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Permutation based decision making under fuzzy environment using Tabu search

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A R T I C L E I N F O

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A B S T R A C T

One of the techniques, which are used for Multiple Criteria Decision Making (MCDM) is the permutation. In the classical form of permutation, it is assumed that weights and decision matrix components are crisp. However, when group decision making is under consideration and decision makers could not agree on a crisp value for weights and decision matrix components, fuzzy numbers should be used. In this article, the fuzzy permutation technique for MCDM problems has been explained. The main deficiency of permutation is its big computational time, so a Tabu Search (TS) based algorithm has been proposed to reduce the computational time. A numerical example has illustrated the proposed approach clearly. Then, some benchmark instances extracted from literature are solved by proposed TS. The analyses of the results show the proper performance of the proposed method.

1. Introduction

Multiple Criteria Decision Making (MCDM) is one of the most prevalent approaches in decision making and is often classified into two categories of multiple attribute decision making (MADM) and multiple objective decision making (MODM). In real world problems, usually crisp numbers for weights of different criteria and decision matrix are not available. For example, when we consider a group decision making problem, these information not available because there are many uncertainties involved. In these cases, fuzzy theory should be mixed with MADM techniques to handle any possible uncertainties. There are literally various methods proposed such as Fuzzy TOPSIS, Fuzzy ELECTRE (I, II, III), Fuzzy AHP, etc. A brief literature review of these methods is mentioned below:

One of well known methods for ranking different alternatives is called analytical hierarchy procedure (AHP), developed primarily by Saaty (1980), and It is used around the world in a wide variety of decision situations, in different fields such as government, business, industry, healthcare, and educational systems. AHP is based on pair wise comparisons between the alternatives from each attribute's point of view. The integration of AHP with the fuzzy theory , fuzzy AHP, was proposed by Chang. Van Laarhoven and Pedrycz (1983) extended the AHP for decision making for different cases conducted in the uncertain and fuzzy environments. Buckley (1985) proposed fuzzy hierarchical analysis and Boender et al. (1989) proposed a MCDM technique with fuzzy pair wise comparisons.

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Cheng (1996) evaluated naval tactical missile systems by fuzzy AHP. Cheng et al. (1999) evaluated attack helicopters by AHP based on linguistic variables for weights of criteria. Kuo et al. (2002) proposed a decision support system for selecting convenience store location through integration of fuzzy AHP and artificial neural network. Kwong and Bai (2003) determined the importance weights for the customer requirements using fuzzy AHP with an extended analysis method. Kahraman et al. (2004) proposed a multi-attribute comparison of catering service companies using fuzzy AHP. Enea and Piazza (2004) focused on the constrained fuzzy AHP for project selection. Kapoor and Tak (2005) provided a methodology for solving common robot selection problems using a modification of the conventional AHP. Erensal et al. (2006) determined key capabilities in technology management using the fuzzy AHP. Percin (2008) proposed a method for using fuzzy AHP in evaluating benefits of information sharing decisions in a supply chain. Ramík and Perzina (2010) introduced an extension of the AHP with feedback between criteria.

The basic idea for Fuzzy TOPSIS is to choose the alternative, which is as close to the positive ideal solution as possible and as far from the negative ideal solution as possible. Tsaur et al. (2002), for instance, implemented a hybrid of AHP with TOPSIS for air force quality ranking. Chen and Tzeng (2004) offered a method of grey related analysis for MADM problems. Abo-Sinna and Amer (2005) proposed an extension of TOPSIS for multi-objective large-scale nonlinear programming problems. Jahanshahloo et al. (2006) proposed the extension of TOPSIS method for decision-making problems with fuzzy data. Chen et al. (2006) developed a fuzzy decision-making method to cope with the supplier selection problem in the supply chain system. Wang and Elhag (2006) developed a non-linear programming (NLP) solution procedure using fuzzy TOPSIS method based on α-cut level concept. Wang and Chang (2007) developed an evaluation procedure based on TOPSIS to help the air force academy in Taiwan choose an optimal initial training aircraft in a fuzzy environment where the vagueness and subjectivity were handled with linguistic terms parameterized by triangular fuzzy numbers. Kuo et al. (2007) presented a fuzzy multi-criteria decision analysis method based on the concepts of positive and negative ideal points. Kahraman et al. (2007) proposed a fuzzy hierarchical TOPSIS model for the multi-criteria evaluation of the industrial robotic systems. Benitez et al. (2007) applied a fuzzy TOPSIS methodology to increase an overall service performance index for evaluating the service quality of three hotels. Li (2007) developed a new fuzzy closeness method for MADM in fuzzy environments, which can be applied for fuzzy TOPSIS. Xu (2007) defined the notations of the positive ideal fuzzy set and negative ideal fuzzy set, which can be used in fuzzy TOPSIS or other fuzzy MADM methods. Abo-Sinna et al. (2008) extended the TOPSIS for large-scale multi-objective non-linear programming problems with block angular structure. Nut and Soner (2008) proposed an integrated approach using AHP and TOPSIS in a fuzzy environment for shipping site selection. Sadi nejad and Khalili (2009) proposed a fuzzy TOPSIS method based on modified preference ratio and fuzzy distance measurement in assessment of traffic police centers performance.

Fuzzy ELECTRE, Elimination ET Choix Traduisant la Réalité or Elimination and Choice Translating Reality, is a method that outranks the alternatives based on pair wise comparison between alternatives under each criterion independently (Kaharman, 2008) and there are literally various versions of this method. Roy (1990) proposed the outranking method and the foundations of ELECTRE method. Goumas and Lygerou (2000) proposed an extension of the PROMETHEE, a very similar method to ELECTRE (III), for decision-making in the fuzzy environment. Tervonen (2004) proposed an inverse approach for ELECTRE III. Qahri Saremi and Montazer (2007) ranked various structures of a website using ELECTRE (III). Almeida (2007) proposed a multi-criteria decision model for outsourcing contracts selection based on utility function and ELECTRE. Chou et al. (2008) proposed a method for ranking irregularities when evaluating alternatives by using ELECTRE. Other authors such as Papadopoulos and Karagiannidis (2008) and Montazer et al. (2009) proposed different methods on fuzzy ELECTRE.

This paper proposes a new approach based on classic permutation method in a fuzzy environment, which makes this method to be more realistic. The proposed model of this paper also concentrates on a metaheuristic solution algorithm to find the final results, efficiently. This method can be applied in many fields such as scheduling for the sequence of production with some criteria with linguistic states, supplier selection, etc. As mentioned before, permutation method considers each combination of alternatives, then calculates the rate of each permutation and finally outranks the alternatives. This process needs a lot of computational time, and it would be intolerable by increasing the number of alternatives. Rinnooy (1976) proved that if the number of alternative increases, then the problem would become NP-hard. This fact has been presented by a permutation based method for solving a sequencing problem. Because of similarity in the concept of a sequencing problem and other kinds of problems with permutation method, we can extend this fact to all permutation problems with great numbers of alternatives. To solve the mentioned problem in a reasonable amount time, a Tabu Search (TS) method has been proposed, which belongs to the metaheuristic algorithms. The proposed solution approach has been compared with the exact solution result in a numerical example. The rest of this paper is structured as follows:

Section 2 describes notations and arithmetic operators used in the paper. In Section 3 methods for outranking fuzzy numbers will be overviewed. Section 4 illustrates the fuzzy permutation method. Section 5 is dedicated to introduce TS, a metaheuristic approach applied in the proposed method to reduce the solution procedure time. In Section 6, a numerical example has been handled. Final section is dedicated to conclusions and future research suggestions.

2. Notations and arithmetic operators

Definition2.1: Let $\widetilde{A}$ be a fuzzy number and this defines as Eq. (1).

$$\widetilde{A} = \{(x_1, \mu_{\widetilde{A}}(x_1)), (x_2, \mu_{\widetilde{A}}(x_2)), \ldots, (x_n, \mu_{\widetilde{A}}(x_n))\}, \tag{1}$$

where each point like, x_i, is a member of $\widetilde{A}$ with a membership degree of $\mu_{\widetilde{A}}(x_i)$.

Definition2.2: The membership function for a triangular fuzzy number is given by Eq. (2).

$$\mu_{\widetilde{A}} = \begin{cases} \dfrac{x-a}{b-a} & (a \le x \le b) \\ \dfrac{c-x}{c-b} & (b \le x \le c), \end{cases} \tag{2}$$

where b is called the mode of the fuzzy number, a and c are called lower and upper limits for $\widetilde{A}$, respectively.

Definition2.3: the membership function for a trapezoidal fuzzy number denoted as Eq. (3).

$$\mu_{\widetilde{A}} = \begin{cases} \dfrac{x-a}{b-a} & (a \le x \le b) \\ 1 & (b \le x \le c) \\ \dfrac{c-x}{c-d} & (c \le x \le d) \end{cases} \tag{3}$$

where a and d are lower and upper limits and $[b,c]$ is the mode interval for $\tilde{A}$. In other words, a triangular fuzzy number (TFN) is a special case of trapezoidal fuzzy number (TrFN) and one can show a TFN like a TrFN as Eq. (4).

$$\tilde{A}=(a,b,b,c) \tag{4}$$

Let $\tilde{A}$ and $\tilde{B}$ be two TrFN parameterized as (a_1, b_1, c_1, d_1) and (a_2, b_2, c_2, d_2). This paper uses the operators below for its calculations.

$$\tilde{A}+\tilde{B}=(a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2) \tag{5}$$

$$\tilde{A}-\tilde{B}=(a_1-d_2, b_1-c_2, c_1-b_2, d_1-a_2) \tag{6}$$

$$\tilde{A}\times\tilde{B}=(a_1\times a_2, b_1\times b_2, c_1\times c_2, d_1\times d_2) \quad \text{if} \quad a_1,a_2,b_1,b_2,c_1,c_2,d_1,d_2\geq 0 \tag{7}$$

$$\tilde{A}\div\tilde{B}=\left(\frac{a_1}{d_2},\frac{b_1}{c_2},\frac{c_1}{b_2},\frac{d_1}{a_2}\right) \quad \text{if} \quad a_1,a_2,b_1,b_2,c_1,c_2,d_1,d_2\geq 0 \tag{8}$$

$$m\times\tilde{A}=(m\times a, m\times b, m\times c, m\times d) \quad \text{if} \quad a,b,c,d\geq 0 \tag{9}$$

$$(\tilde{A})^{-1}=\left(\frac{1}{d},\frac{1}{c},\frac{1}{b},\frac{1}{a}\right) \quad \text{if} \quad a,b,c,d\geq 0 \tag{10}$$

3. Ranking methods

To solve decision making problems, we need a method to distinguish maximum and minimum numbers in a set of fuzzy numbers. In recent years, many methods have been proposed for ranking fuzzy numbers. Some of these methods are as follows:

Dubois and Prade (1983) offered a ranking method for fuzzy numbers in the setting of possibility theory. Bortolan and Degani (1985) proposed a review of some methods for ranking fuzzy subsets. Chen (1985) described a technique for ranking fuzzy numbers with maximizing and minimizing sets. Cheng (1998) offered a new approach for ranking fuzzy numbers by distance method. Chu and Tsao (2002) suggested a method for ranking fuzzy numbers with an area between the centroid point and the original point. Li et al. (2007) proposed a new routine for ranking fuzzy numbers, which is illustrated below and is used in this paper. Let $\tilde{A}$ be a trapezoidal fuzzy number with membership function mentioned in Eq. (3). So the centroid point $(\bar{X}_0, \bar{Y}_0)$ can be found for $\tilde{A}$ by Eq. (11) and Eq. (12):

$$\bar{X}_0=\frac{a^2+b^2-c^2-d^2+ab-cd}{3(-a-b+c+d)} \tag{11}$$

$$\bar{Y}_0=\frac{a+2b+2c+d}{3(a+b+c+d)}. \tag{12}$$

The distance index for $\tilde{A}$ can be calculated as follows,

$$R(\tilde{A}_i)=\sqrt{(\bar{X}_0(\tilde{A}_i))^2+(\bar{Y}_0(\tilde{A}_i))^2}. \tag{13}$$

With this index, one can compare two fuzzy numbers with each other and distinguish which one is greater according to the following rules,

1) $if \quad R(\tilde{A}_i)<R(\tilde{A}_j), then \quad \tilde{A}_i<\tilde{A}_j$

2) $if \quad R(\tilde{A}_i)>R(\tilde{A}_j), then \quad \tilde{A}_i>\tilde{A}_j$

3) $if \quad R(\tilde{A}_i)=R(\tilde{A}_j), then \quad \tilde{A}_i=\tilde{A}_j$

4. Proposed method

Permutation method is one of the techniques for solving MADM problems, which was originally proposed by Jacquet-Lagreze (1969). This method considers every possible permutation of alternatives and calculates its rate as follows,

$$R_t = \sum_{j\in C_{kl}} w_j - \sum_{j\in D_{kl}} w_j \quad , \tag{14}$$

where Ckl is the concordance matrix contains the criteria in which alternative k dominates alternative l and Dkl is the discordance matrix contains criteria in which alternative l dominates alternative k. Eq. (14) computes the rate of t^{th} possible permutation. In the real world cases, the parameters or decision matrix are defined in fuzzy form. The proposed method considers the rate function to handle the ambiguity. As mentioned before, classic permutation method needs great computational time. Moreover, there is another weakness in Eq. (14) to compare two permutations of alternatives in some cases: If two alternatives are the same at the view point of all criteria except two, and if the weights for those two criteria are the same, then the classic permutation method using Eq. (14) encounter a problem for distinguishing the best permutation. To illustrate this deficiency we should define parameters below first:

$i=1,2,\ldots,n$: counter for alternatives
$j=1,2,\ldots,p$: counter for criteria
w_j: weight of j^{th} criterion
D_{ij}: value of i^{th} alternative at the j^{th} criterion's point of view
Suppose two permutations P_1 and P_2

$$P_1 = (\ldots., A_i, A_{i'}, \ldots)$$
$$P_2 = (\ldots., A_{i'}, A_i, \ldots)$$
$$if \;\; \forall \;\; j : D_{ij} = D_{i'j} \quad j = 1,2,\ldots,p \;\; except \;\; j = J \;\; and \;\; j = J'$$
$$for \;\; j = J: \;\; D_{iJ} \rangle D_{i'J} \Rightarrow D_{iJ} = D_{i'J} + a$$
$$for \;\; j = J': \;\; D_{iJ'} \langle D_{i'J'} \Rightarrow D_{i'J'} = D_{iJ'} + b$$
$$a > b$$
$$w_J = w_{J'} \Rightarrow R_{p_1} = R_{p_2} = \sum_{j\in c_{kl}} w_j - \sum_{j\in d_{kl}} w_j$$

R_{p_1} and R_{p_2} are the same because the weights of those two criteria under discussion will neutralize each other in both equations. It means that by utilizing Eq. (14), there is no preference between P_1 and P_2. However, it is obvious that for this deduction we should consider the degree of preference between A_i and $A_{i'}$ at the J^{th} and J'^{th} criteria's point of view. Note that values of J^{th} and J'^{th} attributes have the same scale in the decision matrix. Next example will illustrate the aforementioned deficiency. Suppose that in a real example we have three criteria with related weights given in Table 1 and three alternatives and the decision matrix are given in Table 2, respectively.

Table 1
Weights of criteria

W1	W2	W3
(0.2,0.3,0.4,0.5)	(0.35,0.4,0.5,0.6)	(0.2,0.3,0.4,0.5)

Table 2
Decision matrix

	Criteria		
Alternative	C1 (-)	C2(+)	C3(+)
A1	(200,250,300,350)	(5,5.5,6,6.5)	(200,205,210,215)
A2	(300,350,400,450)	(5,5.5,6,6.5)	(210,215,220,225)
A3	(350,400,450,500)	(3,3.5,4,4.5)	(150,155,160,165)

In Table 2, Cj(-) is the notation of sumptuary criteria and Cj(+) is the notation of revenue criteria. The problem results according to Eq. (13) and Eq. (14) will be (A1, A2, A3) and (A2, A1, A3) with rate of 2.4266. It means that there is no preference between A1 and A2. Table1 shows that the weights of first and third criteria are equal; hence it is obvious that alternative A1 is better than alternative A2 because of the same scale of C1 and C3. This example illustrates the deficiency of classical permutation method. In this paper, a revision approach of fuzzy permutation is proposed to increase the precision of the permutation method in solving problems.

The proposed equation for calculating the rate of each permutation is introduced in Eq. (15). It contains another subject, which leads to more precision in decision making stage.

$$\begin{aligned} R' = {} & p\left(p'\left(\sum_{j\in C_{kl}} w_j - \sum_{j\in D_{kl}} w_j \right) + (1-p')\left(\sum_{j\in C_{kl}} v_j - \sum_{j\in D_{kl}} v_j \right)\right) + \\ & (1-p)\left(p'\left(\max_{j\in C_{kl}} w_j - \max_{j\in D_{kl}} w_j \right) + (1-p')\left(\max_{j\in C_{kl}} v_j - \max_{j\in D_{kl}} v_j \right)\right), \end{aligned} \tag{15}$$

where $v = A \times w$ is weighted of normalized decision matrix.

MADM methods are divided in two types: compensatory and non-compensatory methods. In the compensatory methods of MADM such as TOPSIS, the trade-offs among attributes are permitted, i.e., the high performance of an alternative achieved on one or more criteria can be compensated for the weak performance of the same alternative compared with other criteria. Non-compensatory methods can be thought of as screening devices, with all feasible solutions consisting of those alternatives that fulfill certain standards. These methods do not allow trade-offs among attributes; thus, a single weak attribute may be sufficient to exclude an alternative. Each of these methods has its own beneficiaries, so the idea of employing both of them simultaneously motivated us to propose Eq. (15).The first part of this equation provides compensatory type by coefficient p and the second part provides the other type.

On the other hand, one of the most important issues in MADM techniques such as TOPSIS and ELECTERE use weighted normalized matrix (v) for their computational procedures. Classical permutation method does not benefit from this matrix. Hence, we divide each part of the equation into two sections. v is imported in the second section of each part by coefficient $(1-p')$. The example discussed earlier is considered once more with this equation. We suppose $p = p' = 0.5$ and the result shows that the best permutation is (A1, A2, A3). Next, we present an algorithm to decrease the computational time.

5. TS solution algorithm

Exact methods often face with great difficulty when they encounter hard optimization problems. There are many important applications in engineering, economics, business and science formulated as combinatorial problems and the exact optimal solution cannot be found in reasonable amount of time. These cases should be solved by using metaheuristic algorithms, which would reduce the computational time to find the near optimal solution. There are literally many methods in this field such genetic algorithm, particle swarm optimization, Tabu search (TS), etc. This paper uses TS to decrease the computational time for solving problems.

TS is proposed by Glover (1989, 1990) and it is in a theatrical manner changing the ability of solving problems of practical significance. Many works have been done in the area of applying TS for accelerating the process of permutation method in sequencing problem area (e.g. Nowicki & Smutnicki, 1996; Grabowski & Wodecki, 2004). Grabowski and Pempera (2007) used this method for minimizing make span in a flow shop problem. Liao and Huang (2010) proposed a method for solving a sequencing problem, which uses two TS algorithms simultaneously to solve a kind of sequencing problem by applying permutation method.
Our proposed equation consumes much time to rank alternatives. Hence, TS is suggested to reduce the computational time. We examined other algorithms for this method by statistical tools, and the result shows that the proposed TS algorithm is significantly superior to others. The pseudo code of this method is as follows:

1. Enter fuzzy matrix and fuzzy weight vector, then Generate a random permutation, name as Best_ per, calculate the rate of this permutation and name as Best_ R,
2. Max_Iter=500, Max_STM=5, Iter=1, LTM=n×n,
3. Generate a matrix support on Eq.16 and title as "Random", where *i* and *j* show the row and the column of "Random" matrix, respectively,
4. Calculate the "Random" matrix,
5. $$Random\,(i,\,j) = \frac{rand\,(0,1)}{LTM\,(i,\,j)}\,. \tag{16}$$
6. Let N1and N2 be respectively equal to the row and column, which have the maximum values in random matrix,
7. If exchange of N1 and N2 is in the STM then, go to step 4, else go to step 8,
8. Exchange N1 and N2 and name this permutation as Per, then set this exchange in the STM, delete the last exchange in it (for example M1 and M2) and LTM (M1, M2) =LTM (M1, M2) +1, then Calculate the rate of this permutation,
9. If R is greater than Best_R, then Best_Per=Per, Best_R=R, Iter=Iter+1, else Iter=Iter+1,
10. If Iter is greater than Max_Iter, then print Best_R and Best_Per, else go to step 4.

This pseudo code applies diversification and intensification concepts. LTM notation works as long term memory in the algorithm. In steps 5 and 8, this notation diversifies the method of search. In STM notation, Tabu list is generated. The Tabu move is restrained by STM when the algorithm transfers from step 7 to step 4. STM operates as short -term memory in the procedure of algorithm. This algorithm has been illustrated more clearly in Fig.1.
As mentioned before, TS contains parameters that should be determined at the beginning of the implementation of the procedure. For this problem, the tuning empirical values have been expressed in consequence of some experiments. To determine the suitable parameter values, we employ Taguchi's method, which is the most notable proponent of the use of fractional factorial designs. This method contains a special set of orthogonal arrays to lay out experiments. We use $L_9(3^2)$ Taguchi's orthogonal arrays in our TS parameter tuning. This array can handle two parameters with three levels running nine experiments.
In the first experiment, the performance of the proposed algorithm is studied under different values of the Max_STM and Max_Iter . The Max_STM parameter varies from 5 to 15 with the step size of 5 and Max_Iter changes from 500 to 1500 by the step size of 500. Fig. 2 shows the effect of Max_STM and Max_Iter on the performance of our algorithm. The vertical axis shows the average of objective function value of problem instances which are solved by our algorithm (these instances will be described in section 7), and the horizontal axis shows the level of parameters. The results show that Max_STM has positive effect on the performance of the algorithm.

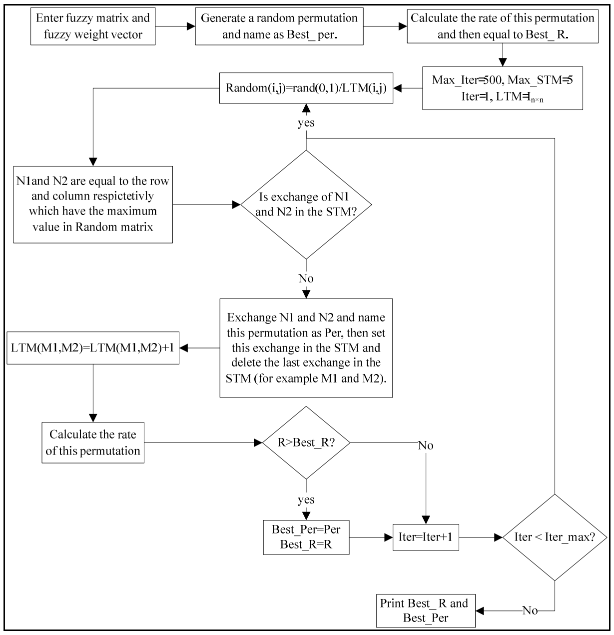

Fig. 1 . The flowchart of the proposed solution method

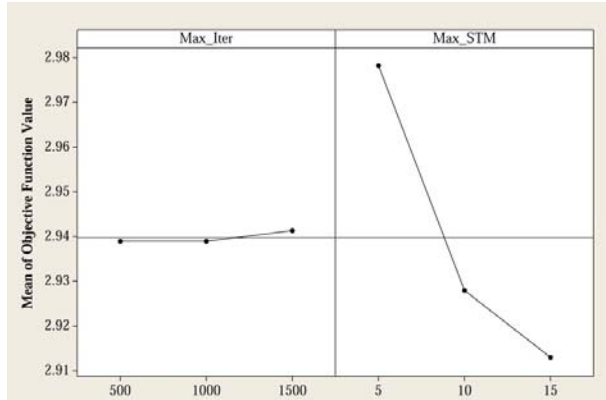

Fig. 2. The effect of parameters of our proposed TS

According to the experimental results; the suitable performance of the proposed TS can be attained by setting maximum short term memory as: $Max\_STM = 5.$ The selected levels for maximum of iteration show that this parameter does not reasonable effect on the performance of algorithm, hence to reduce the TS CPU time, we set this parameter as $Max\_Iter = 500.$

6. Numerical example

Consider a problem of multiple attribute decision making in which a company desires to hire an employee. After initial screening, eight candidates A1, A2,…, A8 remained for more evaluations. Four criteria have been considered: Personality (C1), Emotional steadiness (C2), Self-confidence (C3) and Skill (C4). A committee of three decision makers, D1, D2 and D3 has been formed to perform the interview and choose the most suitable candidate. Decision makers use the linguistic weighting variables to evaluate the importance of criteria as shown in the Table 3. Linguistic variables used to value linguistic terms in fuzzy numbers are shown in Table 4. These data are extracted from Sadi nejad and Khalili (2009).

Table 3
Linguistic variable for importance of each attribute

Linguistic variable	TrFNs
Extremely low (EL)	(0, 0, 0.1, 0.2)
Very low (VL)	(0.1, 0.2, 0.3, 0.4)
Low (L)	(0.2, 0.3, 0.4, 0.5)
Medium low (ML)	(0.3, 0.4, 0.5, 0.6)
Medium (M)	(0.4, 0.5, 0.6, 0.7)
Medium high (MH)	(0.5, 0.6, 0.7, 0.8)
High (H)	(0.6, 0.7, 0.8, 0.9)
Very high (VH)	(0.7, 0.8, 0.9, 1.0)
Extremely high (EH)	(0.8, 0.9, 1.0, 1.0)

Table 4
Linguistic variables for rating of each alternative with respect to each attribute

Linguistic variable	TrFNs
Extremely low (EL)	(0, 0, 1, 2)
Very low (VL)	(1, 2, 3, 4)
Low (L)	(2, 3, 4, 5)
Medium low (ML)	(3, 4, 5, 6)
Medium (M)	(4, 5, 6, 7)
Medium high (MH)	(5, 6, 7, 8)
High (H)	(6, 7, 8, 9)
Very high (VH)	(7, 8, 9, 10)
Extremely high (EH)	(8, 9, 10, 10)

Final decision makers' idea about the importance of each criterion is shown in Table 5.The numerical importance of each criterion is derived from terms in Table 5 and is shown in table 6.

Table 5
The importance weight of criteria

	D1	D2	D3
C1	M	H	ML
C2	VL	L	H
C3	H	MH	VH
C4	EH	VH	EH

Table 6
Final importance of criteria

C1	C2	C3	C4
(0.43, 0.53, 0.63, 0.73)	(0.3, 0.4, 0.5, 0.6)	(0.6, 0.7, 0.8, 0.9)	(0.77, 0.87, 0.97, 1)

The final aggregation of committee's ratings under all criteria for eight candidates has been depicted in Table 7. These data are randomly generated for declaring our proposed algorithm more explicitly.

Table 7
The Final aggregation of committee's ratings

	C1	C2	C3	C4
A1	(2.25, 3, 4, 5)	(2.5, 3.5, 4.5, 5.5)	(3.25, 4.25, 5.25, 6.25)	(2.5,3.25, 4.25, 5.2
A2	(4, 4.75, 5.75, 6.5)	(6.25, 7.25, 8.25, 8.75)	(4, 5, 6, 7)	(4.5,5.25, 6.25, 7
A3	(4, 5, 6, 7)	(2.5, 3.25, 4.25, 5.25)	(7, 8, 9, 9.5)	(5,6,7,7.75)
A4	(3.75,4.75,5.75,6.75)	(2.5,3.5,4.5,5.5)	(3.75,4.75,5.75,6.75)	(4.75,5.75,6.75,7.7
A5	(1.5,2.25,3.25,4.25)	(5.25,6.25,7.25,8.25)	(2,2.75,3.75,4.75)	(5.75,6.75,7.75,8.
A6	(4.75,5.75,6.75,7.75)	(4.25,5.25,6.25,7)	(3.5,4.5,5.5,6.5)	(6,7,8,8.5)
A7	(6,7,8,9)	(2.75,3.5,4.5,5.5)	(6.25,7.25,8.25,8.75)	(2.25,3,4,5)
A8	(3.5,4.25,5.25,6)	(4.5,5.5,6.5,7.25)	(3.5,4.25,5.25,6.25)	(2,3,4,5)

In this step we calculate the rate of each permutation using Eq. (15). The number of permutations used for this problem is 8! =40320. This method consumes much time to rank the alternatives, exactly. Hence, the TS method is applied for reducing computational time. The result for exact solution is $6 \succ 3 \succ 7 \succ 2 \succ 4 \succ 5 \succ 8 \succ 1$ and $R' = 1.8841$ by consuming 90.96 seconds. When the TS solution method is executed, R' increased to 1.8848 and the computational time is 2.94 seconds. The results show that the proposed solution algorithm can find the best solution in less computational time comparing with the exact solution. It seems that the aforementioned example is not enough for presenting the proper performance of proposed TS algorithm. So, next section is dedicated to clarifying suitable execution of this method.

7. Computational experiments

In this section, the proposed approach is applied for a real-world case study derived from Sadi nejad and Khalili (2009). The problem of this case is the assessment of traffic police centers. We use our proposed method to rank the alternatives of the mentioned example. The calculation of the fuzzy rate for this benchmark instance shows that our proposed approach contains higher rates, and can be used for alternative ranking problems. The proposed procedure is applied for some instances. We use 5 instances contains 5, 10, 15, 20, 25 and 30 alternative subsets, which are found by taking the top 5×4, 10×4, 15×4, 20×4, 25×4 and 30×4 sub matrices. Therefore, we can take six benchmark instances for this problem. Comparison of the computed rates with the results of Sadi nejad and Khalili (2009) have been reported in Table 8. All the tests were executed in MATLAB 7.8 and they were run in an experimental computer equipped with 2.99GB of RAM and a Pentium microprocessor running at 2.53 GHz.

Table 8
Computational result for instances

Instance	TS	Time (s)	TOPSIS
5	1.2548	0.98564	0.4374
10	2.1465	1.509257	0.326
15	2.9547	2.228823	0.0082
20	3.3734	3.301115	0.0679
25	3.7499	4.507437	0.0495
30	4.3903	6.072755	0.3566

In this table, first column is for number of alternatives. The second column is dedicated to the rate of the best rank solved by TS solution method. The final column is for the rate of TOPSIS results by applying this paper's proposed method. The comparison between the results shows that our proposed method results better rates than what TOPSIS does.

8. Conclusion

In this research, a new approach has been proposed to rank the alternatives in a multi attribute decision making based on permutation. To transform the procedure more applicable in real problems, a fuzzy approach has been proposed based on classic permutation. Finally, a meta- heuristic based solution method is proposed for the mentioned problem. The analysis of results for some numerical examples shows that the proposed method contains more precise results and simultaneously consumes less computational time. Applying other solution algorithms in this study and also other defuzzifiers in final step can be as future studies. Furthermore, application of this method in other real cases can be another future research in this subject.

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