International Journal of Industrial Engineering Computations 4 (2013) 111–126

Contents lists available at GrowingScience

International Journal of Industrial Engineering Computations

homepage: www.GrowingScience.com/ijiec

Designing reliable supply chain network with disruption risk

Fateme Bozorgi Atoei[a*], Ebrahim Teimory[a], Ali Bozorgi Amiri[b]

[a]*Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran*
[b]*Department of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran*

C H R O N I C L E

Article history:
Received 26 August 2012
Received in revised format
14 September 2012
Accepted October 25 2012
Available online
27 October 2012

Keywords:
Disruption risk
Reliability
Supply chain
Network design

A B S T R A C T

Although supply chains disruptions rarely occur, their negative effects are prolonged and severe. In this paper, we propose a reliable capacitated supply chain network design (RSCND) model by considering random disruptions in both distribution centers and suppliers. The proposed model determines the optimal location of distribution centers (DC) with the highest reliability, the best plan to assign customers to opened DCs and assigns opened DCs to suitable suppliers with lowest transportation cost. In this study, random disruption occurs at the location, capacity of the distribution centers (DCs) and suppliers. It is assumed that a disrupted DC and a disrupted supplier may lose a portion of their capacities, and the rest of the disrupted DC's demand can be supplied by other DCs. In addition, we consider shortage in DCs, which can occur in either normal or disruption conditions and DCs, can support each other in such circumstances.

Unlike other studies in the extent of literature, we use new approach to model the reliability of DCs; we consider a range of reliability instead of using binary variables. In order to solve the proposed model for real-world instances, a Non-dominated Sorting Genetic Algorithm-II (NSGA-II) is applied. Preliminary results of testing the proposed model of this paper on several problems with different sizes provide seem to be promising.

1. Introduction

In a modern society, engineers and technical managers are responsible for planning, designing, manufacturing and operating from a simple product to the most complex systems. Failure of a system could cause disruption at its various levels, which can be considered a threat to society and environment. When a series of facilities are built and deployed, one or a number of them could probably fail at any time. For example, due to bad weather conditions, labor strikes, economic crises, sabotage or terrorist attacks and changes in ownership of the system, it is possible that the entire set of facilities or services fail to perform, properly. For this reason, the reliability in network design of the supply chain has been proposed and, in the recent years, there has been special attention for creating reliable systems. According to Snyder (2003), a system is called reliable if, "in the event of failure of a part or parts of the system, it is still able to perform its duties, effectively".

* Corresponding author.
E-mail: bozorgi.atoei@yahoo.com (F. Bozorgi Atoei)

doi: 10.5267/j.ijiec.2012.010.003

Snyder (2010) states four main reasons to consider supply chain disruptions in recent years. First, several events with undesirable impacts, including the terrorist attacks of September 11, 2001, the west-coast port lockout in 2002 and Hurricane Katrina in 2005 set disruptions into the center of public attention. Second, in recent decades, the popular just-in-time (JIT) philosophy increases supply chains' vulnerability. The system operates effectively when all factors function exactly as expected, but when a disruption happens, system may encounter serious problems in operation. Third, companies are less vertically integrated than in the foretime, and their supply chains are increasingly global; suppliers are placed around the world, some areas that are politically or economically mutable.

These failures and interruptions in production and distribution facilities may lead to additional transportation costs due to existing distance from customers. Therefore, while the goal is to minimize the cost of deployment, facility placement and transport costs, with the possibility of disruptions, convenient and efficient mathematical models can be provided to simultaneously increase the system's reliability. In other words, modeling this class of problems by considering potential disruptions in the system has been considered and the purpose of this problem is that the systems' performance in all conditions, both normal and disrupted occurrence, should be acceptable (Cui, 2010). In this class of problems, studies directly associated with reliable locating of facilities are considered and there is a focus on the modeling or providing solution for it. In addition, in most of these studies, the "reliability issue" on the classic of P-Median Problem and Uncapacitated Fixed charge Location Problem are implemented; for the brevity from now on, they are called UFLP and PMP and reliable locating issues associated with them are respectively called RPMP and RUFLP.

Drezner (1987) investigated the facility location under random disruption risks and proposed two models. In the first one, a reliable PMP was investigated, which considers a given probability for the failure of facilities. The objective was to minimize the expected demand-weighted travel distance. The second model called the (p, q)-center problem considers p facilities, which must be located considering a minimax objective cost function where at most q facilities may fail. In both problems, customers are selected from the nearest non-disrupted facility based on a neighborhood search heuristic approach in both problems.

Lee (2001) proposed an efficient method based on space filling curves to solve the reliable RPMP. This model is a continuous locating model, in which the probability of failure of facilities cannot be independent. Snyder (2003) investigated the issues of RUFLP and RPMP based on the expected and maximum failure costs. Here, locating facilities were performed so that the total system's cost is minimized under the normal operating conditions. Depending on whether a facility fails to work, the system's cost after reallocation of customers does not exceed a predetermined limit of (V*).

Snyder and Daskin (2005) studied RPMP and RUFLP, in which a distribution center (DC) may fail since a disruption can occur with some probability. They assumed that when a DC fails, it cannot operate and serve customers and present customer must be reassigned to a non-disrupted DC. The objective function is the minimization of a weighted sum of nominal costs by overlooking disruptions and the expected expenditures of disruption circumstances where there is an additional transportation cost for disrupted DCs. In their model, customers are assigned to several DCs, one of which is the "original" DC, which serves it under regular situation (without disruption), the others serve it when the primary DC fails and so on. For the sake of simplicity, Snyder and Daskin (2005) assumed that all DCs have the same disruption probability, which allows the expected transportation expenditure to be declared as a linear function of the decision variables. They solve the model by applying Lagrangian relaxation algorithm.

Snyder and Daskin (2006), in another assignment, implemented the scenario planning approach to formulate their previous problem one more time and introduced the concept of stochastic p-robustness where the relative regret was always less than p for any possible scenario. One obvious problem occurs when the size of the problem increases since the scenario approach considers all disruption scenarios and complexity of the resulted problem creates trouble.

Berman et al. (2007) proposed a PMP, in which the objective function was to minimize the demand-weighted transportation expenditure. They considered site dependent disruption probabilities in various DCs. The resulted problem formulation called the median problem with unreliable facilities uses non-linear terms to compute the expected transportation expenditure when disruption happens and the resulted problem was solved using a greedy heuristic. Berman et al. (2009), in other work, assumed that customers do not know which DCs are disrupted and must travel from a DC to another until they find a non-disrupted one and implement a heuristic method to solve the resulted problem.

Cui et al. (2010) proposed another problem formulation for site-dependent disruption probabilities. Unlike the model proposed by Berman et al. (2007), which involves compound multiplied decision variables, the only non-linear term of their model is a product of a single continuous and a single discrete decision variable and continuum approximation (CA) was implemented to formulate the resulted model. Using such approximation, customers are distributed uniformly throughout some geographical areas, and the parameters are presented as a function of the location. Replacing explicit disruption probabilities with probabilities depending on the location, helps to calculate the expected transportation expenditure or distance without using any assignment decision. Lagrangian relaxation was also implemented to solve the model.

Qi et al. (2010) studied the SCND under random disruptions with inventory control decisions. They assumed that when a retailer is disrupted, any inventory on hand at the retailer is unusable and the resulted customers' unmet demands assigned to a retailer are backlogged under a penalty cost. The resulting model was a concave minimization problem and the Lagrangian relaxation algorithm was implemented as a solution strategy.

Li and Ouyang (2010) studied the SCND under random disruption risks, in which the disruption probabilities are given to be site-dependent and correlated, geographically. They applied CA to formulate the resulted model. Lim et al. (2010) proposed the SCND under random disruptions by considering reinforcing selected DCs where disruption probabilities are also site-dependent. They categorized DCs into two groups of unreliable and reliable and implemented the reliable backup DCs assumption to formulate their proposed model. The disruption happens in unreliable DCs and reliable DCs are those, which are improved against disruptions by considering an additional investment and disruptions does not have any impact on them called hardening strategy. Similar to previous works, when a disruption occurs, an unreliable DC totally fails. In their model the customers in disruption situation are assigned to the closest reliable DCs. like many studies in the literature, the Lagrangian relaxation was implemented to solve the resulted problem formulation.

Peng et al. (2011) developed a capacitated version of SCND under random disruptions with stochastic p-robustness criteria and site dependent disruption probabilities. They adopted similar approach developed originally by Snyder and Daskin (2006) and used the scenario approach to model the problem. A hybrid metaheuristic algorithm based on genetics algorithm, local improvement search, and the shortest augmenting path method was proposed to solve the resulted model.

Table 1 summarizes other relevant works, which are categorized based on different groups.

Table1
Literature Review

Author	Year	Model				Disruption Probability						Capacity constraints	Objective Function		Solution		
		Based on the scenario	Based on the assignment level	Based on game theory	Other	scenario	probabilistic non-linear terms	Reliable backup	Continuous approximation	Fixed Probability	Other		min	max	Exact	Huristic	Metahuristic
Drezner	1987				✓					✓			✓			✓	
Lee	2001				✓					✓			✓			✓	
Snyder	2003	✓				✓							✓				✓
Bundschuh et al.	2003												✓		✓		
Snyder & Daskin	2005		✓				✓						✓				✓
Snyder & Daskin	2006a	✓	✓			✓	✓						✓				
Berman et al.	2007			✓			✓						✓			✓	
Shen et al.	2007	✓	✓			✓	✓						✓			✓	
Zhan	2007	✓	✓			✓	✓						✓			✓	
Aryanezhad et al.	2009		✓				✓						✓			✓	
Robert et al.	2009		✓				✓						✓			✓	
Berman et al.	2009			✓					✓				✓			✓	
Berman et al.	2009b			✓							✓		✓			✓	
Lim et al.	2010				✓			✓					✓				✓
Cui ei al.	2010		✓				✓		✓				✓				✓
Li & Ouyang	2010	✓							✓				✓				
Peng	2011	✓									✓	✓	✓			✓	
Jabbarzadeh et al.	2011		✓			✓						✓		✓			✓
Azad	2011				✓			✓				✓	✓			✓	
Azad	2012				✓			✓				✓	✓			✓	
Researcher	2012	✓				✓						✓	✓	✓	✓		✓

After investigating studies in the fields of this research, now, in this section, the problem of planning models will be presented and discussed.

2. The proposed study

In this model, a three level supply chain including customers, distributors and suppliers are considered in which the goal is to minimize costs and maximize reliability.

In this model, there is a potential location for all distributors, which are not assigned to any location. These points are also considered to have potential reliability.

Once a failure occurs in a distribution center, the center would lose part of its capacity; i.e. it would not completely fail and would be able to answer part of the customer's needs. This failure in capability for meeting customer demand has to be supplied by other DC that can respond. It is also possible that some DCs are still compensated by others, in case of no disruption (Support). To express different states of the disruption, various scenarios are considered. Each scenario includes the possibility of a disruption in each supplier and distribution center, which follows a normal distribution. These disorders in each scenario could be different incidents. For example, in the first scenario, it is possible that distributors 1 and 3 and supplier 2 would be disrupted; this disruption could be an earthquake for the first distributor, a flood for the second distributor and a labor strike for the second supplier. Fig. 1 shows the framework of the proposed study.

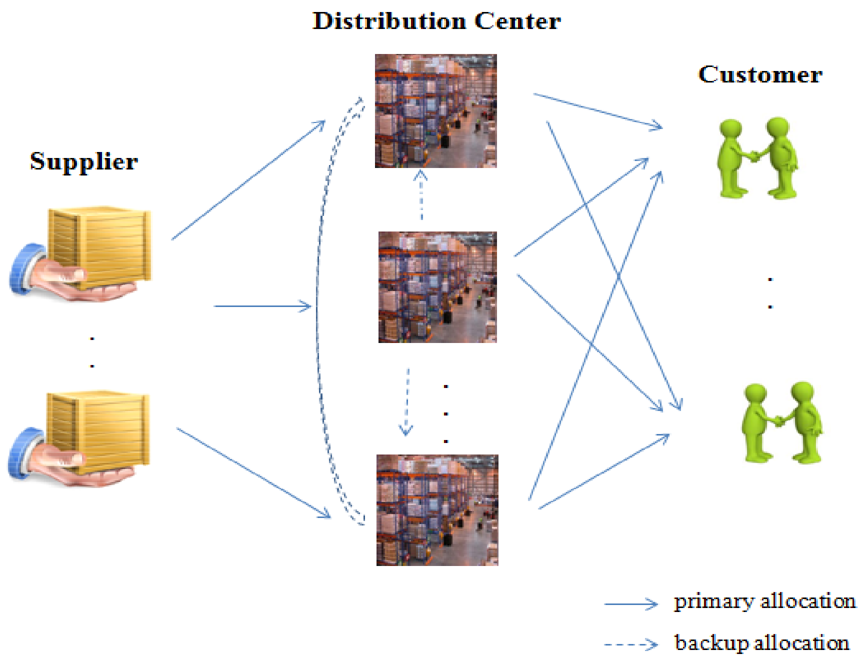

Fig. 1. General Structure of the model

2.1. Assumptions

- Demand is normal and distribution is indeterminate.
- Demands of customers are independent from each other and, as a result, the covariance among retailers with each distributor is zero.
- Current policy is (Q, r).
- The issue is a monoculture model.
- The model is considered for a limited period of time.
- The customer does not keep inventory so there is no need to control the inventory for the customer.
- Customer has no capacity constraints.
- If customer's demand is not fulfilled, there will be a shortage.
- A certain number of places have been considered for setting up distribution centers, in which the decision on opening or closing the facilities would be performed.
- Lack of reliability would be considered in the occurrence of disruption and other factors would not affect reliability.
- Probability of disruption is different and independent for various facilities' locations and for suppliers.
- Suppliers and customers have their own specific places and the DC is just required to be located (discrete locating).
- Distribution and supply centers of suppliers have a limited capacity.

In case of the ordering policy of distribution centers, to calculate the economic order quantity and re-order according to the ordering policy (r, Q), asymptotically approach of the EOQ, which was introduced by Axaster (1996), is used. In the worst scenario, its disruption would be equivalent to 11.8% (Axaster 2006). Zheng (1992) also examined various examples; this approach had high quality approximate responses with the average error of less than 1%.

For this reason, here like many other models with integrate locating (e.g. Daskin et al. 2002, Shen et al. 2003, Miranda & Gridu 2004, Xu et al. 2005, Schneider et al., 2007, Ozsn et al. 2008, Yu & Grossman, 2008; Park et al., 2010), the approximation for the EOQ ordering policy (r, Q) was used.

2.2 Innovation

Random disruptions in the location and capacity of distribution centers and the location and capacity of suppliers have been considered. Due to a disruption, distribution centers would not lose all their capacity and only a fraction of their capacity would be impaired. In the distribution centers, in case of shortage, either in disruption or normal condition, they can typically carry goods to each other (support cover). Capacity percentage of distribution centers affected by disruption is random and follows a normal distribution. The metaheuristic and exact solution algorithm NSGA-2 was used due to being bi-objective. There are two objective functions where the one first is the expected total cost, which includes the fix cost of locating, shipping costs from the supplier to the distribution center, deficiency cost and disruption costs. In addition, the second objective function maximizes the reliability average. Every customer can be assigned to multiple distribution centers according to the distance and costs to supply its needs. Reliability is considered for potential locations in which DC would be located. Demand is uncertain and follows the normal distribution. The proposed model also consider different scenarios. It uses a scenario for entering the possibility of disruption and modeling the problem and it uses the possibility limitation for entering the random variable. The proposed model of this paper determines location for distribution centers and allocates customers to distribution centers. The problem formulation also determines the content of each product to each customer from the each distribution center, the amount of goods sent by the sponsors. Finally, we determine how to allocate the distribution centers to the suppliers and the amount of products received from any of the suppliers to each distribution center are determined.

Index:

I = Suppliers $i = 1, 2 \ldots n$

J= Set of potential distribution centers $j = 1, 2 \ldots n$

j' = Set of distribution centers that are disrupted or deficient $j' \in J$

K = Set of customers $k = 1, 2 \ldots k$

S = Set of scenarios $s = 1, 2 \ldots s$

Parameters

D_k	Demand of the k^{th} customer which has a normal random variable (μ_j , σ_j^2)
f_j	Constant costs for opening and operating *DC*
p	The total number of distribution centers which should be localized
g_{ij}	Fixed cost per shipment from supplier *i* to distribution center *j*
k_{ij}	Transportation cost of each supplier *i* to DC_j

$C_{j\acute{j}}$	Transportation cost unit from each normal DC_j to each deficient $DC_{\acute{j}}$
d_{jk}	Transportation cost from DC_j to k customer
S_j	Fixed cost per order from DC_j
h_j	The annual maintenance cost per unit in DC
I_{js}	Inventory in DC under the scenario of *s*
q_s	The possibility of corruption in scenario *s*
Cap_j	Capacity of DC_j
a_{js}	A fraction of total capacity of DC_j which has been ruined in scenario s
π_j	The penalty cost of shortage in service to DC per unit of demand
b_{js}	The amount of shortage in DC under s scenario
r_{js}	Reliability for the place of *j* under s scenario ($0 \le r_{js} \le 1$)
cap_i	Supplier capacity
a_{is}	The percent of capacity of supplier *i* under scenario *s* which is eliminated due to disruption.

2.3. Decision Variables

2.3.1 Continuous variables

y_{jk}	Percentage of product that customer *k* can allocate to DC_j. $0\le y_{jk} \le 1$
$T_{j\acute{j}s}$	The amount of product which DC_j would receive in case of disruption from $DC_{\acute{j}}$ under the scenario *s*
D_j	The total annual demand which will be allocated to *j* distribution centers
n_j	The number of orders in *j* distribution centers
Y_{ij}	Percentage of *j* distribution center's demand which will be sent to *t* supplier *i* $0\le y_{ij} \le 1$
Z_{ij}	The amount of products that supplier *i* will send to *j* distribution center
$V_{j\acute{j}s}$	The amount of products that DC_j will send to $DC_{\acute{j}}$ under scenario *s* due to disruption or deficiency

Zero – One variable

$$X_j = \begin{cases} 1 & if\ DC_j\ is\ open \\ 0 & otherwise \end{cases}$$

Cost

Fixed cost of locating= $f_j x_j$

Running Inventory Costs

The cost of ordering= $s_j n_j$

Shipping costs from suppliers to DCs = $\sum_i (g_{ij} + k_{ij} Z_{ij}) n_j$

Maintenance cost = $\sum_s q_s I_{js} h$

Disruption costs= $\sum_s q_s \left(\sum_{j\in J_s}\left(\sum_j T_{jj's}C_{jj}X_j\right)\right)$

Shipping cost from located DCs to the customers = $d_{jk}\mu_k y_{jk}$

Shortage costs = $b_{js}\pi_j \sum_s q_s$

Finally, the model will be as follows:

$$Min\left[\sum_j f_j x_j + \left(\sum_j\left(s_j n_j + \sum_i (g_{ij}n_j + k_{ij}Z_{ij}) + \sum_s q_s (I_j)h_j\right) + \sum_s q_s\left(\sum_{j\in J_s}\left(\sum_j T_{jj's}C_{jj}X_j\right)\right)\right) + \sum_j\sum_k d_{jk}\mu_k y_{jk} + \sum_j\sum_s q_s b_{js}\pi_j\right] \tag{1}$$

$$\max \sum_j \sum_s r_{js} X_j \tag{2}$$

subject to:

$$\sum_j X_j = \mathrm{p} \tag{3}$$

$$\sum_j y_{jk} = 1 \qquad \forall\, k \in K \tag{4}$$

$$y_{jk} \le X_j \qquad \forall\, k \in K, j \in J \tag{5}$$

$$\sum_k D_k y_{jk} \le (1-a_{js})cap_j X_j \qquad \forall s, j \tag{6}$$

$$n_j = \sqrt{\frac{D_j h_j}{2S_j}} \qquad \forall\, j \in J \tag{7}$$

$$D_j \ge \sum_k D_k y_{jk} \qquad \forall\, j \in J \tag{8}$$

$$\sum_j D_j Y_{ij} \le (1-a_{is})cap_i \qquad \forall s, i \tag{9}$$

$$Z_{ij} = D_j Y_{ij} \qquad \forall j, i \tag{10}$$

$$\sum_i Z_{ij} + \sum_{j'\ne j} T_{j'js} X_j - D_j - \sum_{j\ne j'} T_{jj's} X_j = I_{js} - b_{js} \qquad \forall s, j \tag{11}$$

$$\sum_{j'} T_{jj's} \le \sum_i Z_{ij} \qquad \forall s, j, j' \ne j \tag{12}$$

$$\sum_i y_{ij} <= 1 \qquad \forall\, j \in J \tag{13}$$

$$Y_{ij} \le X_j \qquad \forall i, j \tag{14}$$

$$Z_{ij} \le M * X_j \qquad \forall\, j \in J \tag{15}$$

$$r_{js} <= 1 - a_{js} \qquad \forall\, j \in J, s\epsilon S \tag{16}$$

$$0 \le y_{jk} \le 1 \tag{17}$$

$$X_j \in \{0,1\} \tag{18}$$

$$T_{jj'} \ge 0 \tag{19}$$

$$Z_{ij} \ge 0 \tag{20}$$

$$0 \le y_{ij} \le 1 \tag{21}$$

$r_{js} \geq 0$ (22)

- ✓ Eq. (1) (the first objective function), the total expected costs include:
  Costs of operation and openness of distribution centers (first statement= $\sum_j f_j x_j$)

  Working inventory cost (second statement $=\sum_j \left(s_j n_j + \sum_i (g_{ij} n_j + k_{ij} Z_{ij}) + \sum_s q_s \, (I_j) h_j\right) + \sum_s q_s \, \left(\sum_{j \in J_s} \left(\sum_j T_{j\acute{j}s} C_{j\acute{j}} X_j\right)\right)$)

  Cost of transportation from DC to the located customer (third statement = $\sum_j \sum_k d_{jk} \mu_k y_{jk}$)

  Penalty cost of deficiency (fourth statement = $\sum_j \sum_s q_s b_{js} \pi_j$)

- ✓ Eq. (2) (second objective function) tries to maximize the reliability of located distribution centers.
- ✓ Eq. (3) says that p is the number of distribution centers, which should be potentially located.
- ✓ Eq. (4) states that any customer could order as much as it wants in order not to be faced with any shortage (it is somehow the client balance equation).
- ✓ Eq. (5) emphasizes that at least a distribution center should be open to have customer allocation.
- ✓ Eq. (6) is the capacity limitation of distributer.
- ✓ Eq. (7) calculates the annul number of orders in each distribution center.
- ✓ Eq. (8) calculates the total annual demands of j distributer centers (due to fluctuations of customer demands, ≥ is considered).
- ✓ Eq. (9) is the limitation of supplier's capacity.
- ✓ Eq. (10) shows the amount of product submissions from each supplier to each distribution center.
- ✓ Eq. (11) is the limitation of the equilibrium in j distribution center and expresses the difference between incoming goods toDC_j and the outputs from it in case of either disruption/shortage or normal situation.
- ✓ Eq. (12) states that the amount of sent products to the healthy distribution center (backup) for the disrupted distribution center can be at the same level that suppliers sent to them.
- ✓ Eq. (13) shows that each distribution center could request products in maximum as much as its needs (because they may be faced with a shortage).
- ✓ Eq. (14) stresses on the point that each distribution center should be open to be allocated to the suppliers.
- ✓ Eq. (15) defines that a distribution center should be open to allow the suppliers send goods to them.
- ✓ Eq. (16) shows that the reliability of each distributer center is in maximum at the level of no disruption in that center.
- ✓ Eqs. (17-22) are limitations of the signs.

Chance Constraint

Note that constrains given by Eq. (6) and Eq. (8) are probability. When we consider normal distribution for demand and using conversion Probability Constraint, we can replace demand with average and this limitation can be rewritten as follows:

Constrains 6:

$$p\left(\frac{\sum_k D_j\, Y_{jk} - \sum_k \mu_k\, Y_{jk}}{\sqrt{\sum_k \sigma^2{}_k {Y_{jk}}^2}} \leq \frac{(1-a_{js})cap_j X_j - \sum_k \mu_k\, Y_{jk}}{\sqrt{\sum_k \sigma^2{}_k {Y_{jk}}^2}}\right) \geq 1-\alpha$$

or

$$\frac{\left((1-a_{js})cap_j X_j - \sum_k \mu_k\, Y_{jk}\right)}{\sqrt{\sum_k \sigma^2{}_k {Y_{jk}}^2}} \geq Z_{1-\alpha} \tag{23}$$

Constrains 8:

$$p\left(D_j \geq \sum_k D_k\, Y_{jk}\right) \geq 1-\alpha \qquad p\left(\frac{D_j - \sum_k \mu_k\, Y_{jk}}{\sqrt{\sum_k \sigma^2{}_k {Y_{jk}}^2}} \geq \frac{\sum_k D_k\, Y_{jk} - \sum_k \mu_k\, Y_{jk}}{\sqrt{\sum_k \sigma^2{}_k {Y_{jk}}^2}}\right) \geq 1-\alpha$$

$$\frac{\left(D_j - \sum_k \mu_k\, Y_{jk}\right)}{\sqrt{\sum_k \sigma^2{}_k {Y_{jk}}^2}} \geq Z_{1-\alpha} \tag{24}$$

Applying linearization, the probability constraint are rewritten as follows,

$$\begin{aligned} \min \Bigg[\sum_j f_j \mathrm{x}_j + \Bigg[\sum_j \Bigg(s_j n_j + \sum_i (g_{ij} n_j + k_{ij} Z_{ij}) + \sum_s q_s\,(I_j) h_j\Bigg) + \sum_s q_s \Bigg(\sum_{j \in J_s}\Bigg(\sum_j E_{jjs} C_{jj}\Bigg)\Bigg)\Bigg] \\ + \sum_j \sum_k d_{jk} \mu_k y_{jk} + \sum_j \sum_s q_s b_{js} \pi_j \Bigg] \end{aligned} \tag{25}$$

$$\max \textstyle\sum_j \sum_s r_{js} X_j \tag{26}$$

subject to:

(3)-(5)

$$\frac{\left((1-a_{js})cap_j X_j - \sum_k \mu_k Y_{jk}\right)}{\sqrt{\sum_k \sigma^2{}_k {Y_{jk}}^2}} \geq Z_{1-\alpha} \qquad \forall s, j \tag{27}$$

(7)

$$\frac{\left(D_j - \sum_k \mu_k Y_{jk}\right)}{\sqrt{\sum_k \sigma^2{}_k {Y_{jk}}^2}} \geq Z_{1-\alpha} \qquad \forall\, j\, \in J \tag{28}$$

(9-22)

3. Solution Approach

According to the existing literature on the issue of locating, the facility, particularly those mentioned by Meggido and Supowit (1984), the proposed model in the simplest case is in the form of locating-allocating without limitation of capacity, which is an NP-hard. Therefore, this model, which is a basis for the development, is an NP-hard. Consequently, using metaheuristic methods to find an approximate solution for large size problems is necessary.

To solve the model, an initial step is to solve the resulted problem using a simple optimization technique such as generalized reduced gradient used in several commercial software packages such as Lingo, GAMS, etc. The proposed model of this paper is solved using Lingo11 to find some optimal solutions. Since the problem formulation is a bi-objective one, we use epsilon constraint method. But, because the current version of this software is capable of solving problems with maximum 50 integer variables, this software cannot be used for medium and large-scale problems and also the current version of this application had other limitations in using continuous variables and limited numbers for calculating the problem. The proposed algorithm is for this multi-objective genetic algorithm with undesirable sorting (NSGA-II) and the following briefly discusses the procedure.

3.1.ε-constraint method

Epsilon constraint method is known as one of the popular techniques for handling multi-objective problems, in which by transferring all but one of the objective functions in each step into constraints, we solve a traditional single objective problem and Pareto frontier can be generated by constraint ε.

$$x^* = \min \; \left\{ \mathrm{f}_1(\mathrm{x}) \middle| \mathrm{x} \in \mathrm{X}, f_2(x) \le \varepsilon_2, \cdots, f_n(x) \le \varepsilon_n \right\}$$

ε -constraint method has the following steps:

1. It selects one of the objective functions as the main function.
2. It solves the problem each time by considering one of the objective functions and getting the optimal amounts of each objective function.
3. It divides the optimum interval between the sub-objective functions into pre-defined values and gaining a values table for $\varepsilon_2 \ldots \varepsilon_n$

3.2. Non-dominated Sorting Genetic Algorithm

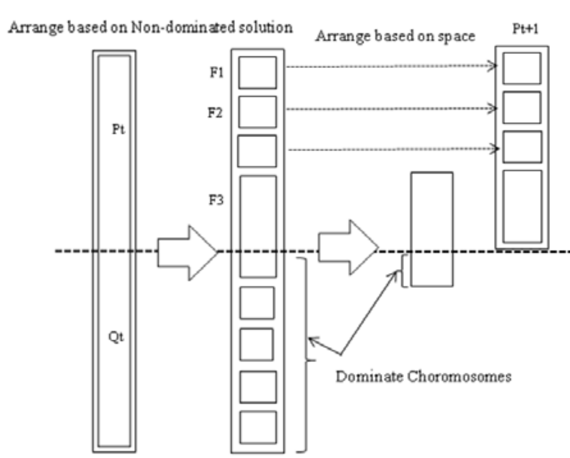

Fig. 2. Non-dominated Sorting Genetic Algorithm

As can be observed, the implementation of this bi-objective optimization model is as follows:

1. Random generation of parent (P_0) to the number of N
2. Arrangement of the initial parent generation based on a non-dominated solution
3. Considering the ranking in proportion to rating of non-domination for each non-dominated response (1 for the best level, 2 for the best level after 1, ...)
4. Generation of (Q_0) to the number of N with using select, coupling and mutation operators
5. According to the first generation produced that contains the chromosomes of the parent and child, the new generation will be produced as follows:

- Combine chromosomes of parent (P_t) and children (Q_t) and generating (R_t) to the number of 2N.
- Arrange generations (R_t) categories based on non-domination and identifying non-dominated fronts $F_1, F_2, \dots, F_l$
- Produce parent generation for the next iteration using the produced non-dominated fronts with total number of N. In this stage, considering the number of needed chromosomes for parent generation (N), initially, the number of chromosomes of parent generation will be selected; if this number is not sufficient, the total number of required parent generation, the fronts 2, 3 … will be used, in this order, to achieve the total amount
- The coupling and mutation operations on the new produced parent generation (P_{t+1}) and generation of children (Q_{t+1}) with the total amount of N
- Repetition of step 5 to obtain the total number of required iterations

4. Computational result

To illustrate the applicability of the model, 6 numerical examples have been presented, all data are provided in the appendix. It should be noted that the related calculations were done using Lingo 11 and MATLAB 2012 software (metaheuristic algorithm was coded using MATLAB software) in a personal computer with Intel core2 and 2 GB RAM. Metaheuristic parameter are optimized using trial and error. Numerical values are given for each variable. It all costs are to ten thousand Rials. Capacity and demand are given in tons.

Table 2
Parameter value

parameter	Value
Supplier capacity (cap_i)	Uniform (750-810)
Constant costs for opening and operating DC (fj)	Uniform(480000-525000)
Fixed cost per order from DC_j (s)	Uniform (650-750)
The annual maintenance cost per unit in DC (h)	Uniform (64-78)
Capacity of DC_j (capj)	Uniform (710-840)
The penalty cost of shortage in service to DC per unit of demand (π_j)	Uniform (100-320)
customer demand (Dk)	normal(280,24)
Probability of each scenario (qs)	Uniform (0-0.5)
Transportation costs from the DCj to customers (d_{jk})	Uniform (10-19)
Transportation costs from the suppliers to the DCj (k_{ij})	Uniform (11-22)
Transportation cost unit from each normal DC_j to each deficient DC_j (C_{jj})	Uniform (41-25)
Fixed cost per shipment from supplier to distribution center(g$_{ij}$)	Uniform (100-250)
fraction of total capacity of DC_j which has been ruined in scenario s (a_{js})	Uniform(0,1)
Fraction of capacity of supplier i under scenario s which is eliminated due to disruption (a_{is})	Uniform(0,1)

The results showed that the Lingo software gives better solutions (except in one case) and is more reliable because of its exact solutions. However, for the large-scale problems (instance 5 and instance 6), the optimal solution is not available and the proposed metaheuristic provides near optimal solution.

Table 4
Comparison between Lingo and NSGA-II solution

	MATLAB (NSGA-II)				Lingo11				
Problem #	Run Time	No. of Pareto solution	objective function 2	objective function 1	State	Run Time	objective function 2	objective function 1	sup/DC/cus/sen./p
1	9":86	1	1.4	1027850.22	local	2':14"	1.65	972008.00	1-2-2-2-2
2	12":64	2	1.61	1518880.10	local	10':46"	1.71	1336614.48	3-4-3-3-3
3	12":26	4	1.61	1521720.88	local	2:25':08"	1.75	1369548.79	4-5-5-4-3
4	13":96	3	2.55	2550805.40	local	9:25':08"	2.74	2270216.80	5-7-6-4-5
5	14":86	4	3.39	2991604.70	unknown	30:15':58"			7-8-6-3-6
6	16":94	4	4.092	4016599.33	unknown	39:20':34"			7-10-8-5-8

As we can observe from the results of Table 4 and as expected, the time spent for solving the metaheuristic algorithm was much shorter than the Lingo software. In the first four problems, in which Lingo reached the local optimal solution, the algorithm NSGA-II reached the optimal solution in much less time. For the last two problems in which Lingo could not find the response after 30 and 39 h, NSGA-II algorithm found the response in a reasonable amount of time. As shown by the objective function values given in Table 4 and Fig. 3 and Fig. 4, the solutions of Lingo software, except for one case in numerical example 3 and in case of the first objective function, provided a better response because Lingo is based on an exact solution method. The comparison between the first and second objective functions for both exact and metaheuristic methods are shown below in the form of figures for the two above mentioned numerical examples.

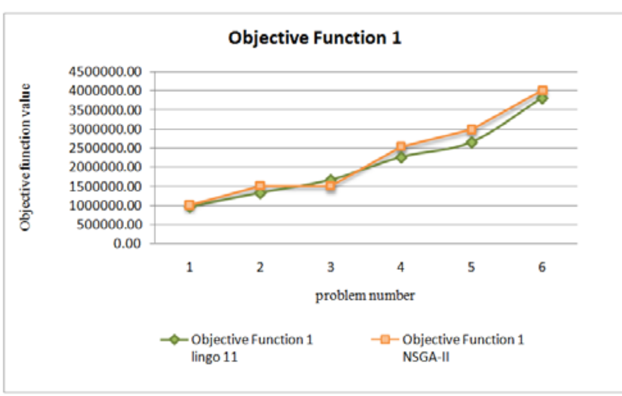

Fig. 3. Compare between Objective Function 1 in Lingo Software & NSGA-II algorithm

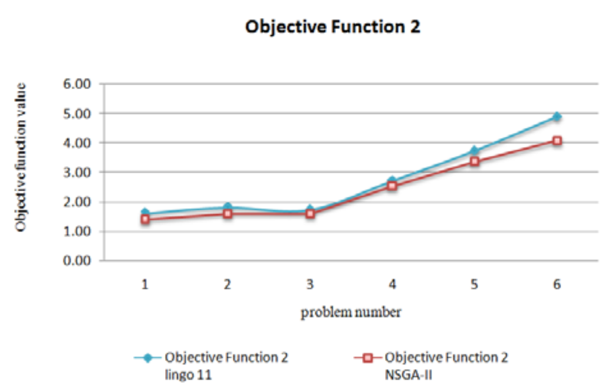

Fig 4. Compare between Objective Function 2 in Lingo Software & NSGA-II

As it was mentioned, in the first objective function, Lingo found the response in each 4 cases and, except in numerical example 3, the response was much better than that of meta-heuristic algorithms and objective function diagrams of Lingo was under the NSGA-II figure because the first objective function was minimized and it should have fewer responses to be optimized. In all 4 cases in which Lingo found the solution, this figure shows the superiority of Lingo's responses and, since the second objective function is maximization, the Lingo's diagram is on top of the NSGA-II algorithm. In this section, a graph of NSGA-2 outputs has been depicted for numerical example number 2 which includes 3 suppliers, 4 distribution centers (3 DCs must be located) and 3 customers. Disruption has occurred in DC1 , DC 3 and supplier 2.

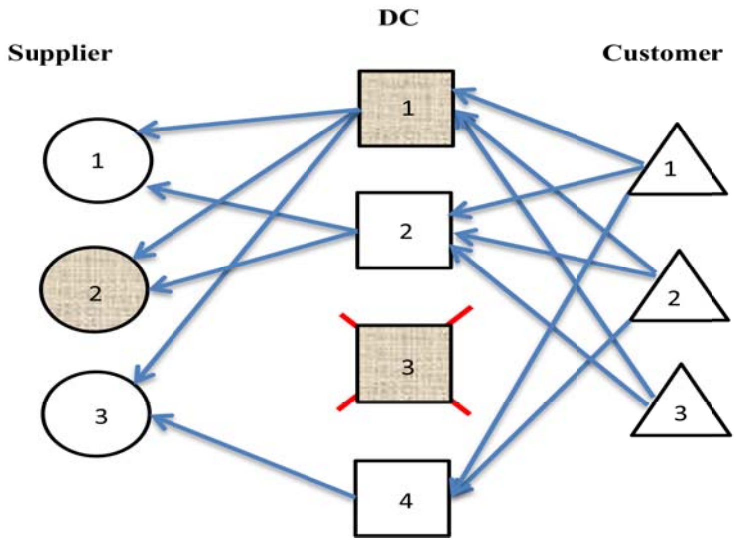

Fig. 5. Final solution for the third problem

According to the Fig. 5, DC 3 has not been opened.

4.1.Sensitivity Analysis

The following figures show the expressed analysis compared with the parametric changes in summary.

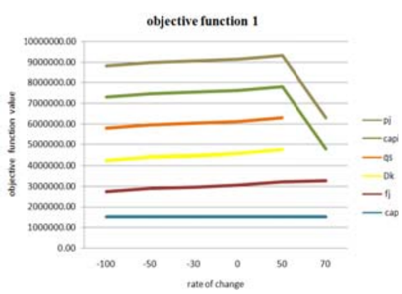

Fig. 6.(a) sensitivity analysis for objective function 1

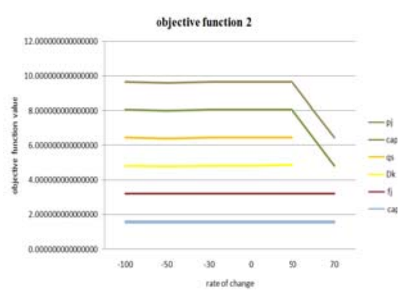

Fig 6.(b) Sensitivity analysis for objective function 2

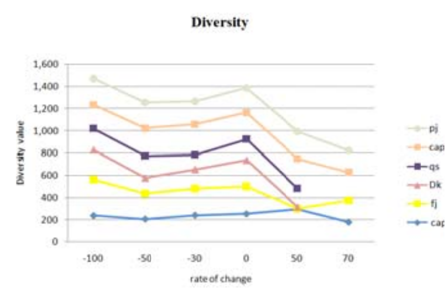

Fig. 7. Sensitivity analysis for Diversity

5. Conclusion

In this paper, we have presented a reliable capacitated supply chain network design (RSCND) model with random disruptions in accordance with the real-world situations. We have assumed that random disruptions takes place in the location and capacity of the distribution centers (DCs) and suppliers. Model simultaneously determined the optimal number and location of DCs with the highest reliability, the assignment of customers to opened DCs and opened DCs to supplier with lowest transportation cost, supporting disrupted DCs or DCs, which faced shortage by other non-disrupted DCs.

Unlike other studies in the extent literature, we have used new approach to model the reliability of DCs and considered reliability as a range. In order to solve the proposed model optimally, first, lingo software and then a Non-dominated Sorting Genetic Algorithm-II (NSGA-II) has been applied. Computational results for several problems with different sizes indicate that the heuristic method is more efficient.

For future research, we suggest five directions as follows:

- Elaborating more effective solution methods to solve the model,
- Considering correlated disruption probabilities for the model,
- Taking into account the disruption parameters based on fuzzy logic,
- Formulating a robust model in the cases of having imperfect data on the disruption probability in SCND,
- Contemplating other cost factors such as reconstruction cost of ruined facilities or destroyed inventory, etc.

Reference

Aryanezhad, M.B, & Jabbarzadeh, A. (2009). An integrated model for location-inventory problem with random disruptions. *Proceedings of the International Conference on Computers & Industrial Engineering*, 39, 687-703.

Azad, N., & Davoudpour, H. (2011). Stochastic distribution network design with reliability, *consideration Journal of Zhejiang University-SCIENCE C (Computers & Electronics). in press*, **doi:**10.1631/jzus.C11a0087.

Azad,N., Saharidis, G. K.D., Davoudpour, H., Malekly,H. & Yektamaram,S.A. (2012) Strategies for protecting supply chain networks against facility and transportation disruptions: an improved Benders decomposition approach . *Springer Science Business Media, Annual Operation research*, DOI 10.1007/s10479-012-1146-x.

Bérubé, J. F., Gendreau, M. & Potvin, J.Y.. (2009). An exact ε-constraint method for bi-objective combinatorial optimization problems: Application to the traveling salesman problem with profits. *European Journal of Operational Research,* 194, 39-50.

Berman, O., Krass, D., & Menezes, M. B. C. (2007). Facility reliability issues in network p-median problems: Strategic centralization and co-location effects. *Operations Research*, 55 (2), 332-350.

Berman, O., Krass, D., & Meneze, M. B. C. (2009). Location Facilities in the Presence of Disruptions and Information. *Decision Science*, 40, 845-868.

Cui,T., Ouyang,Y. & Shen,Z.J.M. (2010). Reliable Facility Location Design under the Risk of Disruptions. *Operations Researc,* 58 (4), 998-1011.

Daskin, M. S. & C. R. Coullard. (2002). An inventory-location model:Formulation, solution algorithm and computational results. *Annals of Operations Research*, 110, 83-106.

Drezner, Z. (1987). Heuristic solution methods for two location problems with unreliable facilities. *Journal of the Operational Research Society,* 38, (6), 509–514.

Lee, S.-D. (2001). On solving unreliable planar location problems. *Computers and Operations Research*, 28(4), 329-344.

Li, X., & Ouyang, Y. (2010). A continuum approximation approach to reliable facility location design under correlated probabilistic disruptions. *Transportation Research Part B: Methodological*, 44(4), 535–548.

Lim, M., Daskin, M. S., Bassamboo, A., & Chopra, S. (2010). A Facility Reliability Problem: Formulation, Properties, and Algorithm. *Naval Research Logistics*, 57(1), 58–70

Miranda, P. A. & R. A. Garrido (2004). Incorporating inventory control decisions into a strategic distribution network design model with stochastic demand. *Transportation Research Part E,* 40, 183–207.

Ozsen, L. & C. R. Coullard. (2008). Capacitated warehouse location model with risk pooling. . *Naval Research Logistics*, 55(4), 295-312.

Park, S., Lee, T. & Sung, C.S. (2010). A three-level supply chain network design model with risk-pooling and lead times. *Transportation Research Part E,* 46, 563–581.

Peng P., Snyder , L., Lim, & Liu. (2011). Reliable logistics networks design with facility disruptions. *Transportation Research*, 45(8), 1190–1211.

Qi, L., Shen, Z.-J., & Snyder, L. V. (2010). The effect of supply disruptions on supply chain design decisions. *Transportation Science*, 44(2), 274–289.

shen,Z.J.M. ,Coullard,C. & Daskin,M.S.,(2003). a joint location-inventory model. *transportation science,* 37, 40-55.

Shu, J. & C. Teo. (2005). stochastictransportation inventory network design problem. *operational Research*, 53(1),48-60.

Snyder, L.V. (2003). Supply chain robustness and reliability: Models and algorithms. *Ph.D. dissertation, Northwestern University, Department of Industrial Engineering & Management Sciences, Evanston, IL.*

Snyder, L. V., & Daskin, M. S. (2005). Reliability models for facility location: The expected failure cost case. *Transportation Science*, 39 (3), 400–416.

Snyder, L. V., Scaparra, P. M., Daskin, M. S., & Church R.L. (2006). Planning for disruptions in supply chain networks. *In Tutorials in Operations Research, INFORMS*, 234–257.

Snyder, L. V., Daskin, M. S. & Teo,C. (2007). The stochastic location model with risk pooling. *European Journal of Operational Research*, 179(3), 1221-1238.

Snyder, L. V., Atan,Z., Peng, P., Rong, Y., Schmitt, A. & Sinsoysal, B. (2010). OR/MS Models for Supply Chain Disruptions: A Review. *social science research network, working papers series*