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Inventory model with different demand rate and different holding cost

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ABSTRACT

This paper deals with the development of an inventory model for time varying demand and constant demand; and time dependent holding cost and constant holding cost for case 1 and case 2 respectively. Previous models incorporating that the holding cost is constant for the entire inventory cycle. Mathematical model has been developed for determining the optimal order quantity, the optimal cycle time and optimal total inventory cost for both cases. Differential calculus is used for finding optimal solution. Numerical examples are given for both cases to validate the proposed model. Sensitivity analysis is carried out to analyze the effect of changes in the optimal solution with respect to change in various parameters.

1. Introduction

It is important to find the optimal stock and optimal cycle time of inventory to meet the future demand. The main objective of inventory management is to minimize the inventory carrying cost. In traditional EOQ model, the demand rate is assumed to be constant. In real life, it is frequently observed that demand for a particular product can be influenced by internal factors such as price, time and availability. The change in the demand in response to inventory or marketing decisions is as demand elasticity. Thus, when the demand rate is constant, the effect of variability of the holding cost of the total inventory cost functions of such models has also been considered. Two types of demand and holding cost have been considered (i) time–dependent demand rate and time-dependent holding cost for case 1 (ii) constant demand rate and constant holding cost for case 2. An algorithm that minimizes the total inventory cost is developed.

Various models have been proposed for constant demand rate with constant holding cost. Teng et al. (2005) developed an EOQ model on optimal pricing and ordering policy under permissible delay in payments by assuming that the selling price is necessarily higher than the purchase cost. They established an appropriate model for a retailer to found its optimal price and lot size, simultaneously,

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when the supplier offered a permissible delay in payment. Goyal (1985) developed an EOQ model under permissible delay in payments but ignored the difference between the selling price and the purchase cost. Aggarwal and Jaggi (1995) extended Goyal's model for deteriorating items and Liao et al. (2000) developed an EOQ model for stock–dependent demand rate under permissible delay in payment. Muhlemann and Valtis-Spanopoulos (1980) investigated the constant rate EOQ model but with variable holding cost expressed as a percentage of the average value of capital investigated in stock. Vander Veen (1967) presented an EOQ inventory system with the holding cost as a nonlinear function of inventory. Weiss (1982) investigated traditional EOQ model with the holding cost per unit modified as a nonlinear function of the length of time an item was held in stock. Goh (1994) presented an EOQ model with general demand and holding cost function and demand rate for an item was considered as a function of existing inventory level and carrying cost per unit was allowed to change.

Alfares (2007) presented the step structure of the holding cost by considering the inventory policy for an item with a stock–level dependent demand rate and a storage-time dependent holding cost. The holding cost per unit of item per unit time was assumed an increasing function of the time spent in storage. The holding cost was assumed to be varying over time in only few inventory models. Giri et al. (1996) presented generalized EOQ model for deteriorating items with shortages, in which both the demand rate and the holding cost were continuous function of time. Datta and Pal (1990) developed an infinite time horizon deterministic inventory system without shortage, which has a level-dependent demand rate up to a certain stock-level and a constant demand for the rest of the cycle. Pal et al. (1993) presented a deterministic inventory model assuming that the demand rate was stock-dependent and that the items deteriorate at a constant rate. The total profit over one production run is maximized by numerically solving two non – linear equations.

In the study of EOQ model, the effect of inflation cannot be ignored. In this direction Hou and Lin (2009) presented a cash flow oriented EOQ model with deteriorating items under permissible delay in payments. In this study, they also considered cash flow as part of their modeling formulation. Tripathi et al. (2010) extended Hou and Lin (2009) model by considering time–dependent demand rate. Liao et al. (2000) presented an inventory model with deteriorating items under inflation and permissible delay in payment presented. Liao et al. (2000) developed an inventory model for initial stock–dependent consumption rate when a delay in payment was permissible. Hou et al. (2006) developed an inventory model for deteriorating items with stock–dependent consumption rate and shortages under inflation and time discounting. Hou (2006) presented a finite planning horizon inventory model for deterioration items with stock–dependent consumption rate and shortages with the effect of inflation and time–value of money on replenishment policy. In this direction, Jaggi et al. (2007) presented a model retailer's optimal ordering policy under two-stage trade credit financing. They also developed an inventory model under two levels of trade credit policy by assuming the demand was a function of credit period offered by the retailer to the customers using discounted cash flow (DCF) approach. A DCF approach permits a proper recognition of the timing of cash flows connected with an inventory system under the trade credit. Dye et al. (2007) investigated inventory and pricing strategies for deteriorating items with shortages using a discounted cash flow approach. They found the optimal inventory and pricing strategies maximizing the net present value of the total profit over the infinite horizon. Chung and Liao (2009) developed an optimal ordering policy of EOQ model under trade credit depending on the ordering quantity from the DCF approach. They discussed the optimal order quantity of the EOQ model that is not only dependent on the inventory policy but also on firm credit policy using discounted cash- flow (DCF) approach and trade credit depending on the quantity ordered.

Researchers in the past have established their inventory lot–size models under trade credit financing by assuming that the demand rate is constant (Jaggi et al., 2011; Hsu, 2012; Roy et al., 2012). Recently, Teng et al. (2012) established an EOQ model with trade credit financing for non–decreasing demand and optimal solution and relevant managerial phenomena was also calculated. An EOQ model with delay in payments and time varying deterioration rate was discussed and developed by Sarkar (2012) where the retailers were allowed a trade-credit offer by suppliers to buy more items with different discount rates on the purchasing cost. Hung (2011) developed an inventory model with generalized type demand, deterioration and backorder rates. Hung (2011) extended their model from ramp type demand rate and Weibull deterioration rate to arbitrary demand rate and arbitrary deterioration rate in the consideration of partial backorder. Khanra et al. (2011) presented an EOQ model for a deteriorating item with time–dependent quadratic demand under permissible delay in payment. In this paper, an effort has been made to analyze an EOQ model for deteriorating item considering quadratic time dependent demand rate and permissible delay in payment. Sana (2010) formulated optimal selling price and lot size with time varying deterioration and partial backlogging. In this work, an EOQ model over an infinite time horizon for perishable item where demand is price dependent and partial backorder permitted is discussed. Deterministic inventory model for deteriorating items with trade credit financing and capacity constraints is developed by Liao and Huang (2010) and they presented an inventory model for optimizing the replenishment cycle time for a single deteriorating item under a permissible delay in payments and constraints on warehouse capacity.

In this paper, we consider the demand rate is time varying and holding cost is time–dependent for case 1; and demand rate and holding cost both are constant for case 2. The main objective of this paper is to obtain minimum total inventory cost (TIC), order quantity and corresponding order cycle for both cases. The remainder of the paper is organized as follows. Relevant notation and assumptions are given in the next section. This is followed by mathematical formulation in the section 3. Algorithm and numerical example is given in section 4 and 5, respectively followed by sensitivity analysis is in section 6. Finally, suggestions and concluding remarks are given in section 7.

2. Notations and Assumptions

The following notations are used throughout the manuscript:

k : ordering cost per order
λ_0 : constant annual demand rate
I(t) : on-hand inventory level at time 't'
h : holding cost of the item for case 2
h(t) : time dependent holding cost of the item at time t, $h(t) \equiv h.t$
T : cycle time
β : demand parameter indicating elasticity, $0 < \beta < 1$
R(t) : time varying demand i.e. $R(t) = \lambda_0 t^{-\beta}$, for case 1 $\lambda_0 > 0$, $0 < \beta < 1$, $0 \le t \le T$
T* : optimal cycle time foe case 1
T** : optimal cycle time foe case 2
TIC : total inventory cost per cycle
TIC* : optimal total inventory cost per cycle for case 1
TIC** : optimal total inventory cost per cycle for case 2
TIC_1* : optimal total inventory cost per cycle for case 1 as $\beta \to 0$
Q : ordering quantity
Q_1* : optimal ordering quantity for case 1
Q_2* : optimal ordering quantity for case 2
Q_{11}* : optimal ordering quantity as $\beta \to 0$

In addition, the following assumptions are being made to develop aforesaid model:

1. The demand rate R(t) is decreasing function of time with increase of 'β' for case 1
2. The demand λ_0 rate is constant for case 2
3. The holding cost is time dependent and holding cost parameter 'h' i.e. $h(t) \equiv h.t$
4. Shortages are not allowed
5. The inventory system under consideration deals with single item

6. The planning horizon in infinite and lead time is zero
7. The demand rate (for case 1) R(t) is decreasing function of time is expressed as

$R(t) = \lambda_0 t^{-\beta}, \lambda_0 > 0, \quad 0 < \beta < 1, \quad 0 \le t \le T$ (1)

3. Mathematical Formulation

The objective is to minimize the total inventory cost (TIC) per unit time, which contains two components: (a) the ordering cost and (b) the holding cost. The ordering cost per unit time is (k/T), since one order is made per cycle. The total holding cost per cycle is the integral of the product of the holding cost h(t) and inventory level I(t) over the whole cycle 'T'.

$$TIC = \frac{k}{T} + \frac{1}{T}\int_0^T h(t).I(t)dt \tag{2}$$

Since the demand rate is equal to the rate of inventory level decrease, the rate of change of inventory level is governed by the following differential equation:

$$\frac{dI(t)}{dt} = -\lambda_0 t^{-\beta}, \lambda_0 > 0, \quad 0 < \beta < 1, \quad 0 \le t \le T \tag{3}$$

The on-hand inventory level at time 't' I(t), can be evaluated on solving (3) with the initial condition I(T) = 0, we obtain

$$I(t) = \frac{\lambda_0}{(1-\beta)}\left(T^{1-\beta} - t^{1-\beta}\right) \tag{4}$$

and the order quantity is

$$Q = \frac{\lambda_0 T^{1-\beta}}{1-\beta}, \quad 0 < \beta < 1 \tag{5}$$

From (3), we obtain

$$T = \left\{\frac{(1-\beta)Q}{\lambda_0}\right\}^{1/(1-\beta)} \tag{6}$$

3.1. Case 1. Time dependent demand rate and Time dependent holding cost

In this case, holding cost *h* is assumed to be an increasing step function of storage time. The holding cost depends on the length of the storage used in this case. The total inventory cost per unit time is expressed as

$$TIC = \frac{k}{T} + \frac{1}{T}\int_0^T h.t.I(t)dt = \frac{k}{T} + \frac{h\lambda_0 T^{2-\beta}}{2(3-\beta)} \tag{7}$$

Using Eq. (6) in Eq. (7) yields,

$$TIC = kQ^{-1/(1-\beta)}\left(\frac{\lambda_0}{1-\beta}\right)^{1/(1-\beta)} + \frac{h\lambda_0}{2(3-\beta)}Q^{(2-\beta)/(1-\beta)}\left(\frac{1-\beta}{\lambda_0}\right)^{(2-\beta)/(1-\beta)} \tag{8}$$

$$\frac{d(TIC)}{dQ} = -\frac{kQ^{-(2-\beta)/(1-\beta)}}{(1-\beta)}\left(\frac{\lambda_0}{1-\beta}\right)^{1/(1-\beta)} + \frac{h\lambda_0(2-\beta)}{2(3-\beta)(1-\beta)}Q^{1/(1-\beta)}\left(\frac{1-\beta}{\lambda_0}\right)^{(2-\beta)/(1-\beta)} \tag{9}$$

$$\frac{d^2(TIC)}{dQ^2} = \left[\frac{k(2-\beta)Q^{-(3-2\beta)/(1-\beta)}}{(1-\beta)^2}\left(\frac{\lambda_0}{1-\beta}\right)^{1/(1-\beta)} + \frac{h\lambda_0(2-\beta)}{2(3-\beta)(1-\beta)^2}Q^{\beta/(1-\beta)}\left(\frac{1-\beta}{\lambda_0}\right)^{(2-\beta)/(1-\beta)}\right] > 0 \tag{10}$$

The optimal (minimum) Q =Q* is obtained by solving $\frac{d(TIC)}{dQ}=0$, for Q, we obtain

$$Q = Q^* = \frac{\lambda_0^{2/(3-\beta)}}{1-\beta}\left\{\frac{2k(3-\beta)}{h(2-\beta)}\right\}^{(1-\beta)/(3-\beta)} \tag{11}$$

If $\beta \rightarrow 0$, the optimal Q = Q*= Q_{11}* reduces to

$$Q = Q_1^* = \lambda_0^{2/3}\left(\frac{3k}{h}\right)^{1/3} \tag{12}$$

and TIC = TIC_1* = $\frac{k}{T}+\frac{h\lambda_0 T^2}{6}$ (13)

3.2 Case 2. Constant demand rate and constant holding cost

The objective is to minimize the total inventory cost (TIC) per unit time. The total inventory cost (TIC) contains the same components as stated in case 1. Since the demand rate is equal to the rate of inventory level decrease, we can describe I(t) by the following differential equation:

$$\frac{dI(t)}{dt} = -\lambda_0 \quad , \lambda_0 > 0 \quad , 0 \le t \le T \tag{14}$$

The solution of (13) with I(T) = 0, is given by

$$I(t) = \lambda_0 (T-t), 0 \le t \le T \tag{15}$$

The order quantity Q = I(0) = $\lambda_0 T$ (16)

Thus, $T = \frac{Q}{\lambda_0}$. (17)

In this case, the TIC per unit time can be expresses as

$$TIC = \frac{k}{T} + \frac{1}{T}\int_0^T h.I(t)dt \tag{18}$$

Substituting I(t) from Eq. (15) into Eq. (18) yields,

$$TIC = \frac{k}{T} + \frac{1}{T}\int_0^T h.\lambda_0 (T-t)dt = \frac{k}{T} + \frac{h\lambda_0 T}{2} \tag{19}$$

Substituting the values of T from Eq. (17) into Eq. (19) yields,

$$TIC(Q) = \frac{k\lambda_0}{Q} + \frac{hQ}{2} \tag{20}$$

Differentiating (20) w.r.t. ‘Q’ two times, we obtain

$$\frac{dTIC}{dQ} = -\frac{k\lambda_0}{Q^2} + \frac{h}{2} \text{ and } \frac{d^2TIC}{dQ^2} = \frac{2k\lambda_0}{Q^3} > 0 \tag{21}$$

The optimal (minimum) Q = Q_2* is obtained by solving $\frac{d(TIC)}{dQ}=0$ from (21) for Q, we obtain

$$Q = Q_2^* = \sqrt{\frac{2k\lambda_0}{h}} \tag{22}$$

4. Numerical Examples

Example 1: Let λ_0 = 500 units/ year, k = \$ 400 per order, β = 0.1.

Table 1

Values of optimal T = T*, Q = Q_1* and TIC = TIC* for different values of 'h'

h	Q = Q_1*	T = T*	TIC = TIC*	h	Q = Q_1*	T = T*	TIC = TIC*
10	358.693	0.615012	992.705	37	238.993	0.391699	1558.661
11	348.239	0.595129	1025.873	38	237.023	0.388113	1573.061
12	338.961	0.577538	1057.119	39	235.120	0.384653	1587.214
13	330.645	0.561816	1086.703	40	233.280	0.381310	1601.132
14	323.127	0.547640	1114.831	41	231.499	0.378076	1614.823
15	316.282	0.534765	1141.672	42	229.774	0.374947	1628.297
16	310.010	0.522996	1167.364	43	228.102	0.371917	1641.563
17	304.232	0.512176	1192.025	44	226.481	0.368982	1654.628
18	298.883	0.502180	1215.752	45	224.907	0.366133	1667.914
19	293.909	0.492903	1238.631	46	223.378	0.363369	1680.186
20	289.268	0.484263	1260.734	47	221.892	0.360684	1692.692
21	284.921	0.476184	1282.124	48	220.447	0.358075	1705.026
22	280.837	0.468606	1302.857	49	219.041	0.355538	1717.192
23	276.989	0.461477	1322.982	50	217.672	0.353070	1729.196
24	273.355	0.454755	1342.541	51	216.338	0.350670	1741.044
25	269.914	0.448399	1361.573	52	215.038	0.348326	1752.741
26	266.648	0.442374	1380.112	53	213.771	0.346047	1764.292
27	263.543	0.436654	1398.190	54	212.534	0.343823	1775.700
28	260.585	0.431212	1415.835	55	211.327	0.341654	1786.971
29	257.763	0.426027	1433.071	56	210.149	0.339538	1798.109
30	255.065	0.421075	1449.922	57	208.997	0.337471	1809.117
31	252.483	0.416341	1466.409	58	207.873	0.335455	1820.000
32	250.007	0.411807	1482.551	59	206.773	0.333483	1830.759
33	247.631	0.407461	1498.366	60	205.697	0.331555	1841.400
34	245.347	0.403287	1513.870	65	200.650	0.322529	1892.934
35	243.150	0.399277	1529.078	70	196.088	0.314392	1941.928
36	241.034	0.395418	1544.004	80	188.128	0.300244	2033.434

Table 2

Values of optimal T = T**, Q = Q_2* and TIC = TIC** for different values of 'h' for example 2 (Case 2)

h	Q = Q_2*	T = T**	TIC =TIC**	h	Q = Q_2*	T = T**	TIC = TIC**
1	632.456	1.264911	632.456	36	105.409	0.210818	3794.733
5	282.843	0.565685	1414.214	37	103.975	0.207950	3847.0768
10	200.000	0.400000	2000.000	38	102.598	0.205200	3898.718
11	190.693	0.381385	2097.618	39	101.274	0.202548	3949.684
12	182.574	0.365148	2190.890	40	100.000	0.200000	4000.000
13	175.412	0.350823	2280.351	41	98.773	0.197546	4049.691
14	169.031	0.338062	2366.432	42	97.590	0.195180	4098.780
15	163.299	0.326599	2449.490	43	96.449	0.192897	4147.288
16	158.114	0.316228	2529.822	44	95.346	0.190693	4195.235
17	153.393	0.306786	2607.681	45	94.281	0.188562	4242.641
18	149.071	0.298142	2683.282	46	93.250	0.186501	4289.522
19	145.095	0.290191	2756.810	47	92.253	0.184506	4335.897
20	141.421	0.282843	2828.427	48	91.287	0.182574	4381.780
21	138.013	0.276026	2898.275	49	90.351	0.180702	4427.189
22	134.840	0.269680	2966.479	50	89.443	0.178885	4472.136
23	131.876	0.263752	3033.150	51	88.561	0.177123	4516.636
24	129.099	0.258199	3098.387	52	87.706	0.175412	4560.702
25	126.491	0.252982	3162.278	53	86.874	0.173749	4604.346
26	124.035	0.248069	3224.903	54	86.0663	0.172133	4647.580
27	121.716	0.243432	3286.335	55	85.280	0.170561	4690.416
28	119.523	0.239046	3346.640	56	84.515	0.169031	4732.864
29	117.444	0.234888	3405.877	57	83.771	0.167542	4774.934
30	115.470	0.230940	3464.102	58	83.045	0.166091	4816.638
31	113.592	0.227185	3521.363	59	82.339	0.164677	4847.983
32	111.803	0.223607	3577.709	60	81.650	0.163299	4898.979
33	110.096	0.220193	3633.180	65	78.446	0.156893	5999.018
34	108.465	0.216930	3687.818	70	75.593	0.151186	5291.499
35	106.904	0.213809	3741.657	80	70.711	0.141421	5656.861

5. Sensitivity Analysis

To find sensitivity analysis, the effect of parameters 'h', 'k' and 'β' on the optimal solution, the set of values of 'h', 'k' and 'β' are assumed to as h = 60 ,55 , 50, 45, 40 , 35, 'k'= 410 , 420, 430 , 440, 450, 460, 470, and 480, and 'β'= 0.2,0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9.

Meanwhile the other parameter values follows those data mentioned above in the numerical example 1. The results of sensitivity analysis are given in Tables 3, 4, and 5.

(i) Sensitivity Analysis: (for case 1).

Table 3
Variation of optimal solution of Q = Q_1*, T = T* and TIC = TIC* with the variation of 'h' and 'β', keeping all the parameters same as in Example 1.

h ↓	β→	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
60	Q	251.757	314.687	403.868	536.492	747.455	1119.077	1900.104	4338.992
	T	0.320905	0.310055	0.299021	0.287824	0.276495	0.265076	0.253622	0.242208
	TIC	1938.962	2048.974	2173.765	2316.230	2480.018	2669.768	2891.445	3155.787
55	Q	258.094	321.866	412.059	545.910	758.374	1131.850	1915.193	4357.007
	T	0.331034	0.320209	0.309196	0.298018	0.286704	0.275296	0.263853	0.252454
	TIC	1879.633	1983.996	2102.222	2237.002	2391.716	2570.654	2779.319	3024.850
50	Q	265.219	329.919	421.223	556.416	770.517	1146.009	1931.86	4376.827
	T	0.342492	0.331715	0.320742	0.309599	0.298319	0.286944	0.275536	0.264176
	TIC	1816.728	1915.182	2091.508	2153.323	2298.596	2466.306	2661.482	2890.633
45	Q	273.324	339.055	431.590	568.265	784.167	1161.867	1950.452	4398.841
	T	0.355629	0.344915	0.334006	0.322925	0.311707	0.300395	0.289052	0.277768
	TIC	1749.637	1841.886	1946.073	2064.459	2199.870	2305.882	2537.0244	2749.103
40	Q	282.679	349.568	443.482	581.811	799.712	1179.854	1971.449	4423.582
	T	0.370909	0.360294	0.349485	0.338504	0.327385	0.316178	0.304949	0.293793
	TIC	1677.565	1763.565	1859.881	1960.045	2094.515	2238.268	2452.032	2599.234
35	Q	293.672	361.882	457.360	597.558	817.710	1200.584	1995.527	4451.800
	T	0.389026	0.378551	0.367902	0.357076	0.346117	0.335078	0.324032	0.313081
	TIC	1599.440	1678.182	1766.772	1867.018	1981.163	2112.021	2263.151	2439.103
30	Q	306.895	376.638	473.923	616.268	838.990	1224.968	2023.688	4484.599
	T	0.411043	0.400804	0.390375	0.379786	0.369077	0.358305	0.347550	0.336927
	TIC	1513.764	1583.152	1665.067	1755.374	1857.914	1975.108	2110.003	2266.474

(ii) Sensitivity Analysis: (for case 1)

Table 4
Variation of optimal solution of Q = Q*, T* and TIC* with the variation of 'h' and 'k', keeping all the parameters same as in Example 1.

h ↓	k→	410	420	430	440	450	460	470	480
60	Q	207.279	208.835	210.366	211.872	213.355	214.815	216.254	217.672
	T	0.33439	0.337181	0.339928	0.342633	0.345299	0.347926	0.350516	0.353069
	TIC	1871.435	1901.212	1930.750	1960.053	1989.123	2017.974	2046.608	2075.037
55	Q	212.953	214.552	216.124	217.672	219.195	220.695	222.173	223.630
	T	0.344576	0.347451	0.350282	0.353069	0.355816	0.358523	0.361192	0.363823
	TIC	1787.094	1845.016	1873.679	1902.117	1930.329	1958.327	1986.114	2013.702
50	Q	219.346	220.993	222.612	224.206	225.775	227.321	228.843	230.343
	T	0.356089	0.359060	0.361985	0.364866	0.367704	0.370502	0.373260	0.375979
	TIC	1757.397	1785.364	1813.102	1840.618	1867.920	1895.011	1921.901	1948.596
45	Q	226.637	228.338	230.012	231.658	233.280	234.876	236.449	237.999
	T	0.369264	0.372345	0.375378	0.378366	0.381309	0.384210	0.387070	0.389890
	TIC	1694.695	1721.663	1748.413	1774.946	1801.274	1827.399	1853.330	1879.072
40	Q	235.074	236.838	238.575	240.283	241.965	243.621	245.252	246.860
	T	0.38457	0.387779	0.390938	0.394049	0.397115	0.400136	0.403114	0.406052
	TIC	1627.245	1653.139	1678.823	1704.302	1729.580	1754.666	1779.566	1804.281
35	Q	245.021	246.860	248.669	250.450	252.203	253.929	255.629	257.305
	T	0.402692	0.406052	0.412618	0.412618	0.415828	0.418991	0.422110	0.425186
	TIC	1554.016	1578.746	1627.605	1627.605	1651.746	1675.705	1699.482	1723.086
30	Q	257.027	258.957	260.855	262.722	264.561	266.372	268.156	269.914
	T	0.424676	0.428219	0.431708	0.435144	0.438529	0.441865	0.445154	0.448398
	TIC	1473.569	1497.020	1520.277	1543.349	1566.241	1588.958	1611.506	1633.887

(iii)Sensitivity Analysis: (for case 2).

Table 5
Variation of optimal solution of Q = Q_2*, T = T** and TIC = TIC** with the variation of 'h' and 'k', keeping all the parameters same as in Example 1.

h ↓	k→	410	420	430	440	450	460	470	480
60	Q	82.664	83.666	84.656	85.635	86.603	87.560	88.506	89.443
	T	0.165328	0.167332	0.169312	0.171270	0.173205	0.175119	0.177012	0.178885
	TIC	4959.838	5019.960	5079.375	5138.090	5196.154	5253.570	5310.371	5366.570
55	Q	86.340	87.386	88.421	89.443	90.453	91.453	92.442	93.420
	T	0.172679	0.174773	0.176841	0.178885	0.180907	0.182906	0.184883	0.186840
	TIC	4748.690	4806.240	4863.126	4919.356	4974.934	5029.908	5084.293	5138.090
50	Q	90.554	91.652	92.736	93.808	94.868	95.917	96.954	97.980
	T	0.181108	0.183303	0.185472	0.187617	0.189737	0.191833	0.193907	0.195959
	TIC	4527.689	4582.576	4636.814	4690.411	4743.412	4795.835	4847.682	4898.982
45	Q	95.452	96.609	97.753	98.883	100.000	101.105	102.198	103.280
	T	0.190904	0.193218	0.195505	0.197765	0.200000	0.202210	0.204396	0.206559
	TIC	4295.349	4347.417	4398.864	4449.722	4500.000	4549.725	4598.914	4647.580
40	Q	101.242	102.470	103.682	104.881	106.066	107.238	108.397	109.544
	T	0.202484	0.204939	0.207364	0.209762	0.212132	0.214476	0.216795	0.219089
	TIC	4049.697	4098.780	4147.292	4195.233	4242.641	4289.523	4335.895	4381.781
35	Q	108.233	109.544	110.841	112.122	113.389	114.642	115.882	117.108
	T	0.216465	0.219089	0.221682	0.224245	0.226779	0.229285	0.231763	0.234216
	TIC	3788.140	3834.058	3879.432	3924.281	3968.624	4012.481	4055.864	4098.780
30	Q	116.904	118.322	119.722	121.106	122.474	123.828	125.166	126.491
	T	0.233809	0.236643	0.239444	0.242212	0.244949	0.247656	0.250333	0.252982
	TIC	3507.136	3549.649	3591.655	3633.181	3674.234	3714.833	3754.998	3794.735

Based on the results we can make the following conclusions,

(a) Based on the observations found from Table 1 we can conclude that the optimal total inventory cost TIC* is directly associated with holding cost whereas the optimal order quantity Q* and optimal cycle time T* are inversely associated with holding cost.

(b) Based on the results of Table 2 we can conclude that the optimal total inventory cost TIC** is directly associated with holding cost 'h' whereas the optimal order quantity Q_2* and optimal cycle time T** is inversely associated with holding cost 'h'.

(c) The observation found from the Table 3 are as follows,

(i) Any increase on 'β' results to an increase in optimal order quantity Q*, TIC*, whereas any decrease on optimal cycle time T* does not change holding cost 'h'.

(ii) Any increase on holding cost 'h' increases the optimal order quantity Q*, optimal cycle time T*, whereas it decreases the total inventory cost TIC*.

(d) From Table 4, it can be easily seen that:

(i) An increase on ordering cost 'k' results to an increase on optimal order quantity Q*, optimal cycle time T* and optimal total inventory cost TIC*, keeping holding cost 'h' constant.

(ii) An increase on holding cost 'h' results to an increase on optimal order quantity Q*, optimal cycle time T*, whereas it decreases the optimal total inventory cost TIC*.

(e) From Table 5, we observe that:

(i) An increase on ordering cost 'k' results to an increase on optimal order quantity Q_2*, optimal cycle time T**, optimal total inventory cost TIC**, keeping holding cost 'h' constant.

(ii) An increase on holding cost 'h' results to an increase of optimal order quantity Q_2*, optimal cycle time T**, whereas it decreases the optimal total inventory cost TIC**, keeping ordering cost 'k' constant.

6. Conclusion and Future Research

In this paper, we have presented a new method for inventory system with time dependent demand and time dependent holding cost by considering two cases. The first case considers constant demand and holding cost is considered constant for the second case. Simple common optimization algorithm has been developed to find optimal solution and the proposed model has been examined using some numerical examples. The preliminary results indicate that the total inventory cost increases when we increase the holding cost 'h'. It has also observed from the sensitivity analysis that the total inventory cost increases with the increase of ordering cost 'k' and 'β' , whereas total inventory cost decreases with the increase of holding cost 'h'

The model presented in this study can be extended in different ways. For instance, the model can be extended for variable ordering costs and non-instantaneous receipt of orders. This model can also extend for deteriorating items as well as shortages, freight charges etc.

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