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Game-theoretic analysis of supply chain coordination under advertising and price dependent demand

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CHRONICLE	ABSTRACT
	Supply chain members cannot act independently and they need to act as a part of a unified system and coordinated with other members. Therefore, a coordination mechanism may be necessary to motivate members to achieve coordination. In this paper, the coordination problem is studied in a two-level supply chain consisting of a supplier and a retailer where demand is a function of price and advertising expenditures in two scenarios. The first scenario is "No coordination", and the other scenario is "coordination with Revenue sharing contract". The models are solved using game theory, Cooperative and Nash equilibrium. Finally, numerical examples are presented indicating that the average expected profit in the second scenario, coordination with revenue sharing, is higher than the first scenario. In addition numerical examples indicate that as price and advertising elasticity to demand increase, profitability of supply chain decreases.

1. Introduction

The intensive competition in today's global market, the availability of various products with short life cycles and increasing customers' expectations have forced many businesses to invest more and to pay more attention to their supply chains. Meanwhile, the recent advances on communication technology and transportation have created incentives for sustainable development and supply chain management practices (Simchi-Levi, 2009). However, since various parts of the supply chains are constantly looking for profit maximization, they pay less attention on the entire supply chain's profitability. Centralized system is more secure for supply chain coordination, but may not be always true. On the other hand, in a decentralized system, supply chain members make decision regardless of their decision's effect on the performance of the other members and the entire supply chain. This is the key issue in supply chain management to develop an appropriate mechanism such that various objectives are aligned, and integrate their activities to optimize the entire system. In recent years, the issue of setting some contracts for supply chain coordination has attracted much attention among researchers. Supply chain contracts are useful tools for communication and coordination among supply chain members in a decentralized system. The contract can be defined as an agreement between the two sectors. The

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contracts imply conditions that offer appropriate information and incentive mechanisms, and ensure all parts of supply chain to achieve coordination and optimal performance of supply chain.

There are literally various types of contracts used in the literature to coordinate the supply chain such as quantity discount contract (Weng, 1995), revenue sharing contract (Cachon, 2003), quantity flexibility contract (Tsay, 1999), and buy back contract (Pasternack, 2008). According to Lovejoy (1999), quantity flexibility contracts have been used by Toyota and by Nippon Otis as a manufacturer of elevator equipment. Connors et al. (1995) mentioned that quantity flexibility contracts could improve sales' figures. Chopra and Meindl (2004) mentioned that quantity flexibility contracts are common for components in the electronic and computer industry. According to Chopra and Meindl (2004), Benetton as a fashion clothing manufacturer used these contracts with its retailers to increase supply chain profits. If buyback contracts, revenue sharing contracts and quantity flexibility contracts are properly implemented, they will increase profit of both parties resulting in a win-win situation (Cachon, 2003; Chopra & Meindl, 2004). However, will these benefits carry over if demand is price dependent? Cachon and Lariviere (2005) studied a vendor managed inventory contract with revenue sharing and demonstrated that the decentralized system could provide less capacity than the integrated system. Hua and Li (2008) proposed a cooperative game model to describe the payment bargaining process between the manufacturer and the retailer, and determined a new consignment contract with revenue sharing.

Pasternack (2008), Kandel (1996) and Emmons and Gilbert (1998) are among some pironners who performed comprehensive studies on the buyback contract. Yao et al. (2008) presented a price dependent demand model, with buyback contract. Leng and Parlar (2010) studied a model for coordinating decentralized supply chain, evaluated buy back contract, lost sales contract and cost sharing contract, and used game theory to solve the model. Emmons and Gilbert (1998) studied a model of price dependent demand where buyback policies would benefit both the retailer and the manufacturer. The authors showed that while the wholesale price is in a certain range, the manufacturer chooses a positive buyback price over not offering a buyback price. Mahajan (2008) used the same price dependent demand model originally developed by Emmons and Gilbert (1998) and considered a revenue sharing contract. They reported an existence of a positive revenue sharing ratio that the manufacturer would prefer. He also used the same model of price dependent demand as Emmons and Gilbert (1998) and considered the quantity flexibility contract. Finally, Wang and Hu (2010) considered a model with effort dependent demand with buy back contract, revenue sharing contract and quantity flexibility contract.

Literature review indicates that in supply chain coordination with price and advertising dependent demand, revenue sharing contract and quantity flexibility contract were rarely considered using game theory approach. In this paper, two scenarios of "without coordination" and "coordination revenue sharing contract" are considered. For each scenario, a model with price and advertising dependent demand is presented for a two-level supply chain consisting of a client and supplier. The models are solved in two solutions using game theory including Cooperative and Nash equilibrium. Numerical results are presented for two cases, to clearly show the effect of coordination on supply chain performance. Numerical results indicate that the contracts can increase profit of supply chain.

This paper is organized as following; in section 2 the model is formulated, in section 3 the solution for the model is presented, in section 4 numerical samples are solved, and finally conclusions are presented in section 5.

2. Research methodology

Consider a two-stage supply chain with a supplier and a retailer. The retailer is a newsvendor who faces an uncertain customer demand D, which depends on price and advertising. The methodology used in this study is described in Fig. 1.
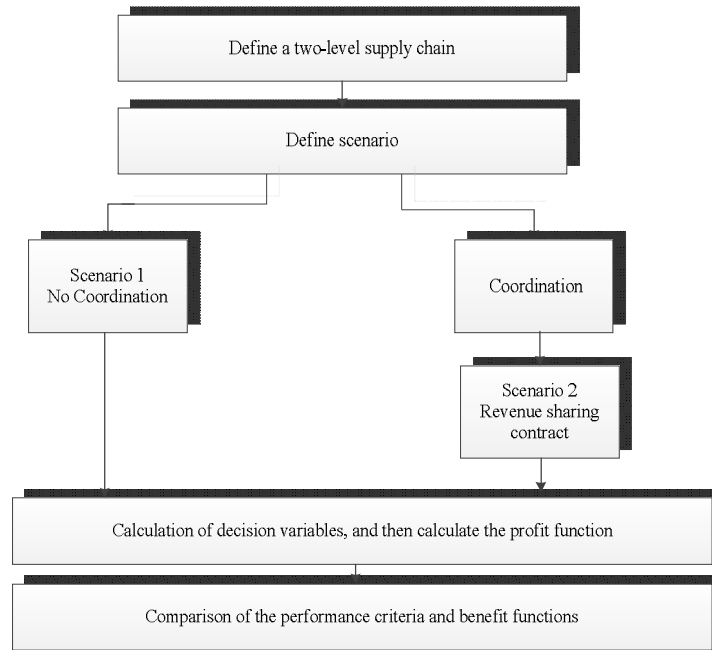
Fig. 1. Procedure for comparison

2.1. Assumptions

- A two-level supply chain consists of a retailer and a supplier.
- The model is in a form of a single period model.
- There is only one single product.
- The demand is stochastic and depends on price and advertising and it is considered the same for comparing all contracts as follows,

$$D(p, E) = \alpha p^{-\beta} E^{\delta} + \varepsilon, \tag{1}$$

where α is the initial demand, β and δ are price and marketing elasticity to demand, respectively and finally ε represents residuals, which has a uniform distribution on the interval *[a, b]*.

2.2. Notation

c: Production cost
$D(p,E)$: Annual demand (depend upon price and advertising)
$E(\pi_s)$: expected profit of supplier
$E(\pi_r)$: expected profit of retailer
$E(\pi_{sc})$: expected profit of whole supply chain *(E(\pi_r) + E(\pi_s))*
q: optimal order quantity
w: wholesale price charged by supplier per unit
p: selling price per unit
E: advertising cost

Decision variables for Revenue sharing contract:

w': wholesale price which is according to Revenue sharing contract and retailer charges at the time of purchase that is lower than w $w' < w$
φ: Ratio of the revenue raised by retailer and shared with supplier $0 \leq \varphi \leq 1$

The unit cost and unit price should follow the following constraints because of the value addition at different stages of S.C:

$$c(\text{production cost}) \leq w(\text{wholesale price}) < p(\text{price of product})$$

3. Mathematical Modeling

The contracts help the supply chain members achieve coordination, which would lead to improved supply chain performance. To realize the importance of coordination by contracts, two scenarios have been explored. The first scenario is "No coordination" in which the S.C. members act independently, and the second scenario is "coordination with Revenue sharing contract". The various performance measures of the first scenario are compared with the case of coordination.

3.1. Scenario1: No coordination

In this case, there is no coordination between the retailer and supplier. The retailer determines his/her optimal order quantity and the supplier provides the order, and does not do any effort to encourage the retailer make any more orders.

3.1.1. Profit function of retailer

In this case, the retailer's revenue is derived from selling the product to customers, and his/her costs are expenses of advertising and expenses of buying the product.

$$\pi_r = \begin{cases} pD - wq - E & a \leq D < q \\ pq - wq - E & q \leq D \leq b \end{cases} \tag{2}$$

As mentioned in the assumptions, the demand function is considered as a function of price and advertising, so Eq. (1) is replaced with D in the retailer's expected profit function.

$$E(\pi_r) = \int_a^q p(\alpha p^{-\beta} E^{\delta} + x) f(x) d(x) + \int_q^b pq f(x) d(x) - wq - E \tag{3}$$

In this problem, x has uniform distribution in the interval $[a, b]$. Thus, in Eq. (3), $f(x)$ equals to $\frac{1}{b-a}$. If integration in Eq. (3) is performed, the retailer's expected profit function will be obtained as following.

$$E(\pi_r) = \frac{1}{b-a}\left[pq(b-q) + p\left[(\alpha p^{-\beta} E^{\delta})(q-a) + \frac{(q-a)^2}{2}\right]\right] - wq - E \tag{4}$$

3.1.2. Profit function of supplier

The supplier provides the retailer's order, so his/her revenue is earned from selling the product to the retailer. The equation is formulated as following:

$$E(\pi_s) = q(w - c) \tag{5}$$

3.2. Scenario 2: coordination with Revenue sharing contract

Based on the revenue sharing contract, the supplier reduces the wholesale price from w to w' and the retailer shares the ratio (φ) of the revenues earned with supplier. Retailer and supplier profit function for this scenario is formulated in Eq. (8) and Eq. (11).

3.2.1. Profit function of retailer

In this case, retailer's revenue is earned from selling product to the client who shares the ratio with supplier, according to the contract. In contrast, the retailer pays less to buy products from supplier.

$$\pi_r = \begin{cases} \varphi p D - w' q - E & a \leq D < q \\ \varphi p q - w' q - E & q \leq D \leq b \end{cases} \tag{6}$$

$$E(\pi_r) = \int_a^q \varphi p \left(\alpha p^{-\beta} E^\delta + x\right) f(x) d(x) + \int_q^b \varphi p q f(x) d(x) - w' q - E \tag{7}$$

Similar to the previous scenario, Eq. (1) is replaced with D and $f(x)$ equals to $\frac{1}{b-a}$. So the retailer's expected profit function is obtained as Eq. (8) derived from Eq. (4).

$$E(\pi_r) = \frac{\varphi}{b - a} \left[pq(b - q) + p \left[(\alpha p^{-\beta} E^\delta)(q - a) + \frac{(q - a)^2}{2} \right] \right] - w' q - E \tag{8}$$

3.2.2. Profit function of supplier

For the revenue sharing contract, the profit function of supplier is:

$$\pi_s = \begin{cases} (1 - \varphi) p D + w' q - cq & a \leq D < q \\ (1 - \varphi) p q + w' q - cq & q \leq D \leq b \end{cases} \tag{9}$$

$$E(\pi_s) = w' q - cq + \int_a^q (1 - \varphi) p \left(\alpha p^{-\beta} E^\delta + x\right) f(x) d(x) + \int_q^b (1 - \varphi) p q f(x) d(x) \tag{10}$$

Eq. (1) is replaced with D and $f(x)$ equals to $\frac{1}{b-a}$, and the supplier's expected profit function will change as per Eq. (11)

$$E(\pi_s) = \frac{(1-\varphi)}{b-a}\left[pq(b-q) + p\left[(\alpha p^{-\beta}E^{\delta})(q-a) + \frac{(q-a)^2}{2}\right]\right] + w'q - cq \tag{11}$$

4. Solution

In this section, two game-theoretic models based on a non-cooperative games including Nash and a cooperative are discussed. Solution methodology is described in Fig. 1.

4.1. Nash game

When the manufacturer and the retailer have the same decision power, they determine their strategies, independently and simultaneously. This situation is called as a Nash game and the solution to this structure is the Nash equilibrium. The results of solving the model for two scenarios are as follows.

4.1.1. Scenario1: No coordination

As illustrated in Fig. 1, the amount of q based on p and E is obtained by take the derivative of the retailer's expected profit function.

By replacing q in the expected profit function of retailer, we can obtain the values of p and E:

$$J(\beta - \delta) - \ln\left(\frac{\alpha(1-\beta - e^J(1-2\beta))}{b(e^J - 1 + w)}\right) - \delta \ln\left(\frac{\delta b(w + e^J - 1)(w(1-\beta) + \beta e^J)}{(1-\beta - e^J(1-2\beta))^2}\right) = 0 \tag{12}$$

$$p^* = e^J \tag{13}$$

$$E^* = \frac{\delta b p^*(w + p^* - 1)(w(1-\beta) + \beta p^*)}{(1-\beta - p^*(1-2\beta))^2} \tag{14}$$

$$q^* = \frac{b(w + \beta p^* - \beta w)}{1 - \beta - p^* + 2\beta p^*} \tag{15}$$

### 4.1.2. Scenario 2: coordination with Revenue sharing contract

In this section, we present the decision variables of retailer for the scenario that retailer and supplier agree to cooperate with Revenue sharing contract. These values are obtained according to the methodology which is shown in Fig. 2 and it is quite similar to previous scenario.

$$H(\beta - \delta) - \ln\left(\frac{\varphi\alpha(1-\beta - e^H(1-2\beta))}{b(\varphi(e^H - 1) + w')}\right) - \delta \ln\left(\frac{\delta b(w' + \varphi(e^H - 1))(w'(1-\beta) + \beta\varphi e^H)}{\varphi(1-\beta - e^H(1-2\beta))^2}\right) = 0 \tag{16}$$

$$p^* = e^H \tag{17}$$

$$q^* = \frac{b(w' + \beta\varphi p^* - \beta w')}{\varphi(1-\beta - p^* + 2\beta p^*)} \tag{18}$$

$$E^* = \frac{\delta b p^* (w' + \varphi(p^* - 1))(w'(1 - \beta) + \beta \varphi p^*)}{\varphi(1 - \beta - p^*(1 - 2\beta))^2} \qquad (19)$$
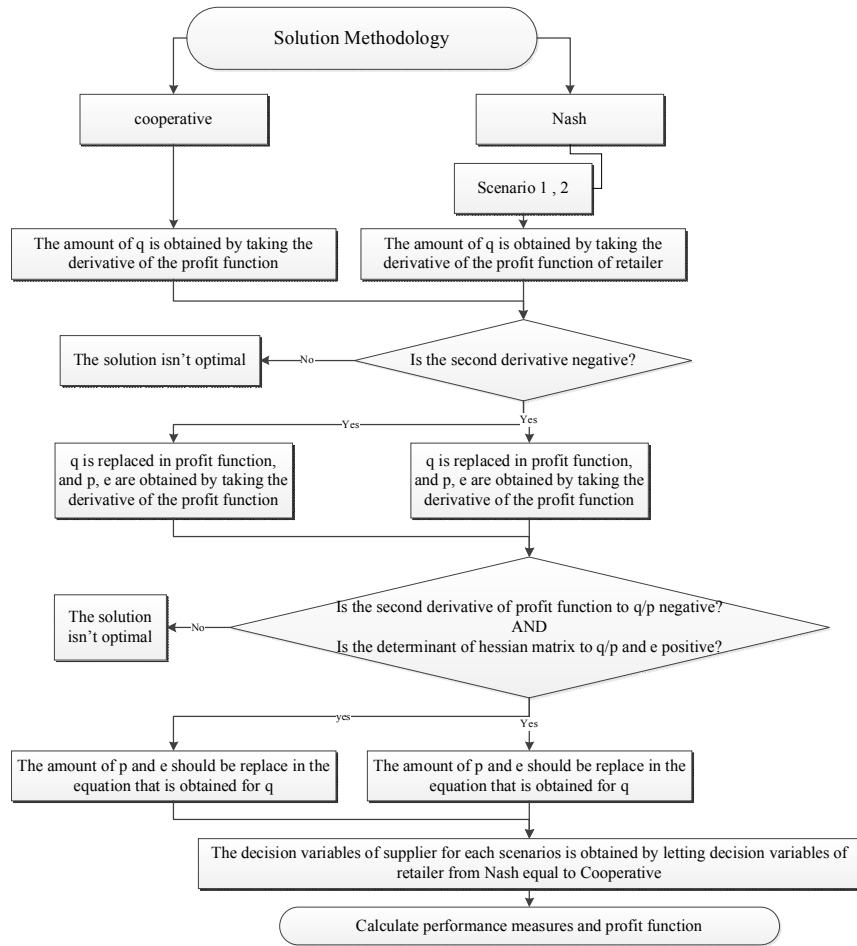
Fig. 2. Solution Methodology

4.2. Cooperative game

In previous subsection, a non-cooperative game was discussed. Now we model the supplier–retailer relationship as a cooperative game in which both channel members agree to cooperate and maximize the profit of the whole system. The cooperation expected profit function is the same as Eq. (20)

$$\pi_{sc} = q(w - c) - wq - e + \frac{1}{b - a}\left[pq(b - q) + p\left[(\alpha p^{-\beta} E^\delta)(q - a) + \frac{(q - a)^2}{2} \right] \right] \qquad (20)$$

$$k(\beta - \delta) - \ln\left(\frac{\alpha\left(1 - \beta - e^k(1 - 2\beta) \right)}{b(e^k - 1 + c)} \right) - \delta \ln\left(\frac{\delta b(c + e^k - 1)(c(1 - \beta) + \beta e^k)}{\left(1 - \beta - e^k(1 - 2\beta) \right)^2} \right) = 0 \qquad (21)$$

$$p^* = e^k \qquad (22)$$

$$q^* = \frac{b(c + \beta p^* - \beta c)}{1 - \beta - p^* + 2\beta p^*} \qquad (23)$$

$$E^* = \frac{\delta b p^* (c + p^* - 1)(c(1 - \beta) + \beta p^*)}{(1 - \beta - p^*(1 - 2\beta))^2} \qquad (24)$$

Proof for the optimality of the obtained solution is presented in the Appendix.

As we know, when the condition of a decentralized supply chain is closer to integration channel, it means supply chain members act more coordinated. Therefore, in this paper, we suppose the retailer's decision variables coincide with the decision variables of integrated channel. As mentioned in Fig. 2, the supplier's decision variables for the second scenario is obtained by letting the retailer's decision variables from Nash game equal to cooperative game as following:

$$w' = \frac{\varphi(\beta e^k - \beta c + 2c\beta e^H - \beta e^H + c - ce^H)}{1 - \beta - e^k(1 - 2\beta)} \qquad (25)$$

5. Numerical Example and Numerical analysis

First, we describe the experimental parameters used in the numerical analysis for the model. Then we analyze these results and get better view over the contract properties for industrial practice. Table 1 shows the improvements provided by contract in a supply chain performance.

Table 1
Optimal solutions of scenario 1 and Scenario 2

β	δ	Scenario 1 (No coordination)						Scenario 2 (Coordination with Revenue sharing)					
		p^*	q^*	E^*	$E(\pi_r)$	$E(\pi_s)$	$E(\pi_{sc})$	w'^*	p^*	q^*	E^*	$E(\pi_r)$	$E(\pi_s)$	$E(\pi_{sc})$
5	10	9.025013	48.07	563.23	16312	48.07	16358	0.8443	7.389	52.357	368.632	62146.4	15757.3	77903.7
	20	7.893187	46.94	982.95	8261.8	46.94	8308.7	0.8447	6.554	51.915	654.389	35581.05	9061.763	44642.813
	30	6.903303	45.62	1284.4	2159.2	45.62	2207.3	0.8452	5.812	51.408	871.174	11729.18	3027.353	14756.533
15	10	6.03756	38.84	102.44	2087.4	38.84	2121.8	0.8501	5.155	46.089	73.1946	7503.634	1921.667	9425.301
	20	5.28039	36.8	175.45	1116.9	36.8	1153.7	0.8509	4.572	45.274	129.468	4465.791	1151.696	5617.487
	30	4.618177	34.41	223.36	291.54	34.41	330.38	0.8518	4.055	44.326	171.556	1658.316	435.7668	2094.0828
25	10	4.039012	30.46	35.648	404.53	30.46	427.48	0.8537	3.597	42.24	29.25	1885.526	491.026	2376.552
	20	3.532479	27.04	58.073	255.47	27.04	282.51	0.855	3.19	40.889	51.4002	1194.815	314.3642	1509.1792
	30	3.089471	22.95	68.292	68.003	22.95	98.464	0.8565	2.829	39.288	67.4838	519.8481	140.1115	659.9596
35	10	2.702021	17.43	11.438	22.4111	17.43	30.685	0.8587	2.509	36.891	13.7459	502.8722	135.7754	638.6476
	20	2.363161	11.4	14.039	19.286	11.4	26.4052	0.8609	2.226	34.548	23.6822	363.9086	99.52701	463.43561
	30	2.066797	3.994	6.9928	7.9258	3.994	25.356	0.8635	1.974	31.689	30.1613	199.8395	56.10347	255.94297

As indicated in the table, the improvements for both of the supplier and retailer can be considerable. For example, when β is 15 and δ is 10, for the case of "No coordination", the whole supply chain profit is 2121.8 and for the other scenario it is 9425.301. So obviously, the expected profit of the whole supply chain increases when contracts are used. The experimental parameters used are as following. First, consider the parameters of α (the primary demand) and β (consumer's price-sensitivity). In the literature, Yao (2008) considered three states for α and β:

$$\alpha = 20, \ \beta = 0.01, 0.1, 1.5, 2 \qquad \alpha = 200, \ \beta = 5, 15, 25, 35, 45 \qquad \alpha = 2000, \ \beta = 1, 10, 100, 200$$

He proved that in case of $\frac{\alpha}{\beta} < 15$, the results are difficult to interpret and if 200 units is added up to the upper bound of $\frac{\alpha}{\beta}$, it will lead to very high retail price and very small order quantity. In this article we select the results calculated by letting $\alpha = 200$ and $\beta = 5, 15, 25, 35$ to analyze the performance of the different scenarios. The consumer's price-sensitivity (δ) is considered to be 10, 20, 30. In addition, we let the production cost as $c=1\$/unit$, the wholesale price as $w=2 \$/unit$ and the decision variable as

$\varepsilon \sim U[0,100]$. For scenario 2 we assume $\varphi = 0.8$. The results used to conduct our analysis, have been tabulated in Table 1 implying the scenario1 and scenario 2.

Let's observe contracts behavior by comparing results. The supplier's profit and retailer's profit in the scenario that contract is considered, is generally larger than the corresponding profit in the case of "No coordination". This means both the retailer and the manufacturer have earned more profit than what they would earn in a "No coordination" case. For each of two scenarios, by increasing of β, retailer's expected profit (π_r) decreases, on the other hand, by increasing of δ, both π_s and π_r decrease, this means if the customer's sensitivity to advertising increases, retailer and supplier will achieve lower profit. Also by increasing β, supplier's profit for the scenario 2 decreases.
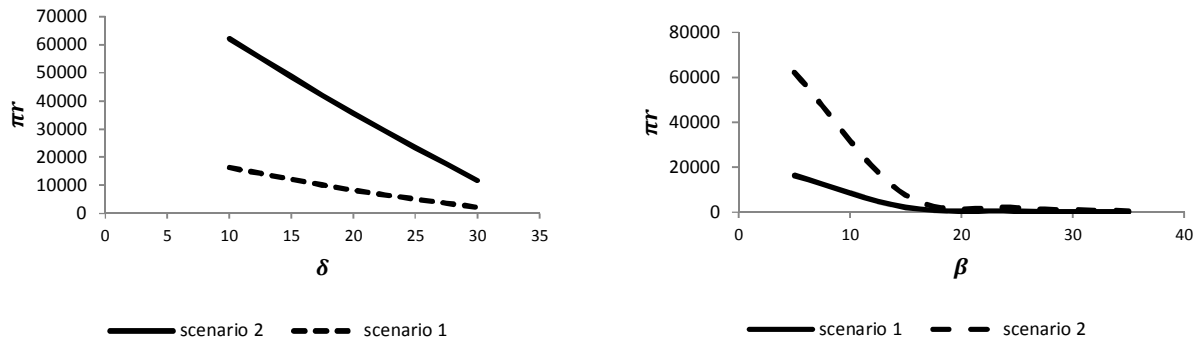

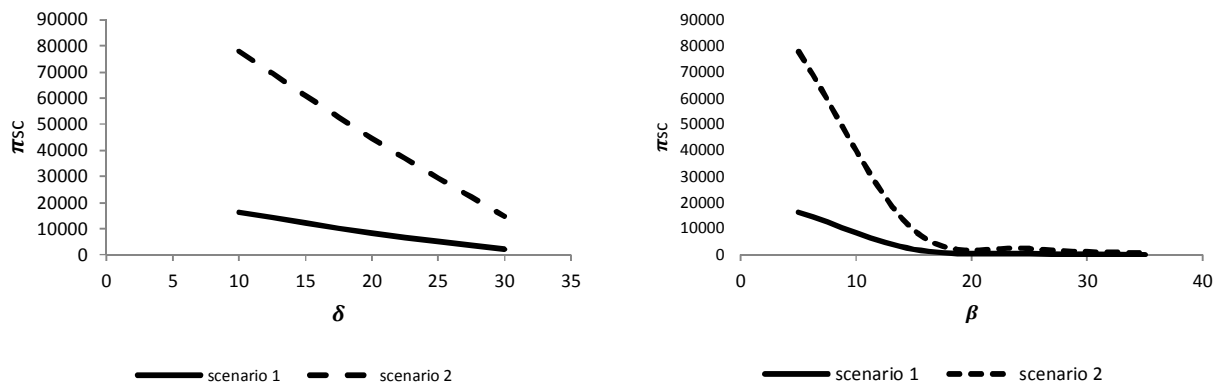Fig 3. The retailer's expected profit function in terms of "β" and "δ"

Fig. 4. The expected whole supply chain profit function in terms of β and δ

According to fig 3, the expected profit function in terms of β and δ for retailer is descending, which means if β and δ rise, the expected profit function of retailer will decrease. However, the sensitivity of functions to δ is a bit more than β. Fig 4 shows the sensitivity of the expected whole supply chain profit function in terms of β and δ, and as we can observe, by increasing β and δ, the whole supply chain profit function decreases.

5. Conclusion

In this paper, we have presented a decentralized two-level retailer-supplier supply chain with price and advertising dependent demand. One form of contracts has been assessed and different scenarios of coordination may be simulated, which may help in quantifying the performance measures and the effectiveness of coordination. The first scenario was "No coordination", and the other scenario was

"coordination with Revenue sharing contract". The preliminary results have indicated that the contract can increase the profit of supply chain members. The proposed models are solved using Game theory approach, Cooperative and Nash equilibrium. Eventually the numerical results have indicated that contracts can make the profit functions much better and the average expected profit in the second scenario -coordination with Revenue Sharing contract- was more than the other scenario.

Since in this paper, the demand is assumed to be dependent upon the retail price and advertisement, we may relax this assumption in future and analyze supply chains where demand depends on price, quality and some other factors. In this paper, we used the exponential function for demand, so in the future; researchers can use the other functions such as Normal and etc. Researchers can extend these models to three-echelon supply chains. They can consider the supply uncertainty that is a very attractive and unfamiliar assumption in this field.

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Appendix

For two scenarios and cooperative channel, we prove the optimality of the obtained solution in 3 steps:
1. The value of q is obtained by taking the derivative of retailer's expected profit function, and it is optimal when the retailer's expected profit function is concave to q.

The value of q is replaced in retailer's expected profit function, so p and E are obtained by taking the derivative of retailer's expected profit function. They are optimal solution when:

2. The retailer's expected profit function is concave respect to p.
3. The hessian matrix determinant is negative definite respect to p and E.

Scenario 1

Step 1.

$$\frac{d^2\pi_r}{dq^2} < 0 \rightarrow \frac{2p-1}{a-b} < 0$$

So the retailer's expected profit function is concaved to q

Step 2

$$Hessian = \begin{pmatrix} \dfrac{\alpha p^{1-\beta}(\beta-1)e^{\delta}(q-a)\beta}{p^2(a-b)} & \dfrac{\alpha p^{1-\beta}(1-\beta)e^{\delta}\delta(q-a)}{pe(b-a)} \\ \dfrac{\alpha p^{1-\beta}(1-\beta)e^{\delta}\delta(q-a)}{pe(b-a)} & \dfrac{\alpha p^{1-\beta}(\delta-1)e^{\delta}\delta(a-q)}{(a-b)e^2} \end{pmatrix} \qquad \frac{d^2\pi_r}{dp^2} < 0 \rightarrow \frac{\alpha p^{1-\beta}(\beta-1)e^{\delta}(q-a)\beta}{p^2(a-b)} < 0$$

Step 3

$$if\ \beta < \delta \rightarrow determinant = -\frac{\delta\alpha^2\left(p^{1-\beta}\right)^2(\beta-1)(\beta-\delta)(a-q)^2 e^{2\delta}}{p^2 e^2(a-b)^2} > 0$$

Thus the hessian matrix in terms of p and E is negative definite.

Scenario 2

Step 1.

$$\frac{d^2\pi_r}{dq^2} < 0 \rightarrow \frac{\varphi(2p-1)}{a-b} < 0$$

So the retailer's expected profit function is concaved to q

Step 2.

$$Hessian = \begin{pmatrix} \dfrac{\varphi\alpha p^{1-\beta}(\beta-1)e^\delta(q-a)\beta}{p^2(a-b)} & \dfrac{\varphi\alpha p^{1-\beta}(1-\beta)e^\delta\delta(q-a)}{pe(b-a)} \\ \dfrac{\varphi\alpha p^{1-\beta}(1-\beta)e^\delta\delta(q-a)}{pe(b-a)} & \dfrac{\varphi\alpha p^{1-\beta}(\delta-1)e^\delta\delta(a-q)}{(a-b)e^2} \end{pmatrix} \qquad \frac{d^2\pi_r}{dp^2} < 0 \rightarrow \frac{\varphi\alpha p^{1-\beta}(\beta-1)e^\delta(q-a)\beta}{p^2(a-b)} < 0$$

Step 3.

$$if\ \beta < \delta \rightarrow determinant = -\frac{\delta\varphi^2\alpha^2\left(p^{1-\beta}\right)^2(\beta-1)(\beta-\delta)(a-q)^2 e^{2\delta}}{p^2 e^2(a-b)^2} > 0$$

Thus the hessian matrix in terms of p and E are negative definite.

Cooperative channel

Step 1.

$$\frac{d^2\pi_r}{dq^2} < 0 \rightarrow \frac{2p-1}{a-b} < 0$$

So, the retailer's expected profit function is concave to q

Step 2.

$$Hessian = \begin{pmatrix} \dfrac{\alpha p^{1-\beta}(\beta-1)e^\delta(q-a)\beta}{p^2(a-b)} & \dfrac{\alpha p^{1-\beta}(1-\beta)e^\delta\delta(q-a)}{pe(b-a)} \\ \dfrac{\alpha p^{1-\beta}(1-\beta)e^\delta\delta(q-a)}{pe(b-a)} & \dfrac{\alpha p^{1-\beta}(\delta-1)e^\delta\delta(a-q)}{(a-b)e^2} \end{pmatrix} \qquad \frac{d^2\pi_r}{dp^2} < 0 \rightarrow \frac{\alpha p^{1-\beta}(\beta-1)e^\delta(q-a)\beta}{p^2(a-b)} < 0$$

Step 3.

$$if\ \beta < \delta \rightarrow determinant = -\frac{\delta\alpha^2\left(p^{1-\beta}\right)^2(\beta-1)(\beta-\delta)(a-q)^2 e^{2\delta}}{p^2 e^2(a-b)^2} > 0$$

Thus the hessian matrix in terms of p and E is negative definite.