International Journal of Industrial Engineering Computations 4 (2013) 547–558

Contents lists available at GrowingScience

International Journal of Industrial Engineering Computations

homepage: www.GrowingScience.com/ijiec

Multi-objective optimization of cutting parameters in turning using grey relational analysis

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C H R O N I C L E

Article history:
Received March 16 2013
Received in revised format May 9 2013
Accepted May 30 2013
Available online June 3 2013

Keywords:
UD-GFRP
Grey relational analysis
Taguchi method
Optimization
Polycrystalline diamond cutting tool

A B S T R A C T

This study presents optimization of performance characteristics in unidirectional glass fiber reinforced plastic composites using Taguchi method and Grey relational analysis. Performance characteristics such as surface roughness and material removal rate are optimized during rough cutting operation. Process parameters including tool nose radius, tool rake angle, feed rate, cutting speed, cutting environment and depth of cut are investigated using mixed L18 orthogonal array. Grey relation analysis is used to optimize the parameters and Principal Component Analysis is used to find the relative significance of performance characteristics. Depth of cut is the factor, which has great influence on surface roughness and material removal rate, followed by feed rate. The percentage contribution of depth of cut is 54.399% and feed rate is 5.355%.

1. Introduction

Environmental FRPs are an important class of materials in advanced structural applications due to their light weight, high modulus and specific strength. In addition, many fiber reinforced plastic composites boast excellent fatigue strength/weight ratios (Davim & Mata, 2004). The fiber reinforced plastic industry, which is one of the fastest growing industries in the world concentrates on the single piece design of complex shapes. However, there are events when the best design calls for the manufacture of a product in parts prior to assembly. The FRP machining methods now in use, utilize the existing machines and tools developed for machining conventional materials. Machines and tools exclusively designed for FRP machining are yet to be developed (Santhanakrishnan, 1989). The machining of FRP is different from that of metal working in many respects, because the behaviour is not only inhomogeneous, but also depends on the fiber and matrix properties, fiber orientation and type of weave (Konig, 1985). It brings about many undesirable results, such as rapid tool wear, rough surface finish, defective sub surface layer with cracks and delamination. Glass fiber reinforced plastic(GFRP), an advanced composite material, is widely used in variety of applications including aircrafts, hose buildings, storage tanks, robots, machine tools and piping. Glass fiber reinforced plastics are extremely

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doi: 10.5267/j.ijiec.2013.06.001

abrasive, thus proper selection of the cutting tool and cutting parameters is very important for a perfect machining process.

Grey theory can provide a solution to a system in which the model is unsure or the information is incomplete (Deng, 1990). It also provides an efficient solution to the uncertainty, multi-input and discrete data problem. Fu et al. (2012) investigated the optimization problem of cutting parameters in high-speed milling on NAK80 mold steel. An experiment based on the technology of Taguchi was performed. Three parameters such as cutting speed, feed rate and depth of cut were selected to minimize the cutting forces. The optimum cutting parameters were obtained by the grey relational analysis. The principal component analysis was applied to evaluate the weights so that their relative significance could be described properly and objectively. Huang and Liao (2003) applied grey relational analysis to determine the optimal selection of machining parameters for the Wire Electrical Discharge Machining (Wire-EDM) process. Kao and Hocheng (2003) developed the application of the grey relational analysis for optimizing the electro polishing of 316L stainless steel with multiple performance characteristics. The processing parameters (temperature, current density, and electrolyte composition) were optimized for multiple performance characteristics (surface roughness and passivating strength).

Sadasiva Rao et al. (2012) work was focused to study the effect of process parameters such as speed, feed and depth of cut and approach angle of the cutter on cutting force, tool life and surface roughness in face milling of Inconel 718. The experiments were designed based on L9 orthogonal array and carried out under dry conditions. Grey relational analysis was used to optimize the multi performance characteristics to minimize the cutting force and surface roughness and maximize the tool life criteria. Refaie et al. (2010) used Taguchi method grey analysis to determine the optimal combination of control parameters in milling. The measures of machining performance were material removal rate and surface roughness. Wang et al. (2006) utilized a hybrid algorithm combining Genetic algorithm (GA) and the Simulated Annealing (SA) to optimize multicriteria high speed milling process.

Jean et al. (2004, 1999) solved the optimization problem with multiple performance characteristics using grey relational analysis. The corresponding weighting value was calculated using fuzzy logics. Lua et al. (2009) found optimization design of the cutting parameters for rough cutting process in high-speed end milling on SKD61 tool steel. The major characteristics indexes for performance selected to evaluate the processes were tool life and metal removal rate and the corresponding cutting parameters were type of milling, spindle speed, feed per tooth and radial depth of cut and axial depth of cut. In this study, Grey relational grade as performance index was specially adopted to determine the optimal combination of cutting parameters. The principal component analysis was applied to evaluate the weighting values corresponding to various performance characteristics so that their relative importance could be properly and objectively described.

Chakradhar and Venu Gopal (2011) investigated the parametric optimization of process parameters for Electrochemical machining of EN-31 steel using grey relation analysis. The process parameters considered were electrolyte concentration, feed rate and applied voltage and were optimized with considerations of multiple performance characteristics including material removal rate, over cut, cylindricity error and surface roughness. Tarang et al. (2002) reported the use of fuzzy logic in the Taguchi method to optimize the submerged arc welding process with multiple performance characteristics. Tsao (2009) proposed the application of Grey–Taguchi method to optimize the milling parameters of aluminium alloy. It was concluded that the grey-Taguchi method is very suitable for solving the flank wear and surface roughness quality problem in milling A6061P-T651 aluminium alloy. In attempt to offer a more adequate treatment to the optimization problems with multiple correlated responses, the Principal Component Analysis (PCA) was considered as a good alternative Wang and Du & Rossi (2000, 2001).

This paper investigates optimization problem of the cutting parameters in turning of unidirectional glass fiber reinforced plastic (UD-GFRP) composite rods. The surface roughness and material removal rate are the response variables. The experiments are performed using Taguchi L_{18} orthogonal array. The grey relational analysis is used to find the optimum process parameters. Principal component analysis is used to find the weight corresponding to different performance characteristics.

2. Experimental Procedure

2.1 *Work Material*

The work material used for the present investigation is unidirectional glass fiber reinforced plastic (UD-GFRP) composite rods. The workpiece material having size of 840 mm in length with 42 mm diameter is used. The material used for the experiments is pultruded unidirectional glass fiber reinforced plastics composite having E-glass as fiber and epoxy as resin. The properties of material used are shown in Table 1.

Table 1
Properties of UD – GFRP

Sr. No	*Particular*	*Value*	*Unit*
1	*Glass Content (by weight)*	*75±5*	*%*
2	*Epoxy Resin content (by weight)*	*25±5*	*%*
3	*Reinforcement, unidirectional*	*'E' Glass Roving*	*----*
4	*Water absorption*	*0.07*	*%*
5	*Density*	*1.95-2.1*	*gm/cc*
6	*Tensile Strength*	*6500*	*Kgf / cm.sq.*
7	*Compression Strength*	*6000*	*Kgf/ cm.sq.*
8	*Shear Strength*	*255 kgf*	*Kgf / cm.sq.*
9	*Modulus of elasticity*	*3200*	*10 Kg/ cm.sq.*
10	*Thermal Conductivity*	*0.30*	*Kcal /Mhc°*
11	*Weight of Rod 840 mm*	*2.300*	*Kgs*
12	*Electrical strength (Radial):*	*3.5*	*KV / mm*
13	*Working Temperature Class:*	*Class 'F' (155)*	*Centigrade*
14	*Martens Heat Distortion*	*210*	*Centigrade*
15	*Temperature* *Test in oil : (1) At 20 C:* *(2) At 100 C:*	*20 KV/cm* *20 KV/cm (50 KV / 25 mm)*	*KV/cm*

Table 2
Properties of PCD tool

Clearance angle	*7º*
Grade	*M10*
Density	*3.80-4.50 g/cm³*
Hardness	*1600 Vickers kg/mm2*
Transverse Rupture strength	*1200-1700 N/mm2*
Thermal conductivity	*150-550 W/mK*
Compressive Strength	*7000-8000 N/mm2*
Thermal Expansion coefficient	*3.2-4.6 10%C*
Young's modulus	*800-900 GPa*
Cutting edge inclination angle Top	*7º*
Front Clearance	*10º*
Tool rake angle	*-6°, 0°, +6°*
Tool nose radius	*0.4 mm, 0.8 mm*

2.2 *Experimental setup*

The experimental work is carried out on a high-precision NH–22 HMT lathe of 11 kW spindle power with maximum speed 3000 rpm. The cutting tool used for the experimentation is polycrystalline diamond tool of different tool rake angle and tool nose radius. The detail of the PCD tool is shown in

Table 2. The surface roughness of the turned surface is measured using a Tokyo Seimitsu Surfcom 130A type instrument. The instrument is set to a cutoff length of 0.8 mm with a transverse length of 4 mm. A tool holder SVJCR steel EN47 is used during the turning operation.

2.3 *Process Parameters of Turning Operation*

In order to identify the process parameters that affect the quality of the turned parts, an Ishikawa cause-effect diagram is constructed as shown in Fig. 1. The Ishikawa cause-effect diagram depicts that the following process parameters may affect the quality of the turned parts:

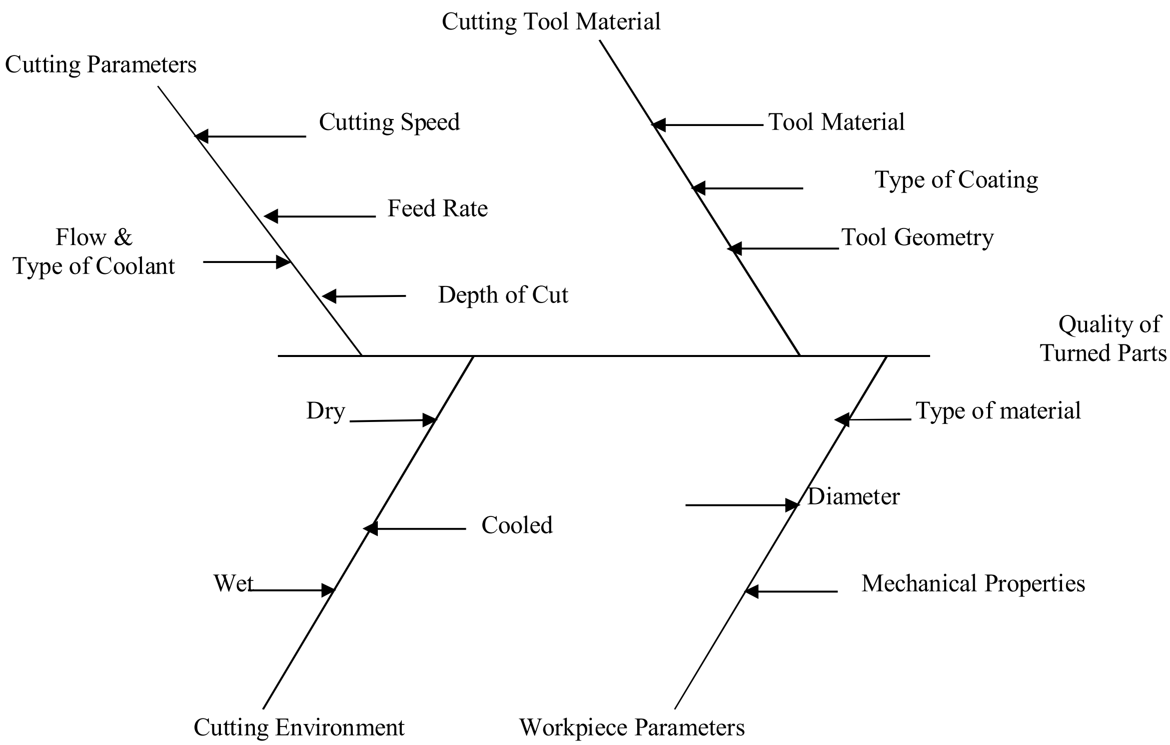

Fig. 1. Ishikawa Cause-Effect Diagram of a Turning Process

- Cutting parameters: cutting speed, feed rate, depth of cut
- Environment parameters: dry, wet, cooled
- Cutting tool parameters: tool geometry, tool material
- Work piece material: metals, composite materials

2.4 Selection of the Machining Parameters and their Levels

In this study, the experimental plan has tool nose radius, tool rake angle, feed rate, cutting speed, cutting environment (dry, wet and cooled) and depth of cut as the controllable variables. On the basis of preliminary experiments conducted by using one variable at a time approach, the feasible range for the machining parameters is selected Table 3 shows the cutting parameters and their levels considered for the experiments. Table 4 shows the L_{18} orthogonal array employed for the experimentation. The Taguchi's mixed level design is selected as it is decided to keep two levels of tool nose radius. The rest five parameters are studied at three levels – denoted by 1, 2 and 3, respectively. Two level parameter has 1 DOF, and the remaining five three level parameters have (5×2=10) DOF, i.e., the total DOF required is 11 [= (1*1+ (5*2)]. Orthogonal array chosen is L_{18} ($2^1 * 3^7$) OA with 17 [= 18-1] DOF. Parameters are assigned using linear graphs. The unassigned columns are treated as error.

Table 3
Process Parameters with Different Operating Levels

Input Parameters	Levels		
	Level 1	Level 2	Level 3
Tool nose Radius / mm	0.4	0.8	NIL
Tool Rake angle / Degree	-6	0	+6
Feed rate / (mm/rev.)	0.05	0.1	0.2
Cutting speed / (m/min.) & rpm	(55.42) 420	(110.84) 840	(159.66) 1210
Cutting environment	Dry (1)	Wet (2)	Cooled (3)
Depth of cut / mm	0.2	0.8	1.4

Table 4
Orthogonal Array L_{18} of Taguchi along with Assigned Value

Expt. No.	Tool Nose Radius /mm (A)	Tool Rake Angle / Degree (B)	Feed Rate / (mm/rev.) (C)	Cutting Speed / (m/min) & rpm (D)	Cutting Environment (E)	Depth of Cut/ mm (F)
1	0.4	-6°	0.05	(55.42) 420	Dry (1)	0.2
2	0.4	-6°	0.1	(110.84) 840	Wet (2)	0.8
3	0.4	-6°	0.2	(159.66) 1210	Cooled (3)	1.4
4	0.4	0°	0.05	(55.42) 420	Wet (2)	0.8
5	0.4	0°	0.1	(110.84) 840	Cooled (3)	1.4
6	0.4	0°	0.2	(159.66) 1210	Dry (1)	0.2
7	0.4	+6°	0.05	(110.84) 840	Dry (1)	1.4
8	0.4	+6°	0.1	(159.66) 1210	Wet (2)	0.2
9	0.4	+6°	0.2	(55.42) 420	Cooled (3)	0.8
10	0.8	-6°	0.05	(159.66) 1210	Cooled (3)	0.8
11	0.8	-6°	0.1	(55.42) 420	Dry (1)	1.4
12	0.8	-6°	0.2	(110.84) 840	Wet (2)	0.2
13	0.8	0°	0.05	(110.84) 840	Cooled (3)	0.2
14	0.8	0°	0.1	(159.66) 1210	Dry (1)	0.8
15	0.8	0°	0.2	(55.42) 420	Wet (2)	1.4
16	0.8	+6°	0.05	(159.66) 1210	Wet (2)	1.4
17	0.8	+6°	0.1	(55.42) 420	Cooled (3)	0.2
18	0.8	+6°	0.2	(110.84) 840	Dry (1)	0.8

3. Taguchi Method

Taguchi's technique allows us to study the variation of process and ultimately to optimise the process variability as well as target, using Signal-to-Noise ratio, which presents the ratio between response mean control factors effect and variation. The Taguchi method is very popular for solving optimization problems in the field of production engineering. The ratio depends on the quality characteristics of the product/process to be optimized. The standard S/N ratios generally used are as follows: - Nominal-the-Best (NB), lower-the-better (LB) and Higher-the-Better (HB). The optimal setting is the parameter combination, which has the highest S/N ratio (Ross, 1988).

Lower-the-better $-10\ Log\frac{1}{n}\sum y^2,$ (1)

Higher-the-better $-10Log\frac{1}{n}\sum\frac{1}{y^2},$ (2)

Nominal-the-best $\frac{S}{N} = 10log\frac{\bar{y}}{S_y^2},$ (3)

where n is the number of observations and y is the observed data.

4. Grey Relation Analysis

Grey relation is the certainty of association among things, or the uncertainty between system factors and the main behavioral factors (Wang et al., 2001). The grey relational analysis is primarily a quantitative analysis on dynamic process of system. It measures the degree of proximity according to similarity or difference among the development situations of factors (Fung, 2003). In order to optimize two machining characteristics simultaneously, GRA is utilized. After selecting process parameters and their ranges, experimental results are obtained using taguchi's design of experiment method. For multiple performance characteristics optimization using GRA, following steps are followed: (1) Conduction of experiments at different setting of parameters based on OA (2) Normalization of raw data of experimental results for all performance characteristic (3) Calculation of quality loss function (4) Calculation of grey relational coefficient (5) Principal component analysis to optimize the corresponding weighting value for each performance characteristics (6) Calculation of grey relational grade using weighting factor for performance characteristics

4.1 Data Normalization

It is the first step in the grey relational analysis. In a data sequence, the original data requires normalization to get a comparable sequence because of different scope and dimension. In this study, a linear normalization of surface roughness and material removal rate is performed in range of 0 to 1. A linear data preprocessing method for raw data can be expressed as

$$x_i^*(k) = \frac{max x_i(k) - x_i(k)}{max x_i(k) - min x_i(k)}, i = 1, 2, \ldots\ldots, m;\ k = 1, 2, \ldots\ldots, n \tag{4}$$

where m is the number of experiments, n is the number of response variables. Where $xi(k)$ is the original sequence of the surface roughness and material removal rate, $xi^*(k)$ is the comparable sequence after data normalization, max $x_i(k)$ and min $x_i(k)$ are the largest value and smallest value of $x_i(k)$. In this paper, m =18, n = 2 is taken.

4.2 Calculation of Quality Loss Function

Δ_{oi} (k) is called the quality loss function, which is the absolute value between the reference sequence $x_o^*(k)$ and the comparability sequence $x_i^*(k)$ as follows,

$$\Delta_{oi}(k) = x_o^*(k) - x_i^*(k) \tag{5}$$

4.3 Calculation of Grey Relational Coefficient

After normalization of the original sequence, the grey relational coefficient is calculated (Ho & Lin, 2003). It can be expressed as

$$\gamma\left(x_o^*(k), x_i^*(k)\right) = \frac{\Delta_{min} + \zeta.\Delta_{\max}}{\Delta_{\mathrm{oi}}(\mathrm{k}) + \zeta.\Delta_{\max}} \tag{6}$$

where ζ is the distinguishing coefficient and ζ ∈[0,1]. ζ is set at 0.5.

$$\Delta_{\min} = \min_{\forall_i} \min_{\forall_k} \Delta_{oi}(k)$$

$$\Delta_{\max} = \max_{\forall_i} \max_{\forall_k} \Delta_{oi}(k)$$

4.4 Calculation of Weights According to Principal Component Analysis

The procedure for finding the weights is denoted as follow

(a) Calculation of correlation coefficient

$$C_{ke} = \frac{Cov(x_i^*(j),\ x_i^*(k))}{\sigma x_i(j) \times \sigma x_i(k)}, \tag{7}$$

where j= 1, 2, …, n

k=1, 2, …, n

where $Cov(x_i^*(j),\ x_i^*(k))$ is the covariance of sequence $x_i^*(j)$ and $x_i^*(k)$, $\sigma x_i(j)$ and $\sigma x_i(k)$ are the standard deviations of sequence of $x_i^*(j)\ and\ x_i^*(k)$ respectively.

(b) Determination of Eigen value and Eigen vector

(c) Calculation of contribution of the performance characteristics to principal component. The Eigen value is arranged in descending order. Only the Eigen value greater than 1 is taken into consideration. Sequence of the Eigen vector corresponding to first principal component gives the contribution of the corresponding performance characteristics to the principal component. Square of elements of eigen vector gives the contribution of each performance characteristics.

4.5 Calculation of Grey Relational Grade

The grey relational grade represents the level of correlation between the reference sequence and Comparability sequence. The grey relational grade is a weighted average of the grey relational coefficients of multi-objective (Tosun & Pihtili, 2003). It is determined as

$$\Psi(x_o^*, x_i^*) = \sum_{k=1}^{n} \omega_k \gamma(x_o^*(k), x_i^*(k)), \tag{8}$$

where ω_k is the weight of the k^{th} performance characteristics and $\sum_{k=1}^{n} \omega_k = 1$.

5. Results and Discussion

Experiments are performed on turning machine according to L_{18} orthogonal array shown in Table 4. Experimental results are listed in Table 5. Table 6 shows the normalized data after preprocessing according to Eq. (4).

Table 5

Test Data Summary for Surface Roughness and Material Removal Rate

Expt. No.	Responses Raw Data Surface Roughness (µm)			Average Ra (µm)	Responses Raw Data Material Removal Rate (mm^3/sec)			Average MRR (mm^3/sec)
	R1	R2	R3		R1	R2	R3	
1	1.38	1.46	1.35	1.397	8.60	8.50	8.70	8.60
2	1.67	1.36	1.33	1.453	145.00	145.02	144.95	144.99
3	3.00	2.79	3.44	3.076	327.58	347.03	347.23	340.61
4	1.31	1.47	1.32	1.366	36.24	36.24	36.24	36.24
5	1.70	1.24	1.65	1.530	249.90	249.96	249.88	249.91
6	2.05	2.93	2.22	2.400	106.02	105.86	105.90	105.93
7	1.61	1.33	1.60	1.513	125.00	124.98	124.98	124.99
8	1.67	1.79	1.45	1.636	52.96	52.99	52.97	52.97
9	2.43	2.20	2.16	2.263	144.97	144.97	145.02	144.99
10	1.38	1.83	1.43	1.547	104.42	104.38	104.40	104.40
11	1.52	1.43	1.87	1.606	125.00	125.00	125.00	125.00
12	2.24	1.90	1.76	1.966	73.57	73.58	73.55	73.57
13	1.57	1.57	1.65	1.597	18.39	18.39	18.39	18.39
14	1.40	1.86	1.63	1.630	208.72	208.92	208.92	208.85
15	2.14	1.80	2.77	2.237	250.09	250.09	250.05	250.08
16	2.12	1.80	1.90	1.940	180.00	180.04	180.00	180.01
17	1.23	1.53	1.70	1.486	18.38	18.38	18.38	18.38
18	1.98	1.66	2.28	1.973	275.93	275.87	275.75	275.85

Table 6
Sequence after Data Preprocessing

No.	Surface Roughness, (μm)	MRR, (mm^3/sec.)
Reference Sequence	1.000	1.0000
Comparability Sequence		
1	0.0181	0
2	0.0509	0.4108
3	1.0000	1.0000
4	0	0.0833
5	0.0959	0.7268
6	0.6047	0.2932
7	0.0860	0.3506
8	0.1579	0.1336
9	0.5246	0.4108
10	0.1058	0.2885
11	0.1404	0.3506
12	0.3509	0.1957
13	0.1351	0.0295
14	0.1544	0.6031
15	0.5094	0.7273
16	0.3357	0.5163
17	0.0702	0.0295
18	0.3550	0.8049

Table 7 gives the quality loss function according to Equation 5. The grey relation coefficient is calculated according to Eq. (6) and it is shown in Table 8.

Table 7
Quality Loss Function

Comparability sequence	Surface Roughness, (μm)	Material Removal Rate, (mm^3/sec.)
No.	$\Delta_{oi}(1)$	$\Delta_{oi}(2)$
i=1	0.9819	1.0000
i=2	0.9491	0.5892
i=3	0	0
i=4	1.0000	0.9167
i=5	0.9041	0.2732
i=6	0.3953	0.7068
i=7	0.9140	0.6494
i=8	0.8421	0.8664
i=9	0.4754	0.5892
i=10	0.8942	0.7115
i=11	0.8596	0.6494
i=12	0.6491	0.8043
i=13	0.8649	0.9705
i=14	0.8456	0.3969
i=15	0.4906	0.2727
i=16	0.6643	0.4837
i=17	0.9298	0.9705
i=18	0.6450	0.1951

Table 8
Grey Relational Coefficients for 18 Comparability Sequence

Comparability Sequence, No.	Surface Roughness, (μm)	Material Removal Rate, (mm^3/sec.)
1	0.3374	0.3333
2	0.3450	0.4591
3	1.0000	1.0000
4	0.3333	0.3529
5	0.3561	0.6467
6	0.5585	0.4143
7	0.3536	0.4350
8	0.3726	0.3659
9	0.5126	0.4591
10	0.3586	0.4127
11	0.3678	0.4350
12	0.4351	0.3833
13	0.3663	0.3400
14	0.3716	0.5575
15	0.5047	0.6471
16	0.4294	0.5083
17	0.3497	0.3400
18	0.4367	0.7194

Table 9
Eigen Values, Eigen Vectors and Accountability Proportion

No.	1^{ST} Principal Component	2^{nd} Principal Component
Eigen values	1.7613	0.2387
Eigen vectors	-0.7071, 0.7071	0.7071, 0.7071
Accountability Proportion (AP)	0.88065	0.11935
Cumulative Accountability Proportion (CAP)	0.88065	1.000

Table 10
Contribution of Response Variables for the First Principal Component

Response variables	Contribution
Surface Roughness	0.5
Material Removal Rate	0.5

Table 11
Grey Relational Grades for 18 Comparability Sequence

Expt. No	Overall Grey Relational Grade	S/N
1	0.3353	-9.4900
2	0.4021	-7.9144
3	1.0000	0
4	0.3431	-9.2916
5	0.5014	-5.9963
6	0.4864	-6.2601
7	0.3943	-8.0835
8	0.3692	-8.6536
9	0.4859	-6.2700
10	0.3856	-8.2761
11	0.4014	-7.9285
12	0.4092	-7.7613
13	0.3532	-9.0408
14	0.4646	-6.6594
15	0.5759	-4.7931
16	0.4688	-6.5793
17	0.3448	-9.2474
18	0.5780	-4.7607

To find out the relative importance of each performance characteristics, the weights are found according to Principal component analysis. Correlation coefficient matrix is found out according to Equation 7. Eigen values and corresponding Eigen vector are calculated. Table 9 shows the eigen values, eigen vector, accountability proportion and Cumulative accountability proportion for two quality indicators according to Equation 8. Table 10 shows the contribution of surface roughness and material removal rate as 0.5 and 0.5 respectively. So the weights for surface roughness and material removal rate are considered as 0.5 each. Table 11 shows the overall grey relational grade and S/N ratio for eighteen experiments. The higher the value of grey relation grade, optimal is the corresponding factors combination. The S/N ratio for overall grey relational grade is calculated using higher the better criteria. It is clear from the experiments that experiment no. 3 has large value of grade. Therefore, it provides best combination for multiple performance characteristics. In order to separate out of effects of each process variable on grey relational grade at different levels using Taguchi methodology. Grey relational graph is plotted as shown in Fig. 2.

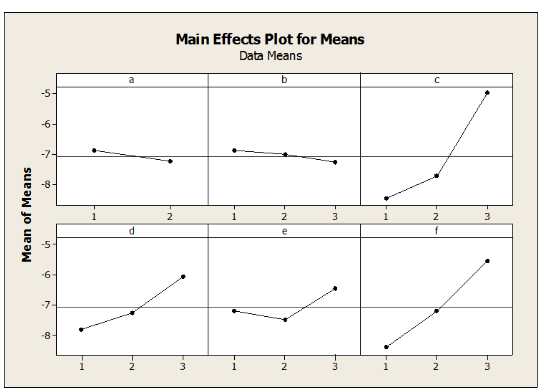

Fig. 2. Effects of Process Parameters on R_a and MRR (Raw Data)

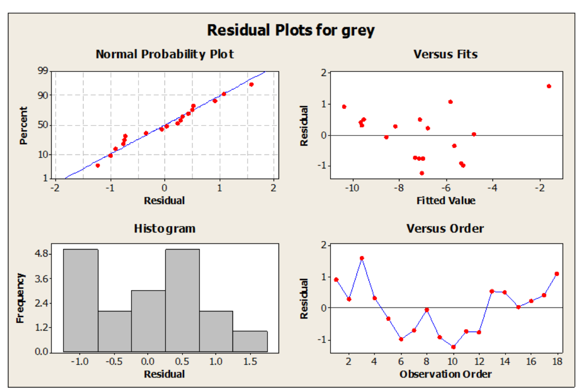

Fig. 3. Residual Plots for R_a and MRR (Raw Data)

Mean value of Grey relational grade is 0.46107. Basically, the larger the Grey relational grade, the better is the multiple performance characteristics. Combination of $A_1B_1C_3D_3E_3$ and F_3 showed larger value of Grey relational grade for factors A, B, C, D, E and F, respectively. Therefore, $A_1B_1C_3D_3E_3F_3$ is optimal parameter combination for two performance characteristics. However, significant contributions of process parameters still need to be known to predict optimal values of performance characteristics.

Residual plots for machining parameters (a) Normal probability plot of residuals for grey raw data (b) Residuals vs. the order of the data, (c) Plot of residuals vs. the fitted values for grey, (d) Histogram are shown in Fig. 3. It can be seen from Fig. 3(a) that all the points on the normal plot lie close to the straight line (mean line). This implies that the data are fairly normal and a little deviation from the normality is observed. It is noticed that the residuals fall on a straight line, which implies that errors are normally distributed. In addition, Figs. 3(b), (c) and (d) revealed that there was no noticeable pattern or unusual structure present in the data. Table 12 shows the average of each response characteristic (raw data) for each level of each factor. The delta statistic is the highest minus the lowest average for each factor. Minitab assigns ranks based on delta values; rank 1 to the highest delta value, rank 2 to the second highest and so on. The ranks indicate the relative importance of each factor to the response. The difference of a factor of a response variable is the change in the response when the factor goes from its level 1 to level 3. The mean response refers to the average value of the performance characteristic for each parameter at different levels. The difference of raw data between level 1 and 3 indicates that feed rate has the highest effect (Δ = max-min = 3.486) followed by depth of cut (Δ = max-min = 2.845) and cutting speed (Δ = max-min = 1.765).

Table 12
Response Table for Means

Level	Tool nose Radius, (mm)	Tool Rake Angle, (°)	Feed Rate, (mm/rev)	Cutting Speed, (m/min)	Cutting Environment	Depth of Cut, (mm)
Level 1	-6.884	-6.895	-8.460	-7.837	-7.197	-8.409
Level 2	-7.227	-7.007	-7.733	-7.259	-7.499	-7.195
Level 3	----	-7.266	-4.974	-6.071	-6.472	-5.563
Delta (max-min)	0.343	0.371	3.486	1.765	1.027	2.845
Rank	6	5	1	3	4	2

Table 13
Analysis of Variance for Grey Relational Grade

Source	SS	DOF	V	F ratio	P value	SS$^{/}$	P (%)
Tool nose radius	0.529	1	0.529	Pooled	0.604	---	---
Tool rake angle	0.434	2	0.217	Pooled	0.887	---	---
Feed rate	40.586	2	20.293	11.48*	0.009	6.806	54.399
Cutting speed	9.722	2	4.861	Pooled	0.142	---	---
Cutting Environment	3.344	2	1.672	Pooled	0.440	---	---
Depth of cut	24.464	2	12.232	6.92*	0.028	0.670	5.355
T	89.691	17				89.691	100.00
e (pooled)	10.611	6	1.768			3.731	29.821

S = 1.32982 R-Sq = 88.17% R-Sq (adj) = 66.48%
Tabulated F-ratio at 95% confidence level $F_{0.05;1;6}$ = 5.99, $F_{0.05;2;6}$ = 5.14

Analysis of variance (ANOVA) of the overall grade is done to show the significant parameters. If the P value for a factor becomes less than 0.05 then that factor is considered as significant factor at 95% confidence level. Statistical software with an analytical tool of ANOVA is used to determine which parameter significantly affects the performance characteristics. The results of ANOVA for the grey relational grades are listed in Table 13. It shows that the two parameters C and F are found to be the major factors with the selected multiple performance characteristics, because their corresponding P ratio is less than 0.05. The percentage error can be used to evaluate if an experiment possesses feasibility and sufficiency or not, since it is related to the uncertain or uncontrollable factors. The percentage error for contribution is 29.821% as shown in Table 13 that indicates that the proposed method as well as the outcome in this study is proven to be highly acceptable.

6. Predicting Optimal Value

The optimal grey relational grade (μGRG) is predicted at the selected optimal setting of process parameters. The significant parameters with optimal levels are already selected as: C3 and F3. The estimated mean of the response characteristic is computed as (Ross, 1988).

$$\mu_{GRG} = \overline{T}_{GRG} + (\overline{C}_3 - \overline{T}_{GRG}) + (\overline{F}_3 - \overline{T}_{GRG}) \tag{9}$$

where $\overline{T}_{GRG}$ = overall mean of grey relational grade = 0.46107 T_{GRG} = overall mean of grey relational grade = 0.46107. C3 and F3 are the mean values of grey relational grade with parameters at optimum levels. From figure 2, $\overline{C}_3 = -4.974, \overline{F}_3 = -5.563$, Hence $\mu_{GRG} = 11.920$. A confidence interval for the predicted mean on a confirmation run is calculated using the Eq. 10 (Ross, 1988).

$$CI_{CE} = \sqrt{F_\alpha(1, f_e)V_e\left[\frac{1}{n_{eff}} + \frac{1}{R}\right]}, \tag{10}$$

where $F\alpha$; (1, fe) = $F_{0.05}$; (1, 6) = 5.99 (Tabulated).
α = risk = 0·05,
fe = error DOF = 6 (Table 13)
N = total number of experiments = 18
Ve = error variance = 1.768 (Table 13)
Total DOF associated with the mean (μ_{GRG}) = 11, Total trial =18, N=18×3 = 54
n_{eff} = effective number of replications = N/{1 + [Total DOF associated in the estimate of mean]} = 54 / (1 + 11) = 4.5
R = number of repetitions for confirmation experiment = 3
A confidence interval for the predicted mean on a confirmation run is ± 2.424
The 95% confidence interval of the predicted optimal grey relational grade is: $[\mu_{GRG} - CI] < \mu_{GRG} < [\mu_{GRG} + CI]$ i.e. $9.496 < \mu_{GRG} < 14.344$
Predicting value for multiple performance characteristics at optimal setting of process parameters are confirmed through experimental results as shown in Table 14.

Table 14
Predicted and Experimental Values at Optimal Setting

Performance Characteristics	*Optimal Combination*	*Predicted Grey Relational Grade*	*Predicted Mean*	*Experimental Value*
Surface Roughness	*A1B1C3D3E3F3*	*11.920*	*2.989*	*3.076*
MRR			*345.39*	*340.61*

7. Conclusions

GRA is applied to determine optimal process parameters for optimization of multiple performance characteristics (surface roughness and material removal rate), which are investigated during rough cutting operation with polycrystalline diamond cutting tool. Using GRA, optimal setting of process parameters for multiple performance characteristics is $A_1B_1C_3D_3E_3F_3$. Corresponding predicted values are confirmed experimentally. Surface roughness (3.076 μm) is achieved with a material removal rate of 340.61mm^3/sec., which is quite acceptable for rough cut. By the average of grey relational grade analysis using Taguchi method, feed rate followed by depth of cut found to be the most influential factors for surface roughness and material removal rate in turning process.

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