q_{[(i+1)_{3j}]}$$
$$+\frac{c_{1j}+c_{2j}}{2}t_j\sum_{i_{3j}=1}^{N_{3j}}Q_{[i_{3j}]}^2+\frac{c_{2j}-c_{1j}}{2}t_j\sum_{i_{3j}=1}^{N_{3j}}Q_{[i_{3j}]}$$

$$+c_{1j}\sum_{i_{3j}=1}^{N_{3j}} Q_{[i_{3j}]}\,[\sum_{l_{3j}=1}^{i_{3j}-1}(t_j Q_{[l_{3j}]}+s)+\sum_{m_{3(j-1)}=1}^{N_{3(j-1)}}(t_{(j-1)}Q_{[m_{3(j-1)}]}+s)$$

$$+2\,t_{PM}+\sum_{i_{1j}=1}^{N_{1j}}\left\{\left(t_jQ_{[i_{1j}]}+s\right)\right\}+\sum_{i_{2j}=1}^{N_{2j}}\left\{\left(t_jQ_{[i_{2j}]}+s\right)\right\}] \tag{4}$$

Then $ToIC_{[3,p]} = ToIC_{[1]} + ToIC_{[2]} + ToIC_{[3]}$ is

$$ToIC_{[3,p]} = \sum_{k=1}^{3}[c_{11}\sum_{i_{k1}=1}^{N_{k1}-1}\left\{\sum_{l_{k1}=1}^{i_{k1}}\left(t_1Q_{[l_{k1}]}+s\right)\right\}Q_{[(i+1)_{k1}]}$$
$$+\frac{c_{11}+c_{21}}{2}t_1\sum_{i_{k1}=1}^{N_{k1}}Q_{[i_{k1}]}^2+\frac{c_{21}-c_{11}}{2}t_1\sum_{i_{k1}=1}^{N_{k1}}Q_{[i_{k1}]}$$
$$+\sum_{j=2}^{p}[\,c_{1j}\sum_{i_{kj}=1}^{N_{kj}-1}\left\{\sum_{l_{kj}=1}^{i_{kj}}\left(t_jQ_{[l_{kj}]}+s\right)\right\}Q_{[(i+1)_{kj}]}$$
$$+\frac{c_{1j}+c_{2j}}{2}t_j\sum_{i_{kj}=1}^{N_{kj}}Q_{[i_{kj}]}^2+\frac{c_{2j}-c_{1j}}{2}t_j\sum_{i_{kj}=1}^{N_{kj}}Q_{[i_{kj}]}$$
$$+\,c_{1j}\sum_{i_{kj}=1}^{N_{kj}}Q_{[i_{kj}]}\,[\sum_{l_{kj}=1}^{i_{kj}-1}(t_jQ_{[l_{kj}]}+s)+\sum_{m_{k(j-1)}=1}^{N_{k(j-1)}}(t_{(j-1)}Q_{[m_{k(j-1)}]}+s)$$
$$+(k-1)t_{PM}+\sum_{n=1}^{k-1}\sum_{i_{nj}=1}^{N_{nj}}\left\{\left(t_jQ_{[i_{nj}]}+s\right)\right\}]] \tag{5}$$

By considering any changes taking place in each production run then the total holding cost for multiple items processed on single machine for $g$ production runs and $g$ PM intervals (see Fig. 4) will become Eq. (6).

$$ToIC_{[g,p]} = \sum_{k=1}^{g}[c_{11}\sum_{i_{k1}=1}^{N_{k1}-1}\left\{\sum_{l_{k1}=1}^{i_{k1}}\left(t_1Q_{[l_{k1}]}+s\right)\right\}Q_{[(i+1)_{k1}]}$$
$$+\frac{c_{11}+c_{21}}{2}t_1\sum_{i_{k1}=1}^{N_{k1}}Q_{[i_{k1}]}^2+\frac{c_{21}-c_{11}}{2}t_1\sum_{i_{k1}=1}^{N_{k1}}Q_{[i_{k1}]}$$
$$+\sum_{j=2}^{p}[\,c_{1j}\sum_{i_{kj}=1}^{N_{kj}-1}\left\{\sum_{l_{kj}=1}^{i_{kj}}\left(t_jQ_{[l_{kj}]}+s\right)\right\}Q_{[(i+1)_{kj}]}$$
$$+\frac{c_{1j}+c_{2j}}{2}t_j\sum_{i_{kj}=1}^{N_{kj}}Q_{[i_{kj}]}^2+\frac{c_{2j}-c_{1j}}{2}t_j\sum_{i_{kj}=1}^{N_{kj}}Q_{[i_{kj}]}$$
$$+\,c_{1j}\sum_{i_{kj}=1}^{N_{kj}}Q_{[i_{kj}]}\,[\sum_{l_{kj}=1}^{i_{kj}-1}(t_jQ_{[l_{kj}]}+s)+\sum_{m_{k(j-1)}=1}^{N_{k(j-1)}}(t_{(j-1)}Q_{[m_{k(j-1)}]}+s)$$
$$+\,(k-1)t_{PM}+\sum_{n=1}^{k-1}\sum_{i_{nj}=1}^{N_{nj}}\left\{\left(t_jQ_{[i_{nj}]}+s\right)\right\}]]. \tag{6}$$

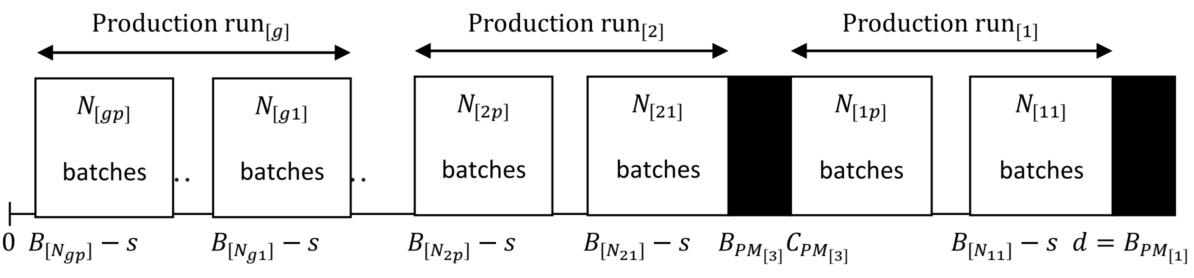


**Fig. 4.** Batches position in multiple items single machine with $g$ production runs in total actual flow time criteria

## 3. ROCOF Function (Jiang & Murthy, 2008)

ROCOF (*rate of occurrence of failures*) characterizes the probability that a failure occurs in the interval $[t,t+\delta t]$. The ROCOF is given by an intensity function

$$\lambda(t) = \lim_{\delta t\to 0}\frac{P\{N(t+\delta t)-N(t)\geq 1\}}{\delta t} \tag{7}$$

where $N(t)$ is the number of failures in the interval [0,t). In an assumption that the probability of two or more failures in the interval [t,t+$\delta t$] is zero as $\delta t \rightarrow 0$, then the intensity function is equal to the derivative of the conditional expected number of failures, so that

$$\lambda(t) = \frac{d}{dt} E\{N(t)\} \tag{8}$$

If the failures are minimally repaired and the time to repair is negligible, then ROCOF function $\lambda(t) = r(t)$, where $r(t)$ is the failure rate function. The cumulative ROCOF function is given by

$$\Lambda(t) = \int_0^t \lambda(t)dt \tag{9}$$

A ROCOF function that has been used extensively is the Weibull ROCOF. The cumulative ROCOF (or the expected total number of failures) is given by the function

$$\Lambda(t) = \left(\frac{t}{\alpha}\right)^{\beta}, \tag{10}$$

with scale parameter $\alpha$ and shape parameter $\beta$.

Let a system (machine) with a Weibull failure time distribution has a shape parameter of $\beta$ = 1.69 and a scale parameter $\alpha$ = 2,857.14, then by ROCOF cumulative function, it can be estimated the first, the second, and so on for failure times. The failure times can be written as follows:

If $\Lambda(t) = \left(\frac{t}{\alpha}\right)^{\beta} = 1$ then $t$ = 2,857.14.

If $\Lambda(t) = \left(\frac{t}{\alpha}\right)^{\beta} = 2$ then $t$ = 4,305.82.

If $\Lambda(t) = \left(\frac{t}{\alpha}\right)^{\beta} = 3$ then $t$ = 5,473.33.

If $\Lambda(t) = \left(\frac{t}{\alpha}\right)^{\beta} = 4$ then $t$ = 6,489.03.

From the calculation above, it can be estimated the time interval between machine failure times, where the time between failures of the machine is decreasing over time. It indicates that the machine has increasing failure rate characteristic or the machine is a deteriorating machine.

## 4. Estimation of non-conforming parts

This research proposes a policy that PM is carried out before an expected first failure time based on cumulative ROCOF function. Based on the policy assumed that the length of the second, the third and so on until $g^{th}$ production run less than or equal the first failure time. In model formulation, length of the first production run (from due date) less than a common due date $d$ to accommodate that the model will let the machine produce non-conforming parts if the cost of rework for non-conforming parts less than the cost of PM and to accommodate if the problem consists of only one production run. Probability of defect parts on in control state $p_1$ and probability of defect part on out of control state $p_2$, where $p_2 > p_1$.

An example of a condition for a case of two production runs and two PMs is shown in Fig. 5. In the second production run there is no non-conforming part, because the out-of-control state takes place only in the first production run, so that the number of non-conforming parts for $k$ = 2 can be formulated as follows:

$$\begin{aligned} M_2 &= p_1 x \text{ number of parts processed in interval } [B_{[N_{2p}]} - s\ C_{[1_{21}]}\,] \\ &\quad + p_1 x \text{ number of parts processed in interval } [B_{[N_{1p}]} - s, B_{[N_{1p}]} - s + \alpha] \\ &\quad + p_2 x \text{ number of parts processed in interval } [B_{[N_{1p}]} - s + \alpha,\ C_{[1_{11}]}\,]. \end{aligned} \tag{11}$$

In the same way with two production run, the total non-conforming parts for $g$ production runs can be formulated as follows:

$$M_g = \sum_{k=2}^{g} p_1 x \text{ number of parts processed in interval } [B_{[N_{kp}]} - s, C_{[1_{k1}]}] + p_1 x \text{ number of parts processed in interval } [B_{[N_{1p}]} - s, B_{[N_{1p}]} - s + \alpha] + p_2 x \text{ number of parts processed in interva } [B_{[N_{1p}]} - s + \alpha, C_{[1_{11}]}]. \quad (12)$$

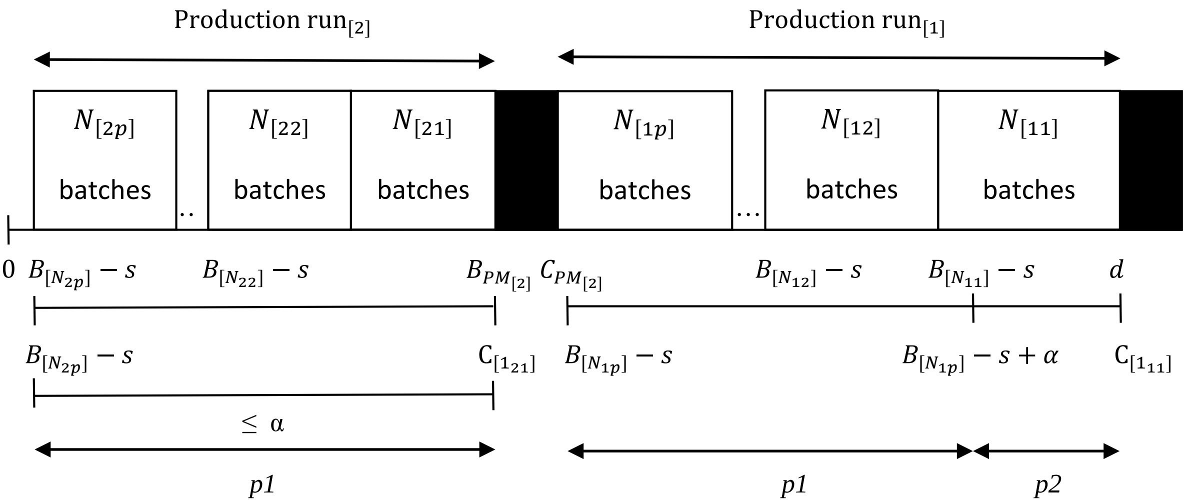


**Fig. 5.** A condition for two production runs and two PM's with expected machine failure time based on cumulative ROCOF

## 5. Model Formulation and Algorithm

### *5.1 Model Formulation*

In order to formulate the integrated batch production and maintenance scheduling model, we use following notations.

Parameters

$q$ : total number of parts scheduled
$q_j$ : number of part type-$j$, where $j=1,2,\ldots, p$
$t_j$ : unit processing time of part type-$j$
$s$ : unit setup time
$c_{1j}$ : unit inventory holding cost of finished part type-$j$ per unit part per time unit
$c_{2j}$ : unit inventory holding cost of the work in process part type-$j$ per unit part per time unit
$c_{PM}$ : unit preventive maintenance (PM) cost
$r$ : unit corrective maintenance (CM) cost
$p_{1j}$ : probability of defect parts on in control state of part type-$j$
$p_{2j}$ : probability of defect parts on out of control state of part type-$j$
$d$ : a common due date

Decision Variables

$L_{[i_{kj}]}$ : batch scheduled in $i^{th}$ position in the $k^{th}$ production run with part type-$j$, where $i_{kj} = 1, 2, \ldots, N_{kj}$, $k = 1, 2, \ldots, g$, $j=1,2,\ldots, p$
$Q_{[i_{kj}]}$ : number of part in batch $L_{[i_{kj}]}$
$N_{kj}$ : number of batch on $k^{th}$ production run with part type-$j$
$B_{[i_{kj}]}$ : beginning time of batch $L_{[i_{kj}]}$
$C_{[i_{kj}]}$ : completion time of batch $L_{[i_{kj}]}$
$M_k$ : number of defect part if planning horizon consists of $k$ production runs
$M$ : total number of defect part in planning horizon

$$X_{[i_{kj}]} = \begin{cases} 1, \text{if } Q_{[i_{kj}]} \neq 0, \\ 0, \text{if } Q_{[i_k]} = 0. \end{cases}, \; i_{kj} = 1, 2, \ldots, N_{kj}, k = 1, 2, \ldots, g, j=1,2,\ldots, p$$

The Objective Function

$TC$ : the total cost consisting of inventory cost in process and complete inventory costs, setup cost, preventive and corrective maintenances cost and rework cost.

We adopt some assumptions in formulating these models, as follow:

1. This integrating model for multiple items single stage process,
2. Setup time is not depending on number of parts in batches or kind of items,
3. Batch position number and preventive maintenance number are counted from due date direction (backward approach),
4. The same load force for machine in setup time and in processing time,
5. The machine cannot interrupted as long as production run,
6. Batch size value is in real positive.

Using those defined notations and based on those assumptions, the integrated batch production and maintenance scheduling to minimize production and maintenance costs on a deteriorating machine in just in time environment for multiple items single machine (Model [MISM]) can be expressed as a mix-integer-non-linear programming as follows:

**Model [MISM]**

$$\text{Minimize } TC = ToIC_{[g,p]} + g\, c_{PM} + c_s \sum_{k=1}^{g} \sum_{j=1}^{p} N_{kj} + E(R) + E(W) \tag{13}$$

subject to:

$$\sum_{k=1}^{g} \sum_{j=1}^{p} \sum_{i_{kj}=1}^{N_{kj}} Q_{[i_{kj}]} = q \tag{14}$$

$$\sum_{k=1}^{g} \sum_{i_{kj}=1}^{N_{kj}} Q_{[i_{kj}]} = q_j, \text{for } j = 1, 2, \ldots, p \tag{15}$$

$$B_{[i_{1j}]} + \sum_{l_{1j}=1}^{i_{1j}} (t_j Q_{[l_{1j}]} + sX_{[l_{1j}]}) + \sum_{m_{1(j-1)}=1}^{N_{1(j-1)}} (t_{(j-1)} Q_{[m_{1(j-1)}]} + sX_{[m_{1(j-1)}]}) = d, \text{for } j = 1, 2, \ldots, p, k = 1 \tag{16}$$

$$B_{[i_{kj}]} + \sum_{l_{kj}=1}^{i_{kj}} (t_j Q_{[l_{kj}]} + sX_{[l_{kj}]}) + \sum_{m_{k(j-1)}=1}^{N_{k(j-1)}} (t_{(j-1)} Q_{[m_{k(j-1)}]} + sX_{[m_{k(j-1)}]}) + (k-1)t_{PM} + \sum_{n=1}^{k-1} \sum_{i_{nj}=1}^{N_{nj}} \left\{ \left( t_j Q_{[i_{nj}]} + sX_{[i_{nj}]} \right) \right\}, \text{for } j = 1, 2, \ldots, p, k = 2, 3, \ldots, g \tag{17}$$

$$\sum_{j=1}^{p} \sum_{i_{kj}=1}^{N_{kj}} Q_{[i_{kj}]} \leq d, \text{for } k = 1 \tag{18}$$

$$\sum_{j=1}^{p} \sum_{i_k=1}^{N_{kj}} Q_{[i_{kj}]} \leq \alpha, \text{for } k = 2, 3, \ldots, g \tag{19}$$

$$B_{PM[1]} = d,$$

$$C_{PM[1]} = d + t_{PM},$$

$$B_{PM[k]} = B_{[1_{k1}]} + t_1 Q_{[1_{k1}]}, \text{for } k = 2, 3, \ldots, g,$$

$$C_{PM[k]} = B_{[1_{k1}]} + t_{PM}, \text{for } k = 2, 3, \ldots, g \tag{20}$$

$$Ns + \left\lceil \frac{d}{\alpha} \right\rceil t_{PM} + \sum_{j=1}^{p} t_j q_j \leq d \tag{21}$$

$$g = \left\lceil \frac{d}{\alpha} \right\rceil \quad (22)$$

$$\begin{aligned} M_k = \textstyle\sum_{l=2}^{k} p_1 x \text{ number of parts processed in interval } [B_{[N_{lp}]} - s, C_{[1_{l1}]}] \\ + p_1 x \text{ number of parts processed in interval } [B_{[N_{1p}]} - s, B_{[N_{1p}]} - s + \alpha] \\ + p_2 x \text{ number of parts processed in interval } [B_{[N_{1p}]} - s + \alpha\, , C_{[1_{11}]}] \end{aligned} \quad (23)$$

$$E(M) = M_k, \text{for } k = 1, 2, \dots, g \quad (24)$$

$$E(W) = c_w\, E(M) \quad (25)$$

$$n_{CM} = \left\lfloor \left\{ \frac{d - (B_{[N_{1p}]} - s + \alpha)}{\alpha} \right\}^{\beta} \right\rfloor \quad (26)$$

$$E(R) = c_r\, n_{CM} \quad (27)$$

$$Q_{[i_{kj}]} \geq 0, \text{for } i_{kj} = 1, 2, \dots, N_{kj}, k = 1, 2, \dots, g, j = 1, 2, \dots, p \quad (28)$$

$$Q_{[i_{kj}]} \leq X_{[i_{kj}]}\, q_j, \text{for } i_{kj} = 1, 2, \dots, N_{kj}, k = 1, 2, \dots, g, j = 1, 2, \dots, p \quad (29)$$

$$N_{kj} \geq 0, \text{for } k = 1, 2, \dots, g, j = 1, 2, \dots, p \quad (30)$$

$$X_{[i_{kj}]} = \begin{cases} 1, jika\ Q_{[i_{kj}]} \neq 0, \\ 0, jika\ Q_{[i_{kj}]} = 0 \end{cases}, i_{kj} = 1, 2, \dots, N_{kj}, k = 1, 2, \dots, g, j = 1, 2, \dots, p \quad (31)$$

Eq. (13) declares an objective function of Model [MISM]. Eqs. (14-15) state the material balance in the shop, where the number of parts in all batches must be equal to the total number of parts that will be scheduled and the number of parts in all batches with type-*j* must be equal to total number of type-*j* scheduled. Eqs. (16-17) state the beginning time of each batch on the first run and the next runs respectively. Eqs. (18-19) state the length of the first run and the next runs (backward approach) respectively. Eq. (20) represents a set of constraints for the beginning and the next of the PM times, with assumption that 1st PM in schedule or the last PM in processing (backward approach) after all batches has been completed at a common due date $d$ to ensure the machine in as good as new condition for the next order. Eq. (21) states the sum of possible all setup time, PM time and processing time not exceed the common due date $d$. Eq. (22) states an estimation of upper bound for number of production run. Eq. (23) states the estimation of non-conforming parts for each run. Eqs. (24-25) state the estimation of total non-conforming parts and total rework cost for non-conforming parts respectively. Eqs. (26-27) state the possible number of CM action with cumulative Weibull ROCOF and the expected cost of CM action respectively. Eq. (28) states non-negativity of batch size. Eq. (29) states a batch size of type-*j* less or equal to all parts type-*j* that will be scheduled. Eq. (30) states the existence of the number of batches with type-*j* in each production run and Eq. (31) states a binary constraint that each batch will have $X_{[i_k k]} = 1$ for non-empty batches, and $X_{[i_k k]} = 0$ for empty batches.

### *5.2 Heuristic*

The model was a mixed integer quadratic category that contains some integer variables and binary variables, so that analytic search solution could not be used for the model because the analytic search requires all variables are continuous and differentiable (Winston, 2004). The algorithm starts by solving the Model [MISM] with relaxation of the corrective maintenance cost and the rework cost for non-conforming parts. After having obtained a production schedule, estimate the expected number of non-conforming parts and the expected number of CM. Next, compute estimated rework cost and estimated corrective maintenance cost, and then compute total cost. This step is done for two, three and so on number of batches until an increasing total cost is found. Write the best total cost for one production run and one PM. This process is carried out for two production runs with two PMs until the best total cost is found for two production runs with two PM. Continue the process up to $g$ production runs with $g$ PMs.

The algorithm solution is the minimization of all the best total costs for every possible number of production runs. Then, write all decision variables for the best algorithm solution.

Characteristic of the Model [MISM] will have near similar aspect to the single item single machine (Zahedi et al. (2014)), except it applied to multiple items case. Halim and Ohta (1993) suggested the type of items that are scheduled with non-decreasing ratio (backward sequence) $\frac{t_j q_j + s}{q_j}$ for $j$ = 1, 2, ..., p or type of item with smaller ratio scheduled in advance in backward sequence, because the schedule will minimize the total actual flow time.

**Algorithm [MISM]**

Step-1. Compute $T = \sum_{j=1}^{p} q_j\ t_j$. Go to Step-2.
Step-2. Set the length of the expected first failure time based on cumulative Weibull ROCOF function as $\alpha$. Go to Step-3.
Step-3. A problem is said as feasible if and only if the total processing time with one setup for every item type and minimum possible PM time doesn't exceed the due date $d$, otherwise the problem is not feasible for a model or if $\sum_{j=1}^{p}(s + q_j\ t_j) + \left(\left\lceil \frac{\sum_{j=1}^{p}(s+q_j\ t_j)}{\alpha} \right\rceil - 1\right) t_{PM} \leq d$, then the problem is feasible; Continue to Step-4. Otherwise the problem is not feasible and then **STOP**.
Step-4. Sort all items type with non-decreasing ratio $\frac{t_1 q_1 + s}{q_1} \leq \frac{t_2 q_2 + s}{q_2} \leq \cdots \leq \frac{t_p q_p + s}{q_p}$. Write the new items type as $j$ = 1, 2, ... $p$. Go to Step-5.
Step-5. Compute $g$ with Eq. (22) and compute $N$ with Eq. (21). For simplification of the model, set $N = N_{kj}$, where $k = 1, 2, \ldots, g$, $j = 1, 2, \ldots, p$. Go to Step-6.
Step-6. Estimate the production run position for every type of items by observing the following ratios:

$$\frac{s+t_p q_p}{\alpha} + \frac{s+t_{p-1} q_{p-1}}{\alpha} + \ \ldots\ + \frac{s+t_1 q_1}{\alpha}, \tag{32}$$

go to Step-7.
Step-7. Substitute the values of $g$, $N_{kj}$, $p_{1j}$, $p_{2j}$, $q_j$, $t_j$, $s$, $d$, $t_{PM}$ to the Model [MISM] and set every item type as a batch or set $X_{[ikj]} = 1$ for $i_{kj} = 1$, $k = 1$, $j = 1,2,\ldots,p$ and set $X_{[ikj]} = 0$ otherwise. Go to Step-8.
Step-8. Solve Model [MISM] with relaxing of the constraints of Eqs. (23-27). Compute estimated rework cost by Eq. (25) and estimated restoration cost by Eq. (27), and computes a total cost to find $TC$, write $TC_{[111]} = TC$ . Go to Step-9.
Step-9. Set $k = 1$. Go to Step-10.
Step-10. Set $j = 1$. Go to Step-11.
Step-11. Check whether item type-$j$ is on the $k^{th}$ production run.
- If item type-$j$ is on the $k^{th}$ production run, go to Step -12.
- Otherwise, if item type-$j$ is not on the $k^{th}$ production run, set $k = k + 1$, go back to Step-10.

Step-12. Set $i_{kj} = 2$. Go to Step -13.
Step-13. Set $X_{[ikj]} = 1$ and add $X_{[ikj]} = 1$ to the Model [MISM]. Go to Step-14.
Step-14. Solve Model [MISM] with relaxing of the constraint of Eqs. (23-27). Compute Estimated rework cost by Eq. (25) and estimated restoration cost by Eq. (27), and compute a total cost to find $TC$, write $TC_{[ikj]} = TC$. Go to Step-15.
Step-15. Observe whether $TC_{[ikj]} < TC_{[(i-1)kj]}$.
- if $TC_{[ikj]} < TC_{[(i-1)kj]}$, observe whether $i_{kj} = N_{kj}$,
  - if $i_{kj} = N_{kj}$, go to Step-16.
  - otherwise, set $i_{kj} = i_{kj} + 1$, go back to Step-12.
- otherwise, write $TC_{[k]}{*} = TC_{[(i-1)kj]}$ and write all of $TC{*}$- related decision

variables, go to Step -17.
Step-16. Write $TC_{[k]}* = TC_{[ikj]}$ and write all of $TC_{[k]}*$-related decision variables, go to Step-17.
Step-17. Observe whether $j= p$,
- if $j= p$, go to Step-18,
- otherwise, set $j = j +1$, go back to Step-10.

Step-18. Observe whether $k = g$,
- if $k = g$, go to Step-25,
- otherwise, go to Step-19.

Step-19. Set $k = k +1$, go to Step-20.
Step-20. Set $i_{kj} = 2$, go to Step-21.
Step-21. Set $X_{[ikj]} = 1$ and add $X_{[ikj]} = 1$ to the Model [MISM]. Go to Step-22.
Step-22. Solve Model [MISM] with relaxing of the constraint of Eqs. (23-27). Compute estimated rework cost by Eq. (25) and estimated restoration cost by Eq. (27), and compute a total cost to find $TC$, write $TC_{[i(k+1)j]} = TC$. Go to Step-23.
Step-23. Observe whether $TC_{[i(k+1)j]} < TC_{[k]}*$,
- if $TC_{[i(k+1)j]} < TC_{[k]}*$, go to Step-24.
- otherwise, set $i_{kj} = i_{kj} +1$, go back to Step-12.

Step-24. Observe whether $k = g$,
- if $k = g$, go to Step-25,
- otherwise, set $k = k +1$, go to Step-9.

Step-25. Write $\{TC_{[k]}*, k = 1, 2, \ldots,g\}$ as a set of the best solutions for every possible number of production run.
Step-26. The best solution of Algorithm [MISM] is Minimum $\{TC_{[k]}*, k = 1, 2, \ldots,g\}$. Write all values of decision variables and then **STOP**.

### *5.3 Numerical Experience*

To clarify how the proposed algorithm work, the following an example is given.

**MISM Problem**

Suppose a problem of multiple items single machine integrated batch production scheduling and maintenance scheduling with three types of items has the following parameters:

The total number of parts $q$ = 200 unit parts, number of parts 1st type $q_1$ = 80, number of parts 2nd type $q_2$ = 50, number of parts 3rd type $q_3$ = 70. Setup time between batches $s$ = 10 minutes. The unit processing time for 1st type, 2nd type, 3rd type are $t_1$ = 20, $t_2$ = 10, $t_3$ = 30 in minutes respectively. The length of PM action (in constant assumption) $t_{PM}$ = 60 minutes = 1 / $\mu$. Constant repair rate $\mu$ = 1/60. The shape parameter of Weibull distribution $\beta$ = 1.69 and scale parameter $\alpha$ = 2,857.14 estimated from machine failure times data. A common dute date $d$ = 5.000. The unit inventory holding cost of finished parts for 1st type, 2nd type, 3rd type are $c_{11}$ = 0.20, $c_{12}$ = 0.40, $c_{13}$ = 0.30 in US$ per unit part per minute. The unit inventory holding cost of in process parts for 1st type, 2nd type, 3rd type are $c_{21}$ = 0.10, $c_{22}$ = 0.10, $c_{23}$ = 0.10 in US$ per unit part per minute. The unit cost for a PM action is $c_{PM}$ = US$ 30. The unit cost for a CM (CM minimal repair) is $c_r$ = US$ 120. The unit cost for setup time is $c_s$ = US$ 3. The unit rework cost for 1st type, 2nd type, 3rd type are $c_{w1}$ = 100, $c_{w2}$ = 100, $c_{w3}$ = 100 in US$ per unit part.

The computational steps to solve the problem are the followings.

Step-1 and Step-2 yield $T = \sum_{j=1}^{p} q_j\, t_j$ = 4,200 and $\alpha$ = 2,857.14.

Step-3. Calculate $\sum_{j=1}^{p}(s + q_j\ t_j) + (\left\lceil \frac{T}{\alpha} \right\rceil - 1)t_{PM}$ = 4,350 < $d$ = 5,000, then the problem is feasible for the Model [MISM].

Step-4. Sort 3 item types with non-decreasing ratio as $\frac{t_2q_2+s}{q_2} = 10.20 \leq \frac{t_1q_1+s}{q_1} = 20.13 \leq \frac{t_3q_3+s}{q_3} = 30.14$. Then the new sequence in backward sequencing is 2nd type, 1st type and 3rd type, so $j$ = 1 (2nd type), $j$ = 2 (1st type) and $j$ = 3 (3rd type), and problem data in the model as Table 1.

**Table 1**
Data problem for Model [MISM]

| $j$ | Type of item | $c_{1j}$ | $c_{2j}$ | $t_j$ | $q_j$ |
|---|---|---|---|---|---|
| 1 | 2 | 0.4 | 0.1 | 10 | 50 |
| 2 | 1 | 0.2 | 0.1 | 20 | 80 |
| 3 | 3 | 0.3 | 0.1 | 30 | 70 |

Step-5. Set $g$ = 2 and $N_{kj} = \left\lfloor \frac{d-(\lceil \frac{T}{x} \rceil -1)t_{PM}-T}{s} \right\rfloor = 74$, where $k$ = 1, 2, $j$=1, 2, 3.

Step-6. Calculate the estimation of the production run position of each type of item to pay attention to the following ratios, $\frac{s+t_3q_3}{\alpha} + \frac{s+t_2q_2}{\alpha} + \frac{s+t_1q_1}{\alpha} = 0.85 + 0.65 + 0.21$.

That is the first production run (backward) will be occupied respectively by $j$ = 1, $j$ = 2 and $j$ = 3, whereas the second production run will be occupied only by item j = 3. The ratios will prove the upper bound of number of production run is $g$ = 2 as shown in Fig. 6.

Step-7. Substitute the values of $g$, $N_{kj}$, $p_{1j}$, $p_{2j}$, $q_j$, $t_j$, $s$, $d$, $t_{PM}$ to the Model [MISM] and set every item type as a batch or set $X_{[ikj]}$ = 1for $i_{kj}$ = 1, $k$ = 1, $j$ = 1, 2,…,p and set $X_{[ikj]}$ = 0 otherwise. Go to Step-8.

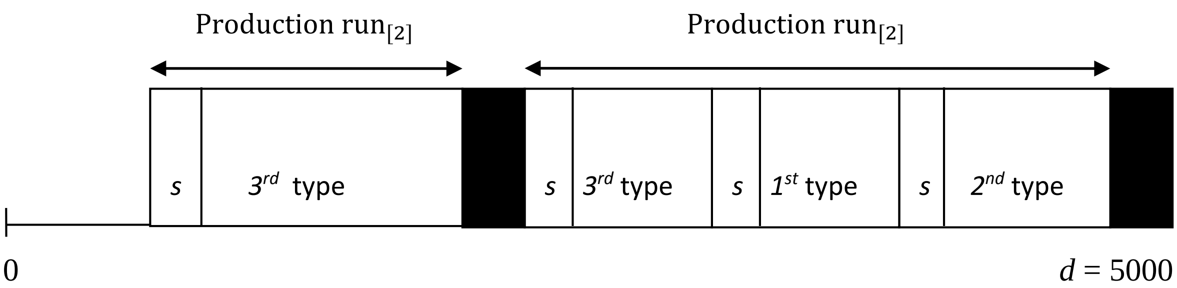


**Fig. 6.** Occupation of the items in each production run

Step-8. Yields $TC_{[111]}$ = 289,368.00. The complete result of Step-8 is shown in Table 2 and Fig. 7.

**Table 2**
Result Step-8 Algorithm [MISM] for the problem

| Batch size $Q_{[1_1j]}$ | | | Number of non- confor ming parts, | Number of C M, | $TC_{[111]}$ |
|---|---|---|---|---|---|
| $Q_{[1_{13}]}$ | $Q_{[1_{12}]}$ | $Q_{[1_{11}]}$ | $M_1$ | $N_{CM}$ | |
| 70 | 50 | 80 | 69.14 | 1 | 289,368.00 |

Step-9 to Step-24, for $k$ = 1 yield the best solution $TC_{[1]}$* = 146,841.30. The decision variables of the best solution for *1st* looping are shown in Table 3.

The *2nd* looping.

Step-9 to Step-24, for $k$ = 2, yield the best solution $TC_{[2]}$* = 142,071.60. The decision variables of the best solution for 2nd looping are shown in Table 4.

Step-25. Yields {$TC_{[k]}$*, $k$ = 1, 2, …,$g$} = {146,841.30, 142,071.60}

Step-26. Yields TC = Min {146,841.30, 142,071.60} = 142,071.60. The complete solution is shown in Table 5 and Fig. 8.

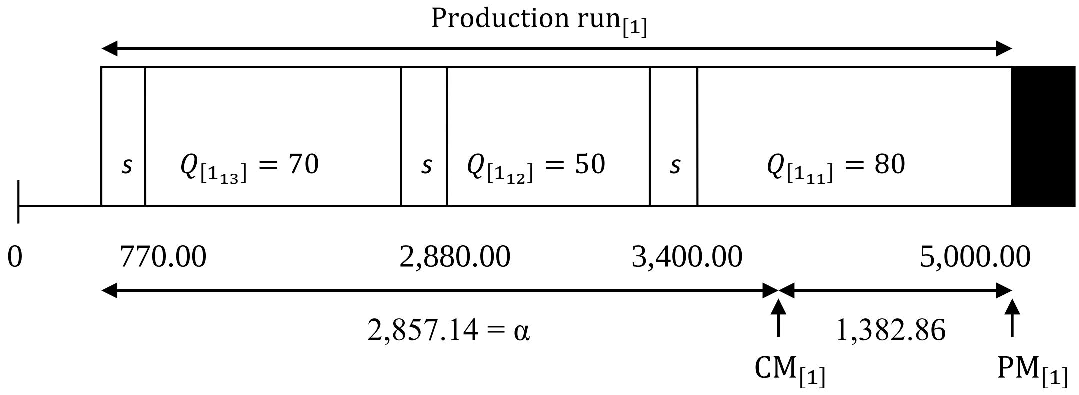


**Fig. 7.** The Gantt-Chart for Step-8 Algorithm [MISM] for the problem

**Table 3**
The decision variable of the best solution for 1st looping ($k$ = 1)

| Number of batch $j$ = 3 | $j$ = 2 | $j$ = 1 | Number of non-conforming parts, $M_1$ | Number of CM, $n_{CM}$ | $TC^{*}_{[1]}$ |
|---|---|---|---|---|---|
| 12 | 10 | 14 | 78.64 | 2 | 146,841.30 |

**Table 4**
The decision variable of the best solution for 2nd looping ($k$ = 2)

| Number of batch: Production run$_{[2]}$ | Production run$_{[1]}$ | | | Number of non-conforming parts, $M_2$ | Number of CM, $n_{CM}$ | $TC^{*}_{[2]}$ |
|---|---|---|---|---|---|---|
| $j$ = 3 | $j$ = 3 | $j$ = 2 | $j$ = 1 | | | |
| 1 | 1 | 10 | 14 | 0 | 0 | 142,071.60 |

**Table 5**
The complete solution of Algorithm [MISM] for the problem

| Batch size $Q_{[i_{kj}]}$: Production run$_{[2]}$ | Production run$_{[1]}$ | | | $M$ | $n_{CM}$ | $TC$ |
|---|---|---|---|---|---|---|
| *j = 3* | *j = 3* | *j = 2* | *j = 1* | | | |
| $Q_{[1_{23}]}$ = 57.00 | $Q_{[1_{13}]}$ =13.00 | $Q_{[1_{12}]}$ = 5.72 | $Q_{[1_{11}]}$ = 6.13 | 0 | 0 | 142,071.60 |
| | | $Q_{[2_{12}]}$ = 5.56 | $Q_{[2_{11}]}$ = 6.07 | | | |
| | | $Q_{[3_{12}]}$ = 5.40 | $Q_{[3_{11}]}$ = 6.00 | | | |
| | | $Q_{[4_{12}]}$ = 4.24 | $Q_{[4_{11}]}$ = 5.94 | | | |
| | | $Q_{[5_{12}]}$ = 5.08 | $Q_{[5_{11}]}$ = 5.88 | | | |
| | | $Q_{[6_{12}]}$ = 4.92 | $Q_{[6_{11}]}$ = 5.81 | | | |
| | | $Q_{[7_{12}]}$ =4.76 | $Q_{[7_{11}]}$ = 5.75 | | | |
| | | $Q_{[8_{12}]}$ = 4.60 | $Q_{[8_{11}]}$ = 5.68 | | | |
| | | $Q_{[9_{12}]}$ = 4.44 | $Q_{[9_{11}]}$ = 5.62 | | | |
| | | $Q_{[10_{12}]}$ = 4.28 | $Q_{[10_{11}]}$ = 5.55 | | | |
| | | | $Q_{[11_{11}]}$ = 5.49 | | | |
| | | | $Q_{[12_{11}]}$ = 5.42 | | | |
| | | | $Q_{[13_{11}]}$ = 5.36 | | | |
| | | | $Q_{[14_{11}]}$ = 5.30 | | | |

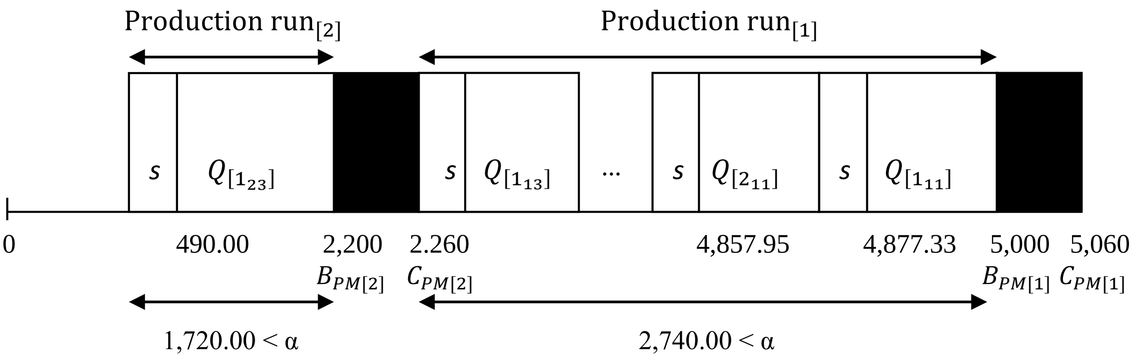


**Fig. 8.** Gantt-Chart of the solution of Algorithm [MISM] for the problem

*5.4 A Comparison between Model Solution and the Practice*

Company X processes an order with constant size of 10 parts for every batch and machine maintenance is performed only when a failure of the machine occurs (reactive maintenance). If the example case is scheduled with constant batch then 200 parts will be divided into 20 batches. The 20 batches are inserted into the Model [MISM] then the total cost for the constant batch size is US$ 152,300.76 (see Table 6). While the method developed will provides total cost of US$ 142,071.60. The method developed in this paper will provide cost efficiency of at least 7.2 %. Other losses that might occur in this practice are the machine failures occur during production activity in progress and late delivery orders to consumers cannot be avoided if machine maintenance time takes long time because of machine breakdown. These losses will be the opportunity costs to the company.

**Table 6**
The decision variable of the best solution for constant batch

| Number of batch | | | | Number of no n- conforming parts, $M_2$ | Number o f CM, $n_{CM}$ | $TC^*_{[2]}$ |
|---|---|---|---|---|---|---|
| Production run$_{[2]}$ | Production run$_{[1]}$ | | | | | |
| $j$ = 3 | $j$ = 3 | $j$ = 2 | $j$ = 1 | | | |
| 6 | 1 | 5 | 8 | 0 | 0 | 152,300.76 |

*5.5 Sensitivity Analysis*

Sensitivity analysis of important parameters of the Model [MISM] is shown in Table 7. It shows the increasing of completed parts inventory holding cost ($c_{1j}$) will decrease number of batches in the best total cost and the total cost will increases fast. The increasing of in process inventory holding cost ($c_{2j}$) will increase number of batches in the best total cost. The increasing of both PM unit cost ($c_{PM}$) and setup unit cost ($c_s$) did not change number of batches in the best total cost, except in increasing total cost value.

## 6. Conclusion

The model integrates batch scheduling and maintenance scheduling to minimize total cost consisting of inventory holding cost, setup cost, maintenance costs and rework cost for non-conforming parts. The problem in the model is divided into two, i. e., to determine batch production schedule and the second is to determine the expected number of corrective maintenance and the expected number of non-conforming parts obtained from determining the production schedule.

The solution is to accommodate a trade off in the following two things. An increase in the number of batch (length of production run) up to a certain limit will minimize the total inventory holding cost. Meanwhile, an increase in the length of production run will imply on an increase in the number of non-conforming parts and in number of corrective maintenance.

**Table 7**
Sensitivity analysis for Model [MISM]

| The change of parameters | Number of batch in each run $Run_{[2]}$ | $Run_{[1]}$ | | | $N*$ | $M$ | $n_{CM}$ | $TC*$ |
|---|---|---|---|---|---|---|---|---|
| | Item-3 | Item-3 | Item-2 | Item-1 | | | | |
| $c_{ij}$ = original values, $j$ =1,2,3 | 1 | 1 | 10 | 14 | 26 | 0.00 | 0 | 142,071.60 |
| $c_{ij}$ = twice of original values | 1 | 1 | 9 | 13 | 24 | 0.00 | 0 | 164,811.20 |
| $c_{ij}$ = three times of original values | 1 | 1 | 9 | 12 | 23 | 0.00 | 0 | 197,465.30 |
| $c_{ij}$ = four times of original values | 1 | 1 | 8 | 12 | 22 | 0.00 | 0 | 211,072.50 |
| $c_{2j}$ = original values | 1 | 1 | 10 | 14 | 26 | 0.00 | 0 | 142,071.60 |
| $c_{2j}$ = twice of original values | 1 | 1 | 11 | 16 | 29 | 1.45 | 1 | 150,942.80 |
| $c_{2j}$ = three times of original values | 1 | 1 | 13 | 17 | 32 | 4.45 | 1 | 159,494.00 |
| $c_{2j}$ = four times of original values | 1 | 2 | 14 | 18 | 35 | 7.45 | 1 | 167,527.20 |
| $c_{PM}$ = original values | 1 | 1 | 10 | 14 | 26 | 0.00 | 0 | 142,071.60 |
| $c_{PM}$ = twice of original values | 1 | 1 | 10 | 14 | 26 | 0.00 | 0 | 142,131.60 |
| $c_{PM}$ = three times of original values | 1 | 1 | 10 | 14 | 26 | 0.00 | 0 | 142,191.60 |
| $c_{PM}$ = four times of original values | 1 | 1 | 10 | 14 | 26 | 0.00 | 0 | 142,251.60 |
| $c_s$ = original values | 1 | 1 | 10 | 14 | 26 | 0.00 | 0 | 142,071.60 |
| $c_s$ = twice of original values | 1 | 1 | 10 | 14 | 26 | 0.00 | 0 | 142,149.60 |
| $c_s$ = three times of original values | 1 | 1 | 10 | 14 | 26 | 0.00 | 0 | 142,227.60 |
| $c_s$ = four times of original values | 1 | 1 | 10 | 14 | 26 | 0.00 | 0 | 142,305.60 |

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