

Integrated quadratic assignment and continuous facility layout problem

Kamran Forghani, Alireza Arshadi khamseh* and Mohammad Mohammadi

Department of Industrial Engineering, Faculty of Engineering, Kharazmi University, Karaj, Iran

ARTICLE INFO

Article history:

Received 12 April 2012
Received in revised format
5 July 2012
Accepted July 6 2012
Available online
8 July 2012

Keywords:

*Integrated facility layout problem
Quadratic assignment problem
Mathematical programming
Heuristic method*

ABSTRACT

In this paper, an integrated layout model has been considered to incorporate intra and inter-department layout. In the proposed model, the arrangement of facilities within the departments is obtained through the QAP and from the other side the continuous layout problem is implemented to find the position and orientation of rectangular shape departments on the planar area. First, a modified version of QAP with fewer binary variables is presented. Afterward the integrated model is formulated based on the developed QAP. In order to evaluate material handling cost precisely, the actual position of machines within the departments (instead of center of departments) is considered. Moreover, other design factors such as aisle distance, single or multi row intra-department layout and orientation of departments have been considered. The mathematical model is formulated as mixed-integer programming (MIP) to minimize total material handling cost. Also due to the complexity of integrated model a heuristic method has been developed to solve large scale problems in a reasonable computational time. Finally, several illustrative numerical examples are selected from the literature to test the model and evaluate the heuristic.

© 2012 Growing Science Ltd. All rights reserved

1. Introduction

Facility layout is an important and complex problem for today's manufacturing systems. A facility layout problem (FLP) is about arranging physical departments and facilities within a manufacturing or service systems. The most commonly considered criterion in developing a FLP is to minimize total material handling distance/cost (Heragu & Kusiak, 1988). According to Tompkins et al. (2003), between 20% and 50% of total operating expenses within manufacturing is attributed to material handling. In a manufacturing system, the material handling function includes transportation of work-in-process (WIP), finished parts, materials and tools between machines or workstations. An efficient layout contributes to reduce production cycles, WIP, Idle times, number of bottlenecks or material handling times and the increase in the production output and productivity (Sule, 1994).

The FLP has attracted extensive attention from industry and academia in last decades. Researches: Kusiak and Heragu (1987), Meller and Gau (1996), Singh and Sharma (2005) and Drira et al. (2007) summarized detailed review, different modeling and solution procedures to the FLP. Due to the

* Corresponding author.

E-mail: arshadi.kh@tmu.ac.ir (A. Arshadi khamseh)

complexity of the problem only limited number of papers have addressed exact solution methods for the FLP; however, the majority of work on the FLP has focused on heuristic and meta heuristic approaches to find good solutions.

In the FLP, it is assumed that the areas of the departments are known, although the exact department shapes are typically not specified in advance. However, it is commonly assumed that departments will take rectangular shapes. The final solution of the layout problem is a block layout that shows the coordinate information including the dimensions and location of each department. Heragu & Kusiak (1991) developed a special case of the FLP, where the length, width, and orientation of the department are known in advance. Exact solution methods based on MIP up to now cannot solve large FLP problems (more than nine departments) in a reasonable time (Konak et al., 2006). From the other side, the QAP can solve the larger size problems, by assuming that all departments are equal-sized and must be allocated to the predetermined locations (candidate points), nevertheless QAP cannot solve large scale problems optimally in a polynomial time when the departments size is above 15 facilities. It is possible to model unequal area departments in the QAP formulation by breaking the departments into small grids with equal areas and assigning large artificial flows among them to prevent the separation of grids of the same department (Kaku et al., 1988; Kusiak & Heragu, 1987; Kochhar et al., 1998). However, this significantly increases the number of integer variables in the formulation so that solving even small problems becomes difficult.

The design of an efficient manufacturing system must take into account a number of issues including the determination of the products to be manufactured, the manufacturing or service processes to be used, the quantity and type of equipment required, and the preliminary process plans. In practice, due to the complicated nature of the integrated problem, these decisions are taken by breaking the main problem into several sub problems and considering interaction among them. For instance, according to Wemmerlöv & Hyer (1986), the design of a cellular manufacturing system (CMS) includes (1) cell formation (CF) – grouping parts with similar design features or processing requirements into part families and associated machines into machine cells, (2) group layout – layout machines within each cell (intra-cell layout) and cells with respect to another (inter-cell layout), (3) group scheduling – scheduling parts and part families for production, and (4) resource allocation – assigning tools and human and materials resources.

Layout types used in manufacturing systems can be classified as process layout, product layout, cellular layout (GT), fixed-position layout, hybrid layout, etc. In these layout types, facilities can be arranged in different shapes such as single-row, multi-row, circular, U-shape, etc. However, enforcement of facilities to get arranged in a pre-specified layout shape may increase the total distance traveled by the materials (Solimanpur et al., 2005). In most of mentioned layout types, especially process layout and cellular layout, the layout problem is an essential part of production process design, but mostly due to overwhelming complexity in the nature of these problems, layout problem is studied as a separate part. Hicks (2006) presented a layout design tool based on genetic algorithm (GA), which can be applied to cellular or non-cellular facilities. They tested the proposed tool using case study from a collaborating capital goods company and concluded that when the layout is considered as brown-field problem there is a reduction of total rectilinear distance travelled of 25% compared to the company's layout, while when the layout is treated as a green-field problem, the total rectilinear distance travelled is reduced by 70%. Tavakkoli-Moghaddam et al. (2007) developed a new mathematical model to solve a facility layout problem in CMSs with stochastic demands. They assumed that both machines and cells (departments) are equal-sized and used an integrated assignment model to allocate cells and machines to predefined locations on how total costs of inter and intra-cell movements can be minimized. Finally, they realized that depending on the attitude of the decision maker towards uncertainty, the optimal layout can be changed significantly. Chan et al. (2006) presented a two-stage approach for solving CF problem as well as cell layout problem, where the first stage is to identify machine cells and part

families and the second stage is to find the linear arrangement of the machine cells as a QAP in order to minimize intercellular movement distance unit. They employed GA to solve the problem.

There are some efforts in the literature that consider the layout problem with other design factors, simultaneously (see Akturk & Turkcan, 2000; Chiang & Lee, 2004; Wu et al., 2007; Taghavi & Murat, 2011). However, in order to handle the complexity of the problem, the authors have to apply some unrealistic assumptions such as: enforcement facilities to get arranged in linear layout, minimization of total number of inter-department (inter-cell) movements due to the exceptional elements in the CMS approach instead of actual inter-department material handling cost, fixed position of departments, equal-sized departments, etc.

Briefly, the traditional models of facility layout design in manufacturing systems, which result in a block diagram, departments (cells) sequence and linear arrangement of facilities, have limitations in revealing the information necessary for designing manufacturing facilities. Therefore, exploration of integrated layout model that concurrently determines intra and inter-department layout is necessary. To fill this gap, in this paper an integrated layout problem will be developed, that incorporates intra and inter-department layout.

In the proposed model the arrangement of facilities within the departments is obtained through the QAP and from the other side, the continuous layout problem is implemented to find the position and orientation of rectangular shape departments on the planar area. In fact in the proposed model the speed of QAP in department level is combined to the flexibility and accuracy of continuous layout in plant floor level. In order to evaluate material handling cost precisely the actual position of machines within the departments (instead of center of departments) is considered. Moreover other design factors such as aisle distance, single or multi row inter department layout and orientation of departments have been considered. Before formulating the integrated model, a modified version of QAP, with fewer binary variables and under conditions is developed (this model can solve small and small-to-medium scale problems in a shorter computational time). Then the integrated model is formulated based on the developed QAP in order to minimize total inter and intra-department material handling cost. In addition, a heuristic algorithm has been proposed to solve large-scale problems in a reasonable computational time. One of the applications of proposed model is in layout design of CMS. Therefore, several illustrative numerical examples are selected from the literature of CMS in order to test the proposed model and evaluate the performance of heuristic method.

2. A new formulation on the QAP

In this section, a modified version of QAP is presented in order to solve small and small-to-medium scale problems in a shorter computational time. The QAP was originally introduced by Koopmans & Beckmann (1957), and generally defined as assignment of M facilities to N candidate points (Fig. 1.a) by minimizing total material handling cost. In this paper, it has been assumed that each department is divided into candidate points, where facilities must be assigned to them, therefore we have a QAP in each department. In the modeling of the QAP, a set of binary variables (z_{ij}) is used to assign the facilities to the candidate points (if $z_{ij} = 1$ then facility i is assigned to candidate point j , and $z_{ij} = 0$ otherwise). The QAP can be formulated as the following integer program with quadratic objective function:

$$\text{QAP: min } \sum_i \sum_{k>i} \sum_j \sum_l c_{ik} \cdot f_{ik} \cdot d_{jl} \cdot z_{ij} \cdot z_{kl} \quad (1)$$

subject to:

$$\sum_i z_{ij} = 1, \quad \forall j, \quad (2)$$

$$\sum_j z_{ij} \leq 1, \quad \forall i, \quad (3)$$

$$z_{ij} \in \{0,1\}, \quad \forall i, j, \quad (4)$$

where c_{ik} is material flow cost per unit distance between facility i and k , f_{ik} is material flow between facility i and k , and d_{jl} is distance between candidate point j and l .

Objective function (1) minimizes the total material handling cost, constraint (2) ensures that each facility is assigned to only one candidate point and constraint (3) assures that each candidate point is occupied by utmost one facility.

There are three main methods used to find the global optimal solution for a given QAP: dynamic programming, cutting plane techniques, and branch and bound procedures. Research has shown that the latter is the most successful for solving instances of the QAP (Commander, 2003). The first attempts to solve the QAP eliminated the quadratic term in the objective function (i.e. $z_{ij} \cdot z_{kl}$), in order to transform the problem into a MIP. The linearization of the objective function is usually achieved by introducing new variables and new linear (and binary) constraints. Here a method from Kaufmann and Broeckx (1978) which has the smallest number of variables and constraints is applied to linearize the QAP. They rearranged objective function (1) to the following expression: $\sum_i \sum_{k>i} z_{ij} \sum_j \sum_l c_{ik} \cdot f_{ik} \cdot d_{jl} \cdot z_{kl}$, then they defined new positive variable as follows: $w_{ij} = z_{ij} \sum_j \sum_l c_{ik} \cdot f_{ik} \cdot d_{jl} \cdot z_{kl}$ and showed that the QAP is equivalent to the following MIP:

$$\text{QAP: Min } \sum_i \sum_j w_{ij} \quad (5)$$

subject to: (2)-(4)

$$\sum_{k>i} \sum_l c_{ij} \cdot f_{ij} \cdot d_{jl} \cdot z_{kl} - w_{ij} \leq (1 - z_{ij}) \sum_{k>i} \sum_l c_{ij} \cdot f_{ij} \cdot d_{jl}, \quad \forall i, j, \quad (6)$$

$$w_{ij} \geq 0, \quad \forall i, j, \quad (7)$$

Sahni and Gonzalez (1976) proved that the QAP belongs to the class of computationally hard problems known as NP-complete (i.e. there is no algorithm which can solve the QAP to optimality in polynomial time), and the problems of size greater than 15 facilities remain nearly intractable.

In order to decrease the number of binary variables, it is assumed that candidate points have been composed of pre-specified columns and rows, such as a grid (see Fig. 1.b, there is 4 columns and 3 rows with 12 candidate points), also following conditions must be met:

- I. Widths of all rows in column m are equal
- II. Heights of all columns in row n are equal

To formulate the problem two sets of binary variables z_{im}^X and z_{im}^Y are used to assign the facilities to the candidate points where z_{im}^X and z_{im}^Y are concerned with the horizontal and vertical facility assignment respectively (e.g. in Fig. 1.b, $z_{53}^X = z_{52}^Y = 1$ represents that M5 is assigned to column 3 and row 2).

The MIP formulation of the grid representation quadratic assignment problem (GRQAP) is as follows:

$$\text{GRQAP: Min } \sum_i \sum_{k>i} c_{ik} \cdot f_{ik} (dx_{ik}^+ + dx_{ik}^- + dy_{ik}^+ + dy_{ik}^-) \quad (8)$$

subject to:

$$dx_{ik}^+ - dx_{ik}^- = \sum_{m=1}^m (z_{im}^X - z_{km}^X) a_m, \quad \forall i < k, \tag{9}$$

$$dy_{ik}^+ - dy_{ik}^- = \sum_{n=1}^n (z_{in}^Y - z_{kn}^Y) b_n, \quad \forall i < k, \tag{10}$$

$$\sum_{m=1}^m z_{im}^X = 1, \quad \forall i, \tag{11}$$

$$\sum_{n=1}^n z_{in}^Y = 1, \quad \forall i, \tag{12}$$

$$\sum_i \gamma_{imn} \leq 1, \quad \forall m, n, \tag{13}$$

$$\gamma_{imn} \geq z_{im}^X + z_{in}^Y - 1, \quad \forall i, m, n, \tag{14}$$

$$\gamma_{imn} \geq 0, \quad \forall i, m, n, \tag{15}$$

$$dx_{ik}^+, dx_{ik}^-, dy_{ik}^+, dy_{ik}^-, \quad \forall i < k, \tag{16}$$

$$z_{im}^X \in \{0,1\}, \quad \forall i, m, \tag{17}$$

$$z_{in}^Y \in \{0,1\}, \quad \forall i, n, \tag{18}$$

where a_m denotes horizontal coordinate of center of column m and b_n denotes vertical coordinate of center of row n with respect to the origin. Objective function (8) minimizes the total material handling cost. Constraints (9) and (10) linearize the absolute value term in the horizontal and vertical distance function, constraints (11) and (12) ensure that each facility is assigned to only one column and row, respectively. Constraint (13) restricts that each candidate point is occupied by utmost one facility. Constraints (14) linearizes the nonlinearity due to the multiplication of two binary variables (where $\gamma_{imn} = z_{im}^X \cdot z_{in}^Y$). Finally constraints (15)-(18) are the logical binary and non-negativity requirements on the decision variables.

Table 1 shows the maximum number of binary and positive variables as well as maximum number of constraints, for both models. Note that in the model of QAP, $|m| \cdot |n|$ is replaced instead of $|j|$ (since $|m| \cdot |n| = |j|$).

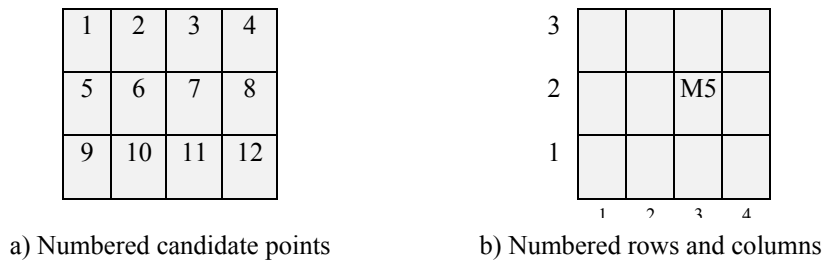


Fig. 1. Two different approaches for assigning facilities to the candidate points

Table 1

Characteristics of the QAP, GRQAP

QAP	No. Binary variables	$ i \cdot j = i \cdot m \cdot n $
	No. Positive variables	$ i \cdot j = i \cdot m \cdot n $
	No. Constraints	$ i \cdot j + i + j = i \cdot m \cdot n + i + m \cdot n $
GRQAP	No. Binary variables	$ i (m + n)$
	No. Positive variables	$ i (2(i - 1) + m \cdot n)$
	No. Constraints	$(i + 1)(m \cdot n + i)$

$|i|$: Number of facilities, $|j| = |m| \cdot |n|$: Number of candidate points, $|m|$: Number of columns, $|n|$: Number of rows

3. Numerical example 1

In this section, several small and small-to-medium scale problems have been selected from the literature in order to compare the GRQAP and QAP. In these examples it is assumed that the dimension of all facilities is 1×1 . All problem instances were solved by using CPLEX 10 on a PC with 2.4 GHz CPU, 2 GB memory, and Windows 7 operating system. The computational results have been reported in Table 2.

Table 2
Computational results of small and small-to-medium scale problems to compare QAP and GRQAP

Problem name	Source	Size $i \times m \times n$	QAP						GRQAP					
			No. binary variables	No. positive variables	No. constraints	CPU time (s)	Obj.	Opt. Gap%	No. binary variables	No. positive variables	No. constraints	CPU time (s)	Obj.	Opt. Gap%
HK4	Heragu & Kusiak(1991)	4×4×1	16	16	24	0.180	40	0%	20	40	40	0.055	40	0%
		4×2×2	16	16	24	0.096	34	0%	16	40	40	0.060	34	0%
LW5	Love & Wong(1976)	5×5×1	25	25	35	0.174	30	0%	30	65	60	0.059	30	0%
		5×3×2	30	30	41	0.181	28	0%	25	70	66	0.168	28	0%
S8H	Simmons(1969)	8×8×1	64	64	80	3.367	200	0%	72	176	144	1.014	200	0%
		8×4×2	64	64	80	5.523	143	0%	48	176	144	3.096	143	0%
		8×3×3	72	72	89	18.384	138	0%	48	184	153	11.965	138	0%
S8H	Simmons(1969)	8×8×1	64	64	80	11.561	488	0%	72	176	144	5.531	488	0%
		8×4×2	64	64	80	15.183	325	0%	48	176	144	7.835	325	0%
		8×3×3	72	72	89	38.504	313	0%	48	184	153	21.877	313	0%
S9	Simmons(1969)	9×9×1	81	81	99	108.180	734	0%	90	225	180	29.485	734	0%
		9×5×2	90	90	109	404.782	490	0%	63	234	190	224.252	490	0%
		9×3×3	81	81	99	138.551	441	0%	54	225	180	80.783	441	0%
S9H	Simmons(1969)	9×9×1	81	81	99	84.852	423	0%	90	225	180	8.141	423	0%
		9×5×2	90	90	109	194.995	294	0%	63	234	190	69.594	294	0%
		9×3×3	81	81	99	90.176	274	0%	54	225	180	38.335	274	0%
A9	Adel El-Baz (2004)	9×9×1	81	81	99	6.018	65239	0%	90	225	180	1.634	65239	0%
		9×5×2	90	90	109	9.874	4854	0%	63	234	190	9.959	4854	0%
		9×3×3	81	81	99	11.064	4818	0%	54	225	180	9.783	4818	0%
S10	Simmons (1969)	10×10×1	100	100	120	373.243	492	0%	110	280	220	16.644	492	0%
		10×5×2	100	100	120	585.348	334	0%	70	280	220	193.374	334	0%
		10×4×3	120	120	142	1614.011	315	0%	70	300	242	970.784	315	0%
LW11	Love & Wong (1976)	11×11×1	121	121	143	4063.60	1207	0%	132	341	264	74.876	1207	0%
		11×6×2	132	132	155	2744.66	795	26.36%	88	352	276	3631.27	793	0%
		11×4×3	132	132	155	4662.41	725	0%	77	352	276	5256.94	725	0%
S11	Simmons (1969)	11×11×1	121	121	143	3509.11	1207	29.68%	132	341	264	105.261	1207	0%
		11×6×2	132	132	155	2889.62	793	26.84%	88	352	276	1485.52	793	0%
		11×4×3	132	132	155	2922.51	725	29.27%	77	352	276	7220.92	725	0%
A12	Adel El-Baz (2004)	12×12×1	144	144	168	1893.83	11055	0%	156	408	312	40.661	11055	0%
		12×6×2	144	144	168	1877.24	8460	41.98%	96	408	312	517.237	8460	0%
		12×4×3	144	144	168	3308.07	8040	15.66%	84	408	312	1239.52	8040	0%

i: number of facilities, m: number of rows, n: number of columns
* in this case solver was interrupted due to insufficient physical memory

All numerical instances were solved optimally by the GRQAP; while the QAP could not solve some cases optimally and the solver was interrupted due to insufficient physical memory. It means that the GRQAP needs relatively less physical memory. In addition, the results show that the GRQAP can solve the problem in shorter computational time than QAP. The average improvement percent in solution time(except the cases which weren't solved optimally) is 52.52%.So it is concluded that the GRQAP can solve small and small-to-medium scale problems about two times as fast as the QAP.

4. Formulating the integrated facility layout problem

In this section, the integrated facility layout problem (IFLP) is formulated, under the following notations:

Indices:

- i, j Index of departments
- k, l Index of facilities

m Index of columns
 n Index of rows

Model parameters:

C number of departments
 w_i width of department i
 h_i height of department i
 M_i number of facilities of department i
 Ro_i number of rows of department i
 Co_i number of columns of department i
 a_{im} length of center of m^{th} column of cell i in x-axis with respect to the left edge of cell i
 b_{in} length of center of n^{th} row of cell i in y-axis with respect to the down edge of cell i
 Lh_{ij} least horizontal distance between department i and department j
 Lv_{ij} least vertical distance between department i and department j
 f_{ijkl} amount of material flow between facility k of department i and facility l of department j
 c_{ijkl} material handling cost per unit flow per unit distance traveled between facility k of department i and facility l of department j
 BM a large number

Decision variables:

(x_i, y_i) coordinates of center of department i
 u_i 1 if department i is located vertically, 0 otherwise
 z_{ikm}^X 1 if facility k is assigned to column m of department i , 0 otherwise
 z_{ikn}^Y 1 if facility k is assigned to row n of department i , 0 otherwise
 $dx_{ijkl}^+ + dx_{ijkl}^-$ horizontal distance between facility k of department i and facility l of department j
 $dy_{ijkl}^+ + dy_{ijkl}^-$ vertical distance between facility k of department i and facility l of department j

The mathematical model:

As it was concluded in the previous section, in contrast with the traditional QAP, the GRQAP can solve problems in a shorter computational time and with less physical memory. After combining GRQAP and continuous layout model and applying some standard linearization methods, the IFLP is formulated as the following MIP model:

IFLP:

$$\min \sum_{i=1}^{c-1} \sum_{j=i+1}^c \sum_{k=1}^{M_i} \sum_{l=1}^{M_j} f_{ijkl} \cdot c_{ijkl} (dx_{ijkl}^+ + dx_{ijkl}^- + dy_{ijkl}^+ + dy_{ijkl}^-) + \sum_{i=1}^c \sum_{k=1}^{M_i-1} \sum_{l=k+1}^{M_i} f_{ikl} \cdot c_{ikl} (dx_{ikl}^+ + dx_{ikl}^- + dy_{ikl}^+ + dy_{ikl}^-) \quad (19)$$

subject to:

$$\sum_{m=1}^{Co_i} (z_{ikm}^X - z_{ilm}^X) a_{im} = dx_{ikl}^+ - dx_{ikl}^-, \quad i = 1, \dots, C, \quad k = 1, \dots, M_i - 1, \quad l = k + 1, \dots, M_i \quad (20)$$

$$\sum_{n=1}^{Ro_i} (z_{ikn}^Y - z_{iln}^Y) b_{in} = dy_{ikl}^+ - dy_{ikl}^-, \quad i = 1, \dots, C, \quad k = 1, \dots, M_i - 1, \quad l = k + 1, \dots, M_i \quad (21)$$

$$x_i - x_j - (1 - u_i) \frac{w_i}{2} + (1 - u_j) \frac{w_j}{2} - u_i \frac{h_i}{2} + u_j \frac{h_j}{2} + \sum_{m=1}^{Co_i} z_{ikm}^X \cdot a_{im} - \sum_{m=1}^{Co_j} z_{jlm}^X \cdot a_{jm} + \xi_{ik} - \xi_{jl} = dx_{ijk}^+ - dx_{ijk}^-, \quad i = 1, \dots, C - 1, \quad j = i + 1, \dots, C, \quad k = 1, \dots, M_i, \quad l = 1, \dots, M_j \quad (22)$$

$$y_i - y_j - u_i \frac{w_i}{2} + u_j \frac{w_j}{2} - (1 - u_i) \frac{h_i}{2} + (1 - u_j) \frac{h_j}{2} + \sum_{n=1}^{Ro_i} z_{ikn}^Y \cdot b_{in} - \sum_{n=1}^{Ro_j} z_{jln}^Y \cdot b_{jn} - \xi_{ik} + \xi_{jl} = dy_{ijk}^+ - dy_{ijk}^-, \quad i = 1, \dots, C - 1, \quad j = i + 1, \dots, C, \quad k = 1, \dots, M_i, \quad l = 1, \dots, M_j \quad (23)$$

$$\sum_{n=1}^{Ro_i} z_{ikn}^Y \cdot b_{in} - \sum_{m=1}^{Co_i} z_{ikm}^X \cdot a_{im} - (1 - u_i) BM \leq \xi_{ik} \leq \sum_{n=1}^{Ro_i} z_{ikn}^Y \cdot b_{in} - \sum_{m=1}^{Co_i} z_{ikm}^X \cdot a_{im} + (1 - u_i) BM, \quad i = 1, \dots, C, \quad k = 1, \dots, M_i \quad (24)$$

$$-u_i \cdot BM \leq \xi_{ik} \leq u_i \cdot BM, \quad i = 1, \dots, C, \quad k = 1, \dots, M_i \quad (25)$$

$$\sum_{m=1}^{Co_i} z_{ikm}^X = 1, \quad i = 1, \dots, C, \quad k = 1, \dots, M_i \quad (26)$$

$$\sum_{n=1}^{Ro_i} z_{ikn}^Y = 1, \quad i = 1, \dots, C, \quad k = 1, \dots, M_i \quad (27)$$

$$\sum_{k=1}^{M_i} \gamma_{ikmn} \leq 1, \quad i = 1, \dots, C, \quad m = 1, \dots, Co_i, \quad n = 1, \dots, Ro_i \quad (28)$$

$$\gamma_{ikmn} \geq z_{ikm}^X + z_{ikn}^Y - 1, \quad i = 1, \dots, C, \quad k = 1, \dots, M_i, \quad m = 1, \dots, Co_i, \quad n = 1, \dots, Ro_i \quad (29)$$

$$x_j - x_i + BM \cdot p_{ij} + BM \cdot v_{ij} \geq \frac{1}{2} [(1 - u_i)w_i + u_i \cdot h_i + (1 - u_j)w_j + u_j \cdot h_j] + Lh_{ij}, \quad i = 1, \dots, C - 1, \quad j = i + 1, \dots, C \quad (30)$$

$$x_i - x_j + BM(1 - p_{ij}) + BM \cdot v_{ij} \geq \frac{1}{2} [(1 - u_i)w_i + u_i \cdot h_i + (1 - u_j)w_j + u_j \cdot h_j] + Lh_{ij}, \quad i = 1, \dots, C - 1, \quad j = i + 1, \dots, C \quad (31)$$

$$y_i - y_j + BM \cdot q_{ij} + BM(1 - v_{ij}) \geq \frac{1}{2} [u_i \cdot w_i + (1 - u_i)h_i + u_j \cdot w_j + (1 - u_j)h_j] + Lv_{ij}, \quad i = 1, \dots, C - 1, \quad j = i + 1, \dots, C \quad (32)$$

$$y_i - y_j + BM(1 - q_{ij}) + BM(1 - v_{ij}) \geq \frac{1}{2} [u_i \cdot w_i + (1 - u_i)h_i + u_j \cdot w_j + (1 - u_j)h_j] + Lv_{ij}, \quad i = 1, \dots, C - 1, \quad j = i + 1, \dots, C \quad (33)$$

$$dx_{ijk}^+, dx_{ijk}^-, dy_{ijk}^+, dy_{ijk}^- \geq 0, \quad i = 1, \dots, C, \quad j = i + 1, \dots, C, \quad k = 1, \dots, M_i, \quad l = 1, \dots, M_j \quad (34)$$

$$\xi_{ik} = \text{free}, \quad i = 1, \dots, C, \quad k = 1, \dots, M_i \quad (35)$$

$$p_{ij}, q_{ij} \in \{0, 1\}, \quad i = 1, \dots, C - 1, \quad j = i + 1, \dots, C \quad (36)$$

$$x_i, y_i \geq 0, \quad i = 1, \dots, C \quad (37)$$

$$u_i \in \{0, 1\}, \quad i = 1, \dots, C \quad (38)$$

$$z_{ikm}^X \in \{0, 1\}, \quad i = 1, \dots, C, \quad k = 1, \dots, M_i, \quad m = 1, \dots, Co_i \quad (39)$$

$$z_{ikn}^Y \in \{0, 1\}, \quad i = 1, \dots, C, \quad k = 1, \dots, M_i, \quad n = 1, \dots, Ro_i \quad (40)$$

$$\gamma_{ikmn} \geq 0, \quad i = 1, \dots, C, \quad k = 1, \dots, M_i, \quad m = 1, \dots, Co_i, \quad n = 1, \dots, Ro_i \quad (41)$$

The objective function (19) minimizes the total material handling cost, including inter and intra-department material handling costs. Note that in the intra-department material handling cost function, the indexes of $iikl$ have been reduced to ikl , (e.g. $f_{iikl} = f_{ikl}$ or $dx_{iikl}^+ = dx_{ikl}^+$). Constraints (20) and (21) linearize the absolute value term in the intra-department rectilinear distance function, constraints (22) and (23) linearize the absolute value term in the inter-department rectilinear distance function, constraints (24) and (25) linearize the quadratic term due to the product of a binary variable and a free expression within the absolute value terms in constraints (22) and (23) i.e. $\xi_{ik} = u_i(\sum_{n=1}^{Ro_i} z_{ikn}^Y \cdot b_{in} - \sum_{m=1}^{Co_i} z_{ikm}^X \cdot a_{im})$, where ξ_{ik} is a free auxiliary variable. This quadratic term has been linearized by applying the following proposition.

Proposition 1. Set of constraints (24) and (25) linearize the following quadratic term: $u_i(\sum_{n=1}^{Ro_i} z_{ikn}^Y \cdot b_{in} - \sum_{m=1}^{Co_i} z_{ikm}^X \cdot a_{im})$.

Proof. This can be shown for each of two possible cases:

Case 1: $u_i = 1$. In this case constraints (24) and (25) are reduced to the following constraints:

$$\sum_{n=1}^{Ro_i} z_{ikn}^Y \cdot b_{in} - \sum_{m=1}^{Co_i} z_{ikm}^X \cdot a_{im} \leq \xi_{ik} \leq \sum_{n=1}^{Ro_i} z_{ikn}^Y \cdot b_{in} - \sum_{m=1}^{Co_i} z_{ikm}^X \cdot a_{im}, \quad i = 1, \dots, C, \quad k = 1, \dots, M_i$$

$$-BM \leq \xi_{ik} \leq BM, \quad i = 1, \dots, C, \quad k = 1, \dots, M_i$$

Therefore it is concluded that ξ_{ik} is equal to $\sum_{n=1}^{Ro_i} z_{ikn}^Y \cdot b_{in} - \sum_{m=1}^{Co_i} z_{ikm}^X \cdot a_{im}$.

Case 2: $u_i = 0$. In this case constraints (24) and (25) are reduced to the following constraints:

$$-BM \leq \xi_{ik} \leq BM, \quad i = 1, \dots, C, \quad k = 1, \dots, M_i$$

$$0 \leq \xi_{ik} \leq 0, \quad i = 1, \dots, C, \quad k = 1, \dots, M_i$$

Thus it is concluded that $\xi_{ik} = 0$. □

Also, Table 3 illustrates how the linearization in Proposition 1 works through four numerical examples.

Table 3
Examples to the linearization stated in proposition 1

	Example 1	Example 2	Example 3	Example 4
$\sum_{n=1}^{Ro_i} z_{ikn}^Y \cdot b_{in}$	9	4	9	4
$\sum_{m=1}^{Co_i} z_{ikm}^X \cdot a_{im}$	6	6	6	6
$\sum_{n=1}^{Ro_i} z_{ikn}^Y \cdot b_{in} - \sum_{m=1}^{Co_i} z_{ikm}^X \cdot a_{im}$	3	-2	3	-2
u_i	1	1	0	0
Imposed constraints	$3 \leq \xi_{ik} \leq 3$ $-BM \leq \xi_{ik} \leq BM$	$-2 \leq \xi_{ik} \leq -2$ $-BM \leq \xi_{ik} \leq BM$	$-BM \leq \xi_{ik} \leq BM$ $0 \leq \xi_{ik} \leq 0$	$-BM \leq \xi_{ik} \leq BM$ $0 \leq \xi_{ik} \leq 0$
Result (ξ_{ik})	3	-2	0	0

The functions of remaining constraints are as follows: Constraints (26) and (27) ensure that each facility is assigned to one column and one row respectively, constraints (28) restricts that each

candidate point in each department can be occupied by utmost one facility, constraint (29) linearizes the nonlinearity of $z_{ikm}^X \cdot z_{ikn}^Y$, where the auxiliary variable γ_{ikmn} is equal to $z_{ikm}^X \cdot z_{ikn}^Y$. Set of constraints (30)-(33) ensure that departments do not overlap, also in these constraints, p_{ij}, q_{ij} and v_{ij} are auxiliary binary variables. Finally, constraints (34)-(41) are the logical binary and non-negativity requirements on the decision variables.

5. Numerical example 2

As it was mentioned before, one of the applications of the proposed model is in the layout design of CMS i.e. after specifying machine cells and part families. To show this application, several small, medium and large scale numerical examples have been selected from the literature of CMS, and the IFLP is applied on them. Note that in these examples expression of ‘cell’ is equivalent to ‘department’ and term of ‘machine’ is equivalent to ‘facility’.

5.1. Small scale problems

The first small scale problem (S1) has been selected from Akturk & Turkcan (2000); they concluded that 3 machine cells must be formed to produce 20 parts. The information corresponds to machine cells and material flow cost ($f_{ijkl} \cdot c_{ijkl}$) among machines has been given in Table 4. It is necessary to mention that this table has been obtained according to demand of parts, processing routes of parts and inter and intra-cell material handling cost, that are exist in the reference paper.

Table 4
The material flowcost among machines (problem S1)

	1-1	1-2	1-3	1-4	1-5	1-6	2-1	2-2	2-3	3-1	3-2	3-3	3-4
1-1	0.0	1104.8	0	0	0	391.3	0	0	0	0	0	0	0
1-2		0	1952.1	1192.9	0	924.8	0	0	0	0	0	0	0
1-3			0	0	0	0	0	0	0	287.6	0	0	0
1-4				0	0	0	0	0	0	0	0	0	0
1-5					0	2104.3	0	0	0	0	0	672.0	0
1-6						0	0	0	0	0	0	0	0
2-1							0	1548.8	0	0	0	368.7	0
2-2								0	2138.5	0	0	0	0
2-3									0	0	0	0	0
3-1										0	2481.1	537.6	0
3-2											0	2019.4	2229.0
3-3												0	1367.2
3-4													0

The problem S1 has been solved for different cell sizes as the following cases: S1-1, S1-2 and S1-3. Table 5 shows the dimension of each case.

Table 5
Dimension of the cells in each case of S1

Cell	S1-1				S1-2				S1-3			
	Width	Height	No. rows	No. columns	Width	Height	No. rows	No. columns	Width	Height	No. rows	No. columns
1	6	1	1	6	3	2	2	3	3	2	2	3
2	3	1	1	3	3	1	1	3	3	1	1	3
3	4	1	1	4	4	1	1	4	2	2	2	2

Table 7 summarizes optimum layout of each problem, including cell layout and layout of machines within the cells. The results revealed that by changing the number of rows and columns in each cell, the total material handling cost has been changed, considerably. For example, in S1-1, the layout of machines within all cells are single row and the resulted material handling cost is 26779.6, while in S1-2 by changing the number of rows and columns the material handling cost has been reduced to 24444.3

with a gap of 9%; or by comparing S1-1 and S1-2, it is concluded that the material handling cost has been reduced about 5%. Therefore, in order to attain better solution, the decision maker should consider different dimension of the cells. The next small scale problem (S2) is selected from Cheng et al. (1966); this problem contains 20 parts and 10 machines. They grouped parts into 3 part families that should be produced using 3 machine cells with 5, 3 and 4 machines in cell 1, 2 and 3 respectively. Table 6 presents the material flow among machines, for the proposed problem. In addition, the unit intra and inter-cell handling cost per unit distance is assumed 5 and 7 units, respectively. This problem has been solved in three cases, including S2-1, S2-2 and S2-3. In S2-1 the layout of machines within all cells is linear and in S2-2 it is multi rows for cell 1 and 3. Also S2-3 is similar to S2-2, 0.5 units for aisle distance.

Table 6
Material flow among machines (Problem S2)

	1-1	1-2	1-3	1-4	1-5	2-1	2-2	2-3	3-1	3-2	3-3	3-4
1-1	0	148	0	0	0	310	0	0	94	0	0	32
1-2		0	246	36	122	44	0	0	0	140	0	24
1-3			0	102	122	0	0	0	0	0	0	68
1-4				0	128	0	0	24	0	0	0	36
1-5					0	0	0	82	0	0	0	142
2-1						0	130	0	18	216	0	0
2-2							0	52	0	18	0	96
2-3								0	0	0	0	662
3-1									0	96	0	0
3-2										0	234	22
3-3											0	342
3-4												0

S2-1, S2-2 and S2-3 were solved optimally and the results have been summarized in Table 7. The results show that, unlike S1, by increasing the number of rows, total material handling cost has been increased.

Table 7
The computational results of small scale problems

Problem	Source	Size	Intra-cell layout	Coordinates of the cells	Cells orientation	Objective value	CPU time (s)
S1-1	Akturk & Turkcan (2000)	3×13	Cell 1: 1-5 1-6 1-1 1-2 1-3 1-4 Cell 2: 2-1 2-2 2-3 Cell 3: 3-4 3-3 3-2 3-1	Cell 1: (0,2.5) Cell 2: (3,0) Cell 3: (1,0.5)	Cell 1: vertical Cell 2: horizontal Cell 3: vertical	26779.6	2.140
S1-2	Akturk & Turkcan (2000)	3×13	Cell 1: 1-5 1-6 1-1 1-3 1-2 1-4 Cell 2: 2-3 2-2 2-1 Cell 3: 3-1 3-2 3-3 3-4	Cell 1: (4,0) Cell 2: (0,0.5) Cell 3: (2,0)	Cell 1: horizontal Cell 2: horizontal Cell 3: vertical	25479.1	11.830
S1-3	Akturk & Turkcan (2000)	3×13	Cell 1: 1-5 1-6 1-1 1-3 1-2 1-4 Cell 2: 2-1 2-2 2-3 Cell 3: 3-4 3-3 3-2 3-1	Cell 1: (2.5,0) Cell 2: (0.5,2.5) Cell 3: (0,0)	Cell 1: horizontal Cell 2: vertical Cell 3: horizontal	24444.3	14.743
S2-1	Cheng et al. (1966)	3×12	Cell 1: 1-4 1-5 1-3 1-2 1-1 Cell 2: 2-3 2-2 2-1 Cell 3: 3-4 3-3 3-2 3-1	Cell 1: (0,0) Cell 2: (2,0) Cell 3: (1,0.5)	Cell 1: vertical Cell 2: vertical Cell 3: vertical	32750	6.796
S2-2	Cheng et al. (1966)	3×12	Cell 1: 1-3 1-4 * 1-2 1-5 1-1 Cell 2: 2-2 2-3 2-1 Cell 3: 3-2 3-1 3-3 3-4	Cell 1: (0.5,2.5) Cell 2: (1.5,0) Cell 3: (0,0.5)	Cell 1: horizontal Cell 2: vertical Cell 3: horizontal	34430	29.322
S2-3	Cheng et al. (1966)	3×12	Cell 1: 1-4 1-3 * 1-5 1-2 1-1 Cell 2: 2-2 2-3 2-1 Cell 3: 3-4 3-2 3-3 3-1	Cell 1: (3,1) Cell 2: (0,2) Cell 3: (0.5,0)	Cell 1: vertical Cell 2: horizontal Cell 3: horizontal	42583	17.218

* Empty candidate point

Each of solutions reported by Adil & Rajamani(2000) and Harhalakis et al. (1990) are studied under two cases. In M1-1 and M2-1, the layout of machines within all cells is single row (linear), and in M1-2 and M2-2 it is multi rows. The solutions of these cases have been summarized in Table 10. The problems were solved with good optimality gap and the results show that M1-2 gives the smallest material handling cost.

The next medium scale problem, M3 has been selected from Ramabhata and Nagi (1996), they specified 4 machine cells with 1, 6, 6 and 2 machines in the cells 1-4 respectively. They developed a CMS model to minimize the number of inter-cell material flow instead of the inter-cell material handling cost, and finally it was realized that the total number of inter-cell movements is 83. Now this problem is solved by the IFLP (considering inter and intra-cell material handling cost). It is assumed that the unit inter and intra-cell handling cost per unit distance are 7 and 5 units, respectively, also width of the aisle distance is assumed 0.5 units. M3 is studied under two cases, linear intra-cell layout (M3-1) and multi row intra-cell layout (M3-2). Table 10 shows the optimum solutions for M3-1 and M3-2. Both cases were solved optimally and finally it was concluded that in this problem multi row layout is better than linear layout with a gap of 10%.

Table 10
The computational results of medium scale problems

Problem	Source	Size	Intra-cell layout	Coordinates of cells	Cells orientation	Objective value	CPU time (s)	Relative gap
M1-1	Adil & Rajamani (2000)	3×20	Cell 1: 1-6 1-3 1-4 1-1 1-2 1-5 Cell 2: 2-6 2-5 2-7 2-4 2-3 2-2 2-1 Cell 3: 3-5 3-4 3-3 3-7 3-2 3-1 3-6	Cell 1: (1,0) Cell 2: (0,0.5) Cell 3: (2,0.5)	Cell 1: vertical Cell 2: vertical Cell 3: vertical	1020	1897.343	0%
M1-2	Adil & Rajamani (2000)	3×20	Cell 1: 1-1 1-2 1-5 1-6 1-3 1-4 Cell 2: 2-6 2-5 2-7 2-2 * 2-4 2-3 2-1 Cell 3: 3-7 3-4 3-1 3-6 3-5 3-3 3-2 *	Cell 1: (2.5,2) Cell 2: (0,1) Cell 3: (3,0)	Cell 1: horizontal Cell 2: vertical Cell 3: horizontal	965	6457.313**	28.74%
M2-1	Harhalakis et al. (1990)	4×20	Cell 1: 1-2 1-5 1-4 1-1 1-3 Cell 2: 2-7 2-6 2-3 2-5 2-4 2-1 2-2 Cell 3: 3-1 3-5 3-3 3-2 3-4 Cell 4: 4-2 4-3 4-1	Cell 1: (2,2) Cell 2: (1,2) Cell 3: (0,0) Cell 4: (3,2)	Cell 1: vertical Cell 2: vertical Cell 3: vertical Cell 4: vertical	1075	9702.79**	9.93%
M2-2	Harhalakis et al.(1990)	4×20	Cell 1: 1-5 1-3 * 1-2 1-1 1-4 Cell 2: * 2-6 2-7 2-3 2-2 2-5 2-1 2-4 Cell 3: 3-4 3-5 3-1 * 3-3 3-2 Cell 4: 4-2 4-3 4-1	Cell 1: (3,1.5) Cell 2: (0,2) Cell 3: (0.5,0) Cell 4: (4.5,0.5)	Cell 1: vertical Cell 2: horizontal Cell 3: horizontal Cell 4: vertical	975	6346.16**	32.17%
M3-1	Ramabhata & Nagi (1996)	4×15	Cell 1: 1-1 Cell 2: 2-4 2-5 2-3 2-1 2-6 2-2 Cell 3: 3-3 3-5 3-4 3-2 3-1 3-6 Cell 4: 4-2 4-1	Cell 1: (1.5,5) Cell 2: (1.5,1) Cell 3: (0,2) Cell 4: (3,0)	Cell 1: horizontal Cell 2: vertical Cell 3: vertical Cell 4: vertical	5192.5	215.678	0%
M3-1	Ramabhata & Nagi (1996)	4×15	Cell 1: 1-1 Cell 2: 2-3 2-6 2-2 2-4 2-5 2-1 Cell 3: 3-3 3-4 3-6 3-5 3-2 3-1 Cell 4: 4-1 4-2	Cell 1: (2.5,3) Cell 2: (2,0.5) Cell 3: (4.5,0.5) Cell 4: (0,0)	Cell 1: horizontal Cell 2: vertical Cell 3: vertical Cell 4: vertical	4657.5	1605.90	0%

* Empty candidate point

**Terminated by the solver due to insufficient physical memory

5.3. Large scale problems

The first large scale problem has been selected from Kazerooni et al. (1997), they specified 7 machine cells with 4, 2, 2, 4, 5, 4 and 3 machines in the cells 1-7, respectively. To see the data sets, including processing routes and demand of parts, in order to specify material flow among machines, refer to Kazerooni et al. (1997). The unit intra and inter-cell material handling cost per unit distance are assumed 5 and 10 units, respectively; also the aisle distance is assumed 0.5 units. This problem has been solved in two conditions, in the first condition i.e. L1-1, the layout of machines within all cells is single row and in the next one i.e. L1-2 the layout of machines in cells 1, 4, 5 and 6 are multi rows and in the remaining cells are single row. The optimum layout of L1-1 and L1-2 has been presented in Table 12. L1-1 was interrupted after 10000 seconds, while L1-2 was terminated by the solver due to insufficient physical memory after 3927 seconds. Amount of material handling cost for L1-1 (i.e. single row layout) is 79910 with an optimality gap of 9.19%; while for L1-2 it is 77385 with an optimality gap of 28.98%. Therefore, it is concluded that for the problem presented by Kazerooni et al. (1997), the multi rows layout is better than single row layout. The next large scale problems (L2 and L3) have been selected from Chan et al. (2006). They studied a two stage CMS problem on an industrial case with 37 machines and 30 parts, where the machine cells and part families are formed in the first stage, and cell sequencing is established in the second stage. They founded two different solutions for their problem (see Table 11) and investigated different cells sequence in order to find the best sequence of the cells how total intercellular part movement being minimized. Finally, they realized that the optimal objective value for the both solutions is equal to 21.

Table 11
Presented machine cells by Chan et al. (2006) as well as machine indexes used in this paper

Problem	Cell number	Machines (Source paper)	Machines (This paper)
L2	1	3, 4, 15, 19, 24, 34, 35, 36, 37	1-1, 1-2, 1-3, 1-4, 1-5, 1-6, 1-7, 1-8, 1-9
	2	7, 8, 9, 10, 11, 12, 13, 16, 18, 20, 21, 26, 31	2-1, 2-2, 2-3, 2-4, 2-5, 2-6, 2-7, 2-8, 2-9, 2-10, 2-11, 2-12, 2-13
	3	14, 17, 22, 27, 28, 29, 30	3-1, 3-2, 3-3, 3-4, 3-5, 3-6, 3-7
	4	1, 2, 5, 6, 23, 25, 32, 33	4-1, 4-2, 4-3, 4-4, 4-5, 4-6, 4-7, 4-8
L3	1	1, 5, 14, 17, 22, 29, 30	1-1, 1-2, 1-3, 1-4, 1-5, 1-6, 1-7
	2	3, 4, 15, 19, 24, 34, 35, 36, 37	2-1, 2-2, 2-3, 2-4, 2-5, 2-6, 2-7, 2-8, 2-9
	3	2, 6, 23, 25, 27, 28, 32, 33	3-1, 3-2, 3-3, 3-4, 3-5, 3-6, 3-7, 3-8
	4	7, 8, 9, 10, 11, 12, 13, 16, 18, 20, 21, 26, 31	4-1, 4-2, 4-3, 4-4, 4-5, 4-6, 4-7, 4-8, 4-9, 4-10, 4-11, 4-12, 4-13

Table 12
The computational results of large scale problems

Problem	Source	Size	Intra-cell layout	Coordinates of cells	Cells orientation	Objective value	CPU time (s)	Relative Gap	
L1-1	Kazerooni et al. (1997)	7×24	Cell 1: 1-4 1-3 1-2 1-1 Cell 2: 2-2 2-1 Cell 3: 3-2 3-1 Cell 4: 4-1 4-2 4-3 4-4	Cell 5: 5-5 5-4 5-3 5-2 5-1 Cell 6: 6-1 6-2 6-3 6-4 Cell 7: 7-3 7-2 7-1	Cell 1: (2,7.5) Cell 2: (3.5,5.5) Cell 3: (6.5,4.5) Cell 4: (0,6) Cell 5: (5,5) Cell 6: (2,3.5) Cell 7: (5,1.5)	Cell 1: horizontal Cell 2: Vertical Cell 3: Vertical Cell 4: horizontal Cell 5: Vertical Cell 6: horizontal Cell 7: horizontal	79910	10000	9.19%
L1-2	Kazerooni et al. (1997)	7×24	Cell 1: 1-3 1-2 1-3 1-2 Cell 2: 2-2 2-1 Cell 3: 3-2 3-1 Cell 4: 4-1 4-4 4-2 4-3	Cell 5: 5-4 5-3 * 5-5 5-2 5-1 Cell 6: 6-3 6-4 6-2 6-1 Cell 7: 7-1 7-2 7-3	Cell 1: (0,11.5) Cell 2: (2.5,7.5) Cell 3: (4,7.5) Cell 4: (0,9) Cell 5: (5,5) Cell 6: (0.5,6.5) Cell 7: (2.5,4.5)	Cell 1: Vertical Cell 2: Vertical Cell 3: Vertical Cell 4: horizontal Cell 5: Horizontal Cell 6: Vertical Cell 7: Vertical	77385	3427.88**	26.36%
L2	Chan et al. (2006)	4×37	Cell 1: 1-3 1-8 1-1 1-2 1-5 1-6 1-4 1-9 1-7 Cell 2: 2-3 2-9 2-11 2-7 2-10 2-8 2-2 2-5 2-6 2-4 2-1 2-12 * * 2-13	Cell 3: 3-5 3-4 3-6 3-3 * 3-7 3-1 3-2 Cell 4: 4-4 4-3 4-1 4-8 4-2 4-5 4-6 4-7	Cell 1: (3,7.5) Cell 2: (4,4) Cell 3: (2.5,0) Cell 4: (0,4.5)	Cell 1: vertical Cell 2: horizontal Cell 3: vertical Cell 4: vertical	182	7123.63**	58.35%
L3	Chan et al. (2006)	4×37	Cell 1: 1-1 1-7 1-8 1-4 1-6 1-3 1-2 1-5 1-0 Cell 2: * 2-4 2-7 2-5 * 2-13 2-9 2-11 2-12 2-3 2-10 2-2 2-6 2-8 2-1	Cell 3: * 3-5 3-4 3-2 3-7 3-6 3-3 3-1 Cell 4: 4-5 4-6 4-7 4-8 4-3 4-4 4-1 4-2	Cell 1: (4,5,4) Cell 2: (0,4) Cell 3: (1.5,1) Cell 4: (5,0)	Cell 1: horizontal Cell 2: horizontal Cell 3: horizontal Cell 4: vertical	184	6151.80**	55.32%

* Empty candidate point **Terminated by the solver due to insufficient physical memory

Now the IFLP is employed to solve the proposed problem by Chan et al. (2006). It is assumed that the unit inter and intra-cell material handling cost per unit distance is 2 and 1 units, respectively, also the aisle distance is assumed 0.5 units.

L2 and L3 were solved and the results have been illustrated in Table 12. Unfortunately, these problems have not been solved with good optimality gap and the relative gaps are 58.35% and 55.32% for L2 and L3, respectively. However, the primary results almost support the equality of L2 and L3, which was reported by Chan et al. (2006).

6. A heuristic method for large scale problems

In order to solve large scale problems in a short amount of time, a heuristic method have been developed. In this heuristic method, the integrated model is decomposed into three sub models. The first submodel (Model F1) creates a primary department layout, the second one (Model F2) uses the outputs of Model F1 to assign facilities to the existent department layout, then the third model (Model F3) uses the outputs of Model F2 to re-design the department layout and improve the inter-department material handling costs. Again, model F2 is implemented to re-assign facilities to the existence layout, in order to minimize total material handling costs. This procedure between Model F2 and Model F3 continues, until no further improvement resulted in the objective function. The stages of proposed heuristic are explained in detail in the next stages.

6.1. Stage 1

In this stage, a primary department layout is created according to the following MIP model.

Model F1:

$$\text{Min } \sum_i \sum_{j>i} FC_{ij} (dx_{ij}^+ + dx_{ij}^- + dy_{ij}^+ + dy_{ij}^-) \quad (42)$$

subject to: (30)-(33) and (36)-(38)

$$x_i - x_j = dx_{ij}^+ - dx_{ij}^-, \quad \forall j > i, \quad (43)$$

$$y_i - y_j = dy_{ij}^+ - dy_{ij}^-, \quad \forall j > i, \quad (44)$$

$$dx_{ij}^+, dx_{ij}^-, dy_{ij}^+, dy_{ij}^- \geq 0, \quad \forall j > i, \quad (45)$$

In the above model, FC_{ij} is total material flow cost between department i and department j , and is calculated as follows: $FC_{ij} = \sum_{k=1}^{M_i} \sum_{l=1}^{M_j} f_{ijkl} \cdot c_{ijkl}$; also $dx_{ij}^+ + dx_{ij}^-$ and $dy_{ij}^+ + dy_{ij}^-$ measure the horizontal and vertical distances between department i and department j , respectively, all remaining parameters are the same before defined. After solving Model F1, total material handling cost is set to $TC = \infty$ and the solutions i.e. \tilde{x}_i , \tilde{y}_i and \tilde{u}_i are placed as the inputs of the next model (Model F2).

6.2. Stage 2

In this stage, facilities are arranged within the existent department layout according to the following MIP model.

Model F2:

$$\text{min } TC_2 = (19)$$

subject to:(20)-(23), (26)-(29), (34) and (39)-(41)

$$\begin{aligned} \tilde{x}_i - \tilde{x}_j + \frac{-w_i + \tilde{u}_i(w_i - h_i) + w_j - \tilde{u}_j(w_j - h_j)}{2} + (1 - \tilde{u}_i) \sum_{m=1}^{Co_i} z_{ikm}^X \cdot a_{im} + \tilde{u}_i \sum_{n=1}^{Ro_i} z_{ikn}^Y \cdot b_{in} \\ - (1 - \tilde{u}_j) \sum_{m=1}^{Co_j} z_{jlm}^X \cdot a_{jm} - \tilde{u}_j \sum_{n=1}^{Ro_j} z_{jln}^Y \cdot b_{jn} = dx_{ijkl}^+ - dx_{ijkl}^-, \quad i = 1, \dots, C, \\ j = i + 1, \dots, C, \quad k = 1, \dots, M_i, \quad l = 1, \dots, M_j \end{aligned} \tag{46}$$

$$\begin{aligned} \tilde{y}_i - \tilde{y}_j + \frac{-h_i + \tilde{u}_i(h_i - w_i) + h_j - \tilde{u}_j(h_j - w_j)}{2} + \tilde{u}_i \sum_{m=1}^{Co_i} z_{ikm}^X \cdot a_{im} + (1 - \tilde{u}_i) \sum_{n=1}^{Ro_i} z_{ikn}^Y \cdot b_{in} \\ - \tilde{u}_j \sum_{m=1}^{Co_j} z_{jlm}^X \cdot a_{jm} - (1 - \tilde{u}_j) \sum_{n=1}^{Ro_j} z_{jln}^Y \cdot b_{jn} = dy_{ijkl}^+ - dy_{ijkl}^-, \quad i = 1, \dots, C, \\ j = i + 1, \dots, C, \quad k = 1, \dots, M_i, \quad l = 1, \dots, M_j \end{aligned} \tag{47}$$

where \tilde{x}_i , \tilde{y}_i and \tilde{u}_i represent existent department layout (obtained by Model F1 or the next model i.e. Model F3). After solving Model F2, if the resulted objective value i.e. TC_2 becomes better than current objective value i.e. TC then $TC = TC_2$ and the main outputs of Model F2 i.e. $\tilde{z}_{ikn}^X, \tilde{z}_{ikn}^Y$, are placed as the inputs of Model F3, otherwise the procedure is terminated.

6.3. Stage 3

In this stage the department layout are re-designed according to Model F3; in fact this model changes the position and orientation of departments in order to improve the inter-department material handling cost. In this submodel there are eight states for each department with respect to other departments that must be considered in the mathematical model, these states have been illustrated in Fig. 2.

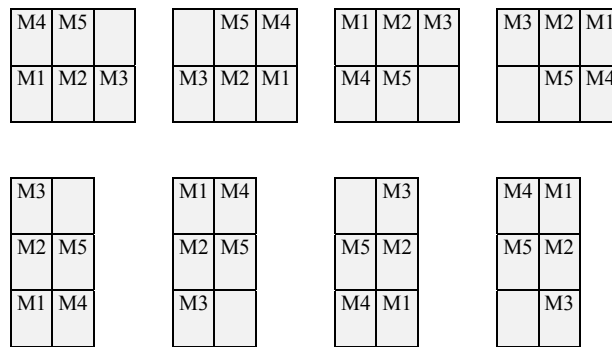


Fig. 2. Eight possible states for placing a department, considering layout of facilities within the department

After applying standard linearization methods Model F3 is formulated as the following MIP model:

Model F3:

$$\min TC_3 = \sum_{i=1}^{C-1} \sum_{j=i+1}^C \sum_{k=1}^{M_i} \sum_{l=1}^{M_j} f_{ijkl} \cdot c_{ijkl} (dx_{ijkl}^+ + dx_{ijkl}^- + dy_{ijkl}^+ + dy_{ijkl}^-) + TC^{Intra} \tag{48}$$

subject to: (30)-(34) and (36)-(38)

$$\begin{aligned}
 x_i - (1 - u_i) \frac{w_i}{2} - u_i \frac{h_i}{2} + A_{ik} + \mu_i^l (h_i - 2B_{ik} + 2A_{ik} - w_i) + u_i^l (w_i - 2A_{ik}) \\
 + u_i (B_{ik} - A_{ik}) - x_j + (1 - u_j) \frac{w_j}{2} + u_j \frac{h_j}{2} - A_{jl} - \mu_j^l (h_j - 2B_{jl} + 2A_{jl} - w_j) \\
 - u_j^l (w_j - 2A_{jl}) - u_j (B_{jl} - A_{jl}) = dx_{ijkl}^+ - dx_{ijkl}^-, \quad i = 1, \dots, C, \\
 j = i + 1, \dots, C, \quad k = 1, \dots, M_i, \quad l = 1, \dots, M_j
 \end{aligned} \tag{49}$$

$$\begin{aligned}
 y_i - (1 - u_i) \frac{h_i}{2} - u_i \frac{w_i}{2} + B_{ik} + \mu_i^u (2B_{ik} - h_i - 2A_{ik} + w_i) + u_i^u (h_i - 2B_{ik}) \\
 + u_i (A_{ik} - B_{ik}) - y_i + (1 - u_i) \frac{h_i}{2} + u_i \frac{w_i}{2} - B_{jl} - \mu_j^u (2B_{jl} - h_j - 2A_{jl} + w_j) \\
 - u_j^u (h_j - 2B_{jl}) - u_j (A_{jl} - B_{jl}) = dy_{ijkl}^+ - dy_{ijkl}^-, \quad i = 1, \dots, C, \\
 j = i + 1, \dots, C, \quad k = 1, \dots, M_i, \quad l = 1, \dots, M_j
 \end{aligned} \tag{50}$$

$$\mu_i^l \leq u_i, \quad i = 1, \dots, C \tag{51}$$

$$\mu_i^l \leq u_i^l, \quad i = 1, \dots, C \tag{52}$$

$$\mu_i^l \geq u_i^l + u_i - 1, \quad i = 1, \dots, C \tag{53}$$

$$\mu_i^u \leq u_i, \quad i = 1, \dots, C \tag{54}$$

$$\mu_i^u \leq u_i^u, \quad i = 1, \dots, C \tag{55}$$

$$\mu_i^u \geq u_i^u + u_i - 1, \quad i = 1, \dots, C \tag{56}$$

$$\mu_i^l, \mu_i^u \geq 0, \quad i = 1, \dots, C \tag{57}$$

Note that, since in Model F3 the layout of facilities within the departments is fixed therefore the intra-department material handling cost is constant and it is calculated as follows: $TC^{Intra} = \sum_{i=1}^C \sum_{k=1}^{M_i-1} \sum_{l=k+1}^{M_i} (|A_{ik} - A_{il}| + |B_{ik} - B_{il}|)$, where $A_{ik} = \sum_{m=1}^{Co_i} \tilde{z}_{ikm}^X \cdot a_{im}$, $B_{ik} = \sum_{n=1}^{Ro_i} \tilde{z}_{ikn}^Y \cdot b_{in}$. Also in the above model u_i, u_i^l and u_i^u are related to the direction of department i . Table 13 illustrates the eight possible states resulted by these variables for a hypothetical facility within a department. After solving Model 3, if the resulted objective value i.e. TC_3 be better than current objective value i.e. TC then $TC = TC_3$ and the main outputs of Model 3 i.e. $\tilde{z}_{ikn}^X, \tilde{z}_{ikn}^Y$, are placed as the inputs of Model F2, otherwise the procedure is terminated. Fig. 3. illustrates the flowchart of proposed heuristic method.

Table 13

The coordinates of a hypothetical facility within a department with respect to the direction of the department

States	u_i	u_i^l	u_i^u	width	height
1	0	0	0	$x_i - \frac{w_i}{2} + \tilde{A}_{ik}$	$y_i - \frac{h_i}{2} + \tilde{B}_{ik}$
2	0	0	1	$x_i - \frac{w_i}{2} + \tilde{A}_{ik}$	$y_i + \frac{h_i}{2} - \tilde{B}_{ik}$
3	0	1	0	$x_i + \frac{w_i}{2} - \tilde{A}_{ik}$	$y_i - \frac{h_i}{2} + \tilde{B}_{ik}$
4	0	1	1	$x_i + \frac{w_i}{2} - \tilde{A}_{ik}$	$y_i + \frac{h_i}{2} - \tilde{B}_{ik}$
5	1	0	0	$x_i - \frac{h_i}{2} + \tilde{B}_{ik}$	$y_i - \frac{w_i}{2} + \tilde{A}_{ik}$
6	1	0	1	$x_i - \frac{h_i}{2} + \tilde{B}_{ik}$	$y_i + \frac{w_i}{2} - \tilde{A}_{ik}$
7	1	1	0	$x_i + \frac{h_i}{2} - \tilde{B}_{ik}$	$y_i - \frac{w_i}{2} + \tilde{A}_{ik}$
8	1	1	1	$x_i + \frac{h_i}{2} - \tilde{B}_{ik}$	$y_i + \frac{w_i}{2} - \tilde{A}_{ik}$

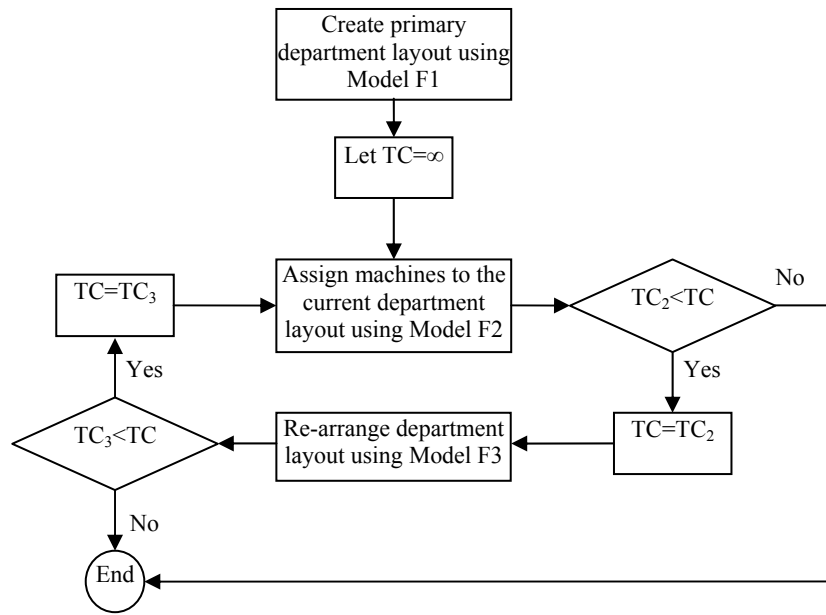


Fig. 3. Flowchart of proposed heuristic method

7. Numerical example.3

In this section, the proposed heuristic method is implemented to solve the same problems presented in section 5. The results are compared against the IFLP. The computational results have been summarized in Table 14. The column of resource time shows the maximum time for each iteration of the heuristic method. Also the error percent column ‘Error (%)’ has been calculated as follows:

$$\left(\frac{Obj^{Heuristic} - Obj^{IFLP}}{Obj^{IFLP}} \right) \times 100.$$

Table 14
The results of the heuristic method

Problem	IFLP			Heuristic method				
	Objective	CPU time (s)	Gap (%)	Resource time (s)	CPU time (s)	Objective	No. iterations	Error (%)
S1-1	26779.6	2.140	0%	-	1.246	26820	5	0.15%
S1-2	25479.1	11.830	0%	-	2.622	25479.1	5	0.00%
S1-3	24444.3	14.743	0%	-	3.495	24923.7	5	1.96%
S2-1	32750	6.796	0%	-	1.212	33986	5	3.77%
S2-2	34430	29.322	0%	-	1.922	34446	7	0.05%
S2-3	42583	17.218	0%	-	1.47	43343	5	1.78%
M1-1	1020	1897.343	0%	-	88.148	1020	5	0.00%
M1-2	965	6457.313*	28.74%	-	400.683	980	5	1.55%
M2-1	1075	9702.79*	9.93%	-	60.807	1060	8	-1.40%
M2-2	975	6346.16*	32.17%	-	759.338	1005	5	3.08%
M3-1	5192.5	215.678	0%	-	8.133	5257.5	3	1.25%
M3-2	4657.5	1605.90	0%	-	21.733	4657.5	5	0.00%
L1-1	79910	10000	9.19%	20	61.864	83155	6	4.06%
L1-2	77385	3427.88*	26.36%	20	67.346	77385	6	0%
L2	182	7123.63*	58.35%	60	254.329	172	7	-5.49%
L3	180	6151.80*	55.32%	100	566.802	175	6	-2.78%

* Interrupted by the solver due to insufficient physical memory

The results show that in cases of S1-2, M1-1, M3-2 and L1-2 the objective value of the proposed heuristic is equal to that of IFLP, and in cases of M2-1, L2, L3 the heuristic method even obtained better solution than the IFLP. Also in the remaining cases, the average error percent between the IFLP

and heuristic method is about 2.2%. Therefore, it is deduced that the proposed heuristic is an efficient method for solving problems in reasonable time.

7. Conclusions

In this paper, an integrated layout model was developed to incorporate inter and intra-department layout. First, a modified version of QAP, namely GRQAP, with fewer binary variables was developed; in contrast to traditional QAP, the GRQAP can solve the problems in a shorter computational time and with less physical memory. Then the IFLP was formulated based on the GRQAP in order to minimize total material handling cost. Several numerical examples were selected from the literature of CMS and the IFLP applied on them. Also due to the complexity of integrated model, an efficient heuristic method was developed in order to solve large scale problems in short amount of time. Finally, it was deduced that in contrast with the ICFLP, the proposed heuristic could even obtain better solution over a short amount of time.

References

- Adel El-Baz, M. (2004). A genetic algorithm for facility layout problems of different manufacturing environments. *Computers & Industrial Engineering*, 47, 233–246.
- Adil, G. K., & Rajamani, D. (2000). The Trade-Off Between intracell and intercell moves in group technology cell formation. *Journal of Manufacturing Systems*, 19 (5), 305-317.
- Akturk, M. S., & Turkcan, A. (2000). Cellular manufacturing system design using a holonistic approach. *International Journal of Production Research*, 38 (10), 2327-2347.
- Chan, F. T., Lau, K., Chan, P., & Choy, K. (2006). Two-stage approach for machine-part grouping and cell layout problems. *Robotics and Computer-Integrated Manufacturing*, 22, 217–238.
- Cheng, C. H., Madan, M. S., & Motwani, J. (1966). Knapsack-based algorithm for designing cellular manufacturing systems. *The Journal of Operation Research Society*, 47 (12), 1648-1476.
- Chiang, C.-P., & Lee, S.-D. (2004). Joint determination of machine cells and linear intercell layout. *Computers & Operations Research*, 31, 1603–1619.
- Commander, C. W. (2003). *A Survey of the Quadratic Assignment Problem, with Applications*. The University of Florida.
- Drira, A., Pierreval, H., & Hajri-Gabouj, S. (2007). Facility layout problems: A survey. *Annual Reviews in Control*, 31, 255–267.
- Harhalakis, G., Nagi, R., & Proth, J. M. (1990). An Efficient Heuristic in Manufacturing Cell Formation for Group Technology Applications. *International Journal of Production Research*, 28 (1), 185-198.
- Heragu, S. S., & Kusiak, A. (1991). Efficient models for the facility layout problem. *European Journal of Operational Research*, 53, 1-13.
- Heragu, S. S., & Kusiak, A. (1988). Machine layout problem in flexible manufacturing systems. *Operations Research*, 36 (2), 258–268.
- Hicks, C. (2006). A Genetic Algorithm tool for optimising cellular or functional layouts in the capital goods industry. *International Journal of Production Economics*, 104, 598–614.
- Kaku, B., Thompson, G., & Baybars, I. (1988). A heuristic method for the multi-story layout problem. *European Journal of Operational Research*, 37, 384–397.
- Kaufmann, L., & Broeckx, F. (1978). An algorithm for the quadratic assignment problem using Benders' decomposition. *European Journal of Operational Research*, 2, 204–211.
- Kazerooni, M. L., Luong, H. S., & Abhary, K. (1997). A genetic algorithm based cell design considering alternative routing. *International Journal of Computational Integrated Manufacturing Systems*, 10 (2), 93–107.
- Kochhar, J. S., Foster, B. T., & Heragu, S. S. (1998). A GENETIC ALGORITHM FOR THE UNEQUAL AREA FACILITY LAYOUT PROBLEM. *Computers & Operations Research*, 25 (7/8), 583-594.

- Konak, A., Kulturel-Konak, S., Norman, B. A., & Smith, A. E. (2006). A new mixed integer programming formulation for facility layout design using flexible bays. *Operations Research Letters*, 34, 660–672.
- Koopmans, T. C., & Beckmann, M. J. (1957). Assignment problems and the location of economic activities. *Econometrica*, 25, 53–76.
- Kusiak, A., & Heragu, S. (1987). The facility layout problem. *European Journal of Operational Research*, 29, 229–251.
- Love, R., & Wong, J. (1976). On solving a one-dimensional allocation problem with integer programming. *Information Processing and Operations Research*, 14 (2), 139–143.
- Meller, R., & Gau, K. (1996). The facility layout problem: recent and emerging trends and perspectives. *Journal of Manufacturing Systems*, 15, 351–366.
- Ramabhatta, V., & Nagi, R. (1996). An integrated formulation of manufacturing cell formation with capacity planning and multiple routings. *Baltzer Journals*, 1-18.
- Sahni, S., & Gonzalez, T. (1976). P-complete Approximation Problems. *Journal of the ACM*, 23, 555-565.
- Simmons, D. (1969). One-dimensional space allocation: An ordering algorithm. *Operations Research*, 17, 812–826.
- Singh, S. P., & Sharma, R. R. (2005). A review of different approaches to the facility layout problems. *The International Journal of Advanced Manufacturing Technology*, 30, 425–433.
- Solimanpur, M., Vrat, P., & Shankar, R. (2005). An ant algorithm for the single row layout problem in flexible manufacturing systems. *Computers & Operations Research*, 3 (32), pp. 583–598.
- Sule, D. R. (1994). *Manufacturing facilities: location, planning, and design (2nd ed.)*. Boston: PWS Publishing Company.
- Taghavi, A., & Murat, A. (2011). A heuristic procedure for the integrated facility layout design and flow assignment problem. *Computers & Industrial Engineering*, 61, 55–63.
- Tavakkoli-Moghaddam, R., Javadian, N., Javadi, B., & Safaei, N. (2007). Design of a facility layout problem in cellular manufacturing systems with stochastic demands. *Applied Mathematics and Computation*, 184, 721–728.
- Tompkins, J. A., White, J. A., Bozer, Y. A., & Tanchoco, J. M. (2003). *Facilities planning (3rd ed.)*. New York: John Wiley & Sons.
- Wemmerlöv, U., & Hyer, N. (1986). Procedures for the part family/machine group identification problem in cellular manufacture. *Journal of Operations Management*, 6, 125-147.
- Wu, X., Chu, C.-H., Wang, Y., & Yan, W. (2007). A genetic algorithm for cellular manufacturing design and layout. *European Journal of Operational Research*, 181, 156–167.