Contents lists available at GrowingScience

International Journal of Industrial Engineering Computations

homepage: www.GrowingScience.com/ijiec

The center location problem with equal weights in the presence of a probabilistic line barrier

M. Amiri-Aref[a*], **N. Javadian**[a], **R. Tavakkoli-Moghaddam**[b] and **M.B. Aryanezhad**[c]

[a]*Department of of Industrial Engineering, Mazandaran University of Science & Technology, Babol, Iran*
[b]*Department of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran*
[c]*Department of Industrial Engineering, Iran University of Science & Technology, Tehran, Iran*

A R T I C L E I N F O

Article history:
Received 1 April 2010
Received in revised form
June, 5, 2011
Accepted 6 June 2011
Available online
9 June 2011

Keywords:
Planar center location problem
Probabilistic line barrier
Rectilinear distance

A B S T R A C T

In this paper, a single facility centre location problem with a line barrier, which is uniformly distributed on a given horizontal route in the plane is proposed. The rectilinear distance metric is considered. The objective function minimizes the maximum expected barrier distance from the new facility to all demand points in the plane. An algorithm to solve the desired problem is proposed where a mixed integer nonlinear programming needs to be solved. The proposed model of this paper is solved using some already existed benchmark problem in the literature and the results are compared with other available methods.

1. Introduction

In the wide range of facility location problems, *center* or *minimax* location problems are recently paid attention by many researchers who are working in the field of the operational research. In this type of problems minimizing the maximum distance from the facility to demand points are the main purpose. Potential applications of this problem are as follows: Warehouse location, public service centers, emergency service centers, military service and so on. The planar center location problem is first introduced by Sylvester (1857). Elzinga and Hearn (1972) efficiently solved the Euclidian center location problems with equal weight. Charalambous (1982) and Hearn and Vijay (1982) solved the same problem with unequal weight, separately. A number of restrictions in location problems were considered by many researchers. *Barrier regions* such as military areas, mountain ranges, big rivers and the lake, are the kind regions where both establishing and travelling are forbidden. For the ease of understanding, see Table 1.

Table 1
Categories of restricted location problems

	forbidden regions	congested regions	barrier regions
Travelling	allowable	allowable with penalty	unallowable
Establishing	unallowable	unallowable	unallowable

* Corresponding author. Tel.:
E-mail: nijavadian@ustmb.ac.ir (N. Javadian)

doi: 10.5267/j.ijiec.2011.06.002

So far most of the research has focused on the Weber problem and suggested different variations and modifications of this problem. Other objective functions, such as the *minimax* or *maximin* objective, have been rarely considered. Many approaches to the center problem in the presence of barriers are based on rectilinear or block norm distances which allow for problem decompositions and discretizations. Aupperle and Keil (1989) presented a polynomial algorithm for restricted Euclidean center location problems. Chakrabarty and Chaudhuri (1990) and Chakrabarty and Chaudhuri (1992) considered a constrained rectilinear distance minimax location problem and presented a Geometric solution approach. Nickel (1998) studied the restricted center location problems under polyhedral gauges. Dearing et al. (2002) considered the rectilinear distances center facility location problem with polyhedral barriers and derived a finite dominating set result for the problem. Segars Jr. (2000) and Dearing and Segars Jr. (2002a,b) extended similar ideas to a more general class of location problems, developed a decomposition approach on which the objective function of a location problem with barriers is convex and optimized the problem using convex optimization methods. After that, Dearing et al. (2005) studied the same problem using the block norm distances in place of the rectilinear distances. Frieß et al. (2005) considered the minimax location problems in the presence of polyhedral barriers with the Euclidean distance. They proposed a solution approach based on propagation of circular wavefronts. Considering such barriers, Bischoff et al. (2009) presented the Euclidean multi-facility location–allocation problem and proposed two heuristics to solve the problem. In the presence of the arbitrary shaped barriers, Savaş et al. (2002) first considered a single finite-size facility location problems with the Manhattan (i.e., rectilinear) distance metric. Based on the work of Savaş et al. (2002), Kelachankuttu et al. (2007) presented the new facility location problem applying a contour line. Sarkar et al. (2007) extended the work of Savaş et al. (2002) for finite facility location problem with only user–facility interactions Nandikonda et al. (2003) considered the rectilinear distance center problem in the presence of arbitrary shaped barriers.

Katz and Cooper (1981) first studied the planar Euclidean Weber problem and one circular barrier. They suggested a heuristic algorithm based on the sequential unconstrained minimization technique (SUMT) for solving this problem. In the same problem, Klamroth (2004) divided the feasible region into some convex regions in which the number of these convex regions is bounded by $O(N^2)$ where N is the number of existing facilities. Found a way of overcoming, Bischoff and Klamroth (2007) took advantage of the Weiszfeld technique and genetic algorithm (GA). Aneja and Parlar (1994) studied the Euclidean Weber problem in the presence of convex or non-convex polyhedral barrier regions. Using simulated annealing, the authors determined some candidate points. They then constructed a visibility graph to evaluate the shortest path between any candidate point and existing facility location. Klamroth (2001a) developed a reduction result for the same problem in which the non-convex barrier location problem reduced to a set of convex location problems. Then, an exact and a heuristic algorithm were presented to solve such a planar location problem with barriers. Klamroth (2001b) considered the Weber location problems in the plane in the presence of line barriers with a finite number of passages. She proved that the time complexity of the problem exponentially grows by increasing the number of passages.

McGarvey and Cavalier (2003) developed the big square small square (BSSS) method to approximate the global optima of the Euclidean Weber problem with the convex polyhedral forbidden regions. To solve planar location problems, Hansen et al. (1981) proposed a branch-and-bound based technique, named BSSS method, originally. In the presence of convex polyhedral barriers, Butt and Cavalier (1996) addressed the Euclidean Weber problem and presented the FORBID heuristic method to decompose the feasible region. Larson and Sadiq (1983) worked on the discretization results for the rectilinear Weber problem with arbitrary shaped barriers. Considering both arbitrary shaped barriers and convex forbidden regions, Batta et al. (1989) generalized the results of Larson and Sadiq (1983). Hamacher and Klamroth (2000) developed a similar discretization for a general class of distance functions.

Wang et al. (2002) formulated a mathematical programming model where facilities were finite-sized shape or point and barriers were rectangular. For line barriers considering various distance functions, Klamroth and Wiecek (2002) proposed an algorithm for multi-criteria location problems. Canbolat and Wesolowsky (2010) presented a solution approach for the rectilinear Weber problem with a probabilistic line barrier. In this paper, the problem considered in Canbolat and Wesolowsky (2010) is extended as the rectilinear *center* location problem with equal weights in the presence of a probabilistic line barrier. Furthermore, an algorithm to solve the desired problem is provided. Although it was years since many researches had worked on facility location problems, there is no work done in the literature. The authors apply the classification *Pos*1/*Pos*2/*Pos*3/*Pos*4/*Pos*5 of location problems as presented in Hamacher and Nickel (1996) (see Hamacher and Nickel (1998) for an overview). *Pos*1 shows the number of new facility should be located. *Pos*2 indicates the solution space of the location problem. *Pos*3 presents the special features of location problems (e.g., *barrier region* or *forbidden region*). *Pos*4 determines the information about the distance function (e.g., l_1 or l_2). *Pos*5 contains the objective function. So, the desired problem in this paper is presented as $1 \,/\, \mathbb{R}^2 \,/\, \mathcal{B} = 1\; probabilistic\; line / D_1^B / \max$.

2. Problem Definition

2.1. Preliminaries definitions

Consider $\mathcal{Ex}$ = {$X_i \in \mathbb{R}^n$: i =1, . . ., I,} be a finite set of existing facility points where I is the number of existing facilities and {B_1, . . ., B_Q} be a finite set of barrier and $\mathcal{B} = \bigcup_{q=1}^{Q} B_q$. The interior of barrier regions, called as int($\mathcal{B}$), is prohibited for the placement of a new facility as well as traveling through int($\mathcal{B}$) is forbidden. Thus, the feasible region, $\mathcal{F} \subseteq \mathbb{R}^n$, for locating and traveling is given by $\mathcal{F} = \mathbb{R}^n \setminus$ int($\mathcal{B}$), (Klamroth, 2002). Whereas the points are in the plane, we have $\mathcal{F} = \mathbb{R}^2 \setminus$ int($\mathcal{B}$). By our definition of $\mathcal{B}$ and $\mathcal{F}$, there is an exception only in *line barriers* which have an empty interior in which travelling will be forbidden also at non-interior points of a barrier. It is assumed that the length of the barrier is constant and the width of barrier is negligible. In addition, the barrier occurs uniformly with known parameters on the barrier route.

To identify the concept of distance function D_p^B, called the p-norm barrier distance, consider two arbitrary points $X, Y \in \mathcal{F}$. So: D_p^B (X,Y) = *inf*{ $l(P_{X\text{-}Y})$: $P_{X\text{-}Y}$ is feasible X,Y path }, where $l(P_{X\text{-}Y})$ is the length of the feasible X-Y path. Let D_p (X,Y) be the p-norm distance between $X, Y \in \mathcal{F}$. Two arbitrary points (i.e., X and $Y \in \mathcal{F}$) are called *p-visible* if the p-norm barrier distance between X, Y is equal to the p-norm distance (i.e., $D_p^B(X,Y) = D_p\,(X,Y)$, that is the presence of barrier has no effect on visibility of two points X, Y. On the other hand, if the p-norm barrier distance between the two points X, Y is greater than the p-norm distance, $D_p^B(X,X_i) > D_p\,(X,X_i)$, then the p-norm distance between $X, Y \in \mathcal{F}$ is called *p-shadow* (i.e., barrier affects on the p-norm distance between two points $X, Y \in \mathcal{F}$). Considering this definition, for one feasible point $X \in \mathcal{F}$, the set of *visible* points is defined as: $visible_p(X)$={$Y \in \mathcal{F}: D_p^B(X,Y) = D_p\,(X,Y)$}. In other word, points from the feasible region, Y, which are *p-visible* with a given feasible point X are called $visible_p$ (X). For a feasible point $X \in \mathcal{F}$ the set of *shadow* points is defined as: $Shadow_p(X)$ = {$Y \in \mathcal{F}: D_p^B\,(X,Y) > D_p\,(X,Y)$}. It means that if distance between X, Y is *p-shadow*, then it becomes barrier distance, (see Klamroth, 2002). In this paper, rectilinear distance metric is emphasized (i.e., p=1). Let I be the existing facilities, the minimax location problems with barriers is generally formulated as:

$$\min_{x,y \in \mathcal{F}} \max_i \{w_i . D_1^B(X,X_i)\}, \qquad (1)$$

in which $X = (x,y)$ is the coordinates of the new facility, $X_i = (x_i, y_i)$ is the coordinates of i-th existing facility and w_i is the weights of existing facilities which are equal, suppose w_i=1, $i=1,\ldots,I$. the objective function is looking for a point to minimize the maximum distance from demand points.

The problem to minimize the maximum expected barrier distance between the optimal and the existing facilities with equal weights is as follows.

$$\min_{x,y\in\mathcal{F}} \max_{i}\{E[D_1^B(X,X_i)]\} \tag{2}$$

Since the *x*-coordinate of the barrier distance function follows uniform distribution function, the *y*-coordinate of the line barrier remains constant at *b*. So, by considering rectilinear metric expression (3) is enhanced.

$$\min_{x,y\in\mathcal{F}} \max_{i}\{\{E[D_1^B(x,x_i)] + |y-y_i|\}\}, \tag{3}$$

On the other word, probability density function of starting point of the line barrier, i.e., X_s, a continuous random variable with known parameters, is as follow:

$$f(X_s) = \begin{Bmatrix} \frac{1}{u_2-u_1} & u_1 \le X_s \le u_2 \\ 0\,; & \text{otherwise} \end{Bmatrix}.$$

Additionally, the ending point of the line barrier, called X_e, can be calculated as: $X_e = X_s + l$ (see Fig.1).

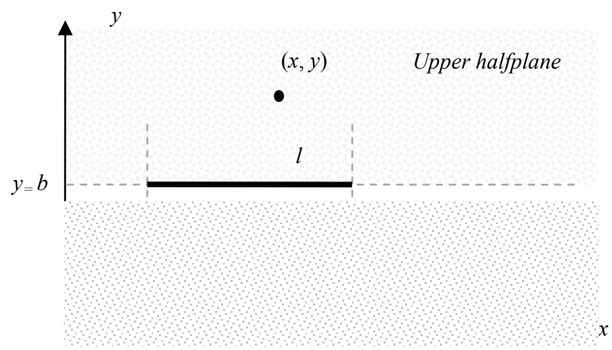

Fig. 1. A probabilistic line barrier.

It is obvious that the barrier affects on the distance while the expression (4) is satisfied.

$$\max\{x-l, x_i-l\} \le X_s \le \min\{x, x_i\}, \quad \forall\, i. \tag{4}$$

The expected barrier distance discussed by Canbolat and Wesolowsky (2010) and it is illustrated as:

$$E[D_1^B(x,x_i)] = \begin{Bmatrix} \frac{(l-|x-x_i|)^2}{2r} + |x-x_i|; & |x-x_i| < l \\ |x-x_i|; & |x-x_i| \ge l \end{Bmatrix}, \quad \forall\, i \tag{6}$$

where $r=u_2-u_1$.

Generally the planar center location problem in the presence of a probabilistic line barrier on a given horizontal route is proposed as:

$$\min_{x,y\in\mathcal{F}} \max_{i}\{\{E[D_1^B(x,x_i)] + |y-y_i|\}\},$$

subject to (6).

3. Model Modification

Suppose *y*-coordinate of the existing facilities are sorted in an increasing order,

$$y_1 \le y_2 \le \cdots \le y_m.$$

Where the horizontal route locate, some facilities may have y-coordinate values smaller than b and some have larger values and also $y_j = b$ is not allowed:

$$y_1 \leq y_2 \leq \cdots \leq y_j < b < y_{j+1} \leq \cdots \leq y_m,$$

where j is the index such that $y_j < b < y_{j+1}$. Let decompose the plane into two half-planes, upper half-plane which is above the barrier route and lower half-plane which is below the barrier route (see Fig. 1). Let $X = (x^*, y^*)$ be the optimal point of the new facility. Based on the solution space decomposition, the set of $\mathcal{E}x$ is divided into two subsets $\mathcal{E}x^{\mathcal{V}} \subseteq \mathcal{E}x$ and $\mathcal{E}x^{\mathcal{S}} \subseteq \mathcal{E}x$, where $\mathcal{E}x^{\mathcal{V}}$ is a subset of the existing facility which are located in the same half-plane, i.e., *visible* points, and $\mathcal{E}x^{\mathcal{S}}$ is a subset of the existing facility which are located in the opposite half-plane, i.e., *shadow* points. Regarding the new facility optimal location, two different cases are generated. Case 1; the location of the new facility optimal point is in the lower half-plane, i.e., $y^* < b$ in which the distance between X and any X_i where $y_i < b$ are *visible* (i.e., $d_1^{\mathcal{B}}(X, X_i) = d_1(X, X_i), i \in \mathcal{E}x^{\mathcal{V}} = \{1, \dots, j\}$) and the distance where $y_i > b$ is not 1-*visible* from existing facility X_i (i.e., $d_1^{\mathcal{B}}(X, X_i) > d_1(X, X_i), i \in \mathcal{E}x^{\mathcal{S}} = \{j+1, \dots, m\}$), so the barrier is in effect. Case 2; the optimal location of the new facility is in the upper half-plane, i.e., $y^* > b$. In which the distance between X and X_i while $y_i > b$ is denoted by $d_1(X, X_i), i \in \mathcal{E}x^{\mathcal{V}} = \{j+1, \dots, m\}$, and otherwise, it is indicated by $d_1^{\mathcal{B}}(X, X_i), i \in \mathcal{E}x^{\mathcal{S}} = \{1, \dots, j\}$. Since the procedures are similar, only Case 1 is considered. Therefore, we can rewrite the rectilinear *center* location problem with a probabilistic line barrier as follow.

$$\begin{aligned}
&\min_{x,y \in \mathcal{F}} \max_{i=1,\dots,I} \{E[d_1^B(X, X_i)]\} \\
&\qquad = \min_{x,y \in \mathcal{F}} \left\{ \max \left\{ \max_{i=1,\dots,j} E\left[d_1\ (X, X_i)\right], \max_{i=j+1,\dots,m} E\left[d_1^{\mathcal{B}}(X, X_i)\right] \right\} \right\} \\
&\qquad = \min_{x,y \in \mathcal{F}} \left\{ \max \left\{ \max_{i=1,\dots,j} \{|x - x_i| + |y - y_i|\}, \max_{i=j+1,\dots,m} \left\{ d_1^{\mathcal{B}}(x, x_i) + |y - y_i| \right\} \right\} \right\} \\
&\qquad = \min_{x,y \in \mathcal{F}} \left\{ \max \left\{ \max_{i=1,\dots,j} \{|x - x_i| + |y - y_i|\}, \max_{i=j+1,\dots,m} \left\{ \frac{(l - |x - x_i|)^2}{2r} . a_i \right.\right.\right. \\
&\qquad\quad \left.\left.\left. + |x - x_i| + |y - y_i| \right\} \right\} \right\}. \qquad (3)
\end{aligned}$$

$$a_i = \begin{cases} 1 & |x - x_i| < l \\ 0 & |x - x_i| \geq l \end{cases}, \qquad i = 1, \dots, I. \qquad (4)$$

4. Solution approach

It is clear that for the line barrier which occurs randomly on a horizontal route, the barrier does not affect on the y-coordinates of the distance between two arbitrary points and only the x-coordinates of the barrier distance may vary. So knowing the value of y^*, the following algorithm is introduced. Assume $f(X_L^*)$ be the objective value of the problem when the new facility optimum location is in the lower halfplane (Case 1), i.e., $y^* < b$ and $f(X_U^*)$ be objective value function of the problem when the new facility optimum location is in the upper halfplane (Case 2), i.e., $y^* > b$. The proposed algorithm represents the solution procedure for this problem. If $y^* < b$, the existing facilities whose $y_i > b$ might be affected by the barrier and $f(X_L^*)$ should be computed. Otherwise, the existing facilities whose $y_i < b$ might be affected by the barrier and $f(X_U^*)$ should be computed.

Algorithm 1
Inputs

- Existing facility coordinates $(x_i, y_i), i = 1, \dots, m$ where $y_1 \leq y_2 \leq \cdots \leq y_m$.
- y^* value form expression (5) or (6).

- A line barrier with
  - the length of l,
  - random x-coordinate (starting point), $x_s = U(u_1, u_2)$,
  - y-coordinate b.

Step 1.

If $y^* < b$ then:

$$f(X_L^*) = \min_{x,y \in \mathcal{F}} \left\{ \max\left\{ \max_{i=1,\dots,j} \{|x - x_i| + |y^* - y_i|\}, \max_{i=j+1,\dots,m} \{d_1^{\mathcal{B}}(x, x_i) + |y^* - y_i|\} \right\} \right\}$$

Else

$$f(X_U^*) = \min_{x,y \in \mathcal{F}} \left\{ \max\left\{ \max_{i=j+1,\dots,m} \{|x - x_i| + |y^* - y_i|\}, \max_{i=1,\dots,j} \{d_1^{\mathcal{B}}(x, x_i) + |y^* - y_i|\} \right\} \right\}$$

End if.

Step 2.

$f(X^*) = \min\{f(X_L^*), f(X_U^*)\}$,

$X^* = \text{argmin}\{f(X^*)\}$.

The value of y^* is easily determined by the following procedure. It is traditionally known that for the planar center location problem with rectilinear distance metric, there are two optimal extreme points:

$$(x_1^*, y_1^*) = \left(\frac{C_1 - C_3}{2}, \frac{C_1 + C_3 + C_5}{2}\right) \quad (5)$$

$$(x_2^*, y_2^*) = \left(\frac{C_2 - C_4}{2}, \frac{C_2 + C_4 - C_5}{2}\right), \quad (6)$$

where $C_1 = \min_i\{x_i + y_i\}$, $C_2 = \max_i\{x_i + y_i\}$, $C_3 = \min_i\{-x_i + y_i\}$, $C_4 = \max_i\{-x_i + y_i\}$ and $C_5 = \max\{C_2 - C_1, C_4 - C_3\}$. In addition, each linear convex combination of two optimal extreme points is an optimal point, i.e., $(x^*, y^*) = \lambda(x_1^*, y_1^*) + (1 - \lambda)(x_2^*, y_2^*), \lambda \in (0,1)$. Note that $y^* \neq b$.

5. An example

To present the reader an outline of the proposed model performance, the sample data provided by Canbolat and Wesolowsky (2010) have been chosen for a facility location problem with line barrier. This location problem consists of 8 demand points with weights $w_i = 1$, $i = 1, \dots, 8$, and a line barrier with length of 4 on the plane. The probabilistic x-coordinate of the line barrier is distributed uniformly with parameters U (0,12) and a constant y-coordinate at $b = 6$. The coordinates of the demand points are provided in Table 2.

Table 2
Existing facility locations

i	1	2	3	4	5	6	7	8
w_i	1	1	1	1	1	1	1	1
a_i	4	12	5	10	7	4	12	7
b_i	2	2	4	4.5	8	9	9.5	11

Since the considered problem is mixed-integer nonlinear programming, LINGO 9.0 is used to find the optimal value of (x^*, y^*). The LINGO software uses the branch-and-bound (B&B) method, which makes it possible to find, at least, the local optima, or at most the optimal solution requiring with more computational time. In this paper, the small-sized problem is solved in a reasonably computational time. Moreover, the *center* problem without the barrier is also solved.

For the given sample problem, we have $(y_1^*, y_2^*) = (5.75, 5.5)$, and for a random value $\lambda = 0.4 \in (0,1)$, we have $y^* = 5.6 < b$. So, $f(X_L^*)$ should be solved (Case 1). The optimal value is $f(X_L^*) = 7.750472 \Rightarrow X^* = \text{argmin}\{f(X^*)\} \Rightarrow x^* = 8.150472$.

Note $f_{nb}(X^*)$ as the objective function value for the center location problem without barrier, the following results are attained.

$f_{nb}(X^*) = 7.750000 \Rightarrow X^* = \text{argmin}\{f_{nb}(X^*)\} \Rightarrow (x^*, y^*) = (8.000000, 5.750000)$.

Fig. 5 graphically illustrates the existing facility points in the plane with a line barrier, the optimum location point with barrier and without barrier.

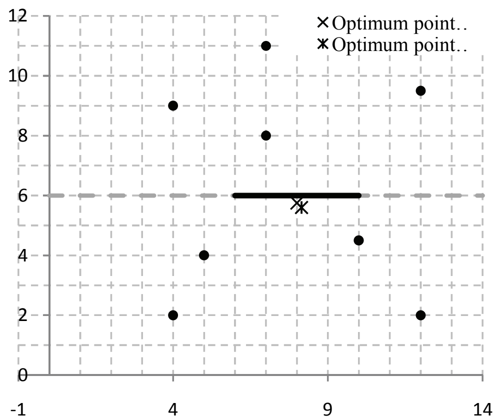

Fig. 2. Typical example

6. Conclusion

In this study, a) a new mathematical formulation model for the planar center location problem with equal weights in the presence of a probabilistic line barrier, b) a solution approach to solve the problem and c) a comparison with the problem without barrier is performed. The objective function of the problem with barrier is greater than the one without barrier, as expected. The problems have been solved by LINGO 9.0 optimization software and the optimum solution was obtained.

As the future research, considering other objective function such as maximin or maxisum, for the hazardous location problems, can be the extension for this problem. The other extension can be applying Euclidian distance functions (*p=2*) and Tchebychev distance functions ($p=\infty$) which both *x*-coordinates and *y*-coordinates should simultaneously be considered.

References

Aneja, Y.P., & Parlar, M. (1994). Algorithms for Weber facility location in the presence of forbidden regions and/or barriers to travel. *Transportation Science*, 28(1), 70–76.

Aupperle, L., & Keil, J. M. (1989) Polynomial algorithms for restricted Euclidean p-centre problems. *Discrete Applied Mathematics*, 23, 25-31.

Batta, R., Ghose, A., & Palekar, U. (1989). Locating facilities on the Manhattan metric with arbitrarily shaped barriers and convex forbidden regions. *Transportation Science*, 23(1), 26-36.

Bischoff, M., Fleischmann, T., & Klamroth, K. (2009). The multi-facility location–allocation problem with polyhedral barriers. *Computers and Operations Research*, 36(5), 1376 –1392.

Bischoff, M., & Klamroth, K. (2007). An efficient solution method for Weber problems with barriers based on genetic algorithms. *European Journal of Operational Research*. 177(1), 22–41.

Butt, S.E., & Cavalier, T.M. (1996). An efficient algorithm for facility location in the presence of forbidden regions. *European Journal of Operational Research*, 90(1), 56–70.

Canbolat, M.S, & Wesolowsky, G.O. (2010). The rectilinear distance Weber problem in the presence of a probabilistic line barrier. *European Journal of Operational Research*, 202(1), 114–121.

Chakrabarty, N. R. & Chaudhuri, P. K. (1990). Geometric solution of a constrained rectilinear distance minimax location problem. *Asia-Pacific Journal of Operations Research*, 7, 163-171.

Chakrabarty, N. R., & Chaudhuri, P. K. (1992). Geometrical solution to some planar constrained minimax problems involving the weighted rectilinear metric, *Asia-Pacific Journal of Operations Research*, 9, 135-144.

Charalambous, C. (1982). Extension of the Elzinga-Hearn algorithm to the weighted case. *Operations Research*, 30, 591-594.

Dearing, P. M., & Segars Jr., R. (2002a). An equivalence result for single facility planar location problems with rectilinear distance and barriers, *Annals of Operations Research*, 111, 89-110.

Dearing, P. M., & Segars Jr., R. (2002b). Solving rectilinear planar location problems with barriers by a polynomial partitioning. *Annals of Operations Research*, 111,111-133.

Dearing, P.M., Hamacher, H.W., & Klamroth, K. (2002). Dominating sets for rectilinear center location problems with polyhedral barriers. *Naval Research Logistics,* 49(7), 647–665.

Dearing, P.M., Klamroth, K., & Segars, R. (2005). Planar location problems with block distance and barriers. *Annals of Operations Research*, 136(1), 117–143.

Elzinga, D. J., & Hearn, D. W. (1972). Geometrical solutions for some minimax location problems. *Transportation Science*, 6, 379-394.

Frieß, L., Klamroth, K., & Sprau, M. (2005). A wavefront approach to center location problems with barriers. *Annals of Operations Research*, 136(1), 35–48.

Hamacher, H.W., & Klamroth, K. (2000). Planar Weber location problems with barriers and block norms. *Annals of Operations Research*, 96, 191–208.

Hamacher, H.W., & Nickel, S. (1996). Multicriteria planar location problems. *European Journal of Operational Research*, 94, 66–86.

Hamacher, H.W., & Nickel, S. (1988). Classification of location problems. *Location Science*, 6, 229–242.

Hansen, P., Peeters, D., & Thisse, J. (1981). On the location of an obnoxious facility. *Sistemi Urbani*, 3, 299–317.

Hearn, D. W., & Vijay, J. (1982). Efficient algorithms for the (weighted) minimum circle problem. *Operations Research*, 30, 777-795.

Katz, I.N., & Cooper, L. (1981). Formulation and the case of Euclidean distance with one forbidden circle. *European Journal of Operational Research*, 6, 166–173.

Kelachankuttu, H., Batta, R., & Nagi, R. (2007). Contour line construction for a new rectangular facility in an existing layout with rectangular departments. *European Journal of Operational Research*, 180, 149–162.

Klamroth, K. (2001a). A reduction result for location problems with polyhedral barriers. *European Journal of Operational Research*, 130(3), 486–497.

Klamroth, K. (2001b). Planar Weber location problems with line barriers. *Optimization*, 49(5-6), 517–527.

Klamroth, K. (2002). *Single-facility location problems with barriers*. Springer series in operations research.

Klamroth, K. (2004). Algebraic properties of location problems with one circular barrier. *European Journal of Operational Research*, 154(1), 20–35.

Klamroth, K., & Wiecek, M.M. (2002). A bi-objective median location problem with a line barrier. *Operations Research*, 50(4), 670–679.

Larson, R.C., & Sadiq, G. (1983). Facility location with the Manhattan metric in the presence of barriers to travel. *Operations Research*. 31, 652–669.

McGarvey, R.G., & Cavalier, T.M. (2003). A global optimal approach to facility location in the presence of forbidden regions. *Computers and Industrial Engineering*, 45(1), 1–15.

Nandikonda, P., Batta, R., & Nagi, R. (2003). Locating a 1-center on a Manhattan plane with 'arbitrarily' shaped barriers. *Annals of Operations Research*, 123(1-4), 157–172.

Nickel, S. (1998). Restricted center problems under polyhedral gauges. *European Journal of Operational Research*, 104, 343-357.

Sarkar, A., Batta, R., & Nagi, R. (2007). Placing a finite size facility with a center objective on a rectangular plane with barriers. *European Journal of Operational Research*, 179(3), 1160–1176.

Savaş, S., Batta, R., & Nagi, R. (2002). Finite-size facility placement in the presence of barriers to rectilinear travel. *Operations Research*, 50(6), 1018–1031.

Segars Jr., R. (2000). Location Problems with Barriers Using Rectilinear Distance. Ph.D. thesis, Dept. of Mathematical Sciences, Clemson University, SC.

Sylvester, J. J. (1857). A question in the geometry of situation. *Quarterly Journal of Pure and Applied Mathematics*, 1, 79-80.

Wang, S.J., Bhadury, J., & Nagi, R. (2002). Supply facility and input/output point locations in the presence of barriers. *Computers and Operations Research*, 29(6), 685–699.