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# Simultaneous optimization of material removal rate and surface roughness for WEDM of WC-Co composite using grey relational analysis along with Taguchi method

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A B S T R A C T

In this paper, wire electrical discharge machining of WC-Co composite is studied. Influence of taper angle, peak current, pulse-on time, pulse-off time, wire tension and dielectric flow rate are investigated for material removal rate (MRR) and surface roughness (SR) during intricate machining of a carbide block. In order to optimize MRR and SR simultaneously, grey relational analysis (GRA) is employed along with Taguchi method. Through GRA, grey relational grade is used as a performance index to determine the optimal setting of process parameters for multiple machining characteristics. Analysis of variance (ANOVA) shows that the taper angle and pulse-on time are the most significant parameters affecting the multiple machining characteristics. Confirmatory results, proves the potential of GRA to optimize process parameters successfully for multi-machining characteristics.



## 1. Introduction

The cemented carbides such as WC-Co are typically used in tool and die industries because of their excellent hardness and strength. It is hard to machine material, produced by sintering of WC powder with the binder (typically Co or Ni) at temperature near the melting point of the metal (Kim et al., 2005). The formation of liquid phase during sintering enhances the densification process and hence increases the hardness of the composite.

Machinability of WC-Co composite is very important in die and tool manufacturing as it may affect different manufacturing phases including product design, process planning, machining operations etc. Processing of WC-Co composite is very difficult with conventional machining method due to its high hardness. Machining of WC-Co, using CBN tools in CNC turning, results in very high cutting forces and very poor surface finish (Liu et al., 200). Ultrasonic machining method (Nath et al., 2009) may yield better machining results as compared to conventional machining process but complex and precise shapes are very difficult to generate. Similar difficulties occur with grinding operation. Electrical discharge machining (EDM) method is the best alternative to machine hard and non-conductive composites (Kucukturk & Cogun, 2010). Wire electrical discharge machining (WEDM) is a special form of EDM, which has the capability to produce intricate shapes and profiles in composite materials with a high degree of accuracy. In WEDM, the erosion mechanism is described as melting and/or evaporation of the surface material by the heat generated in the plasma channel. A spark is

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produced between the wire electrode (usually smaller than 0.3 mm) and workpiece through deionised water, (used as dielectric medium surrounding the workpiece) which erodes the workpiece to produce complex of two and three-dimensional shapes (Çaydaş et al., 2009).
Some unstable machining during EDM of tungsten carbide (Mahdavinejad, 2005) composite has been found. This unstable machining is due to the large difference between the electrical conductivity, melting and evaporation temperature of WC and Co grains. The melting and evaporation temperature are $2800^o$ C, $6000^o$ C for WC and $1320^o$ C, $2700^o$C for Co, respectively. Therefore, discharge energy tends to melt, evaporate and remove cobalt even before the melting of WC. As a result, the WC grains may be released without melting and hence causes unstable machining (Saha et al., 2008). In order to achieve an efficient process planning in machining of WC-Co composite into desired shape, we need accurate machinability data. Jangra et al. (2011) evaluated the effect of various factors and their sub-factors on machinability of WC-Co composite with WEDM using digraph and matrix method. They broadly grouped these factors into work material, machine tool, tool electrode, cutting conditions and geometry to be machined where machinability is measured in terms of material removal rate (MRR). They concluded that the machine tool is the most influencing factor affecting the machinability of WC-Co composite. Low cobalt concentration and small grain size favours the high MRR. In case of cutting conditions, good conductivity and high flow rate of distilled water results in high MRR.
The influence of cobalt concentration and electrical conductivity of the dielectric fluid on WEDM of sintered carbide has been studied by Kim and Kruth (2001). Results revealed that increase of cobalt amount in carbides affect the metal removal rate and worsen the surface quality as greater quantity of solidified metal gets deposited on the eroded surface. Higher electrical conductivity of water yields a higher material removal rate but poorer surface roughness. Lauwers et al. (2006) described the influence of composition and grain size of WC-based cermets on manufacturability by WEDM. It was shown that the cutting rate decreases with increasing grain size and cobalt percentage, which can be explained mainly by the change in thermal conductivity of the material.
An extensive experimental study was conducted by Lee and Li (2001) to investigate the effect of machining parameters such as the electrode materials, electrode polarity, open circuit voltage, peak current, pulse duration, pulse interval and flushing on the machining characteristics, such as MRR, surface finish and relative tool wear in EDM of tungsten carbide. They observed that the MRR generally decreases with the increase of open circuit voltage. For low current setting, the MRR increases with increase in peak current, but becomes constant when machining at higher values of peak current. The surface roughness increases with increasing peak current. Increase in pulse duration results in increase in MRR. Lee and Li (2003) studied surface integrity of EDMed surface of tungsten carbide. They found that the surface roughness is a function of two main parameters, peak current and pulse duration, both of which were settings of the power supply. High peak current and/or long pulse duration produces a rough surface. At high peak current and pulse duration abundance of micro-cracks was observed. Saha et al. (2008) developed a second order multi-variable regression model and a feed forward back-propagation neural network to correlate the input process parameters, such as pulse on time, pulse-off time, peak current and capacitance with the performance measures namely cutting speed and surface roughness while doing WEDM of tungsten carbide-cobalt composite material. Increase in both peak current and capacitance led to increase of cutting speed and surface roughness within the range of investigation. Chen at al. (2010) optimized the WEDM for pure tungsten using an approach that integrates Taguchi's parameter design method, back-propagation neural network, genetic algorithm and engineering optimization concepts. Through ANOVA, the percentage of contribution to the WEDM process, the pulse on time is the most significant controlled factor affecting the cutting speed and surface roughness. Several other researchers (Puertas et al., 2004; Kanagarajan et al., 2006; Kung et al., 2007; etc.) investigated the performance of EDM in processing of tungsten carbide.
The Taguchi method (Ross, 1996; Roy, 2001) is a systematic application of design and analysis of experiments for the purpose of designing and improving product quality. However, the original Taguchi method was designed to optimize the single performance characteristics. According to

Phadke (1989), it is difficult to optimize multi-response in complex process by Taguchi method and engineering judgment is primarily used to resolve such a complicated problem. The grey system theory proposed by Deng (1982) has been proven to be useful for dealing with the problems with poor, insufficient and uncertain information. The grey relational analysis based on this theory can further be effectively adopted for solving the complicated interrelationship among the designated performance characteristics. In recent years, grey relational analysis has become a powerful tool to analyze the processes with multiple performance characteristics. Lin and Lin (2002) combined orthogonal array and grey relational analysis to optimize the electrical discharge machining process with multiple performance characteristics. The results showed that the problem of multiple quality characteristics can be solved, effectively. Tarang et al. (2002) utilized the grey-based Taguchi method to optimize the process parameters of submerged arc welding in hardfacing, considering multiple weld qualities. Huang et al. (2003) successfully optimized the machining parameters in wire EDM using the grey relational analysis along with Taguchi method. Narender singh et al. (2004) adopted grey relational analysis to investigate the EDM parameters on machining of Al-10% SiCp composites. Pan et al. (2007) combined Taguchi method with grey relational analysis to investigate the optimal design of cutting parameters for Nd: YAG laser welding titanium alloy plates. Li and Tsai (2009) optimized the multiple performance characteristics in laser cutting of specially shaped electronic printed circuit board (PCB) carrier substrate of advanced integrated circuit (IC) using grey relational analysis. Tzeng et al. (2009) optimized the CNC turning operation for SKD11 using grey relational analysis. Lu et al. (2009) applied the grey relational analysis coupled with principle component analysis for optimization design of the cutting parameters in high speed end milling. Results showed that the proposed approach can be useful tool to improve the cutting performance of rough cutting processes in high speed end milling process. Based on the above survey we can conclude that the grey relational analysis is a better approach for optimization of multiple response characteristics in different fields. Therefore, grey relational analysis is utilized in this study, for multiple-optimization of the machining characteristics for WEDM of WC-Co composite.

## 2. Experimental set up, design and results

### *2.1 Experimental set up*

In this study, intricate machining of WC-Co composite was performed as shown in Fig. 1. WC-Co composite having 6% cobalt was taken as a work material in the form of rectangular block of thickness 13 mm. The density and hardness of die material was measured as 14.95 g/cm$^3$ and 77 HRC, respectively. The experiments were performed on 5-axis sprint cut (ELPLUS-40) wire-EDM manufactured by Electronica Machine Tool Ltd, India. The range of variable parameters in sprint cut machine tool were as follows: peak current, 10-230 amp.; pulse-on time, 100-131μs; pulse-off time, 0-63μs; wire speed, 1-15 m/min.; wire tension, 1-15N; servo voltage, 10-90 V; dielectric flow rate 0-12 litre per minute (LM$^{-1}$). Zinc coated brass wire of diameter 0.25 mm was used as an electrode because of its good capability to sustain high discharge energy.

**Fig. 1.** Cavities produced in carbide block

*2.2 Experimental design*

In present work, six process parameters namely peak current, pulse-on time, pulse-off time, wire tension, dielectric flow rate and taper angle were selected as input variables during intricate cutting of WC-%6Co composite with WEDM. The experiments were carried out with fixed value of servo voltage at 30V and distilled water was used as a dielectric fluid with a conductivity of 20S. Out of six input parameters, taper angle, which is a geometrical variable was kept at two levels while all five variables were assigned values of three levels. Taper angle was considered to provide some draft angle keeping in mind the concept of die and punch manufacturing. Preliminary experiments were conducted to select the range and values of the machining parameters. These results were discussed by Jangra et al. (2011) in section 5. Table 1 depicts the values of levels of the selected process variables. As the thickness of workpiece material is low (13 mm), therefore, feed rate (or downward movement) of wire was kept constant at a value of 8 m/min. Wire offset was taken at zero value.

**Table 1**
Process variables and their levels

| Symbol | Process Parameters | Level 1 | Level 2 | Level 3 |
|---|---|---|---|---|
| A | Taper angle (degree) | 3 | 1.5 | - |
| B | Peak Current (ampere) | 80 | 100 | 120 |
| C | Pulse-on time (μs) | 108 | 115 | 122 |
| D | Pulse-off time (μs) | 30 | 40 | 50 |
| E | Wire Tension (N) | 6 | 8 | 10 |
| F | Dielectric flow rate ($LM^{-1}$) | 4 | 7 | 10 |

The orthogonal array forms the basis for the experimental analysis in the Taguchi method. The selection of orthogonal array is concerned with the total degree of freedom of process parameters. Total degree of freedom (DOF) associated with six parameters is equal to 11 (1×1+5×2). The degree of freedom for the orthogonal array should be greater than or at least equal to that of the process parameters. Thereby, a $L_{18}$ orthogonal array having degrees of freedom equal to 17 has been considered in present case. The experimental layout is shown in Table 2.

**Table 2**
Orthogonal Array for $L_{18}$

| Exp. No. | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 2 | 2 | 2 | 2 |
| 3 | 1 | 1 | 3 | 3 | 3 | 3 |
| 4 | 1 | 2 | 1 | 1 | 2 | 2 |
| 5 | 1 | 2 | 2 | 2 | 3 | 3 |
| 6 | 1 | 2 | 3 | 3 | 1 | 1 |
| 7 | 1 | 3 | 1 | 2 | 1 | 3 |
| 8 | 1 | 3 | 2 | 3 | 2 | 1 |
| 9 | 1 | 3 | 3 | 1 | 3 | 2 |
| 10 | 2 | 1 | 1 | 3 | 3 | 2 |
| 11 | 2 | 1 | 2 | 1 | 1 | 3 |
| 12 | 2 | 1 | 3 | 2 | 2 | 1 |
| 13 | 2 | 2 | 1 | 2 | 3 | 1 |
| 14 | 2 | 2 | 2 | 3 | 1 | 2 |
| 15 | 2 | 2 | 3 | 1 | 2 | 3 |
| 16 | 2 | 3 | 1 | 3 | 2 | 3 |
| 17 | 2 | 3 | 2 | 1 | 3 | 1 |
| 18 | 2 | 3 | 3 | 2 | 1 | 2 |

2.3 Experimental results

Based on the experimental layout depicted in Table 2, the experiments were performed in random order and each specific experiment was repeated two times. Two machining characteristics namely material removal rate (MRR) and surface roughness (SR) were measured. MRR was measured in mm/min. which was observed from machine tool monitor screen. SR value (in μm) was measured in terms of mean absolute deviation (Ra) using the digital surface tester Mitutoyo 201P. Observed machining characteristics are depicted in Table 3.

**Table 3**
Mean values and S/N ratios of observed results

| Experiment. No. | Trial No. | MRR (mm/min.) | $(S/N\ ratio)_{MRR}$ | SR (μm) | $(S/N\ ratio)_{SR}$ |
|---|---|---|---|---|---|
| 1 | 3 | 1.92 | 5.6646 | 1.43 | -3.1203 |
| 2 | 8 | 2.575 | 8.1938 | 2.025 | -6.1306 |
| 3 | 2 | 2.04 | 6.1773 | 1.91 | -5.6236 |
| 4 | 7 | 1.91 | 5.6117 | 1.27 | -2.0892 |
| 5 | 1 | 2.48 | 7.8755 | 1.805 | -5.1303 |
| 6 | 9 | 1.895 | 5.5369 | 1.88 | -5.4842 |
| 7 | 5 | 1.79 | 4.9394 | 1.125 | -1.0375 |
| 8 | 4 | 1.65 | 4.3324 | 1.43 | -3.1120 |
| 9 | 6 | 2.61 | 8.3297 | 2.565 | -8.1890 |
| 10 | 15 | 0.925 | -0.6958 | 1.195 | -1.5565 |
| 11 | 13 | 1.705 | 4.6209 | 2.015 | -6.0868 |
| 12 | 16 | 1.95 | 5.7427 | 2.89 | -9.2184 |
| 13 | 11 | 1.35 | 2.5888 | 1.505 | -3.5546 |
| 14 | 17 | 1.415 | 2.9681 | 2.085 | -6.3851 |
| 15 | 12 | 1.795 | 5.0763 | 2.405 | -7.6277 |
| 16 | 14 | 0.99 | -0.0993 | 1.1 | -0.8636 |
| 17 | 18 | 1.795 | 5.0731 | 2.235 | -6.9934 |
| 18 | 10 | 1.99 | 5.9504 | 2.505 | -7.98377 |
| Average | | 1.821 | | 1.854 | |

## 3. Optimization of individual machining characteristics

In Taguchi method, the basic method converts the objective parameters to signal-to-noise (S/N) ratio treated as the quality characteristics evaluation index. The least variation and the optimal design are obtained by means of the S/N ratio. The higher the S/N ratio, the more stable the achievable quality. Depending on the required objective characteristics, there are three types of S/N ratio- the lower-the-better, the higher-the-better and the nominal-the-better. In present work, two types of S/N ratio has been used; Higher-the-better for MRR and lower-the-better for SR.

The S/N ratio with a higher-the-better characteristic that can be expressed as follows,

$$\eta_{ij} = -10\log\left\{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_{ij}^2}\right\}. \tag{1}$$

The S/N ratio with a lower-the-better characteristics can be expressed as follows,

$$\eta_{ij} = -10\log\left\{\frac{1}{n}\sum_{i=1}^{n} y_{ij}^2\right\}, \tag{2}$$

where $y_{ij}$ is the $i$th experiment at the $j$th test and $n$ is the total number of tests. Table 3 shows the S/N ratios of measured mean values of MRR and SR.

The response table using Taguchi method is employed here to calculate the effect of each level of process parameter on machining characteristics. It is done by sorting the mean values of machining characteristics corresponding to levels of the process parameter in each column of the orthogonal array, and taking an average on those with same level. For example, the average effect on MRR for parameters A and B at level 1 can be calculated as follows:

$A_1 = (1.92 + 2.575 + 2.04 + 1.91 + 2.48 + 1.895 + 1.79 + 1.65 + 2.61)/9 = 2.097$,
$B_1 = (1.92 + 2.575 + 2.04 + 0.925 + 1.705 + 1.95)/6 = 1.853$.

Using the same method, calculations were performed for each process parameters level and the response Tables were generated for MRR and SR as shown in Table 4.

**Table 4**
Response for mean cutting speed and surface roughness

| | Mean cutting speed | | | | | | | Mean surface roughness | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Level | A | B | C | D | E | F | Level | A | B | C | D | E | F |
| 1 | 2.097 | 1.853 | 1.481 | 1.956 | 1.786 | 1.76 | 1 | 1.716 | 1.911 | 1.271 | 1.987 | 1.840 | 1.895 |
| 2 | 1.546 | 1.808 | 1.937 | 2.023 | 1.812 | 1.904 | 2 | 1.993 | 1.825 | 1.933 | 1.976 | 1.853 | 1.941 |
| 3 | --- | 1.804 | 2.047 | 1.486 | 1.867 | 1.80 | 3 | ---- | 1.827 | 2.359 | 1.60 | 1.869 | 1.727 |

Fig. 2 and Fig. 3 show the S/N ratio plots for MRR and SR. The optimum parameters combination for MRR and SR are $A_1B_1C_3D_2E_1F_2$ and $A_1B_3C_1D_3E_2F_3$ corresponding to the largest values of S/N ratio for all control parameters.

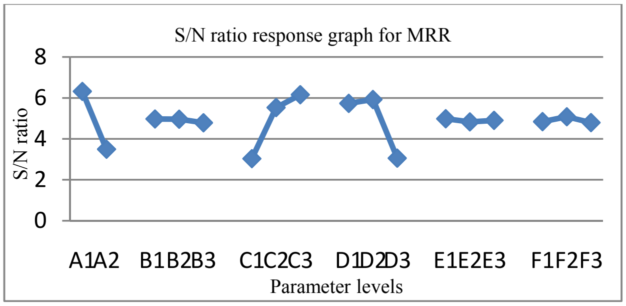


**Fig. 2.** S/N ratio response graph for material removal rate

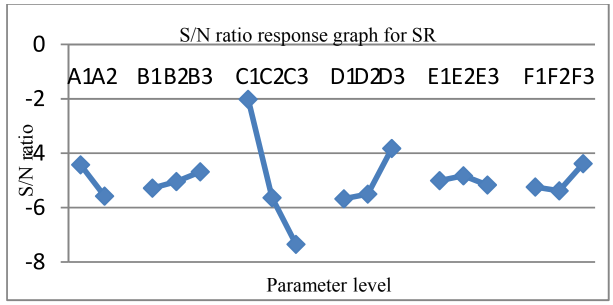


**Fig. 3.** S/N ratio response graph for surface roughness

### *3.1 Predicted optimal results*

In order to predict the optimal values of the machining characteristics, only significant parameters are included which were found utilizing analysis of variance (ANOVA). The optimal values are predicted using the following relationship,

$$\eta_{opt} = \eta_m + \sum_{i=1}^{q} (\eta_i - \eta_m), \tag{3}$$

where $\eta_m$ is the total mean of the machining characteristic under consideration; $\eta_i$ is the mean values at the optimum level (from response Tables) and q is the number of process parameters that significantly affects the machining characteristics.

**Table 5**
ANOVA for mean MRR

| Source | DF | Sum of square | Variance | F | *p*-value | % contribution |
|---|---|---|---|---|---|---|
| A | 1 | 1.36400 | 1.36400 | 81.63 | 0.000* | 54.95 |
| B | 2 | 0.00874 | 0.00437 | 0.26 | 0.778 | 0.17 |
| C | 2 | 1.08010 | 0.54005 | 32.32 | 0.001* | 21.76 |
| D | 2 | 1.02671 | 0.51336 | 30.72 | 0.001* | 20.68 |
| E | 2 | 0.02045 | 0.01023 | 0.61 | 0.573 | 0.0041 |
| F | 2 | 0.06647 | 0.03323 | 1.99 | 0.217 | 1.34 |
| Error | 6 | 0.10026 | 0.01671 | | | 0.67 |
| Total | 17 | 3.66674 | | | | |

DF: degree of freedom; * significant factor;

**Table 6**
ANOVA for mean SR

| Source | DF | Sum of square | Variance | F | *p*-value | % contribution |
|---|---|---|---|---|---|---|
| A | 1 | 0.34583 | 0.34583 | 10.25 | 0.019* | 13.47 |
| B | 2 | 0.02891 | 0.01445 | 0.43 | 0.670 | 0.56 |
| C | 2 | 3.60863 | 1.80432 | 53.47 | 0.000* | 70.29 |
| D | 2 | 0.58176 | 0.29088 | 8.62 | 0.017* | 11.33 |
| E | 2 | 0.00256 | 0.00128 | 0.04 | 0.963 | 0.05 |
| F | 2 | 0.15261 | 0.07630 | 2.26 | 0.185 | 2.90 |
| Error | 6 | 0.20246 | 0.03374 | | | 1.31 |
| Total | 17 | 4.92276 | | | | |

DF: degree of freedom; * significant factor;

Table 5 and 6 depict the ANOVA for MRR and SR, respectively. ANOVA depicts that three process parameters namely taper angle (A), pulse-on time (C) and Pulse-off time (D) are the most significant (since *p-value* ≤ 0.05) parameters affecting the MRR and SR under 95% confidence level. Peak current shows the least contribution. Insignificance of dielectric flow rate and wire tension may be due to the low thickness of workpiece (13mm). In addition, the gap between work surface and upper wire nozzle is very low (1.5mm), which resists the wire electrode vibration even at low wire tension. Since only three parameters (A, C and D) are the most significant factors affecting the MRR, therefore, only these parameters are used to predict the optimal value of MRR. Using Eq. (3), the optimum value is calculated as follows,

$$\eta_{opt} = \eta_m + \sum_{i=1}^{3} (\eta_i - \eta_m) = 1.821 + (2.097 - 1.8210) + (2.047 - 1.821) + (2.023 - 1.821) = 2.52 \text{ mm/min.}$$

Similarly, optimal value for SR is predicted. Confirmatory experiments were conducted for MRR and SR corresponding to their optimal setting of process parameters to validate the used approach. Table 7 displays the predicted and experimental values of MRR and SR.

**Table 7**

Optimal values of individual machining characteristics

| Machining Characteristic | Optimal parameters combination | Significant parameters (at 95% confidence level) | Predicted optimal value | Experimental value |
|---|---|---|---|---|
| MRR | $A_1B_1C_3D_2E_1F_2$ | A, C, D | 2.52 mm/min. | 2.60 mm/min. |
| SR | $A_1B_3C_1D_3E_2F_3$ | A, C, D | 0.88 µm | 0.85 µm |

## 4.Multi-machining characteristics optimization using grey relational analysis

In order to optimize the MRR and SR simultaneously using grey relational analysis (GRA), the following steps were followed:

- Convert the experimental data into S/N values,
- Normalize the S/N ratio,
- Perform the grey relational generating and calculate the grey relational coefficient,
- Calculate the grey relational grade by using the weighing factor for the performance characteristics,
- Analyse the experimental results using the grey relational grade and statistical analysis of variance (ANOVA),
- Select the optimal levels of process parameters,
- Conduct the confirmation experiment to verify the optimal process parameter settings.

### *4.1 Grey relational analysis*

Grey data processing must be performed before calculating the grey correlation coefficients. In this study, a linear normalization of the experimental results (S/N ratios) for MRR and SR were performed in the range of 0 and 1, which is also called the grey relational generating. A linear data pre-processing method for the S/N ratio can be expressed as follows,

$$x_i^*(k) = \frac{x_i^0(k) - \min x_i^0(k)}{\max x_i^0(k) - \min x_i^0(k)}, \tag{4}$$

where $x_i^*(k)$ is the sequence after the data processing; $x_i^o(k)$ is the original sequence of S/N ratio, $i$ = 1, 2, 3…., $m$ and $k$ =1, 2,…., $n$ *with* $m$=18 and $n$=2; max $x_i^o(k)$ is the largest value of $x_i^o(k)$; min $x_i^o(k)$ is the smallest value of $x_i^o(k)$. Table 8 shows the normalized S/N ratio for the MRR and SR. The outcomes are denoted as $x^*_0(k)$ and $x^*_i(k)$ for reference sequence and comparability sequence, respectively. Basically, the larger normalized S/N ratio corresponds to the better performance and the best-normalized S/N ratio is equal to unity.

**Table 8**
The sequence after data pre-processing

| No. | MRR | SR |
|---|---|---|
| Reference sequence | 1.0000 | 1.0000 |
| Comparability sequence | | |
| 1 | 0.7047 | 0.7299 |
| 2 | 0.9849 | 0.3696 |
| 3 | 0.7615 | 0.4303 |
| 4 | 0.6989 | 0.8533 |
| 5 | 0.9497 | 0.4893 |
| 6 | 0.6906 | 0.4470 |
| 7 | 0.6244 | 0.9792 |
| 8 | 0.5571 | 0.7309 |
| 9 | 1.0000 | 0.1232 |
| 10 | 0.0000 | 0.9171 |
| 11 | 0.5891 | 0.3748 |
| 12 | 0.7134 | 0.0000 |
| 13 | 0.3639 | 0.6779 |
| 14 | 0.4059 | 0.3391 |
| 15 | 0.6395 | 0.1904 |
| 16 | 0.0661 | 1.0000 |
| 17 | 0.6392 | 0.2663 |
| 18 | 0.7364 | 0.1478 |

Next, the grey relational coefficient was calculated to express the relationship between the best (reference) and the actual normalized S/N ratio. The grey relational coefficient is expressed as follows,

$$\gamma(x_o^*(k).x_i^*(k)) = \frac{\Delta_{min} + \zeta.\Delta_{max}}{\Delta_{oi}(k)+\zeta.\Delta_{max}} \qquad 0\ \gamma\big(x_o^*(k).x_i^*(k)\big) \leq 1 \tag{5}$$

where $\Delta_{oi}\ (k)$ is the deviation sequence of reference sequence $x_o^*(k)$ and comparability sequence $x_i^*(k)$, i.e. $\Delta_{oi}\ (k)$ = $|\ x_o^*(k) - x_i^*(k)\ |$ is the absolute value of the difference between $x_o^*(k)$ and $x_i^*(k)$,
$\Delta_{min}$= min. min. $\Delta_{oi}\ (k)$,
$\Delta_{max}$= max. max. $\Delta_{oi}\ (k)$,
$\zeta$ is the distinguishing coefficients $\zeta \in |0, 1|$. $\zeta$ is set as 0.5 in this study. The purpose of defining this coefficient is to show the relational degree between the reference sequences $x_o^*(k)$ and the comparability of 18 sequences $x_i^*(k)$. where *i* = 1, 2, 3…., *m* and *k* =1, 2,…., *n* with *m*=18 and *n*=2 in this study. Using Table 9, the deviation sequence $\Delta_{01}$ can be calculated as follows:
$\Delta_{o1}(1) =|\ 1.0000 - 0.7047\ |=\ 0.2953,$
$\Delta_{o1}(2) =|\ 1.0000\ - 0.7299\ |=\ 0.2701,$
Therefore, $\Delta_{o1}$= $(0.2953, 0.2701)$.
The same calculating method was performed for *i* =1-18, and the results of all $\Delta_{oi}$ for *i* =1-18 are listed in Table 8. Investigating the data presented in Table 9, we can find that $\Delta_{max}(k)$ and $\Delta_{min}(k)$ are as follows:
$\Delta_{max} =\ \Delta_{o10}\ (1) = \Delta_{o12}\ (2) = 1.000,$
$\Delta_{min} =\ \Delta_{o9}\ (1) = \Delta_{o16}\ (2) =\ 0.000,$
According to Table 9 and Eq. (5), the grey relational coefficient $\gamma\big(x_o^*(k).x_i^*(k)\big)$ can be calculated as follows:

$$\gamma\big(x_o^*(1).x_1^*(1)\big) = \frac{0.0000\ +\ 0.5\ \times 1.0000}{0.2953 +\ 0.5\ \times 1.0000} = 0.6287,$$

$$\gamma\big(x_o^*(2).x_1^*(2)\big) = \frac{0.0000\ +\ 0.5\ \times 1.0000}{0.2701 +\ 0.5\ \times 1.0000} = 0.6493,$$

thus $\gamma\big(x_o^*(k).x_1^*(k)\big)$ = (0.6287, 0.6493), *k*=1-2.

**Table 9**
The deviation sequences

| Deviation sequences | $\Delta_{o1}(1)$ | $\Delta_{o1}(2)$ |
|---|---|---|
| No. 1, $i$ =1 | 0.2953 | 0.2701 |
| No. 2, $i$ =2 | 0.0151 | 0.6304 |
| No. 3, $i$ =3 | 0.2385 | 0.5697 |
| No. 4, $i$ =4 | 0.3011 | 0.1467 |
| No. 5, $i$ =5 | 0.0503 | 0.5107 |
| No. 6, $i$ =6 | 0.3094 | 0.5530 |
| No. 7, $i$ =7 | 0.3756 | 0.0208 |
| No. 8, $i$ =8 | 0.4429 | 0.2691 |
| No. 9, $i$ =9 | 0.0000 | 0.8768 |
| No. 10, $i$ =10 | 1.0000 | 0.0829 |
| No. 11, $i$ =11 | 0.4109 | 0.6252 |
| No. 12, $i$ =12 | 0.2866 | 1.0000 |
| No. 13, $i$ =13 | 0.6361 | 0.3221 |
| No. 14, $i$ =14 | 0.5941 | 0.6609 |
| No. 15, $i$ =15 | 0.3605 | 0.8096 |
| No. 16, $i$ =16 | 0.9339 | 0.0000 |
| No. 17, $i$ =17 | 0.3608 | 0.7337 |
| No. 18, $i$ =18 | 0.2636 | 0.8522 |

Similar procedure is applied for $i$ = 1-18 and the results are summarized in Table 10.

**Table 10**
The calculated grey relational coefficient for 18 comparability sequences

| No. (Comparability sequence) | MRR | SR | Grey relation grade |
|---|---|---|---|
| 1 | 0.6287 | 0.6493 | 0.6390 |
| 2 | 0.9708 | 0.4423 | 0.7065 |
| 3 | 0.6771 | 0.4674 | 0.5722 |
| 4 | 0.6241 | 0.7732 | 0.6986 |
| 5 | 0.9086 | 0.4947 | 0.7016 |
| 6 | 0.6177 | 0.4748 | 0.5463 |
| 7 | 0.5710 | 0.9600 | 0.7655 |
| 8 | 0.5303 | 0.6501 | 0.5902 |
| 9 | 1.0000 | 0.3632 | 0.6816 |
| 10 | 0.3333 | 0.8577 | 0.5955 |
| 11 | 0.5489 | 0.4444 | 0.4966 |
| 12 | 0.6356 | 0.3333 | 0.4845 |
| 13 | 0.4401 | 0.6082 | 0.5242 |
| 14 | 0.4570 | 0.4307 | 0.4439 |
| 15 | 0.5811 | 0.3818 | 0.4814 |
| 16 | 0.3487 | 1.0000 | 0.6743 |
| 17 | 0.5808 | 0.4053 | 0.4931 |
| 18 | 0.6548 | 0.3698 | 0.5123 |

The grey relational grade is a weighting-sum of the grey relational coefficients. The overall evaluation of multiple performance characteristics is based on the grey relational grade and it is defined as follows,

$$\Gamma(x_o^*.x_i^*) = \sum_{k=1}^{n} \beta_k \, \gamma(x_o^*(k).x_i^*(k)) \tag{6}$$

where $\beta_k$ represents the weighting value of the k$^{th}$ performance characteristics, and $\sum_{k=1}^{n} \beta_k = 1$. Using the same weighting values of MRR and SR as were assigned in utility analysis (i.e.$w_1$=$w_2$=0.5), grey relational grade $\Gamma(x_o^*.x_i^*)$ is calculated as depicted in Table 10.

### *4.2 Optimal level of process parameters*

Optimization of the multiple performance characteristics can be converted into optimization of single grey relational grade. It is clearly observed from Table 10 for grey relational grade, the process parameters setting of experiment no.7 has the highest grey relational grade. Thus, the seventh experiment gives the best multiple performance characteristics among the 18 experiments using GRA. To separate out the effect of each process variable on the grey relational grade at different levels, response graph for grey relational grade is constructed using the Taguchi methodology as shown in Fig. 4.

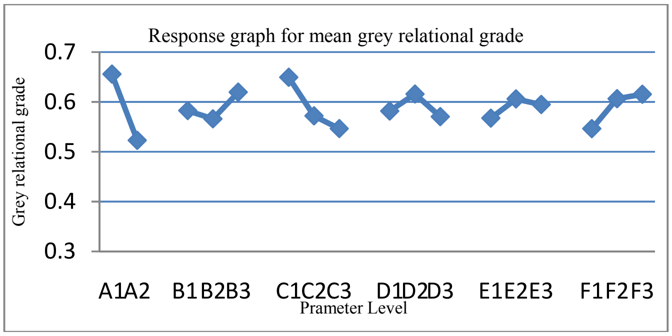


**Fig. 4.** Response graph for mean grey relational grade

Basically, the larger the grey relational grade, the better is the multiple performance characteristics. The combination of $A_1B_3C_1D_2E_2F_3$ shows larger value of the grey relational grade for the factors A, B, C, D, E and F, respectively. Therefore, $A_1(3^0)$, $B_3$(120 ampere), $C_1$(108µs), $D_2$(40µs), $E_2$(8N) and $F_3(10LM^{-1})$ is the optimal parameter combination for multi-machining characteristics.

*4.3 Predicted optimal results*

The optimal value of the machining characteristics has been predicted using the same procedure as discussed in previous section. ANOVA results given in Table 11 depict that the taper angle (A) and pulse-on time (C) are the most significant factors affecting the grade values under 95% confidence level (because $p \leq 0.05$), while dielectric flow rate (F) affects it under 90% confidence level. Therefore, only most significant process parameters i.e. A and C have been considered to predict the optimal values of machining characteristics using Eq. (3).

**Table 11**

ANOVA for grey relational grade

| Source | DF | Sum of square | Variance | F | *p*-value | % contribution |
|---|---|---|---|---|---|---|
| A | 1 | 0.07944 | 0.07944 | 47.22 | 0.00* | 67.82 |
| B | 2 | 0.00901 | 0.00451 | 2.68 | 0.147 | 3.85 |
| C | 2 | 0.03462 | 0.01731 | 10.29 | 0.012* | 14.77 |
| D | 2 | 0.00669 | 0.00334 | 1.99 | 0.218 | 2.85 |
| E | 2 | 0.00475 | 0.00237 | 1.41 | 0.314 | 2.02 |
| F | 2 | 0.01696 | 0.00848 | 5.04 | 0.052 | 7.24 |
| Error | 6 | 0.01009 | 0.00168 | | | 1.43 |
| Total | 17 | 0.16157 | | | | |

DF: degree of freedom; * Significant at 95% confidence level

The percentage error between experimental values and predicted values for MRR and SR using GRA are 2.2 and 0.35, respectively. Therefore, GRA process parameters can be successfully optimized for multiple machining characteristics during WEDM of WC-6%Co composite. Table 12 shows the predicted and experimental results for MRR and SR at a single optimal setting of process parameters using GRA.

**Table 12**

Predicted and confirmatory values of machining characteristics at single optimal setting

| Machining Characteristic | Optimal parameters combination | Predicted optimal value | Experimental value | % error |
|---|---|---|---|---|
| MRR | $A_1B_3C_1D_2E_2F_3$ | 1.76mm/min. | 1.80mm/min. | 2.2 |
| SR | $A_1B_3C_1D_2E_2F_3$ | 1.13µm | 1.126µm | 0.35 |

## 5. Summary of results

Using Taguchi method, process parameters were optimized individually for MRR and SR. The percentage error between experimental values and predicted results are less than 4% for both machining characteristics. Therefore, process parameters are successfully optimized for individual characteristics using Taguchi method. The optimal setting of process parameters for multiple machining characteristics, using GRA is $A_1$ $B_3$ $C_1$ $D_2$ $E_2$ $F_3$. Using ANOVA, two process parameters namely taper angle (A), pulse-on time (C) were found significant affecting the grey relational grade, significantly. The percentage error between experimental values and predicted values for MRR and SR using GRA are 2.2 and 0.35 respectively. Therefore, using GRA, process parameters can be successfully optimized for multiple machining characteristics during WEDM of WC-6%Co composite. Table 13 summarizes the results for individual and multiple machining characteristics.

**Table 13**
Summary and comparison of results

| Method | Optimization technique | Optimal parameters combination | Predicted optimal value |
|---|---|---|---|
| Individual characteristics optimization | Taguchi Method | $A_1$ $B_1$ $C_3$ $D_2$ $E_1$ $F_3$ (MRR)<br>$A_1$ $B_3$ $C_1$ $D_2$ $E_2$ $F_3$ (SR) | MRR= 2.52m/min.<br>SR= 0.88μm |
| Multiple characteristic optimization | Grey relational analysis | $A_1$ $B_3$ $C_1$ $D_2$ $E_2$ $F_3$ | MRR=1.76mm/min<br>SR= 1.13μm |

## 6. Conclusions

In present work, wire electrical discharge machining (WEDM) for WC-Co composite has been studied. Grey relational analysis (GRA), along with Taguchi method were used to optimize the material removal rate (MRR) and surface roughness (SR), simultaneously. Based on the results and discussions, the following conclusions are made:

- Using Taguchi method, MRR and SR were optimized individually. Two different optimal settings of process parameters were found for MRR and SR. The optimal predicted values for MRR and SR are 2.52mm/min. and 0.88μm. Using ANOVA on experimental results, three process parameters namely taper angle (A), pulse-on time (C) and pulse-off time (D) were found the most significant affecting the MRR and SR under 95% confidence level.
- In case of GRA, grey relational grade was used as a performance index to determine the optimal combination of process parameters for multiple machining characteristics. Equal weights were assigned to both the machining characteristics in calculating the grey relational grade. However, with a different set of weights, a different set of optimal parameters for machining characteristics will result. The optimal set predicted will be closer to the optimal set predicted for single characteristic with the largest weight.
- Using ANOVA, only two parameters namely taper angle (A) and pulse-on time (C) were affecting the grey relational grade. The percentage error between experimental values and predicted values for MRR and SR using GRA are 2.2 and 0.35, respectively. Therefore, using GRA, process parameters can be successfully optimized for multiple machining characteristics during WEDM of WC-6%Co. The optimal combination of the process parameters, using GRA for multi-machining characteristics is set to $A_1$ ($3^0$), $B_3$ (120 ampere), $C_1$ (108μs), $D_2$ (40μs), $E_2$ (8N) and $F_3$ (10 $LM^{-1}$).
- GRA can be extended to more number of machining characteristics, provided accurate weights for different characteristics to calculate grade values. Thus, the solutions from this method will be useful for tool manufacturer who are willing to search for an optimal solution of process parameters.

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