A multi-objective Pareto ant colony algorithm for the Multi-Depot Vehicle Routing problem with Backhauls

Jhon Jairo Santa Chávez[a], John Willmer Escobar[b*] and Mauricio Granada Echeverri[c]

[a]*Programa de Ingeniería Eléctrica, Universidad Libre Seccional Pereira and Universidad Tecnológica de Pereira, Colombia*
[b]*Department of Civil and Industrial Engineering, Pontificia Universidad Javeriana Cali, Colombia*
[c] *Programa de Ingeniería Eléctrica, Universidad Tecnológica de Pereira, Colombia*

C H R O N I C L E	A B S T R A C T
	This paper presents a multiobjective ant colony algorithm for the Multi-Depot Vehicle Routing Problem with Backhauls (MDVRPB) where three objectives of traveled distance, traveling times and total consumption of energy are minimized. An ant colony algorithm is proposed to solve the MDVRPB. The solution scheme allows one to find a set of ordered solutions in Pareto fronts by considering the concept of dominance. The effectiveness of the proposed approach is examined by considering a set of instances adapted from the literature. The computational results show high quality results within short computing times.

1. Introduction

The Multi-Depot Vehicle Routing Problem with Backhauls (MDVRPB) is an operational problem of the supply chain management. The MDVRPB considers a supply chain involving two echelons: depots and customers. The MDVRPB is an NP-hard problem, since it is a generalization of the two well-known NP-hard problems: the Multi-Depot Vehicle Routing Problem (MDVRP) (for further details see Escobar et al. 2014a) and the Vehicle Routing Problem with Backhauls (VRPB). The MDVRPB has many realistic applications in Transportation and Logistics. The features of the customers, depots and vehicles, as well as different operating constraints on the performed routes, leads to different variants of MDVRPB: (i) simultaneous collecting and dispatching of products; (ii) collecting first, following the delivery of products; and (iii) collecting after of the delivering process. We have considered a specific version for which the collection of products must be performed after the backhaul customers have been served.

* Corresponding author.
E-mail: jwescobar@javerianacali.edu.co (J. W. Escobar)

The MDVRPB can be defined as the following graph theory problem. Let $G = (V, A)$ be a complete undirected graph, where $V = \{1....v\}$ is the set of vertices, and A is the set of edges. The set V is partitioned into two subsets: the set of customers $N = \{1....n\}$ and the set of potential depots $M = \{1....m\}$. Additionally, the set N is divided into a subset of Linehaul nodes (Linehaul customers - L), and the Backhaul nodes (Backhaul customers – B). Therefore, $N = L \cup B$. The Linehaul customers ask for delivering products while Backhaul customers require the collection of products. Each customer has a nonnegative amount D_i ($j \in N$) of product to be delivered ($i \in L$) or to be picked up ($i \in B$). Each depot has a fictitious demand. i.e. $D_i = 0$, with $i \in M$. A set of K identical vehicles with a given capacity Q_k is initially placed at each depot. It must be clarified that all vehicles are not necessarily used. For each edge $(i, j) \in A$, a nonnegative cost c_{ij} is associated, where $c_{ii} = +\infty$ for each edge $(i, i) \in A, i \in M$. As the symmetrical condition is assumed, for each edge $(i, j) \in V$ the cost of c_{ij} is equal to c_{ji}, for all $i \neq j$. In the MDVRPB, all deposits are not necessarily used. The main goal of the MDVRPB is to find a set of r performed routes by imposing the following constraints:

- Each vehicle must start from a depot m, and return to the same depot;
- Each customer must be visited exactly once.
- The sum of the demands of customers belonging to a determined route must not exceed the vehicle capacity Q;
- For each performed route, the Linehaul customers must precede the Backhaul customers for each sequence;
- The flow between depots is not allowed.

The goal of the MDVRPB is to determine the routes to be performed from the selected depots to the customers by a fleet of homogeneous vehicles in order to satisfy the demand of the customers (products to be collected or products to be delivered). The objective functions considered for the multiobjective version of the MDVRPB is to minimize the total traveled distance, the total time and the consumed energy. The first objective is the common function considered in the literature related to the vehicle routing problems. The second objective is obtained by the allowed speed on each edge. In particular, we have considered a random speed between 30 km /hr to 90 km/hr for the complete graph on the benchmarking set of instances. Finally, the third objective is adopted from the idea of gas emission and consumption of energy introduced by Bektaş and Laporte (2011) and Demir et al. (2014).

In particular, the consumption of energy (P_{ij}) for each edge (i, j) is calculated following the idea introduced by Bektaş and Laporte (2011):

$$P_{ij} \approx \propto_{ij} (\xi + f_{ij})d_{ij} + \beta v_{ij}^2 d_{ij}, \tag{1}$$

where $\propto_{ij}$ is a parameter of the edge (i, j), ξ is the mass of the empty vehicle in kg; f_{ij} is the quantity of product carried from i to j in kg, and d_{ij} corresponds to the distance between i and j in m. In addition, β is a constant and v_{ij}^2 corresponds to the speed of the vehicle on edge (i, j). $\propto_{ij}$ and β are obtained as follow:

$$\propto_{ij} = a + g \sin \theta_{ij} + g C_r \cos \theta_{ij}, \tag{2}$$
$$\beta = 0.5 C_d A \rho, \tag{3}$$

where a corresponds to acceleration of the vehicle in m/s^2, g is the acceleration of gravity in m/s^2, θ_{ij} is the angle of the edge (i, j), C_r is the friction coefficient between the vehicle and the road, C_d is the drag coefficient, A is the frontal area of the vehicle in m^2 and ρ is the density of the air in kg/m^3.

Salhi and Nagy (1999) proposed a heuristic approach to solve the MDVRPB, which is is based on the idea of "Border Customers", defined as the customers geographically located in a middle point between two depots. Min et al. (1992) introduced a version of the MDVRPB by considering collection process after delivery of products. A unified heuristic for different vehicle routing problems with backhaul was presented by Ropke and Pisinger (2006). In this work, a version of mixed pickup and delivery with multiple depots was considered. Two algorithms based on ant colony for the multi depot vehicle routing problem with mixed pickup and delivery were presented by Wade and Salhi (2004) and Wade and Salhi (2001). Finally, genetic algorithms for solving the MDVRPB were proposed by Chunyu and Xiaobo (2009) and Chunyu et al. (2009).

Surveys of existing methods for multi-objective problems were presented in Jozefowiez et al. (2008) and Zhou et al. (2011). In Jozefowiez et al. (2008), the authors examined multiobjective versions of several variants of the Vehicle Routing Problem (VRP) in terms of their objectives, their characteristics and the types of proposed algorithms to solve them. A survey of the state of the art of the multi-objective evolutionary algorithms was proposed by Zhou et al. (2011). This papers covers algorithms frameworks for multiobjective combinatorial problems during the last eight years. However, in the literature reviewed, there are few works considering the multi-objective version of the MDVRPB. Multiobjective metaheuristic approaches for combinatorial problems were presented in Doerner et al. (2004), Liu et al. (2006) and Lau et al. (2009). A multiobjective methodology by Pareto Ant Colony Optimization for solving a portfolio problem was introduced by Doerner et al. (2004). A multi-objective mixed zero-one integer-programming model for the vehicle routing problem with balanced workload and delivery time was introduced by Liu et al. (2006). In this work, a heuristic-based solution method was developed. A fuzzy multi-objective evolutionary algorithm for the problem of optimization of vehicle routing problems with multiple depots, multiple customers, and multiple products was proposed by Lau et al. (2009). In this work, two objectives were considered: minimization of the traveling distance and also the traveling time.

Approaches for multi-objective versions of the VRPB have been proposed by Anbuudayasankar et al. (2012), García-Nájera et al. (2015) and Yalcın & Erginel (2015). Three heuristics approaches for solving a bi-objective vehicle routing problem with forced backhauls were introduced by Anbuudayasankar et al. (2012). In particular, two heuristics are based on the well-known savings algorithm and the third heuristic is based on a Genetic Algorithm (GA). Finally, an evolutionary approach and a fuzzy programming for the multi-objective vehicle routing problems with backhauls were presented by García-Nájera et al. (2015) and Yalcın and Erginel (2015), respectively. Other multiobjective algorithms proposed for solving related logistic combinatorial problems could be consulted in Nezhad et al. (2013), Mortezaei and JabalAmeli (2011), Mohammadi et al. (2011), Rao and Patel (2014), Yazdian and Shahanaghi (2011), Escobar et al. (2013), Escobar et al. (2014b), Escobar et al. (2015) and Bolaños et al. (2015).

This paper proposes a multiobjective algorithm based on an Ant Colony System to solve the MDVRPB with collection of products exclusively after delivery of products. The efficiency of the proposed algorithm has been compared by 33 modified instances taken from the literature. The set of instances originally was proposed by Salhi and Nagy (1999). In particular, information about of mitigation of gas emissions, consumption of energy and traveling times has been added to the set of instances.

The main contribution of the paper is to propose an effective algorithm for the solution of a multiobjective version of the multi-depot vehicle routing problem with backhauls (MDVRPB). The proposed algorithm is a novel metaheuristic approach, which obtains good results within short computing times. This version of the MDVRPB has not been considered in the previous revised literature. Therefore, no attempt has been proposed for solving this variant of the problem.

This paper is organized as follows: Section 2 provides a description of the general framework of the proposed approach. Section 3 gives a detailed description of the proposed multi-objective metaheuristic approach for the MDVRRPB. Section 4 described the analysis of computational results. Finally, section 5 shows the conclusions and future research.

2. Metaheuristic algorithm based on ant colony system

The Ant Colony metaheuristic is based on the natural behavior of ants searching food. The logical tendency of each ant is to reduce the effort and time required to obtain food. This goal is achieved by reducing the distance, time and energy between two specific points for collecting food. The ants are individuals with relatively simple features. However, they perform highly complex work in a simple way. The success lies in the interaction of many individuals with the environment and by indirect communication between them through chemicals substances called pheromones (Gutjahr, 2002). This behavior is emulated in artificial intelligence to find good quality solutions to combinatorial optimization problems characterized by a wide space of solutions.

2.1 Selecting edges

The proposed algorithm considers an ant as the emulation of a vehicle in the process of performing a route. An artificial ant begins its travel from a depot selected randomly to a first customer. The decision of the next client to be visited is based on heuristic preferences biased by the distance, time and energy among nodes and the emulation of the natural pheromone. A probabilistic transition rule defines the likelihood that the ant k (one ant for each available vehicle), placed at node i, decides to move to the node j (selecting the edge $(i, j) \in A$). The rule is defined according to the Eq. (4).

$$p_k(i.j) = \frac{[\tau_{ij}]^\alpha \cdot [\eta_{ij}]^\beta}{\sum_{u \epsilon S}[\tau_{ij}]^\alpha \cdot [\eta_{ij}]^\beta},$$

(4)

where τ_{ij} is the amount of pheromone on the edge $(i.j) \in A$, and η_{ij} is the heuristic information of $(i.j)$, i.e. the distance, time and energy to travel from the node i to the node j. The level of importance on the rule decision from both the heuristic information and the amount of pheromone is given by parameters α and β, respectively. Generally, an off-line process must fit these parameters. Note that this probability distribution is affected by the parameters α and β, which determine the influence of the trails and the visibility, respectively. Finally, S is the set of neighborhoods of node i, which have not visited yet by the ant k.

2.2 Parameters α and β

In particular, if $\alpha = 0$, the nearest neighborhoods from a given node i have more probability to be selected (well-known algorithm of the gradient with multiple starting points). If $\beta = 0$, only the level of pheromone is considered for selecting the neighborhoods. This selection allows solutions with low quality respect to the three considered objective functions, specifically if $\alpha > 1$. Indeed, this situation allows finding local optima of an ant algorithm, because all the ants follow the same way generating sub optimal solutions.

A proper updated pheromone matrix directly impacts the diversity on the decision-making process by an ant for selecting the following edges. A certain level of pheromone is deposited on an edge, when an ant has traveled across it. The level of pheromone is evaporated with the time. An additional positive feedback of the amount of pheromone is used to reinforce future components of good solutions. For a

better solution, more pheromone is provided for the considered edges belonging to the best solutions found so far.

Eq. (5) determines the rule for increasing the quantity of pheromone

$$\Delta\tau_{ij}^k = \begin{cases} \dfrac{1}{C(s_k)} & \text{if the edge } (i.j) \text{ is used by the ant } k \text{ on a performed route} \\ 0 & \text{otherwise} \end{cases} \qquad (5)$$

where $\Delta\tau_{ij}^k$ is the amount of pheromone deposited at the edges visited by the ant k, $C(s_k)$ is the total cost of the solution generated by the ant k, i.e. the total distance, time and energy consumed of the performed route by the ant k. The edges visited by all the ants in the current solution receive an extra contribution of pheromone.

In addition, a pheromone evaporation process is used to prevent an unlimited increasing of trails and to forget low quality solutions respect to the three objectives by the performed k routes. The evaporation level is the same for all pheromone trails, eliminating a predefined percentage of the current value $\tau_{ij}(t)$ for each edge $(i.j)$, through a predefined rate ρ, with $0 \le \rho \le 1$. Eq. (6) describes the general rule for updating the pheromone matrix for all the edges $(i.j) \in A$, at each iteration t.

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \sum_A \Delta\tau_{ij}^k \qquad (6)$$

2.3 Ant System (AS)

In the AS, an artificial colony of ants cooperates to find good solutions for discrete, static, and dynamic combinatorial optimization problems. Three different variants for the AS have been proposed: Ant-Density, Ant-Quantity and Ant-Cycle (Wade & Salhi, 2001). In the first two variants, the pheromone-updated process is performed after each selection edge; while in the third variant (Ant-Cycle), the amount of pheromone is only updated once the ants have completed all the routes. The first two versions generally obtain worse results than the third version (Wade & Salhi, 2001). Therefore, we consider the process for updating the pheromone by assigning a fixed amount of it on each edge at each iteration. Eq. (7) is generally used to determine the quantity of pheromone for each edge (i,j), where K is the number of ants, i.e. the number of available vehicles, and C_{nn} is the tour length obtained at the constructive solution procedure.

$$\tau_{ij} = \dfrac{K}{C_{nn}} \qquad (7)$$

3. Proposed approach for the multi-objective MDVRPB

The proposed approach is based on a Pareto Ant Colony Optimization (PACO). In particular, we have extended and enriched the main idea proposed by Doerner et al. (2004) for the problem of optimization of a financial portfolio to the Multi-objective Multi Depot Vehicle Routing Problem with Backhauls. The proposed approach take into consideration the rule described in Eq. (4). In this case, we have considered three matrices of pheromones for each objective function. These matrices are combined by using three objective weights w_d (for the distance objective), w_t (for the traveling time objective) and w_e (for the objective function energy); i.e., the individual preferences of each ant, which are randomly initialized for each ant. The sum of the value of these factors must be equal to 1. Note, that the domain of the three objective weights are random numbers equally distributed from the domain [0,1].

In the constructive initial solution, each ant tries to make a feasible route by applying a pseudo-random proportional rule using the heuristic information η_{ij} and the pheromone information $\tau_{ij}(t)$. Once a feasible solution has been found, the three objective functions are calculated. If the solution S is feasible, it is stored. The global pheromone updated process is performed by using the best solution found of the current iteration for each objective. The types of feasible solutions constructed are explained in the following section.

3.1 Types of constructed routes

Let us define the total number of depots as m: i.e., $M_1, M_2 \ldots M_m$, the total number of linehaul customers as nl: $L_1, L_2, \ldots, L_{nl}$ and the number of backhaul customers as nb: $B_1, B_2, \ldots, B_{nb}$. The proposed algorithm performs the following routes for a system with three (3) depots: M_1. M_2 and M_3; with seven (7) linehaul customers: $L_1, L_2, L_3, L_4, L_5, L_6$ and L_7; and seven (7) backhaul customers: B_1, B_2, B_3, B_4, B_5, B_6 and B_7:

- **A linehaul – backhaul route:** The Fig. 1 describes this type of route. Its main feature is that the linehaul and the backhaul sequences are reaching the capacity of the vehicle.

- **A single linehaul route:** This route contains only linehaul customers. This route is shown in the Fig. 2.

- **A single backhaul route:** This type of route is depicted in the Fig. 3. It contains only backhaul customers.
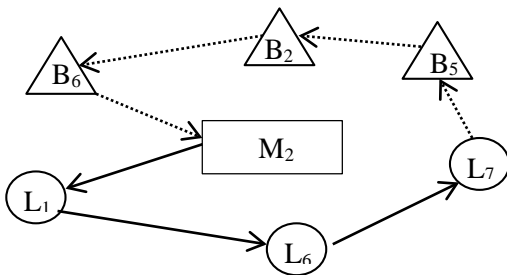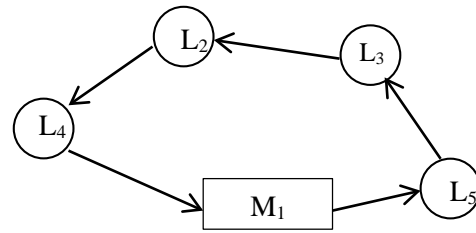

Fig. 1. Linehaul – backhaul route
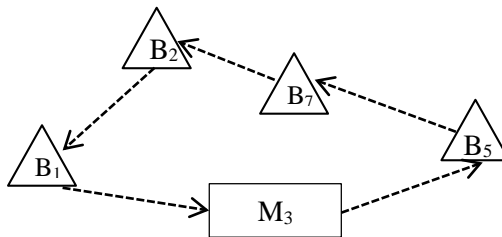

Fig. 2. Single linehaul route

Fig. 3. Single backhaul route

An alternative of solution is conformed by a combination of the three described types of routes. In the proposed algorithm, the construction of a multi-depot solution requires that an ant performs as many subroutes as needed to visit all the customers, and each sub-route comprises a tour considering linehaul and backhaul customers. A possible alternative of solution is shown in the Fig. 4. The dashed line represents the entire tour performed by an ant.
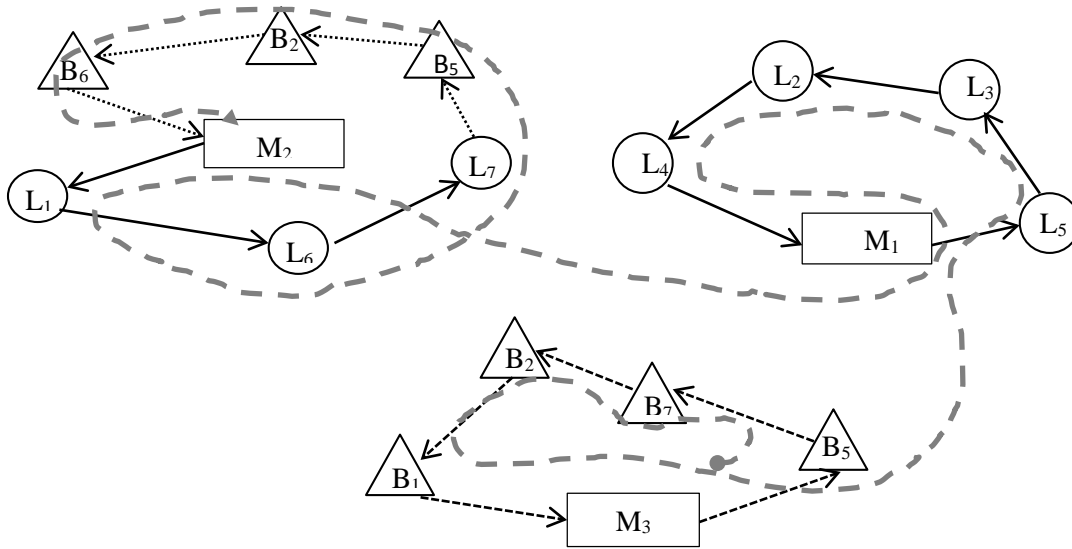
Fig. 4. Alternative route

3.2 Decision rule

Given the pheromone information, the set of objectives $o = \{1,2,..,O\}$ and the set of all the feasible edges to be selected $\Omega(x,y) = \{(i,j) \in A\}$; the edge to be added to the current route performed by an ant is selected according to the following pseudo-random-proportional rule:

$$
(i,j) = \begin{cases} \max\limits_{(i,j) \in \Omega(x)} \left\{ \left[\sum_{o=1}^{O} w_o \tau_{ij} \right]^{\propto} \cdot \eta_{ij}{}^{\beta} \right\} & if \ q < q_0, \\[2ex] \overline{(i,j)}, & otherwise \end{cases}
\tag{8}
$$

where q is a random number between $[0,1)$, q_0 is a given parameter between $[0,1)$ representing the probability to select the best next edge, and $\overline{(i,j)}$ is the edge to be selected according to the Eq. (4).

3.3 Encoding Solution

Fig. 5 shows the structure used for the distance, time and energy matrices. In particular, we have used a three-dimensional array with dimensions $(n+1) \times (n+1) \times (m)$ for each depot. For each matrix, each element C_{ij} corresponds to the distance, the traveling time or the energy of the edge $(i.j)$.
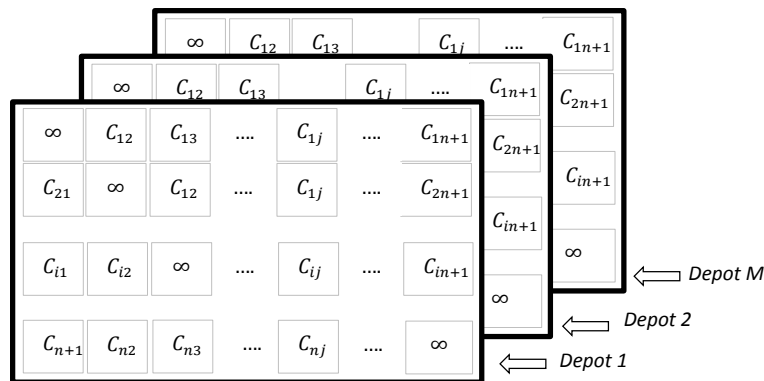

Fig. 5. Encoding for the distance, traveling time and energy matrices

The Visibility Matrix (Fig. 6) is obtained from each element of the matrix elements of distance, time and energy. It contains the inverse matrix of the distance, time and energy. This matrix is symmetrical and is not modified during the execution of the algorithm.
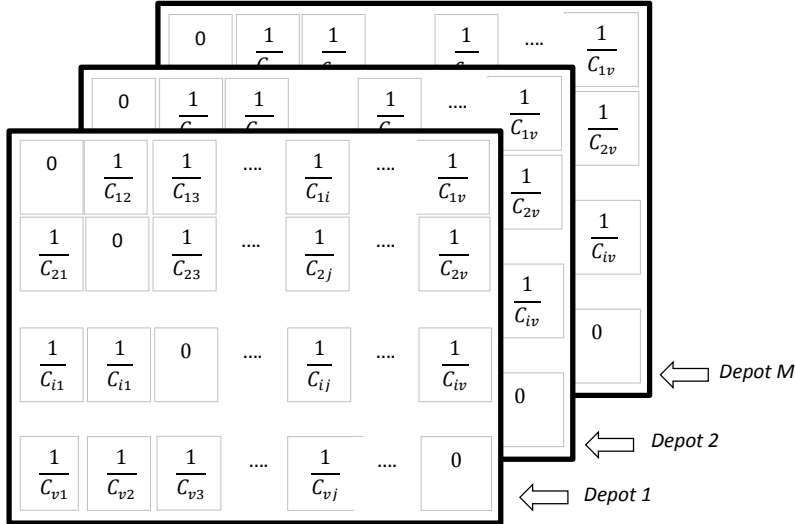

Fig. 6. Encoding of the Visibility matrix

Finally, we proceed to generate the pheromone matrix (Fig. 7) with all positions initialized with the same amount of pheromone. The values of the pheromone matrix are generated with the Eq. (7), and the updating process is restricted to thus elements in the route traveled by each ant.
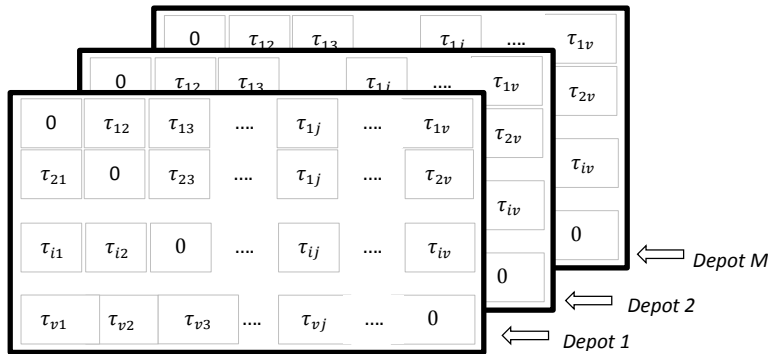

Fig. 7. Encoding the pheromone matrix

The proposed algorithm avoids flow between depots. Therefore, distance, time and energy among depots are not considered. Indeed, the proposed approach uses a new vehicle to continue visiting other customers. The visibility matrix or net matrix is replaced by a squared array of random values that provides diversity in the searching of the new depot to be added to the current solution. The total number of depots gives the dimension of the visibility matrix, and its values are initialized using the Eq. (7). The nodes are updated only if they belonging to the route traveled by a vehicle. In addition, a vector of not visited customers is required. It contains the feasible customers to be selected.

The solution vector is also required. Its first element is the number of a depot, which is randomly selected. It is followed by a sequence of customer until a new depot has been selected again. However, for each sequence of customers, a Linehaul customer must not appear between two Backhaul customers, and vice versa. The Figure 8 shows an example of a route of the proposed solution.
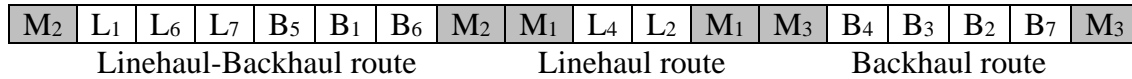
| M$_2$ | L$_1$ | L$_6$ | L$_7$ | B$_5$ | B$_1$ | B$_6$ | M$_2$ | M$_1$ | L$_4$ | L$_2$ | M$_1$ | M$_3$ | B$_4$ | B$_3$ | B$_2$ | B$_7$ | M$_3$ |

  Linehaul-Backhaul route   Linehaul route   Backhaul route

Fig. 8. Encoding the solution Vector

3.4 Pseudocode for the proposed algorithm

Procedure PACO-MDVRPB (depots. customers. vehicleNumber. vehicleCapacity)

      Initialization of PACO-MDVRPB; /* Generate k ants, Initialize the Pheromone matrixes
        Calculate Distance, Time and Energy Matrices
        Initialize Net Matrices
        Initialize Pheromone Contribution Array
        Initialize Net Contribution Array
        Iterations = 0
     while iterations < 100 do
        ants = 0
        create k empty routes
        determine the objective weigh (w_o) for each objective o randomly
        while ants < k do
            // Starting MDVRPB procedure
            initialDepot = choose(depots)
                If anyLinehaul(customers) **then** Process Linehaul customers
                If anyBackhaul(customers) **then** Process Backhaul customers
                select an edge (i, j) by using equation (8)
                evaporate(pheromones)
                update(pheromones)
                ants++
        End while
      End while

4. Computational results

The proposed approach has been tested on an adapted benchmarking set, which is available in http://unilibrepereira.edu.co/backhauls/. In particular, 33 benchmarking instances are used to test the effectiveness of the proposed approach. The proposed algorithm has been implemented in MatLab, and computational experiments have been run on a PC with Core i5 1.4 GHz processor and 8GB of RAM. The algorithm has been executed for 10 runs over 100 iterations, reporting the average results and the best results of all runs.

The parameters used to execute the complete set of instances are the following: Iterations =100, $\alpha = 1$, $\beta = 3$, $\rho = 0.01$, and the contribution factor of the visited edge = Number of customers / Best solution found so far.

4.1 Description of the Instances

The proposed algorithm has been evaluated in 33 instances MDVRPB adapted from Salhi and Nagy (1999). The respective set has a diverse number of Linehaul and Backhaul customers (from 50 to 250), a number of depots between 2 – 5, and a homogeneous set of vehicles are considered. Euclidean distance for each edge is determined. The set of benchmarking instances has been used to solve different variants of the MDVRPB. However in the reviewed literature, we have not found papers that solve the problem MDVRPB with Backhauls at the end of the routes.

The matrices of energy have been performed as follows: We have defined three types of vehicles depending of the load (Type 1-T1 with less than 10 ton, Type 2-T2 between 10 and 20 ton, and Type 3-T3 between 20 and 35 ton). The parameters of the frontal area of each vehicle, weigh of the vehicle, weigh of the load, values of C_d and C_r are described in Table 1.

Table 1

Energy parameters (Source: http://www.kenworthcolombia.com/)

	Type 1-T1	Type 2-T2	Type 3-T3
Frontal Area (m²)	7	9	11.44
Weight of the vehicle (kg)	6000	16000	17000
Weight of the load (kg)	10000	20000	35000
C_d	0.76	0.85	0.95
C_r	0.01	0.0125	0.015

In addition, the following parameters are considered to obtain the energy matrices: acceleration = 0 m/s2, gravity = 9.807, angle of the road = 0º, air density at 20º ρ = 1.2041 kg / m³. Finally, the distances and the speed for each edge are calculated with the coordinates X and Y and the matrices of traveling time. Table 2 shows the type of vehicle used for each instance.

Table 2

Type of used vehicle for each instance

Instance Number	Instance	Vehicle Capacity (units)	Type of Vehicle
1, 2, 3	GJ01H, GJ01Q, GJ01T	80	1
4, 5, 6, 7, 8 ,9	GJ02H, GJ02Q, GJ02T GJ03H, GJ03Q,GJ03T	160	2
10, 11, 12, 16, 17, 18, 19, 20, 21	GJ04H, GJ04Q, GJ04T, GJ06H, GJ06Q, GJ05T, GJ06Q, GJ06T, GJ07H, GJ07Q, GJ07T	100	1
13, 14, 15	GJ05H, GJ05Q, GJ05T	200	2
22, 23, 24 , 25 , 26, 27, 28, 29, 30, 31, 32, 33	GJ08H, GJ08Q, GJ08T, GJ09H, GJ09Q, GJ09T, GJ10H, GJ10Q, GJ10T, GJ11H, GJ11Q, GJ11T	500	3

In addition, the correlation of the objectives for each instance is calculated in order to justify the consideration of the three objectives at the same time. As an example, Fig. 9, Fig. 10 and Fig. 11 show the correlation of the three objectives for instance 3. Note that there are low correlations factors for the considered objectives. Therefore, the objectives are independent and a Pareto Front must be calculated.
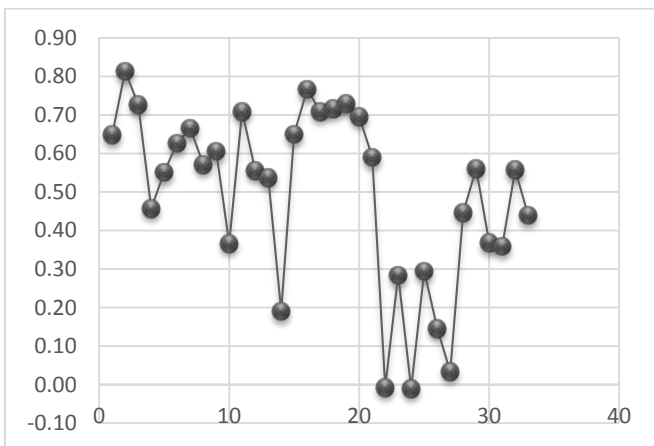
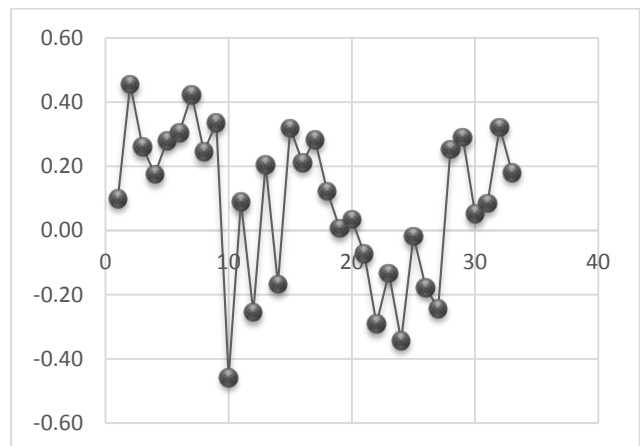**Fig. 9.** Correlation between Distance and Time   **Fig.10.** Correlation between Time and Energy
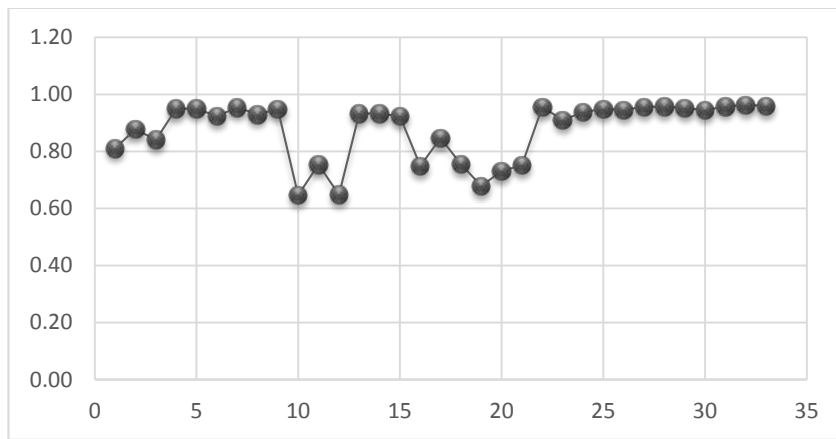
Fig. 11. Correlation between Distance and Energy

4.2 Obtained Results

We report the computing time and the values reached by the metaheuristic objective proposed

Table 3
Obtained results over 10 executions with 100 iterations of the proposed algorithm

Instance Number	Instance	Proposed Methodology with PACO – Distance				Proposed Methodology with PACO – Time				Proposed Methodology with PACO – Energy			
		Best Solution Distance (km)	Objective Function Time (min)	Objective Function Energy (kw/h)	Computing Time (sec)	Objective Function Distance (km)	Best Solution Time (min)	Objective Function Energy (kw/h)	Computing Time (sec)	Objective Function Distance (km)	Objective Function Time (min)	Best Solution Energy (kw/h)	Computing Time (sec)
1	GJ01H	674.6	828.9	449.5	3.57	804.6	716	646.6	2.64	675.7	833.7	446.2	2.46
2	GJ01Q	776.5	868.9	528.9	3.33	865.1	790.6	669.5	2.49	802.1	977.8	512.9	2.42
3	GJ01T	762.6	848.9	533.2	2.9	886.4	751.7	728	1.38	771.5	922.9	522.1	1.56
4	GJ02H	639.4	675.8	1034	1.06	739.6	651.7	1280.4	1.73	651.8	732.3	1022.7	0.75
5	GJ02Q	675.9	681.3	1136.2	1.31	745.4	646.8	1319.5	2.02	676.7	754.5	1081.3	1.02
6	GJ02T	678.2	698.6	1124.4	2.4	744.2	632.8	1314.8	2.57	680.7	743.7	1077.2	1.57
7	GJ03H	836.5	969	1322.5	4.82	858.8	820.3	1465.1	4.36	840.3	982.2	1321.2	2.05
8	GJ03Q	929	994.7	1499.2	3.29	984.4	907.2	1692.4	6.16	948.9	1150.1	1463.1	3.7
9	GJ03T	909.5	1060.1	1416.1	5.65	956.4	911.6	1645.4	6.61	916	1096.7	1409.9	2.87
10	GJ04H	1203	1307.3	845.6	3.46	1360.5	1170.4	1109.7	2.7	1226.5	1536.7	778.6	3.94
11	GJ04Q	1287.2	1492.1	870.2	2.93	1439.2	1357.4	1116.3	2.76	1312.8	1647.9	840.5	0.98
12	GJ04T	1269.3	1515.5	848	1.41	1391.8	1258.2	1087.8	3.6	1276.2	1563.2	835.8	1.36
13	GJ05H	992.4	1055.2	1639.9	1.62	1096.1	987.7	1909.7	2.14	1024.5	1195.7	1619.9	2.05
14	GJ05Q	1063.5	1245.1	1664.2	3.19	1124.5	1083.9	1901	1.71	1076.3	1303.6	1656.7	1.8
15	GJ05T	1026.8	1035.9	1710.9	4.51	1100.7	963.4	1930.9	1.86	1059.9	1252.6	1664.2	3.28
16	GJ06H	1148.1	1199.6	838.4	4.63	1268.2	1120.9	1011.9	3.12	1222	1564.2	770	3.79
17	GJ06Q	1205.6	1223	878.7	6.73	1269.3	1117.2	1011.4	5.8	1301.2	1540	867.6	3.53
18	GJ06T	1203	1423.4	803.1	6.65	1274	1207.6	963.6	3.71	1240.7	1576	784.7	3.28
19	GJ07H	1100.6	1142.7	808.6	6.32	1231.9	1070.6	995.3	4.75	1144.4	1445	736.6	2.74
20	GJ07Q	1198.6	1392.4	817.9	8.64	1332.7	1151.6	1083.7	4.33	1200.4	1397.7	807.2	5.69
21	GJ07T	1196	1364.4	824.3	7.86	1315.1	1174.6	1049.4	4.91	1200.3	1524.9	758.9	6.68
22	GJ08H	5114.7	5933.9	13481.2	11	5432	4858.5	15515.4	13.88	5154.8	6091.3	13457.3	6.9
23	GJ08Q	5763.4	5835.7	15757.3	20.36	6119.5	5473.9	17584.9	18.8	5904.2	6819.7	15527.1	14.38
24	GJ08T	5879	5779.8	16475.1	24.54	5984.5	5593	17030.9	13.42	5936.9	6953.6	15472.4	13.42
25	GJ09H	5106.8	5624.9	13805.7	13.9	5160.7	4707.6	14694.2	12.53	5112.1	5665.7	13802.7	9.3
26	GJ09Q	5889.9	6279.6	16017.6	11.45	6571.8	5759.2	19018.3	12.2	5906.8	6365	15968	10.99
27	GJ09T	6135.2	6955.2	16281.4	15.31	6505.4	6152.4	18347.4	15.68	6325.3	7989.1	16064	10.28
28	GJ10H	4814.2	5458.5	12859.8	18.14	4888.1	4449	13759.5	21.65	4959.8	5865.1	12808.1	17.03
29	GJ10Q	5387.3	5727.3	14550.6	24.91	6274.3	5434.1	18245.8	30.17	5596.6	6732.9	14453.3	28.9
30	GJ10T	5305.4	5914	14201.1	55.1	5458.3	5249.7	15321.9	27.53	5379.6	6535	13896.9	33.05
31	GJ11H	4725.9	5226.8	12664	28.01	5286.7	4768.7	15107.8	25.88	4725.9	5226.8	12664	24.59
32	GJ11Q	5279.7	5716.9	14208.3	57.18	5882.5	5293.2	16788.1	50.16	5366.7	6500.5	13804.1	22.91
33	GJ11T	5149.7	5577.4	13862.7	47.72	5629.9	5040	16203.9	50.63	5184.4	5836.2	13715.6	18.85
Average		**2585.7**					**2532.5**					**5836.7**	

Fig. 12, Fig. 13 and Fig. 14 show the results found for the instance 3 of the Pareto Fronts by using the criterion Min – Min. Note that each figure shows the extreme values of each Pareto Front.
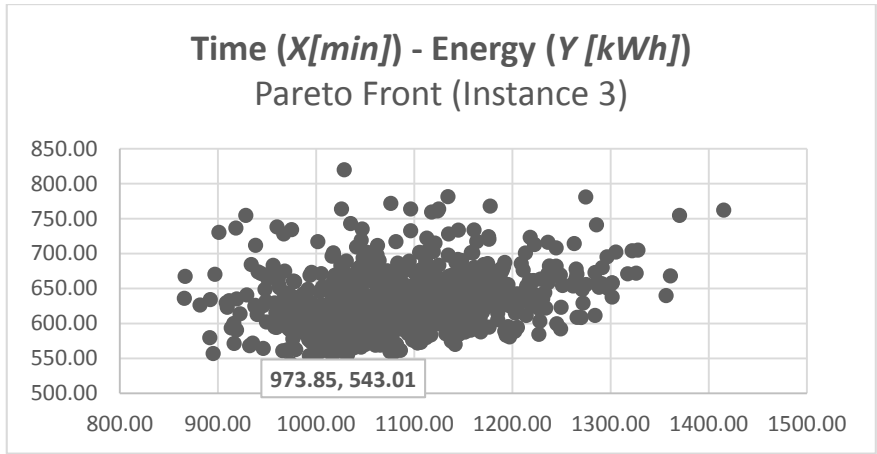
Time (*X[min]*) - Energy (*Y [kWh]*)
Pareto Front (Instance 3)

973.85, 543.01

Fig. 12. Pareto Ant Colony Optimization: Time - Energy

Distance (*X[m]*) - Energy (*Y[kWh]*)
Pareto Front (Instance 3)

797.53, 556.79

814.89, 543.01
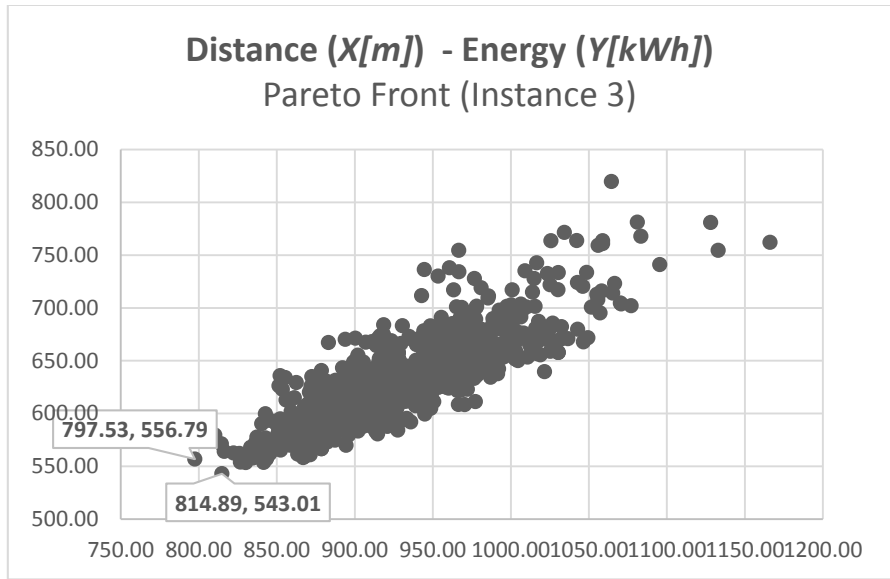
Fig. 13. Pareto Ant Colony Optimization: Distance - Energy

Distance (X[m]) - Time (Y[min])
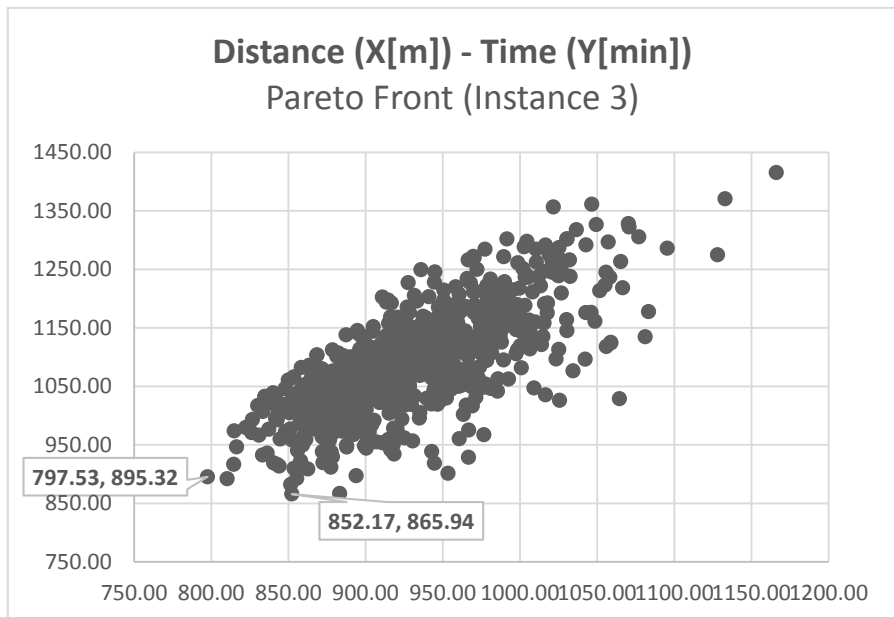Pareto Front (Instance 3)

797.53, 895.32

852.17, 865.94

Fig. 14. Pareto Ant Colony Optimization: Distance - Time

5. Concluding remarks

Multi-objective combinatorial techniques play a decisive role in the field of the vehicle routing problems. Recent researchers have proposed several approaches to solve several variants of the multi-objective vehicle routing problems classified as NP-hard problems.

In this paper, an effective Pareto Ant Colony Optimization has been used to provide an efficient approach for the Multi-objective Multi-Depot Vehicle Routing Problem with Backhauls (MDVRPB). In particular, three objectives of traveled distance, traveling times and total consumption of energy have been minimized. In addition, we have used multiple pheromone matrices and random weights for each objective. The performance of the proposed algorithm has been evaluated by considering adapted instances from the literature.

The proposed methodology could be extended to other vehicle routing problems with many or few constraints and/or objectives. In addition, new heuristic information could be added easily to the proposed approach. We suggest proving the proposed approach to other routing problems such as the Multi Depot Vehicle Routing Problem (MDVRP), the Periodic Location Routing Problem (PLRP), the Muti-Depot Vehicle Routing Problem with Heterogeneous Fleet (HMDVRP), among others.

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