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(Q, R) inventory model with service level constraint and variable lead time in fuzzy-stochastic environment

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In today's global marketplace, individual firms do not compete as independent entities rather as an integral part of a supply chain. Uncertainty is the main attribute in managing the supply chains. Accordingly, we develop a (Q, R) inventory model with service level constraint and variable lead-time in fuzzy-stochastic environment. In addition, the triangular fuzzy numbers counts upon lead-time are used to construct fuzzy-stochastic lead-time demand. Using credibility criterion, the expected shortages are calculated. Without loss of generality, we assume that all the observed values of the fuzzy random variable, representing the demand are triangular fuzzy numbers. Consequently, the value of total expected cost in the fuzzy sense is derived using the expected value criterion or credibility criterion. To determine an optimal policy, a numerical technique is presented and the results are analyzed using scan and zoom for constraint optimization. Finally, in order to demonstrate the accuracy and effectiveness of the proposed model, numerical example and sensitivity analysis are also included.

1. Introduction

In real world, many decisions are made under uncertainty since there are many important parameters that affect the decisions have unknown probability of occurrence. There have been many kinds of mathematical programming models developed to solve decision-making problems under uncertain conditions. Inventory control is one of the main issues in logistic and supply chain management in which various types of uncertain and imprecise parameters exist. The (Q, R) model is widely used to explore the continuous review inventory control problem. Till now, there has been sizeable literature in this area published in all variety of journals and magazines. In most of the earlier literature dealing with inventory problems in deterministic, probabilistic or fuzzy environment, researchers have considered lead-time as constant, stochastic or fuzzy. Therefore, it is not subject to control. According to Tersine (1982) lead-time usually consists of the following components: order preparation, order transit, supplier lead time, delivery time and setup time; in other words it is controllable. Liao and Shyu (1991) first formulated an inventory model in which lead-time is a unique decision variable and the order quantity is predetermined. After that, many scholars worked in this area and developed this field (cf. Ben-Daya & Raouf, 1994; Ouyang & Wu, 1997; Ouyang & Chang, 2000).

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Shortage in inventory is an unavoidable phenomenon due to uncertainty of prevailing environment. Conventional models of inventory systems assume that shortages are either fully backlogged or all sales are lost. In effort to represent the diversity of customer responses to shortages, several scholars have generalized the notion of shortage such that portion of shortages are backlogged while the remaining shortages incur lost sales penalties (cf. Montgomery et al., 1973; Hariga & Ben-Daya, 1999; Wu & Tsai, 2001). But Ouyang et al. (1997) considered the shortage cost as intangible component such as loss of goodwill and potential delay to the other parts of the inventory system, and hence, it is difficult to determine an extra value for the shortage cost. Instead of having shortage cost term in the objective function, they incorporated a service level constraint. Consequently, the shortage level per cycle is bounded. Chang et al. (2004) developed a lead-time reduction model based on continuous review inventory systems in which the uncertainty of demand during lead-time is dealt with a probabilistic fuzzy set and annual average demand by a fuzzy number only. Chang et al. (2006) presented the same work considering lead-time demand as fuzzy random variable instead of probabilistic fuzzy set.

In aforesaid models, the average annual demand has taken as crisp or fuzzy number. However, in actual market, it is very difficult to determine a precise value of demand. In this situation, management needs to collect the demand information from experts. When the experts' opinion are imprecise, like demand is about some fixed quantity and that fixed quantity is randomly chosen, then the demand can be vaguely expressed. Based on this argument, Dutta et al. (2007) developed continuous review inventory model with constant lead-time in a mixed environment by incorporating fuzzy random variable as the customer demand. They treated lead-time demand as triangular fuzzy number and computed expected shortage using interval valued possibilistic mean. Dutta et al. (2007) took lead-time as constant but didn't control it.

Based on above discussion, the present study seeks to extend Dutta et al. (2007) model by incorporating service level constraint and variable lead-time under fuzzy-stochastic environment. In addition, lead-time demand is taken as fuzzy random rather fuzzy number as in Dutta et al. (2007) model. The triangular fuzzy numbers counts upon lead-time are used to construct a lead-time demand. Since the annual demand is a fuzzy random variable, the associated cost function is also a fuzzy random variable. Consequently, the value of total expected cost is derived using the expected value criterion or credibility criterion. For the proposed model, we provide a solution procedure to find the optimal lead-time and the optimal order quantity along with the reorder point such that the total expected annual cost in the fuzzy stochastic sense has a minimum value.

The rest of this paper is organized as follows. In Section 2, we cover some preliminaries of fuzzy random variables. In section 3 notations and assumptions are given which are used to develop the proposed model. Section 4 proposes a fuzzy expected value model. Solution methodology is developed in section 5. Numerical example is provided in section 6 to demonstrate the effectiveness with sensitivity analysis. Section 7 furnishes conclusion.

2. Preliminary

2.1 Possibility Measure, Necessity Measure and Credibility measure

Let Θ be a nonempty set, P (Θ) the power set of Θ and Pos a possibility measure. Then the triplet (Θ, P(Θ), Pos) is called a possibility space. In this paper, the fuzzy variable is defined as follows.

Definition [Liu & Liu, 2001]: A fuzzy variable is defined as a function from a possibility space (Θ, P(Θ), Pos) to the set of real numbers. Let ξ be a fuzzy variable defined on the possibility space (Θ, P(Θ), Pos). Then its membership function μ is derived from the possibility measure through

$$\mu(x) = \text{Pos}\ \{\theta \in \Theta \mid \xi(\theta) = x\}\ ,\ x \in R.$$

Definition [Liu & Liu, 2001]: Let ξ be a fuzzy variable with membership function μ. Then for any Borel set B of real numbers,

$$Pos\{\xi \in B\} = \sup_{x \in B} \mu(x), \tag{1}$$

$$Nec\{\xi \in B\} = 1 - Pos\{\xi \in B^c\} = 1 - \sup_{x \in B^c} \mu(x), \tag{2}$$

$$Cr\{\xi \in B\} = \frac{1}{2}\left(Pos\{\xi \in B\} + Nec\{\xi \in B\}\right). \tag{3}$$

According to Liu and Iwamura (1998)

$$Pos(\xi_1 * \xi_2) = \left\{\sup\left(\min\left(\mu_{\xi_1}(x), \mu_{\xi_2}(y)\right)\right), x, y \in \mathbb{R} \text{ and } x * y\right\}$$

Also, * is any of the relation $>, <, =, \leq, \geq$.

Triangle fuzzy number (TFN): A TFN ξ is specified by three parameters (a_1, a_2, a_3) where $a_1 < a_2 < a_3$ and is characterized by possibility distribution μ_ξ, given by

$$\mu_\xi(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{for } a_1 \leq x \leq a_2, \\ \dfrac{x - a_3}{a_2 - a_3} & \text{for } a_2 \leq x \leq a_3, \\ 0 & \text{otherwise.} \end{cases}$$

If ξ be a TFN and *t* be any crisp number. Then according to Liu and Iwamura (1998) we define possibility measure and necessity measure as follows:

$$Pos(\xi \leq t) = \begin{cases} 0 & \text{for } t < a_1 \\ \left(\dfrac{t - a_1}{a_2 - a_1}\right) & \text{for } a_1 \leq t \leq a_2 \\ 1 & \text{otherwise} \end{cases} \tag{4}$$

$$Nec(\xi \leq t) = \begin{cases} 1 & \text{for } a_3 < t \\ 1 - \left(\dfrac{t - a_3}{a_2 - a_3}\right) & \text{for } a_2 \leq t \leq a_3 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Definition [Liu (2007)]**:** The credibility distribution Φ : R → [0, 1] of a fuzzy variable ξ is defined by

$$\Phi(t) = \left\{x \in \Theta \middle| \xi(x) \leq t\right\} \tag{6}$$

If ξ be a triangle fuzzy variable and *t* be any crisp number. Then credibility distribution of TFN is given by

$$\Phi(t)=\begin{cases}0 & \text{for } t<a_1\\ \left(\dfrac{t-a_1}{2(a_2-a_1)}\right) & \text{for } a_1\le t\le a_2\\ \left(\dfrac{t+a_3-2a_2}{2(a_3-a_2)}\right) & \text{for } a_2\le t\le a_3\\ 1 & \text{otherwise}\end{cases} \tag{7}$$

Definition [Liu & Liu, 2001]**:** Let ξ be a fuzzy variable then the expected value $E[\xi]$ is defined as

$$E[\xi]=\int_0^{+\infty} Cr\{\xi\ge r\}dr-\int_{-\infty}^{0} Cr\{\xi\le r\}dr \tag{8}$$

provided that at least one of the two integral is finite.

Proposition 2.1.1 [Liu (2007)]**:** Let ξ is a fuzzy variable with credibility distribution Φ. If

$$\lim_{t\to-\infty}\Phi(t)=0,\quad \lim_{t\to\infty}\Phi(t)=1$$

and the Lebesgue-Stieltjes integral $\int_{-\infty}^{\infty} t\, d\Phi(t)$ is finite, then we have

$$E[\xi]=\int_{-\infty}^{\infty} t\, d\Phi(t) \tag{9}$$

Making use of (7) and (9) we determine the expected value of $\xi=(a_1,a_2,a_3)$ to be

$$E[\xi]=\frac{a_1+2a_2+a_3}{4} \tag{10}$$

Now, the definitions of fuzzy random variable and its expected value and variance operators are presented. For more theoretical results on fuzzy random variables, see Gil et al. (2006) and Liu and Liu (2003).

2.2 Fuzzy random variable (FRV), its expectation and variance

Definition [Liu & Liu (2003)]**:** Suppose that (Ω, A, P) is a probability space and F_v is a collection of fuzzy variables defined on possibility space (Θ, P(Θ), Pos). A fuzzy random variable is a mapping $\mathrm{X}:\Omega\to F_v$ such that for any Borel subset B of $\Re$, $Pos\{\mathrm{X}(\omega)\in B\}$ is a measurable function of ω.

For any fuzzy random variable X on Ω, for each $\omega\in\Omega$, the expected value of the fuzzy variable $\mathrm{X}(\omega)$ is denoted by $E[\mathrm{X}(\omega)]$ which has been proved to be a measurable function of ω (see [Liu & Liu, 2003]), i.e., it is a random variable. Given the expected value of the fuzzy random variable, X is defined as the mathematical expectation of the random variable $E[\mathrm{X}(\omega)]$.

Definition [Liu & Liu, 2003]**:** Let X be fuzzy random variable defined on a probability space (Ω, A, P). Then, the expected value of X is defined as

$$E[\xi]=\int_{\Omega}\left[\int_0^{+\infty} Cr\{\xi(\omega)\ge r\}dr-\int_{-\infty}^{0} Cr\{\xi(\omega)\le r\}dr\right]\Pr(d\omega) \tag{11}$$

Example 2.2.1: Consider the triangular fuzzy random variable X, defined as

$$\mathrm{X}=\begin{cases} V_1=(2,6,10) \text{ with probability } 0.3 \\ V_2=(5,9,13) \text{ with probability } 0.7 \end{cases}$$

We need to compute the expected values of fuzzy variables $\mathrm{X}(V_1)$ and $\mathrm{X}(V_2)$, respectively, i.e. $E[\mathrm{X}(V_1)]=(2+2\times 6+10)/4=6$ and $E[\mathrm{X}(V_2)]=(5+2\times 9+13)/4=9$. Finally, by above definition, the expected value of X is $E[\mathrm{X}]=0.3\cdot E[\mathrm{X}(V_1)] + 0.7\cdot E[\mathrm{X}(V_2)]=8.1$.

Definition [Liu and Liu, 2003]**:** Let X be fuzzy random variable defined on a probability space (Ω, A, P) with expected value *e*. Then, the variance of X is defined as

$$\mathrm{Var}[\mathrm{X}]=E\left[(\mathrm{X}-e)^2\right] \tag{12}$$

where $e=E[\mathrm{X}]$.

Example 2.2.2: Consider the triangular fuzzy random variable X, which is defined in example 2.2.1. Let us calculate the variance of X. We know that the fuzzy random variable X takes fuzzy variables $\mathrm{X}(V_1)=(2,6,10)$ with probability 0.3 and $\mathrm{X}(V_2)=(5,9,13)$ with probability 0.7. From example 2.2.1, we have $e = E[\mathrm{X}] = 8.1$. Then, $\mathrm{Var}[\mathrm{X}]=0.3\cdot E\left[(\mathrm{X}(V_1)-8.1)^2\right]+0.7\cdot E\left[(\mathrm{X}(V_2)-8.1)^2\right]$

To obtain Var [X], we need to calculate $E\left[(\mathrm{X}(V_1)-8.1)^2\right]$ and $E\left[(\mathrm{X}(V_2)-8.1)^2\right]$, Denoting $Y_1=\mathrm{X}(V_1)-8.1$ we will calculate $Cr\{Y_1^2\geq t\}$ where $t\geq 0$.

$Cr\{Y_1^2\geq t\}=\max\{Cr\{Y_1\geq\sqrt{t}\},Cr\{Y_1\leq-\sqrt{t}\}\}=Cr\{Y_1\leq-\sqrt{t}\}$ Hence,

$$Cr\{Y_1^2\geq t\}=\begin{cases} \frac{1}{2}\left[2-\frac{1.9+\sqrt{t}}{4}\right] & \text{for } 0\leq t\leq 2.1^2 \\ \frac{1}{2}\left[\frac{6.1-\sqrt{t}}{4}\right] & \text{for } 2.1^2\leq t\leq 6.1^2 \\ 0 & \text{otherwise} \end{cases}$$

Therefore, from Eq. (8), we obtain $E\left[(\mathrm{X}(V_1)-8.1)^2\right]=E\left[Y_1^2\right]$ as follows:

$$E\left[Y_1^2\right]=\int_0^\infty Cr\{Y_1^2\geq t\}\,dt=\int_0^{2.1^2}\frac{1}{2}\left[2-\frac{1.9+\sqrt{t}}{4}\right]dt+\int_{2.1^2}^{6.1^2}\frac{1}{2}\left[\frac{6.1-\sqrt{t}}{4}\right]dt = 9.46.$$

Similarly, we obtain $E\left[(\mathrm{X}(V_2)-8.1)^2\right]=E\left[Y_2^2\right]=4.90$.

Thus, $\mathrm{Var}[\mathrm{X}]=0.3\cdot E\left[(\mathrm{X}(V_1)-8.1)^2\right]+0.7\cdot E\left[(\mathrm{X}(V_2)-8.1)^2\right] = 6.27.$

In next section, we present the notations and assumptions used in developing the model.

3. Notation and assumptions

We use the following notation and assumptions to develop inventory models with crashing component lead-time and service level constraint.

A	:	Fixed ordering cost per order
$\widehat{\widehat{D}}$	:	Demand rate in units per year which is fuzzy random in nature.
h	:	Inventory holding cost per item per year
L	:	Lead-time (decision variable) that has n mutually independent components. The i-th component has a minimum duration a_i and normal duration b_i with a crashing cost c_i per unit time under the assumption $c_1 \le c_2 \le ... \le c_n$. The components of L are crashed one at a time starting from the component of least c_i and so forth. Hence, the range for L is from $\sum_{j=1}^{n} a_j$ to $\sum_{j=1}^{n} b_j$
L_r	:	Length of lead-time with components 1; 2; ... ; r crashed to their minimum durations. We define $L_n = \sum_{j=1}^{n} a_j$ and $L_r = L_n + \sum_{k=r+1}^{n} (b_k - a_k)$ for $r = 0,1,...,n-1$. Since $b_r > a_r$, it follows that $L_{r-1} > L_r$ for $r = 1,2,...,n$.
Q	:	Order quantity (decision variable).
$C(L)$	:	Lead-time crashing cost per cycle for a given $L \in [L_r, L_{r-1}]$ is given by $C(L) = c_r (L_{r-1} - L) + \sum_{k=1}^{r-1} c_k (b_k - a_k)$.
R	:	Reorder point (decision variable) given by R = expected demand during lead-time + safety stock. Inventory is continuously reviewed. Replenishments are made whenever the inventory level falls to the reorder point R.
X	:	Fuzzy random demand during lead-time.
α	:	Proportion of demand that is not met from the stock; hence $(1-\alpha)$ is the service level.
x^+	:	Maximum value of x and 0, i.e. $x^+ = \max\{x, 0\}$
$E(\bullet)$	:	Expected value of $(\bullet)$

4. Mathematical formulation

Ouyang and Wu (1997) proposed a continuously review inventory model with service level constraint which is mathematically represented by

$$\min\ C(Q,R,L) = A\frac{D}{Q} + h\left[\frac{Q}{2} + R - E(Y) + (1-\beta)E(Y-R)^+\right] + \frac{D}{Q}C(L) \tag{13}$$

$$\text{subject to } \frac{E(Y-R)^+}{Q} \le \alpha, \tag{14}$$

where $E(Y-R)^+ = \int_R^\infty (y-R) f(y)\,dy = \int_R^\infty y f(y)\,dy - R\int_R^\infty f(y)\,dy$.

Here the demand is taken as random variable. As demand is influenced by many factors, it is very difficult to determine a precise value of demand rate for actual market. In this case, management collects the demand information from experts. When the experts' opinion are imprecise, like demand is about some fixed quantity and that fixed quantity is chosen, randomly then the demand can be vaguely expressed. Therefore, total annual demand is treated as fuzzy random variable (FRV). When the parameter $\hat{\tilde{D}}$ becomes FRV, the objective function in Eq. (13) is also a FRV and can be written as

$$\min \tilde{C}(Q,R,L) = A\frac{\hat{\tilde{D}}}{Q} + h\left[\frac{Q}{2} + R - E(X) + (1-\beta)E(X-R)^{+}\right] + \frac{\hat{\tilde{D}}}{Q}C(L) \tag{15}$$

As the lead-time is variable and the total annual demand is characterized by fuzzy randomly, the value of lead-time demand (LTD) may have variation depending upon the length of lead-time in this uncertain environment. Generally, the demand estimation during the lead-time period is based on the DM's imprecise apperception from his intrinsic understanding and hence it can be imprecisely expressed. Therefore, based on the variable lead-time L we consider fuzzy random LTD X as

$$\mathrm{X} = \begin{cases} V_1 = (m_1 L, m_2 L, m_3 L) \text{ with probability } p_1 \\ V_2 = (m_4 L, m_5 L, m_6 L) \text{ with probability } p_2, \end{cases} \tag{16}$$

where $p_1 + p_2 = 1,\ 0 < m_1 < m_2 < m_3,\ 0 < m_4 < m_5 < m_6$ are confirmed by decision maker which reflect a kind of fuzzy apperception from his intrinsic understanding.

Using Eq. (7) the credibility distributions of V_1 and V_2 are respectively as follows,

$$\Phi_1(t) = \begin{cases} 0 & \text{for } t < m_1 L \\ \left(\dfrac{t - m_1 L}{2(m_2 - m_1)L}\right) & \text{for } m_1 L \le t \le m_2 L \\ \left(\dfrac{t + m_3 L - 2m_2 L}{2(m_3 - m_2)L}\right) & \text{for } m_2 L \le t \le m_3 L \\ 1 & \text{otherwise} \end{cases} \tag{17}$$

and

$$\Phi_2(t) = \begin{cases} 0 & \text{for } t < m_4 L \\ \left(\dfrac{t - m_4 L}{2(m_5 - m_4)L}\right) & \text{for } m_4 L \le t \le m_5 L \\ \left(\dfrac{t + m_6 L - 2m_5 L}{2(m_6 - m_5)L}\right) & \text{for } m_5 L \le t \le m_6 L \\ 1 & \text{otherwise} \end{cases} \tag{18}$$

We assume reorder point induced by expectation and variance of LTD X i.e.

$$R = E[X] + k\sigma_X \tag{19}$$

where k is safety factor, $\sigma_X = \sqrt{Var(X)}$, $E[X]$ and $Var(X)$ are expectation and variance of fuzzy random LTD X respectively. Moreover, the expected shortage is given by

$$E(X-R)^{+} = \sum_{i=1}^{2}\left[\int_R^{\infty}(t-R)\,d\Phi_i(t)\right]\times p_i \tag{20}$$

The expected value of this fuzzy random lead-time demand i.e. $E\left[X\right]$ can be obtained by using Eq. (11). Denoting $X_1 = X - R$ the objective function of the model in fuzzy stochastic environment can be rewritten as

$$\min\ \widetilde{C}\left(Q,k,L\right) = A\frac{\widehat{\widetilde{D}}}{Q} + h\left[\frac{Q}{2} + k\sigma_X + \left(1-\beta\right)E\left(X_1\right)^+\right] + \frac{\widehat{\widetilde{D}}}{Q}C\left(L\right) \tag{21}$$

subject to $\dfrac{E\left(X_1\right)^+}{Q} \le \alpha$

Using linearity of operator E, the expected value of the objective function $\widetilde{C}\left(Q,k,L\right)$ is given by

$$E\left[\widetilde{C}\left(Q,k,L\right)\right] = \left(A + C\left(L\right)\right)\frac{E\left[\widehat{\widetilde{D}}\right]}{Q} + h\left[\frac{Q}{2} + k\sigma_X + \left(1-\beta\right)E\left(X_1\right)^+\right] \tag{22}$$

Then the above optimization problem with service level constraint is reduced to the following optimization problem,

$$\min\ E\left[\widetilde{C}\left(Q,k,L\right)\right] \quad \text{subject to } E\left(X_1\right)^+ \le \alpha Q \tag{23}$$

It is difficult to obtain the mathematical expression for $E\left(X_1\right)^+$ therefore, in order to determine an optimal policy we have to make use of numerical technique for constraint optimization. In this paper, the optimal solution is found by using constrained scan and zoom method proposed by Venkataraman (2009). The algorithm, incorporating constrained scan and zoom method, for finding optimal solution is developed in the next section.

5. Solution Methodology

5.1 Constrained scan and zoom method

It is simple but more effective method in identifying the neighbourhood of the optimum. It is based on the objective function and does not require derivative computation. The method will go through a systematic search for the minimum in the design space.

Primary input for this method is the extent of the design region based on prescribed values of the lower and upper limits of the decision variables. In this method, initially the solution is searched over a coarse grid over several levels where each level entails a fine grid than the previous level. The best value, at the end of scanning in a particular level becomes centre of the search for the next level with half the extent of the previous level. This indicates that we basically zooming around the best solution in terms of significant figures. The algorithm searches for the minimum over given number of levels with a diminishing design region.

The following computational algorithm embedding constraint scan and zoom is developed to come up with the optimal solution of $\left(Q,k,L\right)$ denoted by $\left(Q^*,k^*,L^*\right)$. For the ease of notation we denote $f\left(x\right) = E\left[\widetilde{C}\left(Q,k,L\right)\right]$ and $g\left(x\right) = E\left(X_1\right)^+ - \alpha Q \le 0$ where $x = \left(Q,k,L\right)$.

5.2 Computational Algorithm

Step 1: For each $L \in \left[L_r, L_{r-1}\right]$, r = 1, 2, …, n, perform step 2.1 to step 2.4.

Step 2.1: For zoom level i = 1

Grid design region $\left(x_{ci}-\Delta x_i,\ x_{ci}+\Delta x_i\right)$ using M number of points where x_{ci} = centre of design region Δx_i = extent of design region.

Step 2.2: For j = 1, 2, 3, …, M compute $f_{ij}=f\left(x_{ij}\right)$ and $g_{ij}=g\left(x_{ij}\right)$

Store f_limit = max(f_{ij}) and g_limit = max(g_{ij}).

Step 2.3: For i = 2, 3, …, N

Grid design region using M points

For j = 1, 2, 3, …, M start scanning

If f_{ij} < f_limit and g_{ij} ≤ g_limit

fsol = f_{ij}

gsol = g_{ij}

xsol = x_{ij}

store best values

Xbest (i) = xsol

Fbest (i) = fsol

Gbest (i) = gsol

Step 2.4: If the design is not changing perturb f_limit and g_limit; M = M + 15;

Go to step 2.3

Else $x_{ci}=Xbest\left(i-1\right)$; $\Delta x_i=0.5*\Delta x_{i-1}$

f_limit = 0.5*f_limit

g_limit = 0.5*g_limit

i = i + 1

Go to step 2.3

If $\left|\text{Fbest}\left(\text{i}\right)-\text{Fbest}\left(\text{i-1}\right)\right|$ < tolerance: exit

Step 3: Find the value of $\underset{r=1,2,..n}{Min}\ E\left[\widetilde{C}\left(Q_r,k_r,L_r\right)\right]$ and set $E\left[\widetilde{C}\left(Q^*,k^*,L^*\right)\right]$ = $\underset{r=1,2,..n}{Min}\ E\left[\widetilde{C}\left(Q_r,k_r,L_r\right)\right]$, Compute the value of R^* using Eq. (19).

6. Numerical example

In order to demonstrate the proposed solution procedure let us consider a situation in which a retailer has demand information as shown in Table 1.

Table 1
Demand Information

Demand	Probability
D_1 = (575, 625, 725)	0.15
D_2 = (550, 600, 650)	0.19
D_3 = (495, 580, 690)	0.27
D_4 = (550, 600, 645)	0.22
D_5 = (570, 590, 610)	0.17

Also, the retailer estimates the demand during lead-time, which is fuzzy random variable X, as

$$X = \begin{cases} V_1 = (9.8L, 11.9L, 14.4L) & \text{with probability } 0.6 \\ V_2 = (11.5L, 13.7L, 16.5L) & \text{with probability } 0.4 \end{cases}$$

Let the service level be 0.95, i.e. $1 - \alpha = 0.05 \Rightarrow \alpha = 0.05$. Other related parameters are as follows:

A = \$200 per order, h = \$15 per unit per year, $\beta = 0.6$, and the lead-time has four components with the data shown in Table 2.

Table 2
Lead-time data for different components

Lead-time component (r)	1	2	3
Normal duration b_r (days)	20	20	16
Minimum duration a_r (days)	6	6	9
Unit crashing cost c_r (\$/day)	0.4	1.2	5.0

The obtained results listed in Table 3.

Table 3
Optimum result for $L \in [L_3, L_0] = [21, 56]$

L	$E[X]$	σ_X	$E\left[\widetilde{C}(Q,k,L)\right]$	Q	R
56	101.92	12.59	2030.82	134.30	109.6
55	100.10	12.37	2031.63	135.78	107.65
54	98.28	12.14	2031.26	135.97	105.69
53	96.46	11.92	2030.36	135.62	103.73
52	94.64	11.69	2029.43	135.19	101.77
51	92.82	11.47	2028.44	134.67	99.82
50	91.00	11.24	2027.32	133.91	97.86
49	89.18	11.02	2026.45	133.42	95.9
48	87.36	10.79	2025.86	133.31	93.94
47	85.54	10.57	2024.32	131.37	91.99
46	83.72	10.34	2023.41	130.17	90.03
45	81.90	10.12	2022.77	129.44	88.07
44	80.08	9.89	2022.23	128.85	86.11
43	78.26	9.67	2021.74	128.28	84.16
42	**76.44**	**9.44**	**2021.34**	**127.28**	**82.20**
41	74.62	9.22	2024.75	126.86	80.24
40	72.8	8.99	2028.22	126.44	78.28
39	70.98	8.77	2031.92	125.62	76.33
38	69.16	8.54	2035.53	125.25	74.37
37	67.34	8.32	2039.18	124.89	72.42
36	65.52	8.09	2042.86	124.55	70.45
35	63.7	7.87	2043.94	130.52	68.5
34	61.88	7.64	2050.27	123.97	66.54
33	60.06	7.42	2053.94	123.79	64.59
32	58.24	7.19	2057.60	123.62	62.63
31	56.42	6.97	2061.95	122.88	60.67
30	54.60	6.75	2065.94	122.52	58.72
29	52.78	6.52	2069.81	122.29	56.76
28	50.96	6.30	2074.57	121.49	54.80
27	49.14	6.07	2087.57	140.52	52.84
26	47.32	5.85	2118.36	121.98	50.89
25	45.50	5.62	2133.53	126.01	48.93
24	43.68	5.40	2153.92	126.68	46.97
23	41.86	5.17	2184.72	122.17	45.01
22	40.04	4.95	2206.92	122.18	43.06
21	38.22	4.72	2304.98	188.64	39.21

In order to determine optimal values of decision variables programs have been developed in MATLAB for finding expectation and variance of fuzzy random demand and for implementing the above algorithm. The values for Scan and Zoom parameters were set as follows: N = Number of Zoom level = 20, M = Number of grid points = 15. From the above table the optimal order quantity is 127.28 units, optimal reorder point 82.20 units, optimal lead time 6 weeks and corresponding minimum total expected cost is $ 2021.34. To show the efficiency of proposed solution methodology, we have depicted demand during lead-time (L^* = 6) and shortages in Fig. 1.

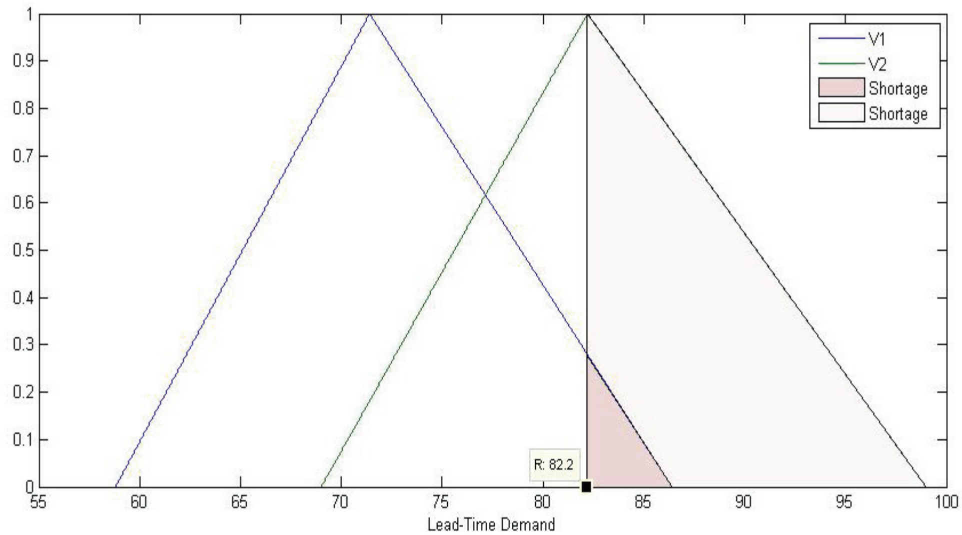

Fig. 1. demand during lead-time (L^* = 6) and shortages

Sensitivity: Sensitivity analysis is made with respect to α for different value of β. The results are presented in the Table 4. From the table it is interesting to observe that when α increases, i.e. when the upper bound of the expected stockout level increases then the management decreases the order quantity Q and reorder point to reduce his holding cost. Moreover, Q increases and EC decreases relative to the increases in β, which is apparent.

Table 4
Sensitivity analysis with respect to α for different value of β

α		β 0.4	0.6	0.8
0.01	Q^*	133.69	133.97	134.25
	R^*	84.28	84.25	84.25
	L^* (weeks)	6	6	6
	EC	2054.89	2050.87	2046.84
0.02	Q^*	132.11	133.69	134.02
	R^*	82.20	82.20	84.24
	L^* (weeks)	6	6	6
	EC	2027.70	2022.94	2017.58
0.05	Q^*	126.61	127.28	nf
	R^*	82.20	82.20	
	L^* (weeks)	6	6	
	EC	2027.01	2021.34	

$EC = E\left[\widetilde{C}\left(Q^*, k^*, L^*\right)\right]$, nf = not feasible.

7. Conclusion

According to the model of Dutta et al. (2007), a new model with service level constraint and controllable lead time in fuzzy stochastic-environment has been proposed. For the first time, the expected shortages are calculated using credibility distribution by treating lead-time demand as fuzzy stochastic. Without loss of generality we have assumed that all the observed values of the fuzzy random variable are triangular fuzzy numbers. The optimization process carried out by employing fresh numerical technique namely Constraint Scan and Zoom Method. The extended model is more effective as it can solve the specific problems in the related field. In future research on this problem, it would be interesting to consider other parameters as fuzzy or fuzzy stochastic.

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