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Retailer's optimal ordering policies with cash discount and progressive payment scheme derived algebraically

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A B S T R A C T

This study presents optimal ordering policies for retailer when supplier offers cash discount and two progressive payment schemes for paying of purchasing cost. If the retailer pays the outstanding amount before or at first trade credit period M, the supplier provides $\mathrm{r}_1$cash discount and does not charge any interest. If the retailer pays after M but before or at the second trade period N offered by the supplier, the supplier provides r_2 cash discount and charges interest on unpaid balance at the rate Ic_1 . If retailer pays the balance after N, $(\mathrm{N} > M)$ then the supplier does not provide any cash discount but charges interest on unpaid balance at the rate Ic_2. The primary objective of this paper is to minimize the total cost of inventory system. This paper develops an algebraic approach to determine the optimal cycle time, optimal order quantity and optimal relevant cost. Numerical example are also presented to illustrate the result of propose model and solution procedure developed.

1. Introduction

In classical economic order quantity model, it is tacitly assumed that the retailer must pay to the supplier for the items as soon as consignment received by him. However, this assumption may not hold in some cases. In real life situation, the supplier often offers the retailers a fixed time period to settle the account. During this period retailers can sell the items and accumulate revenue and earn interest before the end of permissible fixed period. Many researchers investigate this problem under various conditions.

Goyal (1985) developed the economic order quantity (EOQ) model under the conditions of permissible delay in payments. Chand and Ward (1987) investigated Goyal's model (1985) in the light of classical economic order quantity model. While Shah *et al.* (1988) investigated the same model incorporating shortages. Mondal and Phaujdar (1989) extended this issue by considering the interest earned from the sales revenue. Shah (1993a, b) also developed EOQ models for perishable items where delay in payment is permissible. Aggarwal and Jaggi (1995) extend Goyal's model (1985) to the case of exponential deterioration. Shah and Sreehari (1996) presented an extended issue of EOQ model where the delay in payment is permitted and the capacity of own warehouse is limited. Later Jamal *et al.* (1997) extend Aggrawal and Jaggi model by assuming shortage is allowed. Teng (2002) revise Goyal's model by more realistic assumption that unit selling price is higher than unit cost price. Huang (2003) extend Goyal model in the direction of two level of trade credits, in which

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supplier offers a permissible delay period in the paying of purchasing cost to his/her retailer and the retailer is also intern offer a permissible delay period to his/her customer to develop retailer's replenishment policy. Recently Goyal et. al (2007) and Soni and Shah (2008) developed an inventory model in which the supplier offers two opportunity of trade credit to his/her customer with the case of constant and stock-dependent demand

A newer application of the cash discount has emerged in recent years. Some supplier are now offering instantaneous cash discount on products if the customer will pay in cash rather than using a credit card. Traditionally, the cash discount now become a marketing and customer relations strategy that is early return of invested capital which reduce credit expense, obtain faster payment of products and stimulate more sales. Many published paper related to the inventory policy under cash discount and payment delay can be found in Arcelus *et al.* (2001), Chang (2002), Ouyang *et al.* (2002), Huang and Chung (2003), Recently Ouyang *et al.* (2005) developed a EOQ model with deterioration items and one opportunity of cash discount. Later Chung (2008) extend Ouyang's model in the case of deterioration rate θ is not sufficient small with cash discount policy. While Huang *et al.* (2007) developed an EPQ model with cash discount and permissible delay in payments derived algebraically.

On the other hand an algebraically technique become an easy approach to determine optimal solutions for those student who have little knowledge of calculus, may feel difficult to prove optimality condition with second order derivatives. Many research papers published on various inventory problems by applying second order derivative test to prove optimality conditions. Therefore, an algebraic approach can be also used to derive this problem. In this direction, Grubbstrom and Eedem (1999) and Cardenas-Barron (2001) showed that the formulae for the EOQ and EPQ with backlogging and shortages respectively can be derived without differential calculus. Yan and Wee (2002) developed algebraically the optimal replenishment policy for the integrated vendor-buyer inventory system without derivatives. Wu and Ouyang (2003) modified Yan and Wee (2002) in the light of shortages using algebraic method. Recently, Huang (2006) developed retailer's replenishment policy under two levels of trade credit and limited storage space derived without derivatives.

This paper extends Goyal *et al.* (2007) model to allow cash discount and applying algebraic method to obtain optimal solution. As a result, our proposed research paper here is that the supplier provides not only cash discount but also permissible delay period for settlement of payment. In addition, we develop an algebraic approach to determine optimal cycle time, optimal order quantity and optimal total relevant cost under said conditions. Here the objective function is to optimize a total cost of inventory systems for the retailer. Numerical examples are illustrated to managerial insight to proposed problem with help of an algorithm.

2. Assumptions and notations

The following assumptions and notations will be used throughout:

2.1 *Assumptions*

1) The inventory system deals with the single item.
2) Demand rate D, is known and constant.
3) Time horizon is infinite.
4) Lead-time is zero, shortages are not allowed.
5) Replenishment rate is infinite.
6) The interest charges and cash discount policies are as follows:

   - If retailer pays by offered period M, then the supplier provides $r_1$cash discount and does not charge any interest.

- If the retailer pays after the offered period M, but before or at offered period N $(N > M)$, he can keep the difference in the unit sales price and unit purchase price in an interest bearing account at the rate of I_e/unit/year. During the period$[M, N]$, the supplier provides r_2 cash discount and charges interest on unpaid balance at rate Ic_1 .
- If the retailer pays after the offered period N, then the supplier does not provide any cash discount but charges interest on unpaid balance at rate Ic_2. Here the interest rate $Ic_2 > Ic_1$.

2.2 Notations

1) h Inventory holding cost/unit/year excluding interest charges
2) P Selling price/unit
3) c Unit purchase cost, with $c < P$
4) A Ordering cost/order
5) T Replenishment cycle
6) M First offered credit period in settling the account without any charges
7) N Second permissible credit period in settling the account with interest charges Ic_2 on un-paid balance and $N > M$
8) I_e Interest earned/\$/year
9) r_1 Cash discount rate offered on credit period M.
10) r_2 Cash discount rate offered on credit period N and $(r_1 > r_2)$. $0 \le r < 1$
11) D Demand rate per year.
12) Q Order quantity, $Q > 0$.
13) IHC Inventory holding cost/time unit
14) OC Ordering cost/time unit
15) PC Purchasing cost/time unit.
16) Ic_1 Interest charged per \$ in stock per year by the supplier when retailer pays during $[M, N]$
17) Ic_2 Interest charged per \$ in stock per year by the supplier when retailer pays during $[N, T]$
18) $Q(t)$ On-hand inventory at time $t(0 \le t \le T)$.

3. Mathematical model

The inventory level $Q(t)$is depleted due to demand. Hence, the rate of change of inventory

$$\frac{dQ(t)}{dt} = -D, \quad 0 \le t \le T \tag{1}$$

with boundary conditions: $Q(0) = Q$ and $Q(T) = 0$.
The solution of the above differential Eq. (1) is,

$$Q(t) = D(T - t), 0 \le t \le T \tag{2}$$

This at $t = 0$ gives the order quantity,

$$Q = DT. \tag{3}$$

The ordering cost, inventory holding cost and purchasing cost of the system are as follow:

$$\text{Annual ordering cost (OC)} = \frac{A}{T} \tag{4}$$

$$\text{Annual inventory holding cost (IHC)} = \frac{h}{T}\int_0^T Q(t)dt = \frac{hDT}{2} \tag{5}$$

$$\begin{aligned} \text{Annual purchasing cost } &= c(1 - r_1)D \text{ for case } T \le M \\ &= c(1 - r_2)D \text{ for case } M < T < N \\ &= cD \quad\quad\quad \text{ for case } T \ge N \end{aligned} \tag{6}$$

Following the assumptions regarding the interest charged and interest earned, based on the length of the cycle time T, three cases may arise:
Case 1: When $T \le M$ (Fig.1)
Case 2: When $M < T < N$ (Fig.2)
Case 3: When $T \ge N$ (Fig.3)

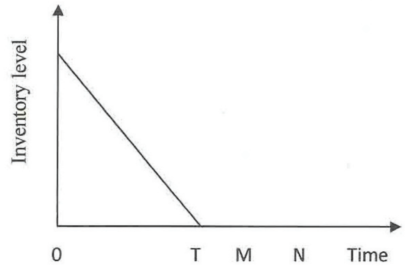

Fig. 1. *Case 1: $T \le M$*

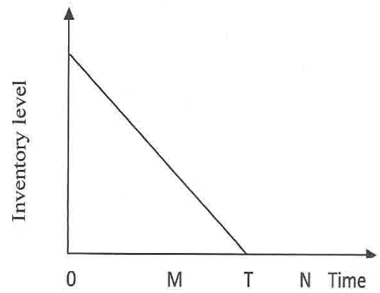

Fig. 2. *Case 2 $M < T < N$*

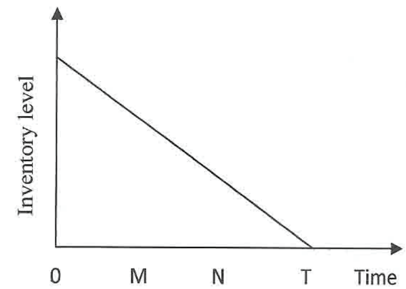

Fig. 3. Case 3 $T \ge N$

Case 1: When $T \le M$ (see Fig. 1)

Here, the retailer sells DT units during $[0, T]$ and paying for cDT units in full to the supplier at time $M \ge T$ with zero interest charged i.e.

$$IC_1 = 0 \tag{7}$$

During the period $[0, M]$ the retailer sells DT units and deposits the revenue into the interest bearing account that earns $I_e/\$/year$. For the period $[T, M]$, the retailer deposits the revenue into the account that earns I_e/\$/year. Therefore interest earned, IE_1, per year is given by

$$IE_1 = \frac{PI_e}{T}\left[\int_0^T Dtdt + Q(M-T)\right] = PI_eD\left(M - \frac{T}{2}\right) \tag{8}$$

Using equations 4, 5, 6,7, 8 then total cost, $TC_1(T)$ per time unit of an inventory system:

$$TC_1(T) = OC + IHC + PC + IC_1 - IE_1 = \frac{A}{T} + \frac{hDT}{2} + c(1-r_1)D - PI_eD\left(M - \frac{T}{2}\right) \tag{9}$$

Case 2. $M < T < N$ (See Fig. 2)

The retailer sells units and deposits the revenue into an interest earning account at an interest rate I_e/\$/year during$[0, M]$. Hence interest earned, IE_2 during $[0, M]$ is

$$IE_2 = PI_e\int_0^M Dtdt = PI_e\frac{DM^2}{2}. \tag{10}$$

Retailer purchases Q-units at time $t = 0$ and pays at the rate of c \$/unit to the supplier during $[0, M]$. The retailer sells DM-units at sell price \$$P$/unit. Therefore, he has generated revenue of PDM plus the interest earned, IE_2 during $[0, M]$ which arises two cases:

Sub-cases 2.1 Let $PDM + IE_2 \ge c(1-r_2)DT$

The retailer has an adequate amount in his account to pay all Q-unit purchase cost at time M. Therefore, retailer gets r_1 cash discount at M. Then interest charges:

$$IC_{2.1} = 0 \tag{11}$$

In addition, the interest earned is as follows,

$$IE_{2.1} = \frac{IE_2}{T}. \tag{12}$$

Using Eqs. (4-6), Eq. (11) and Eq.(12) the total cost, $TC_2(T)$ per time unit of an inventory system is given by

$$TC_{2.1}(T) = OC + IHC + PC + IC_{2.1} - IE_{2.1}\ TC_{2.1} = \frac{A}{T} + \frac{hDT}{2} + c(1-r_1)D - PI_e\frac{DM^2}{2T} \tag{13}$$

Sub-cases 2.2 Let $PDM + IE_2 < c(1-r_2)DT$

The retailer pays interest on un-paid balance $U_1 = c(1-r_2)DT - PDM\left(1 + \frac{I_eM}{2}\right)$ at the rate of Ic_1 at time M to the supplier, thereafter; the retailer gradually reduces the amount of loan due to constant sales and revenue received. Then the interest paid $IC_{2.2}$ per time unit is given by

$$IC_{2.2} = Ic_1 \frac{U_1\left(\frac{U_1}{PD}\right)}{2T} = \frac{Ic_1}{2PDT} U_1^2 \tag{14}$$

and the interest earned

$$IE_{2.2} = \frac{IE_2}{T} \tag{15}$$

Using Eqs. (4-6) and Eq. (14-15) the total cost $TC_{2.2}(T)$, per time of an inventory system is given by

$$TC_{2.2} = OC + IHC + PC + IC_{2.2} - IE_{2.2} = \frac{A}{T} + \frac{hDT}{2} + c(1-r_2)D + \frac{Ic_1}{2PDT} U_1^2 - PI_e \frac{DM^2}{2T} \tag{16}$$

Case 3 $\quad T \geq N$ (See Fig. 3)

Based on the total purchase cost, cQ, total money $PDM + IE_2$ in account at M and total money $PDN + IE_2$ at N, three sub-cases may arise,

Sub-case 3.1. Let $PDM + IE_2 \geq c(1-r_2)DT$

Here retailer will pay the total purchase cost at M and there is no interest charge. So, this Sub-case become the same as sub-case 2.1

Sub-case 3.2 Let $PDM + IE_2 < c(1-r_2)DT$ and

$PD(N-M) + PI_e D \frac{(N-M)^2}{2} \geq c(1-r_2)DT - (PDM + IE_2)$

The retailer does not have sufficient balance to settle his/her account at time M, but he/she can pay off the total purchase cost before or at N. Hence retailer only pays $PDM + PI_e D \frac{M^2}{2}$ at M and the supplier start to charge the retailer the un-paid balance $\left[cDT - \left\{PDM + PI_e D \frac{M^2}{2}\right\}\right]$ with interest Ic_1 at time M. So, this sub-case becomes similar to Sub-case 2.2

Sub-case 3.3 Let $PDM + IE_2 < c(1-r_2)DT$ and

$PD(N-M) + PI_e D \frac{(N-M)^2}{2} < c(1-r_2)DT - (PDM + IE_2)$

Here, the retailer does not have sufficient balance in his account to pay off total purchase cost at N. He will do payment of $PDM + IE_2$ at M and $PD(N-M) + PI_e D \frac{(N-M)^2}{2}$ at N. Therefore, he has to pay interest charges on the un-paid balance $U_2 = cDT - PDM\left(1 + \frac{I_e M}{2}\right)$ with interest Ic_1 during $[M, N]$ and un-paid balance $U_3 = cDT - PDM\left(1 + \frac{I_e M}{2}\right) - PD(N-M)\left[1 + \frac{I_e(N-M)}{2}\right]$ with interest rate Ic_2 during $[N, T]$.

Therefore, total interest charges; $IC_{3.3}$ per time is

$$IC_{3.3} = \frac{U_2 Ic_1 (N-M)}{T} + Ic_2 U_3 \frac{\left[\frac{U_3}{PD}\right]}{(2T)}$$
$$= \frac{U_2 Ic_1 (N-M)}{T} + \frac{Ic_2 D}{2PT}\left[cT - PN - \frac{PI_e}{2}\{M^2 + (N-M)^2\}\right]^2, \tag{17}$$

and total interest earned per time unit is

$$IE_{3.3}(T) = \frac{IE_2}{T}. \tag{18}$$

Using Eqs. (4-6) and Eqs. (17-18) the total cost $TC_{3.3}(T)$ per time unit of an inventory system is given by

$TC_{3.3}(T) = OC + IHC + PC + IC_{3.3} - IE_{3.3}$

$$TC_{3.3}(T) = \frac{A}{T} + \frac{hDT}{2} + cD + \frac{U_2 I c_1 (N-M)}{T} + \frac{I c_2 D}{2PT}\left[cT - PN - \frac{PI_e}{2}\{M^2 + (N-M)^2\}\right]^2 - \frac{PI_e DM^2}{2T} \quad (19)$$

4. Optimal solution using algebraic method

We can rewrite

$$TC_1(T) = \frac{D(h+PI_e)}{2T}\left\{T - \sqrt{\frac{2A}{D(h+PI_e)}}\right\}^2 + \left\{\sqrt{D(h+PI_e)2A} + D\{c(1-r_2) - PI_e\}\right\}. \quad (20)$$

According to Eq. (20) the minimum of $TC_1(T)$ is obtained when the quadratic non-negative term, depending on T, is made equal to zero. Therefore, the optimum value T_1^* is

$$T_1^* = \sqrt{\frac{2A}{D(h+PI_e)}} \quad \text{If } 2A > 0 \quad (21)$$

Therefore, Eq. (20) has a minimum value for the optimal value of T_1^* reducing $TC_1(T)$ to

$$TC_1(T_1^*) = \sqrt{D(h+PI_e)2A} + D\{c(1-r_2) - PI_e\}. \quad (22)$$

Similarly, we can derive

$$TC_{2.1}(T) = \frac{hD}{2T}\left\{T - \sqrt{\frac{2A - PI_e DM^2}{hD}}\right\}^2 + \left\{\sqrt{hD\{2A - PI_e DM^2\}} + c(1-r_1)D\right\}. \quad (23)$$

According to Eq. (23) the minimum of $TC_{2.1}(T)$ is obtained when the quadratic non-negative term, depending on T, is equal to zero. Therefore, the optimum value $T_{2.1}^*$ is

$$T_{2.1}^* = \sqrt{\frac{2A - PI_e DM^2}{hD}} \quad \text{If } 2A - PI_e DM^2 > 0 \quad (24)$$

Therefore, Eq. (23) has a minimum value for the optimal value of $T_{2.1}^*$ reducing $TC_{2.1}(T)$ to

$$TC_{2.1}(TC_{2.1}^*) = \left\{\sqrt{hD(2A - PI_e DM^2)} + c(1-r_1)D\right\}. \quad (25)$$

Similarly,

$$TC_{2.2}(T) = \frac{D\left(h + \frac{IC_1 c^2}{P}(1-r_2)^2\right)}{2T}\left\{T - \sqrt{\frac{2A + PDM^2\left\{IC_1\left(1 + \frac{I_e M}{2}\right)^2 - I_e\right\}}{D\left\{h + \frac{IC_1 c^2}{P}(1-r_2)^2\right\}}}\right\}^2 + \left\{\sqrt{D\left(h + \frac{Ic_1 c^2}{P}(1-r_2)^2\right)\left\{2A + PDM^2\left(Ic_1\left(1 + \frac{I_e M}{2}\right)^2 - I_e\right)\right\}} + cD(1-r_2)\left\{c - Ic_1 M\left(1 + \frac{I_e M}{2}\right)\right\}\right\} \quad (26)$$

According to Eq. (26) the minimum of $TC_{2.2}(T)$ is obtained when the quadratic non-negative term, depending on T, is equal to zero. Therefore, the optimum value $T_{2.2}^*$ is

$$TC_{2.2}(T) = \sqrt{\frac{2A + PDM^2\left\{Ic_1\left(1+\frac{I_eM}{2}\right)^2 - I_e\right\}}{D\left\{h + \frac{Ic_1c^2}{P}(1-r_2)^2\right\}}}$$

If $\quad 2A + PDM^2\left\{Ic_1\left(1+\frac{I_eM}{2}\right)^2 - I_e\right\} > 0.$ (27)

Therefore, Eq. (26) has a minimum value for the optimal value of $T_{2.2}^*$ reducing $TC_{2.2}(T)$ to

$$TC_{2.2}(T_{2.2}^*) = \sqrt{D\left(h + \frac{Ic_1c^2}{P}(1-r_2)^2\right)\left\{2A + PDM^2\left(Ic_1\left(1+\frac{I_eM}{2}\right)^2 - I_e\right)\right\}} + cD(1-r_2)\left\{1 - Ic_1M\left(1+\frac{I_eM}{2}\right)\right\} \tag{28}$$

Similarly,

$$TC_{3.3}(T) = \frac{D\left(h+\frac{Ic_2c^2}{P}\right)}{2T}\left\{- \sqrt{\frac{2A - 2PDMIc_1(N-M)\left(1+\frac{I_eM}{2}\right) + \frac{Ic_2D}{P}\left[PN + \frac{PI_e}{2}\{M^2+(N-M)^2\}\right]^2 - PI_eDM^2}{D\left(h+\frac{Ic_2c^2}{P}\right)}}\right\}^2 + \left\{\sqrt{D\left(h+\frac{Ic_2c^2}{P}\right)\left[2A - 2PDMIc_1(N-M)\left(1+\frac{I_eM}{2}\right) + \frac{Ic_2D}{P}\left[PN + \frac{PI_e}{2}\{M^2+(N-M)^2\}\right]^2 - PI_eDM^2\right]}\right\} + cD\left\{1 + Ic_1(N-M) - Ic_2N - \frac{Ic_2I_e}{2}\{M^2+(N-M)^2\}\right\} \tag{29}$$

Eq. (29) represents that the minimum of $TC_{3.3}(T)$ is obtained when the quadratic non-negative term, depending on T, is made equal to zero. Therefore, the optimum value $T_{3.3}^*$ is

$$TC_{3.3}(T) = \sqrt{\frac{2A - 2PDMIc_1(N-M)\left(1+\frac{I_eM}{2}\right) + \frac{Ic_2D}{P}\left[PN + \frac{PI_e}{2}\{M^2+(N-M)^2\}\right]^2 - PI_eDM^2}{D\left(h+\frac{Ic_2c^2}{P}\right)}}$$

If $2A - 2PDMIc_1(N-M)\left(1+\frac{I_eM}{2}\right) + \frac{Ic_2D}{P}\left[PN + \frac{PI_e}{2}\{M^2+(N-M)^2\}\right]^2 - PI_eDM^2 > 0$ (30)

Therefore, Eq. (29) has a minimum value for the optimal value of $T_{3.3}^*$ reducing $TC_{3.3}(T)$ to

$$TC_{3.3}(T_{3.3}^*) = \sqrt{D\left(h+\frac{Ic_2c^2}{P}\right)\left[2A - 2PDMIc_1(N-M)\left(1+\frac{I_eM}{2}\right) + \frac{Ic_2D}{P}\left[PN + \frac{PI_e}{2}\{M^2+(N-M)^2\}\right]^2 - PI_eL\right.} + cD\left\{1 + Ic_1(N-M) - Ic_2N - \frac{Ic_2I_e}{2}\{M^2+(N-M)^2\}\right\} \tag{31}$$

5. Algorithm

Step 1: Compute $T = T_1$ from the case 1,
Step 2: If $T_1 < M$ go to step 9, otherwise, go to step 3,
Step 3: If $M < T < N$ go to step 4, otherwise, go to step 8,
Step 4: If $pDM + IE_2 > c(1-r_2)DT$ go to step 6, otherwise, go to step 5,
Step 5: If $pDM + IE_2 > c(1-r_2)DT$ and $pD(N-M) + pIeD\frac{(N-M)^2}{2} \geq c(1-r_2)DT - (pDM + IE_2)$ go to step 7, otherwise go to step 8.
Step 6: Compute $T = T_{2.1}$ from sub-case 2.1 or sub-case 3.1 go to step 9,
Step 7: Compute $T = T_{2.2}$ from sub-case 2.2 or sub-case 3.2 go to step 9,
Step 8: Compute $T = T_{3.3}$ from sub-case 3.3 go to step 9,
Step 9: Compute TC go to step 10,
Step 10: Compute $TC(T)$ min $TC(T)$,
Step 11: End.

6. Numerical example

$c = \$8, P = \$10, I_e = .3\%, Ic_1 = .5\%, Ic_2 = .6\%, M = .08$year, $N = .16$year, $D = 1200$units/year $h = 5$/units/year.

Table 1
The optimal solution for different values of A and r

$\boldsymbol{A}$($)	$\boldsymbol{T_i}$ (Year)	$\boldsymbol{TC_i}$ ($)	$\boldsymbol{TC_i}$ ($)	$\boldsymbol{Q_i}$
10	$r_1 = .30$	$T_1 = .0456$	$TC_1 = 6870.18$	$Q_1 = 54.72$
	$r_1 = .34$		$TC_1 = 6486.18$	
	$r_1 = .38$		$TC_1 = 6102.18$	
30	$r_1 = .30$	$T_1 = .0791$	$TC_1 = 7190.95$	$Q_1 = 94.92$
	$r_1 = .34$		$TC_1 = 6806.95$	
	$r_1 = .38$		$TC_1 = 6422.95$	
60	$r_1 = .30$	$T_{2.1} = .1271$	$TC_{2.1} = 7482.73$	$Q_{2.1} = 152.52$
	$r_1 = .34$		$TC_{2.1} = 7098.73$	
	$r_1 = .38$		$TC_{2.1} = 6714.73$	
75	$r_1 = .30$	$T_{2.1} = .1455$	$TC_{2.1} = 7592.79$	$Q_{2.1} = 174.6$
	$r_1 = .34$		$TC_{2.1} = 7208.79$	
	$r_1 = .38$		$TC_{2.1} = 6824.79$	
100	$r_2 = .20$	$T_{2.2} = .1599$	$TC_{2.2} = 8721.62$	$Q_{2.2} = 191.88$
	$r_2 = .24$	$T_{2.2} = .1622$	$TC_{2.2} = 8333.87$	$Q_{2.2} = 194.64$
	$r_2 = .28$	$T_{2.2} = .1645$	$TC_{2.2} = 7946.85$	$Q_{2.2} = 197.4$
120	$r_2 = .20$	$T_{2.2} = .1741$	$TC_{2.2} = 8841.39$	$Q_{2.2} = 208.92$
	$r_2 = .24$	$T_{2.2} = .1766$	$TC_{2.2} = 8451.93$	$Q_{2.2} = 211.92$
	$r_2 = .28$	$T_{2.2} = .1791$	$TC_{2.2} = 8063.26$	$Q_{2.2} = 214.92$
180		$T_{3.3} = .2055$	$TC_{3.3} = 13494.2$	$Q_{3.3} = 246.6$
200		$T_{3.3} = .2145$	$TC_{3.3} = 13589.44$	$Q_{3.3} = 257.4$

For fixed value of A and increase in cash discount rate r, the cycle time and ordering quantity increase but total relevant cost decreases. As the value of A increases, the cycle time, total relevant cost and ordering quantity also increase.

7. Sensitive analysis

Now in this section we study the effects of changes in the values of the system parameter A on the optimal cycle time, total relevant cost and economic order quantity derived by the proposed method. The sensitive analysis is performed here by changing the value of A by $-20\%, +20\%$ taking A only at a time and keeping remaining other parameters unchanged.

Table 2
Sensitivity analysis

$\boldsymbol{A}$	% change in the $\boldsymbol{A}$	% change in the $\boldsymbol{T}^*$	% change in $\boldsymbol{TC}(\boldsymbol{T}^*)$ at $\boldsymbol{r_i}$	% change in $\boldsymbol{TC}(\boldsymbol{T}^*)$ at $\boldsymbol{r_i}$	% change in $\boldsymbol{TC}(\boldsymbol{T}^*)$ at $\boldsymbol{r_i}$	% change in $\boldsymbol{Q}^*$
10	8	$T_1 = 0.0408$	6823.92	6439.92	6055.92	48.96
	12	$T_1 = 0.05$	6912.00	6528.00	6144.00	60.00
30	24	$T_1 = 0.0707$	7111.00	6727.00	6343.00	84.84
	36	$T_{2.1} = 0.0903$	7262.00	6878.00	6494.00	108.36
60	48	$T_{2.1} = 0.1103$	7381.64	6997.64	6613.64	132.36
	72	$T_{2.1} = 0.1420$	7571.91	7187.91	6803.91	170.40
75	60	$T_{2.1} = 0.1217$	7482.73	7098.73	6714.73	146.04
	90	$T_{2.1} = 0.1617$	7690.45	7306.45	6922.45	194.04
100	80	$T_{2.2} = 0.1444$ $T_{2.2} = 0.1516$ $T_{2.2} = 0.1536$	8590.17	8025.00	7819.73	173.28 181.92 184.32
	120	$T_{2.2} = 0.1741$ $T_{2.2} = 0.1766$ $T_{2.2} = 0.1791$	8841.38	8451.92	8063.26	208.92 211.92 214.92
120	96	$T_{2.2} = 0.1569$ $T_{2.2} = 0.1648$ $T_{2.2} = 0.1669$	8696.37	8309.76	7923.01	188.28 197.76 200.28
	144	$T_{3.3} = 0.1888$	13804.30			225.96
180	144	$T_{3.3} = 0.1888$	13804.30			225.96
	216	$T_{3.3} = 0.2214$	15718.96			265.68
200	160	$T_{3.3} = 0.1961$	11523.27			235.32
	240	$T_{3.3} = 0.2314$	21354.27			277.68

For fixed value of A and increases in cash discount rate r, the cycle time, and ordering quantity increase but total relevant cost decreases. As the value of A increases, then cycle time, total relevant cost and ordering quantity also increase. Now in this section we study the effects of changes in the values of the system parameter C, P, h, M and N on the optimal cycle time, total relevant cost and economic order quantity derived by the proposed method. The sensitive analysis is performed here by changing the value of C, P, h, M and N ($A = \$180$ only for N) by $-20\%, +20\%$ taking one parameter only at a time and keeping remaining other parameters unchanged.

Table 3
Sensitivity analysis of the optimal solution

Parameters	% change in the parameters	% change in the $\boldsymbol{T}^*$	% change in $\boldsymbol{TC}(\boldsymbol{T}^*)$ at $\boldsymbol{r_i}$	% change in $\boldsymbol{TC}(\boldsymbol{T}^*)$ at $\boldsymbol{r_i}$	% change in $\boldsymbol{TC}(\boldsymbol{T}^*)$ at $\boldsymbol{r_i}$	% change in $\boldsymbol{Q}^*$
C	6.4	0.0456	5526.18	5218.98	4911.78	54.72
	9.6	0.0456	8214.18	7753.38	7292.58	54.72
P	8.0	0.0475	6911.17	6527.17	6143.17	57.00
	12.0	0.0440	6828.71	6444.71	6060.71	52.80
h	4.0	0.0488	6841.88	6457.88	6073.88	58.56
	6.0	0.0430	6896.76	6512.76	6128.76	51.60
M	0.064	0.0456	6927.78	6543.78	6159.78	54.72
	0.096	0.0456	6812.58	6428.58	6044.58	54.72
N	0.128	0.1962	27896.85			235.44
	0.192	0.2174	19756.36			260.88

When value of c, P, h, M and N increases then the optimal cycle time (stable for c and M but decreases for P and h, and increases for N), total relevant cost (increases for c and h but decreases for P, M and N) and optimal order quantity (stable for c and M but decreases for P and h, and increases for N) for the fixed value of the parameters total relevant cost decreases.

7. Conclusions

In the present paper, we determined the optimal ordering policies for retailers with cash discount and permissible delay period solved by algebraic method. Here supplier offers to his/her retailer two credit periods and two different cash discounts on said credit periods to settlement the account of purchasing goods. By using the numerical examples, sensitive analysis is performed to study the effects of the changes of the parameter values of A and C, P, h, M and N on the optimal cycle time, optimal order quantity and total relevant cost respectively.

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