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Pricing for freight carriers in a competitive environment: A game theory approach

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A B S T R A C T

In recent years, development of freight transport industry has led to fierce competition among transportation companies and therefore carrier-pricing issue has received more attention by researchers**.** This paper studies pricing and fleet management decisions for full-truckload freight carriers, which compete on a road network. We propose a game theory approach under two scenarios. In the first, we model the non-cooperative game wherein the carriers announce their prices simultaneously in competition; in the second, we allow the carriers to share their information and announce their prices while participating in cooperation. We show that carriers can reach the highest profit level in the latter scenario; subsequently, a bargaining game is discussed as a scheme to share the extra joint profit.

1. Introduction

A transportation network consists of origin and destination locations including manufacturing facilities, warehouses, distribution centers, wholesales, and retail outlets, wherein each lane connects a source location to a destination location. Freight carriers provide transportation service for shippers, which are usually transportation management departments. In other words, freight carriers are profitable companies whose input is the demand of shippers and their output are transportation services (Harker, 1985).

Today, pricing policies, as a main component of activities in manufacturing and service companies, have received a great attention by managers. The reason is that “price” is one of the most effective variables through which managers can encourage or discourage demand in a short period. In the literature, two main approaches are introduced for pricing: posted-price mechanism and price-discovery mechanism. In posted-price mechanism, the seller determines the prices and announces them to the market. The goods and commodities are then sold according to announced prices. In price-discovery mechanism, the prices are offered through a suggestion process such as auction or bidding (Elmaghraby & Keskinocak, 2003).

This paper focuses on road transportation freight carriers. According to the American Trucking Association (ATA) report in 2005, the industry has 68.9% share of the total volume of freight transported in U.S. Road transport industry produced 623$ billion annual revenue by conveying 10.7 billion tons in 2005, which was 84.3% of the nation’s freight income. Road freight carriers are classified into two major sectors: Full truckload (FTL) and less than truckload (LTL). In full

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truckload sector addressed in this paper, competition is fierce due to relatively low capital entry requirements. Therefore, offering lower prices may increase the demand of carriers over different lanes. However, proper prices are obtained by considering the downstream effects of fleet repositioning along with profit maximizing problem (Topaloglu & Powell, 2007).

There are few researches in the literature having investigated the pricing issue for full truckload carriers. King and Topaloglu (2007) studied the pricing problem for a FTL carrier based on posted-price mechanism. Their model incorporated the fleet management decisions by considering vehicle repositioning in load pricing problem. Topaloglu and Powell (2007) extended the King and Topaloglu's work by proposing a stochastic approach for searching the best prices. Although the models presented the above works are able to consider the fleet capacity, but they ignore the fierce competition between carriers, which can make their results unreliable. Figliozzi et al. (2007) addressed the dynamic pricing problem for FTL carriers based on an auction mechanism. Their model considers the environment competition effects through offering different competitive options for the carriers. They applied a sequential costing and pricing approach to find the best solutions.

On the other hand, intense competition, low profit margins and high operating costs of carriers in FTL industry, have excluded many small transportation companies. As a result, companies tend to merge with each other to take advantage of scale savings, operational cost reductions and order density increments with the goal of surviving in the market. A more effective option than merging is cooperation among competitor carriers. Through cooperation, freight carriers are able to integrate and coordinate their planning and decision making processes and therefore utilize the full capacity of their fleets. However, at each level of cooperation, there are some potential benefits for participants, but increasing the degree of cooperation often leads to many challenges. In other words, although all participants in the cooperation are moving towards a common goal, but each is also guided according to its own interests. Therefore, any mechanism for managing the cooperation among different companies has to provide the desirability of individual members as well as collective desirability of the cooperation (Ozener, 2008).

Cooperation among different decision makers of a freight transportation network has been studied under the title of "collaborative logistics" with the goal of increasing efficiency and reducing the hidden costs (Ergun et al., 2007). Ozener and Ergun (2008) studied cooperation opportunities in a road freight transport network. They investigated the horizontal cooperation with full information among shippers to reduce costs of transportation through a carrier. Ozener et al. (2010) addressed the issue of cooperation among freight carriers and proposed a mechanism based on bilateral order exchange using a game theoretic approach to manage the cooperation. Hoghtalen et al. (2007) examined cooperation among airline carriers. He proposed a mechanism based on capacity exchange prices to manage carriers' behavior in the cooperation. The results showed that the benefits of cooperation among carriers increase by the network capacity and fleet size. Agarwal and Ergun (2010) studied the cooperation of marine freight carriers from a decentralized perspective. In addition to short-term capacity allocation between carriers, long-term decisions on designing the networks are also examined. The focus of all above articles is on cost reduction, but the effects of pricing decisions and competitive market are neglected.

To our best knowledge, previous works on carrier pricing in FTL sector have focused on the problem of one carrier and they have not considered the effect of competitors' decisions. The only paper that has tried to propose competitive options is based on the auction mechanism (Figliozzi et al. ,2007). In this paper, we study pricing decisions for FTL carriers, which compete on a common road network. The posted-price mechanism is employed and the fleet decisions, which are most critical for FTL carriers, are considered along with pricing. In order to model the carriers' competition, we use a non-cooperative game theory approach with simultaneous moves. In addition, we study the cooperation among FTL carriers with the goal of identifying and utilizing the synergies among rival carriers using a cooperative game approach and a bargaining scheme is presented to share the extra joint profit among the participants.

The rest of this paper is organized as follows. In Section 2, the problem of interest is described and formulated as a generalized Nash equilibrium problem (GNEP). Then a relaxed GNEP is proposed as an approximation approach and a solution approach for the non-cooperative game problem is developed. Section 3 models the cooperation among freight carriers and presents a bargaining scheme to share the extra joint benefit. Section 4 provides numerical experiments and result analysis. Finally, Section 5 is devoted to concluding remarks.

2. Non-cooperative game model

Consider two rival freight carriers service over a common road network. Each node of this network represents a location center, which can be the origin or destination of a freight transportation order and the arcs of the network represent the fleet movement lanes. The carriers compete over this network to capture the origin-destination demands for freight delivery services from a shipper to a receiver. They have potentially different costs for servicing over an arc due to their different equipment technology and their orders density. We propose a game theoretic approach to model the pricing behavior of the freight carriers in such a network. Carriers are allowed to set the price of transportation services over different arcs of the network in order to maximize their profits. The demand of each carrier is assumed as a function of its own prices as well as the prices of the rival firm. When a freight transportation order is received, the carrier has to decide whether to respond this order or not, with regard to related costs including fleet hauling cost and fleet repositioning cost. Repositioning or deadheading refers to empty truck movement decreases the capacity utilization of a carrier and therefore increases its operational costs. The demand rejected by a carrier is assumed as lost sales. Since the customers are not strategic, we suppose that lost sales do not cause any cost. We assume that both carriers are Nash agents in such economy and they are profit optimizers with pricing power.

2.1. Notations

The following parameter and variable definitions are used in this paper.

Parameters:

l : set of the existing locations in transportation network
A : set of the existing arcs in transportation network
V: set of the existing carriers service on the network, $V = \{v_1, v_2\}$
c_{ij}^{v}: cost of service on arc *ij* for carrier *v*
θ^{v}: deadheading cost coefficient
D_{ij}^{v}: amount of potential demand for carrier *v* over arc *ij*
α^{v}: self- price sensitivity coefficient of demand for carrier *v*
β^{v}: rival-price sensitivity coefficient of demand for carrier *v*

Variables:

p_{ij}^{v}: price charged by carrier *v* for transporting goods over arc *ij*
p_{ij}^{-v}: price charged by the rival carrier for transporting goods over arc *ij*
y_{ij}^{v}: number of transportation services served by carrier *v* over arc *ij*
z_{ij}^{v}: number of vehicle repositions of carrier *v* over arc *ij*

We assume that transportation demand is a linear function of carrier's price p_{ij}^{v} and the rival's price p_{ij}^{-v} . Thus, the demand for carrier *v* over arc *ij* is defined as follows.

$$d_{ij}^{v}(p_{ij}^{v}, p_{ij}^{-v}) = D_{ij}^{v} - \alpha^{v}.p_{ij}^{v} + \beta^{v}.p_{ij}^{-v}.$$

The parameter D_{ij}^{v} reflects a forecasted demand, which is not identical for the two carriers due to some distinctions such as local reputation, brand, service quality, etc. This potential demand is then reduced by the carrier's self price p_{ij}^{v} and is increased by the rival price p_{ij}^{-v} with elasticity coefficients α^v and β^v, respectively. Note that the coefficient of self-price sensitivity is greater than rival-price sensitivity is $0 \le \beta^v \le \alpha^v$. The proposed model uses a linear demand function since it is tractable and often satisfactorily fits to the given data set. Fig. 1 depicts a schema of a freight transportation network addressed in this paper. $d_{ij}^{v_1}$ reflects the demand of carrier v_1 on the arc ij, and $d_{ef}^{v_2}$ denotes the demand of carrier v_2 over the arc ef.

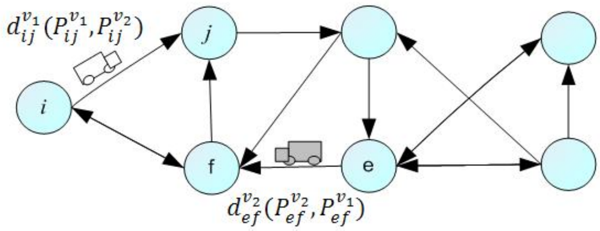

Fig.1. Network representation of the problem

2.2. Mathematical model

The mathematical model of carrier v is defined as follows,

$$Max\ \pi_{v \in V} = \sum_{ij \in A} (p_{ij}^v - c_{ij}^v).y_{ij}^v - \theta^v . \sum_{ij \in A} c_{ij}^v .z_{ij}^v \tag{1}$$

subject to

$$\sum_{j \in l} y_{ji}^v + \sum_{j \in l} z_{ji}^v - \sum_{j \in l} y_{ij}^v - \sum_{j \in l} z_{ij}^v = 0\,, \qquad \forall i \in l \tag{2}$$

$$y_{ij}^v - d_{ij}^v(p_{ij}^v, p_{ij}^{-v}) \le 0, \qquad \forall ij \in A \tag{3}$$

$$y_{ij}^v, z_{ij}^v \in Z_+,\ p_{ij}^v \in R_+ \qquad \forall ij \in A \tag{4}$$

Objective function (1) maximizes the total profit of carrier v. Constraint (2) ensures the flow balance. Constraint (3) guarantees that number of transportation services for carrier v over arc ij is at most equal to its demand on that arc. Finally, constraint (4) determines the domain of decision variables. The problem of interest is to find the optimal prices, the number of transportation services, and the number of vehicle repositions over different arcs of the road network in equilibrium point of the Nash game. Constraint (3) is a joint constraint, which models the interaction among the rival carriers. This constraint causes the game to take the form of generalized Nash equilibrium problem (GNEP) (Facchinei & Kanzow, 2007).

2.3. Solution algorithm

In this section, we propose an algorithm to solve the mixed integer GNEP. The algorithm we use is the modified Nonlinear Gauss–Seidel method presented by Facchinei et al. (2007). Nonlinear Gauss–Seidel method is a decomposition approach using the grasp logic. When the algorithm converges, the solution of GNEP is obtained. The statement of this algorithm for our model is as follows:

Step1: Initialization

Set $x^0 = (p^{0,v_1}, y^{0,v_1}, z^{0,v_1}, p^{0,v_2}, y^{0,v_2}, z^{0,v_2})$ as an initial feasible solution for the game. Let $k := 0$ (k represents the iteration counter), and set τ^k such that $\tau^k \ge 0$.

Step2: Computation

Consider the price of player 2 (carrier 2) as a constant equals to p^{k,v_2} and compute the optimal strategies for player 1 (carrier 1), $x^{k+1,v_1*} = (p^{k+1,v_1*}, y^{k+1,v_1*}, z^{k+1,v_1*})$ by solving the following model,

$$\max \pi_{v_1}(p^{v_1}, y^{v_1}, z^{v_1}, p^{k,v_2}) + \tau^k.\left\| p^{v_1} - p^{k,v_1} \right\|^2 + \tau^k.\left\| y^{v_1} - y^{k,v_1} \right\|^2 + \tau^k.\left\| z^{v_1} - z^{k,v_1} \right\|^2 \tag{11}$$

subject to $(p^{v_1}, y^{v_1}, z^{v_1}) \in X_{v_1}(p^{v_1}, y^{v_1}, z^{v_1}, p^{k,v_2})$.

Step3: Adaption

Consider the price of player 1 (carrier 1) as a constant equals to p^{k+1,v_1}; then compute the optimal strategies of player 2 (carrier 2), $x^{k+1,v_1*} = (p^{k+1,v_1*}, y^{k+1,v_1*}, z^{k+1,v_1*})$ by solving the following model,

$$\max \pi_{v_2}(p^{v_2}, y^{v_2}, z^{v_2}, p^{k+1,v_1}) + \tau^k.\left\| p^{v_2} - p^{k,v_2} \right\|^2 + \tau^k.\left\| y^{v_2} - y^{k,v_2} \right\|^2 + \tau^k.\left\| z^{v_2} - z^{k,v_2} \right\|^2 \tag{12}$$

subject to $(p^{v_2}, y^{v_2}, z^{v_2}) \in X_{v_2}(p^{v_2}, y^{v_2}, z^{v_2}, p^{k+1,v_1})$.

Step4: Convergence verification

If $\left| p^{k+1,v_1} - p^{k,v_1} \right| \le \varepsilon, \left| y^{k+1,v_1} - y^{k,v_1} \right| \le \varepsilon, \left| z^{k+1,v_1} - z^{k,v_1} \right| \le \varepsilon$ and $\left| p^{k+1,v_2} - p^{k,v_2} \right| \le \varepsilon$, $\left| y^{k+1,v_2} - y^{k,v_2} \right| \le \varepsilon$, $\left| z^{k+1,v_2} - z^{k,v_2} \right| \le \varepsilon$ for all $ij \in A$ and $\varepsilon > 0$ (a pre-specified tolerance), then stop; otherwise set $k = k + 1$ and go back to Step 2.

We coded the algorithm in GAMS and solved each subproblem using SBB solver for each iteration. The SBB solver works based on the Branch and Bound (B&B) algorithm, which is believed to be an efficient algorithm to solve integer problems with a concave continuous relaxed form (Linderoth & Savelsbergh, 1999; Tawarmalani & Sahinidis, 2004).

2.4. Approximation method

As the players' models are mixed integer nonlinear programs, we use a relaxation method to approximate the equilibrium solutions. Through relaxation of integer constraints, a GNEP with continuous and differentiable space is achieved. Equilibrium conditions for such a game can be formulated as a quasi variational inequality problem (Harker & Pang, 1990). Equilibrium conditions are obtained by considering the optimality conditions of the system assuming that the two carriers are faced with the optimization problem (1)-(4). So the system–equilibrium condition of both carriers can be mathematically expressed as follows,

$$\begin{gathered}\sum_{v\in V}\{ \sum_{ij\in A}(y_{ij}^{v*}).(p_{ij}^{v} - p_{ij}^{v*}) + \sum_{ij\in A}(p_{ij}^{v*} - c_{ij}^{v}).(y_{ij}^{v} - y_{ij}^{v*}) - \sum_{ij\in A}\theta^{v}.c_{ij}^{v}(z_{ij}^{v} - z_{ij}^{v*})\} \le 0 \\ (y_{ij}^{v}, z_{ij}^{v}, p_{ij}^{v}) \in X_{v}(y_{ij}^{v}, z_{ij}^{v}, p_{ij}^{v}, p_{ij}^{-v}),\end{gathered} \tag{6}$$

Using the Karush–Kuhn–Tucker (KKT) conditions for player's optimization problems, a necessary condition for a point x to be a solution of the GNEP is that it satisfies, together with suitable multipliers, the mixed KKT system of the two players (Facchinei & Kanzow, 2007). In order to derive the mixed KKT system, the Lagrange function of each carrier can be expressed as:

$$\begin{aligned}L_{v}(p_{ij}^{v}, y_{ij}^{v}, z_{ij}^{v}; p_{ij}^{-v}; \lambda_i^{v}, \lambda_i'^{v}, \gamma_{ij}^{v}, \delta_{ij}^{v}, \delta_{ij}'^{v}, \delta_{ij}''^{v}) &= \sum_{ij\in A}\{(p_{ij}^{v} - c_{ij}^{v}).y_{ij}^{v} - \theta^{v}.c_{ij}^{v}.z_{ij}^{v}\} \\ &+ \sum_{i\in l}\lambda_i^{v}.(\sum_{j\in l}\{y_{ji}^{v} + z_{ji}^{v} - y_{ij}^{v} - z_{ij}^{v}\}) + \sum_{i\in l}\lambda_i'^{v}.(\sum_{j\in l}\{y_{ij}^{v} + z_{ij}^{v} - y_{ji}^{v} - z_{ji}^{v}\}) \\ &+ \sum_{ij\in A}\gamma_{ij}^{v}.(D_{ij}^{v} - \alpha^{v}.p_{ij}^{v} + \beta^{v}.p_{ij}^{-v} - y_{ij}^{v}) + \sum_{ij\in A}\delta_{ij}^{v}.p_{ij}^{v} + \sum_{ij\in A}\delta_{ij}'^{v}.y_{ij}^{v} + \sum_{ij\in A}\delta_{ij}''^{v}.z_{ij}^{v},\end{aligned} \tag{7}$$

where $\delta_{ij}''^{v}, \delta_{ij}'^{v}, \delta_{ij}^{v}, \gamma_{ij}^{v}, \lambda_i'^{v}, \lambda_i^{v}$ are the Lagrange multipliers respectively, associated with constraints (2)-(4). Since all of the constraints are linear, Slater's constraint qualification holds (Bazaraa et al., 1993) and we can inspect the KKT conditions of the mathematical program (1)-(4). The KKT stationary is as follows,

$$\nabla L_v(p_{ij}^v, y_{ij}^v, z_{ij}^v; p_{ij}^{-v}; \lambda_i^v, \lambda_i'^v, \gamma_{ij}^v, \delta_{ij}^v, \delta_{ij}'^v, \delta_{ij}''^v) = 0,$$

$$\begin{aligned} & y_{ij}^v - \alpha^v.\gamma_{ij}^v + \delta_{ij}^v = 0, \\ & p_{ij}^v - c_{ij}^v - \lambda_i^v + \lambda_i'^v + \lambda_j^v - \lambda_j'^v - \gamma_{ij}^v + \delta_{ij}'^v = 0, \\ & -\theta^v.c_{ij}^v - \lambda_i^v + \lambda_i'^v + \lambda_j^v - \lambda_j'^v + \delta_{ij}''^v = 0. \end{aligned} \tag{8}$$

The complementary slackness accompanying the primal and dual feasibility conditions are as follows,

$$\begin{aligned} & \lambda_i^v.(\sum_{j\in l} y_{ji}^v + \sum_{j\in l} z_{ji}^v - \sum_{j\in l} y_{ij}^v - \sum_{j\in l} z_{ij}^v) = 0, \\ & \lambda_i'^v.(\sum_{j\in l} y_{ij}^v + \sum_{j\in l} z_{ij}^v - \sum_{j\in l} y_{ji}^v - \sum_{j\in l} z_{ji}^v) = 0, \\ & \gamma_{ij}^v.(D_{ij}^v - \alpha^v.p_{ij}^v + \beta^v.p_{ij}^{-v} - y_{ij}^v) = 0, \\ & \delta_{ij}^v.p_{ij}^v = 0, \\ & \delta_{ij}'^v.y_{ij}^v = 0, \\ & \delta_{ij}''^v.z_{ij}^v = 0, \\ & p_{ij}^v, y_{ij}^v, z_{ij}^v, \lambda_i^v, \lambda_i'^v, \gamma_{ij}^v, \delta_{ij}^v, \delta_{ij}'^v, \delta_{ij}''^v \ge 0. \end{aligned} \tag{9}$$

The KKT conditions can be combined into the following linear complementary vectors. One can take advantage of such a complementary formulation for which efficient algorithms and commercial solvers exist (Harker & Pang, 1990).

$$0 \le \begin{pmatrix} p_{ij}^v \\ y_{ij}^v \\ z_{ij}^v \\ \lambda_i^v \\ \lambda_i'^v \\ \gamma_{ij}^v \\ \delta_{ij}^v \\ \delta_{ij}'^v \\ \delta_{ij}''^v \end{pmatrix} = X^v \perp F_v(X) = \begin{pmatrix} y_{ij}^v - \alpha^v.\gamma_{ij}^v + \delta_{ij}^v \\ p_{ij}^v - c_{ij}^v - \lambda_i^v + \lambda_i'^v + \lambda_j^v - \lambda_j'^v - \gamma_{ij}^v + \delta_{ij}'^v \\ -\theta^v.c_{ij}^v - \lambda_i^v + \lambda_i'^v + \lambda_j^v - \lambda_j'^v + \delta_{ij}''^v \\ \sum_{j\in l} y_{ji}^v + \sum_{j\in l} z_{ji}^v - \sum_{j\in l} y_{ij}^v - \sum_{j\in l} z_{ij}^v \\ \sum_{j\in l} y_{ij}^v + \sum_{j\in l} z_{ij}^v - \sum_{j\in l} y_{ji}^v - \sum_{j\in l} z_{ji}^v \\ D_{ij}^v - \alpha^v.p_{ij}^v + \beta^v.p_{ij}^{-v} - y_{ij}^v \\ p_{ij}^v \\ y_{ij}^v \\ z_{ij}^v \end{pmatrix} \ge 0, \tag{10}$$

Solving the linear complementary problem (10) can result in a candidate solution for GNEP. Hence, the benefit function of each carrier is pseudo concave and the KKT sufficient conditions hold in the problem space (See Appendix), and therefore the obtained candidate solution will be the global solution of the carrier pricing game. This linear complementary problem is formulated in GAMS and the resulted problem is solved using the PATH solver as a Newton based algorithm (Ferris & Munson, 2000).

3. Cooperative game model and bargaining problem

In this section, we focus on a cooperative game structure in which both competitor carriers agree to make decisions jointly such that the total profit would be maximized. We assume that they only share the related information and not their fleet capacity. Other assumptions are similar to the non-cooperative scenario. The demand is price sensitive and defined as a linear function of self prices as well as rival prices. The problem of interest is to find the optimal strategies of two carriers service on a common road network cooperatively and share the extra joint profit. Hence, we have the following optimization problem,

$$\max\ \pi_{co} = \sum_{v\in V}\{\sum_{ij\in A}(p_{ij}^{v}-c_{ij}^{v}).y_{ij}^{v}-\theta^{v}.\sum_{ij\in A}c_{ij}^{v}.z_{ij}^{v}\} \tag{13}$$

subject to

$$\sum_{j\in l}y_{ji}^{v}+\sum_{j\in l}z_{ji}^{v}-\sum_{j\in l}y_{ij}^{v}-\sum_{j\in l}z_{ij}^{v}=0 \qquad \forall i\in l, v\in V \tag{14}$$

$$y_{ij}^{v}-d_{ij}^{v}(p_{ij}^{v},p_{ij}^{-v})\le 0 \qquad \forall ij\in A, v\in V \tag{15}$$

$$y_{ij}^{v},z_{ij}^{v}\in Z_{+},\ p_{ij}^{v}\in R_{+} \qquad \forall ij\in A, v\in V. \tag{16}$$

The objective of the cooperative game is the summation of the carrier's individual benefit functions (Eq.(13)). The solution must guarantee all the constraints of two carriers (constraints (14)-(16)). Regarding to the cooperative game theory assumptions, the cooperation of two competitor carriers is meaningful if the solution is in the *core* of the game. In other words, both sides would participate in the cooperation only if their individual profits are higher than those of non-cooperative case. Thus, the core of such cooperative game can be defined as follows.

$$\pi_{co}^{*}\ge\pi_{v_1}^{*}+\pi_{v_2}^{*}, \tag{17}$$

$$\pi_{v_1}^{co}\ge\pi_{v_1}^{*}, \tag{18}$$

$$\pi_{v_2}^{co}\ge\pi_{v_2}^{*}\ , \tag{19}$$

where $\pi_{v_1}^{*}$ and $\pi_{v_2}^{*}$ are the maximum benefits that carriers v_1 and v_2 can earn in non-cooperative case, respectively. $\pi_{v_1}^{co}$ and $\pi_{v_2}^{co}$ represent the shares of carriers v_1 and v_2 from the total joint benefit, respectively. Here we propose a Nash (1950) bargaining scheme in which carriers offer a set of suggestions and discuss on them until an agreement is achieved. The extra joint benefit is calculated as:

$$\nabla\pi_{v_1,v_2}=\pi_{co}^{*}-(\pi_{v_1}^{*}+\pi_{v_2}^{*}). \tag{20}$$

Since the equilibrium point of the carriers' non-cooperative game is always a feasible solution for the cooperative model, thus $\nabla\pi_{v_1,v_2}>0$. We also assume that

$$\nabla\pi_{v_2}=\pi_{v_2}^{co}-\pi_{v_2}^{*}\ge 0,\ \nabla\pi_{v_1}=\pi_{v_1}^{co}-\pi_{c_1}^{*}\ge 0. \tag{21}$$

According to Nash bargaining, the bargaining outcome is obtained by maximizing the product of individual utilities over the feasible solution. Consider the following utility functions for the two carriers:

$$U_{v_1}(\nabla\pi_{v_1})=(\nabla\pi_{v_1})^{\lambda_1},\ U_{v_2}(\nabla\pi_{v_2})=(\nabla\pi_{v_2})^{\lambda_2};\quad \lambda_1,\ \lambda_2>0 \tag{22}$$

where λ_1 and λ_2 are respectively the risk attitudes of carriers v_1 and v_2. The risk attitude of each carrier shows the amount of risk confronted that carrier, if the cooperation is collapsed. Therefore, the optimal share of each carrier can be obtained by solving Eqs.(23)-(25).

$$\max\quad U = U_{v_1}(\nabla\pi_{v_1}).U_{v_2}(\nabla\pi_{v_2})=(\nabla\pi_{v_1})^{\lambda_1}.(\nabla\pi_{v_2})^{\lambda_2} \tag{23}$$

$$\nabla\pi_{v_1}+\nabla\pi_{v_2}=\nabla\pi_{v_1,v_2} \tag{24}$$

$$\nabla\pi_{v_1},\ \nabla\pi_{v_2}\ge 0 \tag{25}$$

The optimal solution of problem (23)-(25) occurs with $\nabla\pi_{v_1}=\frac{\lambda_1}{\lambda_1+\lambda_2}\nabla\pi_{v_1,v_2}$, $\nabla\pi_{v_2}=\frac{\lambda_2}{\lambda_1+\lambda_2}\nabla\pi_{v_1,v_2}$.

In other words, in Nash bargaining solution, each carrier's share of extra joint profit is equal to its risk attitude ratio. When both carriers have the same risk attitude ($\lambda_1 = \lambda_2$), Nash's model predicts that they will equally split the joint extra-profits, which is common knowledge in the bargaining literature. Otherwise, if one carrier's attitude is to take more risk, this carrier will get more from the joint extra-profit. Therefore, the total benefit of each carrier in the cooperative game is obtained as follows:

$$\pi_{v_1}^{co} = \pi_{v_1}^{*} + \nabla \pi_{v_1},\ \pi_{v_2}^{co} = \pi_{v_2}^{*} + \nabla \pi_{v_2} \tag{26}$$

4.Numerical experiments and discussion

In this section, we apply the modified Nonlinear Gauss–Seidel method to several numerical examples and provide a discussion of the results. In order to develop test networks, we generate a set of instances in a 100*100 square mile region by varying the number of locations and the average number of lanes incident to a location. Lanes are created by randomly picking an origin and destination. The cost of traveling between locations with a full truckload, c_{ij}^{v}, is equal to the Euclidean distance between the locations multiplying to a service level coefficient, σ^{v}. The service level coefficient reflects the technology of equipment and fleet for each carrier. We assume that carrier 2 tolerate more transportation cost traveling over the same lane such that $\sigma^1 = 1$ and $\sigma^2 = 1.05$. The repositioning cost coefficient for both carriers is equal to 0.5. The number of potential demands over different network lanes are generated by a uniform distribution U(40,60) for both carriers. The demand function parameters are assumed to be the same for two carriers such that $\alpha = 0.85$ and $\beta = 0.65$. In the following, we first discuss the results and sensitivity analysis for an example and then summarize the results for instances of different sizes.

Example 1: Consider two rival carriers service over a common transportation network consisting of 5 nodes (locations) and 20 arcs (lanes). We solve the example using the modified Gauss–Seidel method in GAMS 29.9.2. Fig. 2 and Fig. 3 illustrate the output results in terms of the prices and realized demands of the two carriers in the Nash equilibrium point of their game. As we can observe from these figures, the realized demands for each carrier are matched with the announced prices over different network lanes. For example, on the arcs 12, 14, 15, 16, 17, 19, and 20, carrier 1 can devote the market demand to himself by announcing lower prices compared with prices declared by carrier 2. In contrast, on arc 18, carrier 2 can gain more demands through lower price offered.

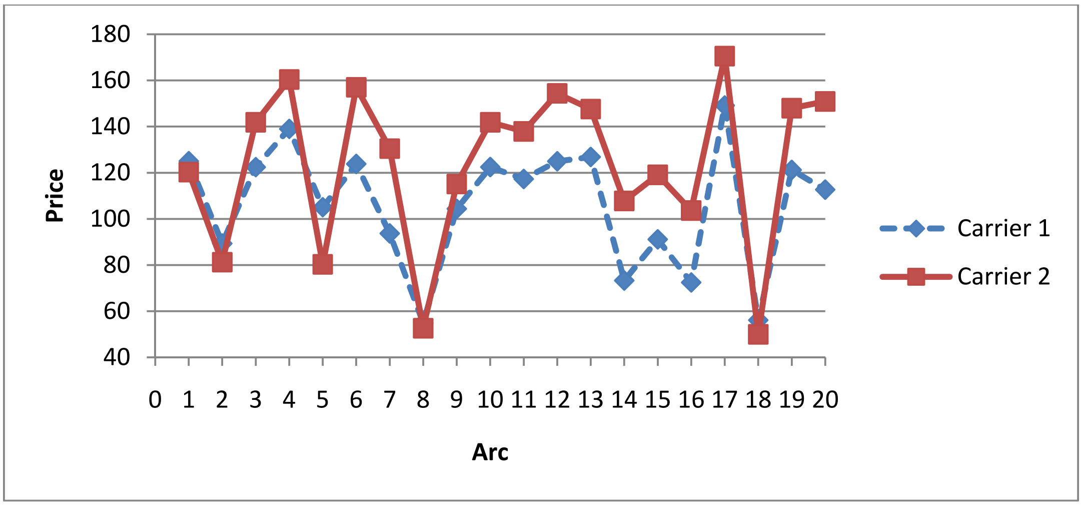

Fig.2 carriers' prices in Nash equilibrium point

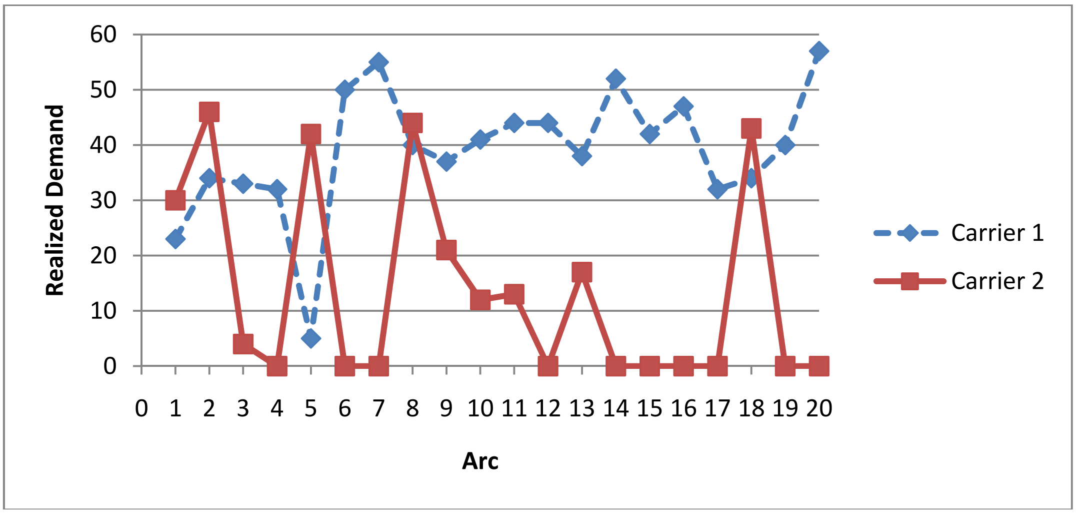

Fig.3 carriers' realized demands in Nash equilibrium point

The benefit values for carrier 1 and 2 are 24173.51 and 22006.40, respectively. We investigate the effects of demand function parameters α and β. Fig. 4 shows the results of example 1 for $\alpha = 0.85$ by varying β. As expected, as β rises, the carrier increases his prices to capture more demands and therefore the benefit is increased. Fig. 5 shows the results of example 1 for $\beta = 0.65$ by varying α. As expected, the carrier decreases his prices by decreasing α to capture more demands, and therefore the benefit is declined.

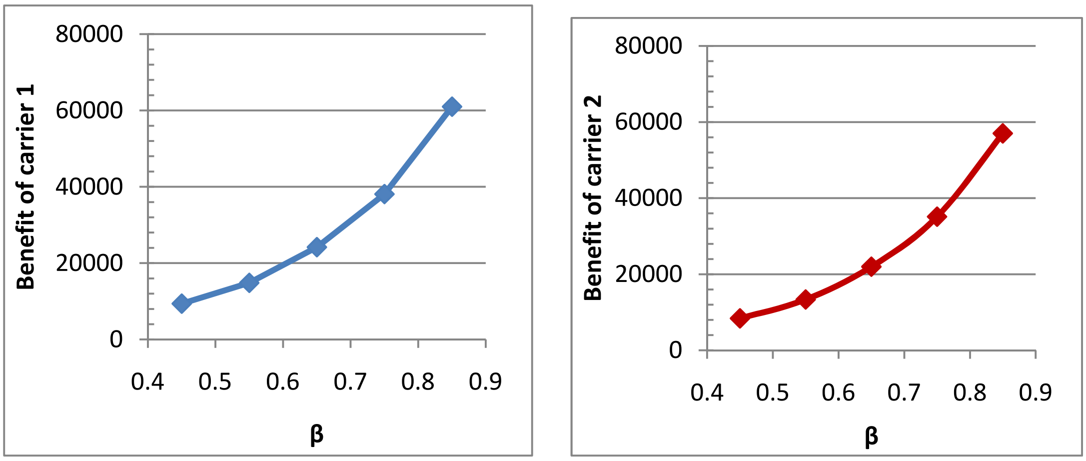

Fig.4 sensitivity analysis of β

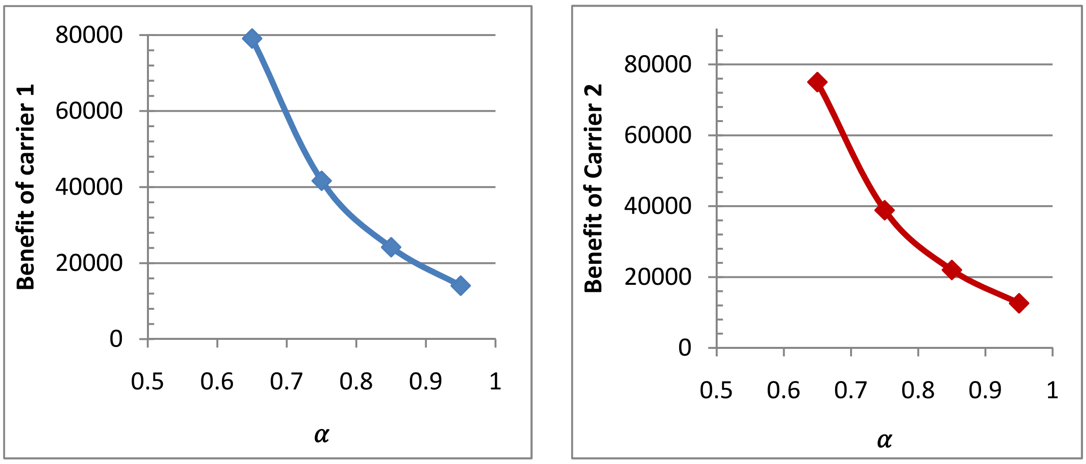

Fig. 5 sensitivity analysis of α

We model the cooperative game mathematical program of Example 1 in GAMS 29.9.2 using SBB solver. The total joint benefit obtained is 75046.52, which shows 62.51% improvements in total benefit of carriers. The extra joint benefit of 28866.61 units is then shared among carriers based on their risk attitudes. We have applied the solution algorithm and the relaxation approach to test problems of different sizes and the results are summarized in Table 1. As can be seen, the results of relaxed GNEP have a small gap from mixed integer GNEP's results, and even for larger instances, the gap becomes smaller. It is concluded that the relaxed problem can create a good approximation of the mixed integer GNEP in a reasonable amount of time, and the use of relaxed method leads to a save time for large-scale instances, significantly.

Table1

Results of GNEP for different instances

Problem size		Mixed integer GNEP			Relaxed GNEP			Gap (%)	
l	*A*	Benefit of carrier 1	Benefit of carrier 2	Time (s)	Benefit of carrier 1	Benefit of carrier 2	Time (s)	Carrier 1	Carrier 2
5	20	24173.51	22006.40	65	24249.63	22052.91	8	0.31	0.27
10	90	82568.45	68490.32	273	82743.56	68628.34	16	0.21	0.20
15	210	202387.22	134184.73	566	202677.96	134408.30	32	0.14	0.17
20	380	421264.36	271484.96	810	421214.74	271422.35	54	0.01	0.02
25	600	477769.03	399450.97	2520	478080.23	399396.64	120	0.01	0.01
30	870	698215.31	586379.59	6612	698190.82	586351.00	180	0.003	0.005

The cooperative game scenario has been examined using problem instances of different sizes and results are shown in Table 2. Results justify that cooperation among rival carriers can lead to a significant improvements (more than 40%) in total benefit achieved. The carriers can then negotiate on their risk attitudes to allocate the extra joint benefit.

Table 2

Comparison of total benefit in two game scenarios

Problem size		Total benefit without cooperation	Total benefit with cooperation	Improvement (%)
l	*A*			
5	20	46179.91	75046.52	62.51
10	90	151058.77	224719.89	48.76
15	210	336571.95	474321.52	40.93
20	380	692749.32	1025503.28	48.03
25	600	877220	1420596.23	61.94
30	870	1284594.9	2079854.49	61.91

5. Concluding remarks

This paper studied pricing issue for freight carriers in full truckload industry which compete on a common road network. A review of literature indicated that the existing works on FTL carrier pricing have ignored the influence of the fierce competition in this industry. In this paper, we proposed a game theory approach to model the competition among freight carriers. The fleet management decisions including the number of transportation services, and the number of vehicle repositions over different arcs of the road network were also considered.

The carrier pricing and fleet management problem has been investigated under two scenarios. In the first, a non-cooperative game was modeled wherein the carriers announce their prices simultaneously in competition; in the second, we allowed the carriers to share their information and announce their prices while participating in cooperation. In non-cooperative case, we faced with a generalized Nash equilibrium problem with mixed integer space. We proposed a modified Nonlinear Gauss–Seidel method to solve this GNEP and a relaxation approach was presented to approximate the equilibrium solution. In cooperative case, we showed that carriers could reach the highest profit level. Subsequently, a Nash bargaining approach was applied as a scheme to share the extra joint profit.

Appendix

This appendix contains the proof of pseudo concavity for carriers' benefit function. Due to the fact that the constraints are all linear, the pseudo concavity of the benefit functions can prove the KKT sufficient conditions.

Theorem 1: The function $f(x)$ is strictly pseudo concave if we have:

$$\forall x_1, x_2 \in X, x_1 \neq x_2; \qquad f'_{x_2-x_1}(x_1) \leq 0 \Rightarrow f(x_2) < f(x_1) \tag{27}$$

Theorem 2: The function $f(x)$ is pseudo concave if it is strictly pseudo concave.

Each player has a benefit function as follows:

$$\pi_v = \sum_{ij\in A}(p_{ij}^v - c_{ij}^v).y_{ij}^v - \theta^v.\sum_{ij\in A} c_{ij}^v.z_{ij}^v = \sum_{ij\in A} p_{ij}^v.y_{ij}^v + \sum_{ij\in A} c_{ij}^v.y_{ij}^v - \sum_{ij\in A}\theta^v c_{ij}^v.z_{ij}^v \tag{28}$$

As it is clear, the second and third terms of the benefit function are linear and the only nonlinear function which has to be concave is the first term. Since the sum of several concave functions would be a concave function, thus it is sufficient to prove that $\sum_{ij\in A} p_{ij}^v.y_{ij}^v$ is pseudo concave. Let $f(p,y) = p.y$ and for every $(p_1, y_1), (p_2, y_2) \in R_+^2$, $p_1 \neq p_2$, $y_1 \neq y_2$, we have $f(p_2,y_2) \geq f(p_1,y_1)$ (the indices were eliminated for simplicity), then it is sufficient to prove $f'_{\vec{d}}(p_1,y_1) > 0$ where $\vec{d} = (p_2, y_2) - (p_1, y_1)$.

$$\forall (p_1, x_1), (p_2, x_2) \in R^2 \quad , p_2.y_2 \geq p_1 y_1 \quad \Rightarrow \quad p_2 \geq \frac{p_1 y_1}{y_2}. \tag{29}$$

We assume that $p_2 = \frac{p_1 y_1}{y_2}$, and then we have to show that

$$f'_{\vec{d}}(p_1, y_1) > 0, \quad \vec{d} = (p_2, y_2) - (p_1, y_1) \quad \equiv \nabla f(p_1, y_1).[(p_2, y_2) - (p_1, y_1)] > 0$$
$$\equiv \begin{pmatrix} y_1 \\ p_1 \end{pmatrix} . \begin{pmatrix} p_2 - p_1 \\ y_2 - y_1 \end{pmatrix} > 0 \quad \equiv \quad y_1.(p_2 - p_1) + p_1.(y_2 - y_1) > 0. \tag{30}$$

Now, we can prove that the inequality (30) holds using $p_2 = \frac{p_1 y_1}{y_2}$.

$$y_1.(\frac{p_1.y_1}{y_2} - p_1) + p_1.(y_2 - y_1) > 0 \quad \Rightarrow \tag{31}$$

$$y_1.(\frac{p_1.y_1 - p_1.y_2}{y_2}) + p_1.(y_2 - y_1) > 0 \quad \Rightarrow \tag{32}$$

$$y_1.(p_1.y_1 - p_1.y_2) + p_1.y_2.(y_2 - y_1) > 0 \quad \Rightarrow \tag{33}$$

$$p_1.y_1^2 + p_1.y_2^2 - 2p_1.y_1.y_2 > 0 \quad \Rightarrow \tag{34}$$

$$y_1^2 + y_2^2 - 2y_1.y_2 > 0 \quad \Rightarrow (y_1 - y_2)^2 > 0 \tag{35}$$

Since $y_1 \neq y_2$, Thus, Eq. (35) always holds and for $p_2 = \frac{p_1 y_1}{y_2}$ the function $f(p,y) = p.y$ is strictly pseudo concave. Eq. (35) also holds when $p_2 \geq \frac{p_1 y_1}{y_2}$ since $p_2 > p_1$ according to Eq. (30). Therefore, $f(p,y) = p.y$ is always strictly pseudo-concave. In addition, $\sum_{ij\in A} p_{ij}^v.y_{ij}^v$ consists of separate terms and one can easily prove the pseudo concavity by using induction.

Let k be the number of terms in $\sum_{ij\in A} p_{ij}^{v}.y_{ij}^{v}$. Eq. (35) holds for $k=1$; now we assume it holds for $k=n$ and show that it shall also hold for $k=n+1$. Note that Eq. (35) can be written for $k=n$ and $k=n+1$ as follows:

$$k=n:\ \sum_{i=1}^{n} p_2^i.y_2^i \geq \sum_{i=1}^{n} p_1^i.y_1^i \quad \Rightarrow \quad \sum_{i=1}^{n} y_1^i.(p_2^i - p_1^i) + p_1^i.(y_2^i - y_i^1) > 0 \tag{36}$$

$$k=n+1:\ \sum_{i=1}^{n+1} p_2^i.y_2^i \geq \sum_{i=1}^{n+1} p_1^i.y_1^i \quad \Rightarrow \quad \sum_{i=1}^{n+1} y_1^i.(p_2^i - p_1^i) + p_1^i.(y_2^i - y_1^i) > 0 \tag{37}$$

Based on the assumptions of Eq.(36) and Eq. (37), we can conclude that $p_2^{n+1}.y_2^{n+1} \geq p_1^{+1}.y_1^{n+1}$ and based on Eq. (31) and Eq. (35) we have $y_1^{n+1}.(p_2^{n+1} - p_1^{n+1}) + p_1^{n+1}.(y_2^{n+1} - y_1^{n+1}) > 0$. Thus, by adding the term $y_1^{n+1}.(p_2^{n+1} - p_1^{n+1}) + p_1^{n+1}.(y_2^{n+1} - y_1^{n+1}) > 0$ to the right hand side of the Eq. (36), the right hand side of the Eq. (37) is resulted and this completes the proof.

References

Agarwal, R., & Ergun, O. (2010). **N**etwork design and allocation mechanisms for carrier alliances in liner shipping**.** *Operations Research***,** 58(6), 1726-1742.

ATA (2005). Trucking activity report, Technical report, American Trucking Association, Arlington, VA. http://www.trucking.org.

Bazaraa, M.S., Sherali, H.D., & Shetty, C.M. (1993). *Nonlinear Programming: Theory and Algorithms*. 2nd ed, John Wiley & Sons, Inc.

Elmaghraby, W., & Keskinocak, P. (2003). *Combinatorial Auctions in Procurement. In: The Practice of Supply Chain Management*, eds. Billington C., Harrison T., Lee H., Neale J., Kluwer.

Ergun, O., Kuyzu, G., & Savelsbergh, M. (2007). Shipper collaboration. *Computers & Operations Research*, 1551-1560.

Facchinei, F., & Kanzow, C. (2007). Generalized Nash equilibrium problems. *4OR*, 5, 173–210.

Facchinei, F., Piccialli, V., & Sciandrone, M. (2007). On a class of generalized Nash equilibrium problems. DIS Technical Report, “Sapienza” Università di Roma, Rome.

Ferris, M. C., & Munson, T. S. (2000). Complementarity problems in gams and the path solver. *Journal of Economic Dynamics and Control*, 24, 165-188.

Figliozzi, M. A., Mahmassani, H. S., & Jaillet P. (2007). Pricing in Dynamic Vehicle Routing problems. *Transportation Science*, 41(3), 302-318.

King, G. J., & Topaloglu, H. (2007). Incorporating the pricing decisions into the dynamic fleet management problem. *Journal of Operation Research Society*, 58, 1065-1074.

Harker, P.T. (1985). The state of the art in the predictive analysis of freight transport systems. Transport Reviews, 5(2), 143-164.

Harker, P.H., & Pang, J.S. (1990). Finite dimensional variational inequality and nonlinear complementarity problems: a survey of theory, algorithms and applications. *Mathematical Programming*, 48, 161-220.

Houghtalen, L., Ergun, O., & Sokol, J., (2007). Designing mechanisms for the management of carrier alliances. *Wiley Encyclopedia of Operations Research and Management Science.*

Linderoth, J.T., & Savelsbergh, M.W.P. (1999). Computational study of search strategies for mixed integer programming. *INFORMS Journal on Computing*, 11(2), 173-187.

Ozener, O.O. (2008). Collaboration in transportation, PhD thesis, H. Milton Stewart School of Industrial and Systems Engineering, Georgia Institute of Technology.

Ozener, O.O., & Ergun O. (2008). Alocating costs in a collaborative transportation procurement network. *Transportation Science*, 42(2), 146-165.

Ozener, O.O., Ergun, O., & Savelsbergh, M. (2010). Lane-exchange mechanisms for truckload carrier collaboration. *Transportation Science*, 2010.

Tawarmalani, M., & Sahinidis, N.V., (2004). Global optimization of mixed integer nonlinear programs: A theoretical and computational study. *Mathematical Programming*, 99, 563-591.

Topaloglu, H., & Powell, W. (2007). Incorporating pricing decisions into stochastic dynamic fleet management problem. *Transportation Science*, 41(3), 281-301.

Nash, J. F. Jr. (1950). The bargaining problem. *Econometrica*, 18(2), 155-162.