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The optimal design of differentiated subsidy policies for new energy vehicle firms by considering the difference in market share and endurance mileage

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A B S T R A C T

To promote the development of the new energy vehicle industry, China has introduced many subsidy policies. Existing policies rarely consider the difference in market share of new energy vehicle enterprises, but the difference in market share will directly affect consumer demand, and then affect the development of the new energy vehicle industry. Therefore, we consider the difference of the market share and endurance of the new energy vehicle firms at the same time, establishing a Stackelberg model and considering three most common subsidy forms, which are quantity-based subsidies, price-based subsidies, and endurance-based subsidies to derive the optimal differentiated subsidies of the government. The analysis of this paper shows that for the new energy vehicle firms with different market share and endurance, differentiated subsidies can achieve higher social welfare. In addition, for any subsidy form of the sale-based subsidies, price-based subsidies, and endurance-based subsidies, the final cost for consumers will be reduced, and the total sales quantity of the new energy vehicles in the market will increase, but the profits of firms and the sales quantity of one firm could increase or decrease. Lastly, under the assumption of this paper, the optimal price of both firms and social welfare are the same under the three aforementioned subsidy forms.

1. Introduction

In the context of the comprehensive green transition of China's economic and social development, China is trying to reach the ambitious goals of peaking carbon dioxide emissions and going carbon neutral. As one of the top three industries with high carbon emissions in China, the transportation industry is the key industry to control carbon emissions. Fuel vehicles are one of the main reasons for high carbon emissions in the road transportation industry. Compared with traditional fuel vehicles, new energy vehicles have some advantages such as low carbon and environmental protection. Therefore, the key to controlling road traffic carbon emissions is to control the travel of fuel vehicles. Government subsidies are a major factor in the growth of new energy vehicle sales (Zheng et al. 2022). Since 2009, the Chinese government has introduced several subsidy policies to help the development of the new energy vehicle industry, including sales subsidies, price subsidies, and endurance subsidies. Under the incentive of government subsidies, the number of new energy vehicles in China has increased from 0.91 million in 2016 to 18.21 million in 2023[1]. Although the number of new energy vehicles in China has shown a rapid growth trend, there is still a lot of room for improvement in the proportion of new energy vehicles in the total vehicle industry. By 2022, new energy vehicles only account for 4.10% of the total vehicles[2]. At present, the government still provides a variety of subsidy policies to increase the proportion of new energy vehicles in the total vehicle industry. However, the high subsidies have put enormous financial pressure on the government. According to incomplete statistics, the cumulative total of China's new energy

[1] https://www.gov.cn/zhengce/jiedu/tujie/202310/content_6908249.htm
[2] http://www.gov.cn/xinwen/2023-01/11/content_5736176.htm

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vehicle subsidies has exceeded 200 billion yuan[3]. Therefore, since 2018, the government subsidy policy has entered a period of adjustment and gradually increased the subsidy threshold for the endurance mileage of new energy vehicles. For example, the government began to provide 9100 yuan for new energy vehicles according to the endurance mileage of 300km, based on the effect of energy conservation and emission reduction, and comprehensive consideration of production costs, scale effects, technological progress, and other factors. For those with an endurance mileage greater than 400km, the subsidy is 12,600 yuan per vehicle[4]. This makes new energy vehicles with low endurance no longer enjoy subsidy policies. The increase in the subsidy threshold can not only ease the financial pressure of the government, but also reduce the negative impact of direct subsidy cancellation on firms (Liu et al. 2022) and promote the high-quality development of the new energy vehicle industry.

At the same time, the market share of different new energy vehicle firms in China varies greatly, with annual sales of more than 1 million firms (such as BYD), and annual sales of less than 0.3 million firms (such as Xiaopeng). Interestingly, the sales volume of Aion, which has an endurance mileage of more than 1,000 kilometers, is only 0.27 million in 2022. While BYD, which has a sales volume of more than 1.79 million vehicles in 2022, has an endurance mileage of only about 600-700 kilometers[5]. Therefore, the difference in market share greatly affects the purchasing decisions of consumers, and the generally adopted non-differential subsidy policy may hinder the development of emerging excellent firms. But the purpose of government subsidies is to enable consumers to buy better quality vehicles at lower prices, which is possible that enterprises with higher endurance mileage, but less influence can get more government subsidies. Therefore, when designing subsidy policies for new energy vehicles, in addition to considering the difference in the endurance mileage of firm products, the government should also consider the difference in the market share of firms, and further improve the efficiency of subsidy policies.

Based on this, we both consider the differences in market share and product endurance of firms and explore the government's differentiated subsidy policy design for different new energy vehicle firms. Specifically, we consider two new energy vehicle firms with different market share and endurance and analyze the optimal pricing decision of the firms and the differentiated subsidy design of the government based on the Stackelberg game model. In addition, we also consider the sales subsidy, price subsidy, and endurance subsidy respectively, and make a comparative analysis of the optimal pricing, sales volume, profit, and overall social welfare of firms under the above four models.

The results show that: (1) Differentiated subsidies provided by the government to new energy vehicle firms with different market shares and endurance can achieve higher social welfare; (2) No matter which new energy vehicle subsidy mode is provided by the government, the final payment cost of consumers will decrease, but the profits of firms will not necessarily increase; (3) No matter which new energy vehicle subsidy mode is provided by the government, the total sales volume in the market will increase, but the sales volume of a single firm may increase or decrease; (4) Under the assumptions of this paper, the optimal sales volume of firms under the three subsidy models of sales subsidy, price subsidy and endurance subsidy is the same as the overall social welfare. Thus, the total amount of subsidies and implementation effect of the three subsidy models are equivalent under the optimal state.

The article is organized as follows. Section 2 reviews the related literature. Section 3 presents the pricing decision model without government subsidies, and Section 4 presents the pricing decision model under three different government subsidies. In Section 5, we analyze and discuss the result of four cases of government subsidies. Finally, we conclude Section 6.

2. Literature Review

This paper aims to study how to design the subsidy policy of new energy vehicles. This section will introduce literature that is highly relevant to the research content of this paper. According to the different subsidy objects, it will be specifically expanded from three parts, which are production subsidies, consumption subsidies and mixed subsidies.

2.1 Production subsidies

Production subsidies mainly include sales subsidies and endurance subsidies. Wu et al. (2022) applied a multi-stage differential model to explore the environmental benefits of government subsidies for new energy vehicle sales, and the results showed that subsidies had a positive impact on the air quality. Zhang and Dou (2022) found that direct subsidies for the purchase of electric vehicles can more effectively promote the adoption of new energy vehicles. At the same time, Li et al. (2019) show that compared with consumer purchase subsidies, production subsidies have a greater impact on the expansion of the new energy vehicle market. Shao et al. (2021) argue that production research and development subsidies have an incentive effect on new energy vehicle firms. At the same time, Zhu et al. (2021) also believe that if subsidies are provided according to the endurance mileage, the research and development of the optimal endurance mileage technology for new energy vehicles will be improved. Similarly, Zhong et al. (2023) argue that endurance subsidies always obtained better social

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[3] http://opinion.people.com.cn/n1/2023/0106/c427456-32601358.html
[4] https://www.gov.cn/zhengce/zhengceku/2021-12/31/content_5665857.htm
[5] http://data.cpcaauto.com/

welfare than sales subsidies. In addition, there are also scholars who study subsidies for different entities of the supply chain. Xin and Xu (2022) found that it is more beneficial to subsidize the manufacturer than to subsidize the retailer by constructing the Stackelberg game model dominated by the manufacturer of the supply chain. At the same time, mixed subsidies are more conducive to the innovation and development of the supply chain. What's more, Zhao et al. (2020) subdivided the effects of government subsidies on the supply chain, and the results showed that subsidies had a greater impact on the financial performance of upstream enterprises.

2.2 Consumption subsidies

Consumer subsidies are mainly subsidies for the purchase price of new energy vehicles. Bao et al. (2020) studied the game action of duopoly automobile manufacturers to produce new energy vehicles and fuel vehicles and found that the price of new energy vehicles is very dependent on government subsidies. Yu et al. (2022) point out that government purchase subsidies to consumers are more cost-effective than subsidies to new energy vehicle firms. Similarly, Sun et al. (2022) studied production subsidies and consumption subsidies respectively and found that consumption subsidies could bring better profits and social welfare than production-end subsidies. Zhang and Huang (2021) also argue that consumption subsidies are better than production subsidies in both environmental and economic terms. Using the Hotelling model, Zhang et al. (2020) found that consumer subsidies can reduce the purchase cost of consumers and promote the enterprises to produce vehicles with higher energy efficiency. In addition, consumer subsidies are always conducive to the environment, but under certain conditions, they will also damage the consumer surplus and the profits of enterprises. The study found that in order to maximize social welfare, the government should give priority to subsidizing consumers, followed by sales subsidies and endurance subsidies. It is interesting that Chemama et al. (2019) show that when the government provides consumption subsidies, the adoption of a single subsidy policy can better stimulate the sales volume of new energy vehicles, but when the government adopts a flexible subsidy policy, firms can obtain better profits.

2.3 Mixed subsidies

As the subsidy strategy combining different subsidy methods is more effective than the single subsidy strategy (Zhang and Huang, 2021), some scholars studied the hybrid subsidy strategy considering both production and consumption subsidy modes. The mixed subsidy model can give two kinds of subsidies to the same object or different object in the same period. It can also be for the same subsidy object, in different periods to give different subsidy models. Li et al. (2023), based on the dynamic interaction between the government, firms, and consumers, study the effects of sales subsidies and endurance subsidies, dynamic subsidies and static subsidies on the new energy vehicle industry and environmental protection, and verify that both sales subsidies and endurance subsidies can promote the demand for new energy vehicles. Among them, sales subsidies stimulate the sales of new energy vehicles in the short term, and endurance subsidies are also effective in promoting the popularity of electric vehicles and less pressure on the government. In addition, dynamic subsidy schemes are better than static subsidy schemes in terms of subsidy efficiency. At the same time, Shi et al. (2022) suggested that a combination of consumption subsidies and subsidies for the construction of charging stations is the best policy when the government budget is minimized. Based on the data of listed enterprises, Wang et al. (2023) that in terms of encouraging enterprises to improve innovation, the combination of subsidy tools is superior to the single subsidy, and the government tends to provide combination subsidies for high-tech enterprises. Kong et al. (2020) adopted a system dynamics model considering the interaction between the government, enterprises, and consumers. It proved that only the combination of purchase subsidy scheme, product subsidy, and carbon emission trading can optimize the effect on the promotion of electric vehicles. Sun et al. (2022) adopted the Stackelberg game, Nash equilibrium and optimal profit model to solve the problem of how the government should formulate optimal subsidy strategies in different research and development stages of new energy vehicles. To be specific, Zhang and Cai (2020) suggest that subsidies should be given to enterprises in the early stage of new energy vehicle research and development, and subsidies should be focused on the construction of new energy vehicle infrastructure before the new energy vehicle is listed. After the vehicle enters the market, the government should provide subsidies to consumers of new energy vehicles, so as to better expand the market of new energy vehicles.

2.4 Summary of literature

It can be seen from the existing literature that scholars have conducted a lot of research on issues related to government subsidies for new energy vehicles. At present, existing studies considering differentiated subsidies for new energy vehicles mainly focus on the differences in subsidy objects (Sun et al. 2022), subsidy sequence (Zhang and Cai 2020), and endurance of new energy vehicles (Zhu et al. 2021). Although the research on new energy vehicle subsidy policies has achieved great research results, there are few research literatures that both consider the difference in market share and product endurance in the design of differentiated subsidy policies for new energy vehicles. Therefore, the main purpose of this paper is to study how the government designs differentiated subsidies for new energy vehicle enterprises with different market shares. Especially, Levi et al. (2017) argue that in many cases, a single subsidy can achieve the best social welfare solution, but when firms have market entry costs, the adoption of non-differentiated subsidy policies may hinder the development of new and outstanding firms.

To sum up, this paper will both consider the differences in market share and product endurance of firms and explore the government's differentiated subsidy policy design for different new energy vehicle firms.

3. Pricing decision model without government subsidies

This paper assumes that there are two new energy vehicle firms with different market shares in the market, which are divided into strong firms (i=s) and weak firms (i=w). Strong firms are the early firms to enter the field of new energy vehicles, generally large scale and high market share, while weak firms are those who are ready to enter or have just entered the new energy automobile industry for a short time, relatively have small scale compared with strong firms, and their market share is low. According to the Stackelberg game model, since the market share of the strong firm is higher than that of the weak firm, this paper assumes that the strong firm is the leader who decides its sales price first, and the weak firm is the follower who decides its sales price later. The specific decision order is shown in Fig. 1.

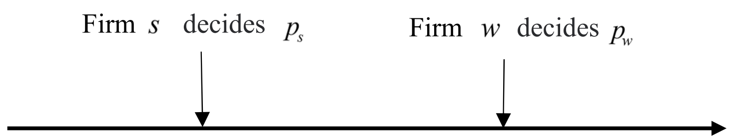

Fig. 1. The order of decision making without government subsidies

In this section, we first analyze the optimal price decisions of the two firms without considering government subsidies. The influence of government subsidies on the pricing decisions of the two firms and the optimal design of government subsidies will be considered in the next section. According to the above model setting, the prices of strong and weak firms are respectively expressed as:

$$p_s = \eta a - q_s - b_w q_w + \theta h_s \tag{1}$$
$$p_w = (1-\eta)a - q_w - b_s q_s + \theta h_w \tag{2}$$

Where, η is the market share of strong firms (η > 0.5), while the market share of weak firms is expressed as 1- η, a is the market capacity of new energy vehicles. p_i represents the firm's pricing of new energy vehicles ($p_i > 0$). q_i represents the firm's sales volume of new energy vehicles ($q_i > 0$). b_i represents the impact of the firm's sales volume on the pricing of another firm, which b_s represents the impact of the firm s (strong firm) sales volume on the firm w (weak firm). Correspondingly, b_w represents the impact of the firm w sales volume on the firm s . We assume that the impact on firm s is greater than the impact on firm w. h_i represents the endurance mileage of new energy vehicles of the firm ($h_i > 0$), and θ represents the monetization unit coefficient of the endurance mileage of new energy vehicles. For ease of calculation, we use t a_i represents the market share of the firm i , which $a_s = \eta a$ and $a_w = (1-\eta)a$. We assume $a_s > a_w$. Based on formulas (1) and (2), the sales volume of firm s and firm w can be expressed as:

$$q_s = \frac{a_s - p_s + \theta h_s - b_w(a_w - p_w + \theta h_w)}{1-b_s b_w}$$
$$q_w = \frac{a_w - p_w + \theta h_w - b_s(a_s - p_s + \theta h_s)}{1-b_s b_w}$$

Without considering government subsidies, the company's income comes entirely from sales profits. We assumed that the goal of both the firm s and the firm w is to maximize the profits of the firm itself. For firm s and firm w, the objective function can be expressed as:

$$\max_{(p_s)} \pi_s = p_s q_s = \frac{1}{1-b_s b_w} p_s(a_s - p_s + \theta h_s - b_w(a_w - p_w + \theta h_w))$$
$$\max_{(p_w)} \pi_w = p_w q_w = \frac{1}{1-b_s b_w} p_w(a_w - p_w + \theta h_w - b_s(a_s - p_s + \theta h_s))$$

According to the Stackelberg game model, we assume that the firm s as the leader first decides the price, and the firm w as the follower decides the price later. The optimal pricing of the two firms can be solved by backward induction.

Proposition 1. Without considering subsidies, the optimal pricing of firm s and firm w is $p_s^* = \frac{1}{2}(a_s + \theta h_s) - \frac{b_w}{2(2-b_s b_w)}(a_w + \theta h_w)$ and $p_w^* = \left(\frac{1}{2} - \frac{b_s b_w}{4(2-b_s b_w)}\right)(a_w + \theta h_w) \; - \frac{b_s}{4}(a_s + \theta h_s)$.

Proposition 1 shows that the optimal pricing of the firm s and the firm w is not only related to the market share and endurance of the firm itself, but also related to the market share and endurance of other firms in the market. For one firm, the larger its market share and endurance, the higher its optimal pricing. On the contrary, the greater the market share and endurance of the firm w, the lower the optimal pricing of the firm s. The same goes for firm w.

Corollary 1. Without the consideration of the government subsidies, the optimal sales volume of the firm s and the firm w is $q_s^* = \frac{(2-b_s b_w)(a_s+\theta h_s) - b_w(a_w+\theta h_w)}{4(1-b_s b_w)}$ and $q_w^* = \frac{-b_s(a_s+\theta h_s) + \frac{4-3b_s b_w}{2-b_s b_w}(a_w+\theta h_w)}{4(1-b_s b_w)}$. The profits of the firm s and the firm w are $\pi_s = p_s^* q_s^* = \frac{2(1-b_s b_w)}{2-b_s b_w} q_s^{*2}$ and $\pi_w = p_w^* q_w^* = (1-b_s b_w) q_w^{*2}$.

Corollary 1 shows that the optimal sales volume of the firm s and the firm w is also related to the endurance and market share of the firm itself and other competing firms. The sales volume of the firm s and the firm w is positively correlated with the endurance mileage and market share of their own firms, and negatively correlated with the endurance mileage and market share of competing firms. It can be seen that firms can increase sales and profits by improving their own endurance and market share.

Corollary 2. p_s^* and p_w^* is decreasing in b_s and b_w.

Corollary 2 shows that the optimal pricing of both firms without subsidies will decrease with the increase of b_s and b_w, which indicates that when the influence of one firm on other competing firms increases, the prices of both firms will decrease.

Corollary 3. (1) q_s^* is increasing in b_s; When $\frac{a_s+\theta h_s}{a_w+\theta h_w} > \frac{1-2b_s b_w}{b_s}$, q_s^* is increasing in b_w;

(2) When $\frac{a_s+\theta h_s}{a_w+\theta h_w} < \frac{b_s(6-8b_s b_w+3b_s^2 b_w^2)}{(2-b_s b_w)^2}$, q_w^* is increasing in b_s; When $\frac{a_s+\theta h_s}{a_w+\theta h_w} < \frac{6-8b_s b_w+3b_s^2 b_w^2}{b_s(2-b_s b_w)^2}$, q_w^* is increasing in b_w.

Corollary 3 shows that with the increase of the influence of the firm s on the firm w, the sales volume of the firm s will also increase, and the profit of the firm s will increase. When the ratio of the sum of the market share and the endurance mileage of the firm s to that of the firm w is greater than the critical value $\frac{1-2b_s b_w}{b_s}$, the sales volume of the firm s will still increase in the influence of the firm w on the firm s. When the ratio of the market share between the firm s and the firm w is less than the critical value, the sales volume of the firm will decrease with the increase of the influence of the firm on the firm. Similarly, when the ratio of market share between firms is less than the critical value $\frac{b_s(6-8b_s b_w+3b_s^2 b_w^2)}{(2-b_s b_w)^2}$, the sales volume of firms w will increase with the increase of firm influence. When the ratio of market share between firms is less than the critical value $\frac{6-8b_s b_w+3b_s^2 b_w^2}{b_s(2-b_s b_w)^2}$, the profits and sales volume of firm w increase with the increase of corporate influence. When the ratio of market share between firm s and firm w is greater than the critical value, the sales volume of firm w decreases with the increase of corporate influence. Therefore, no matter how the market share, endurance and influence of firm s and the firm w at the present stage are, with the continuous improvement of firm s influence on firm w, the optimal sales volume of firm s will also increase, and the profits will increase. But at this point, the sales of firm w may increase or decrease. Interestingly, an increase in corporate influence does not necessarily ensure an increase in firm w sales and profits; in the above cases, it can reduce the profits of firm w. Social welfare consists of the total profits of new energy vehicle firms, consumer surplus and the amount of new energy vehicle subsidies from the government. In this paper, CS_i is used to represent the consumer surplus corresponding to new energy firms. When subsidies are not considered, the corresponding consumer surplus of firm s and the firm w is respectively $CS_s = \frac{1-b_s b_w}{2} q_s^{*2}$ and $CS_w = \frac{1-b_s b_w}{2} q_w^{*2}$, and the total social welfare is $W = \pi_s + \pi_w + CS_s + CS_w = \frac{2(1-b_s b_w)}{(2-b_s b_w)} q_s^{*2} + \frac{(1-b_s b_w)}{2} q_s^{*2} + \frac{3}{2}(1- \quad b_s b_w) q_w^{*2}$.

By observing the expression of CS_i and W, it can be found that the improvement of endurance will increase the consumer surplus of firm s customers, but at the same time, the consumer surplus of firm w will decrease. Similarly, the improvement of firm w endurance will increase the consumer surplus of firm w customers and reduce the consumer surplus of firm s customers.

4. Pricing decision model by considering the government subsidies

This section considers where government subsidies exist. At this time, the decision order of the government, the firm s and the firm w is that the government first makes subsidy policies, and then the firm s decide their prices according to the subsidies, and finally the firm considers the decisions of the government and firm s to make their own prices. The decision order is shown in Fig. 2.

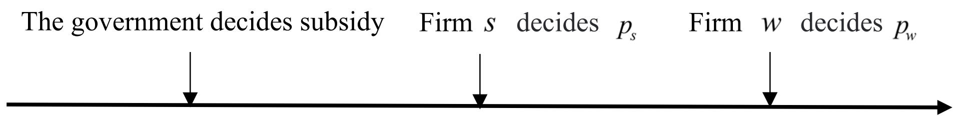

Fig. 2. The order of decision making under government subsidies

There are many kinds of government subsidy models for new energy vehicle firms, including quantity-based subsidies for new energy vehicle firms, price-based subsidies for new energy vehicles to consumers, and endurance-based subsidies for new energy vehicle firms. We will consider the subsidy based on sales volume, the subsidy based on price, and the subsidy based on endurance, which are respectively recorded as various subsidy models including sales volume subsidy s_i^1, price subsidy s_i^2 and endurance subsidy s_i^3.

4.1 Sale-based subsidies policy for new energy vehicle

The new energy vehicle sales subsidy is that the government subsidizes the number of new energy vehicles sold by the new energy vehicle firms. We assume that the government subsidizes s_i^1 to the firms for each new energy vehicle sold by the new energy vehicle firms. At this time, the profits of the firm s and the firm w can be expressed as:

$$\pi_s^1 = (p_s^1 + s_s^1)\cdot q_s^1 = (p_s^1 + s_s^1)\cdot \frac{(a_s + \theta h_s) - p_s^1 + b_w p_w^1 - b_w(a_w + \theta h_w)}{1 - b_s b_w}$$

$$\pi_w^1 = (p_w^1 + s_w^1)\cdot q_w^1 = (p_w^1 + s_w^1)\cdot \frac{(a_w + \theta h_w) - p_w^1 + b_s p_s^1 - b_s(a_s + \theta h_s)}{1 - b_s b_w}$$

At this time, social welfare is composed of the sum of corporate profits and the sum of consumer surplus minus the sum of government subsidy expenditure, so social welfare is expressed as:

$$W_1 = \frac{2(1 - b_s b_w)}{(2 - b_s b_w)}(q_s^1)^2 + \frac{1 - b_s b_w}{2}(q_s^1)^2 + \frac{3}{2}(1 - b_s b_w)(q_w^1)^2 - (s_s q_s^1 + s_w q_w^1)$$

The goal of government is to maximize social welfare, while the goal of the firm is to maximize its own profits. According to the decision order, the government first gives the optimal subsidies s_i^{1*} under the current market conditions according to the market share and endurance of the firms in the market. Then the firm s decides its own optimal price according to the government subsidy situation, and then the firm w decides its own optimal price after knowing the government subsidy and the firm s decision. Through backward induction, we can find out the optimal pricing of firm s and the firm w respectively in the sales subsidy policy is: $p_s^{1*} = \frac{1}{2}(a_s + \theta h_s - s_s^1) - \frac{b_w}{2(2 - b_s b_w)}(a_w + \theta h_w + s_w^1)$ and $p_w^{1*} = \left(\frac{1}{2} - \frac{b_s b_w}{4(2 - b_s b_w)}\right)(a_w + \theta h_w) - \frac{b_s}{4}(a_s + \theta h_s + s_s^1) - (\frac{1}{2} + \frac{b_s b_w}{4(2 - b_s b_w)})s_w^1$.

Proposition 2. For any $s_i^1 > 0$, there has $p_s^{1*} < p_s^*$ and $p_w^{1*} < p_w^*$.

Proposition 2 shows that when the sales subsidy policy is adopted, the price of firm s and the firm w is lower than that without subsidies, because $p_s^* - p_s^{1*} = \frac{1}{2}s_s^1 + \frac{b_w}{2(2 - b_s b_w)}s_w^1 > 0$ and $p_w^* - p_w^{1*} = \frac{b_s}{4}s_s^1 + (\frac{1}{2} + \frac{b_s b_w}{4(2 - b_s b_w)})s_w^1$. Subsidies will enable consumers to obtain a lower purchase price than the original one, which increase the consumer surplus. The optimal sales volume of firm s and the firm w are respectively $q_s^{1*}=\dfrac{(2-b_sb_w)(a_s+\theta h_s+s_s^1)-b_w(a_w+\theta h_w+s_w^1)}{4(1-b_sb_w)}$ and

$$q_w^{1*}=\frac{-b_s(a_s+s_s^1+\theta h_s)+(2-\dfrac{b_sb_w}{2-b_sb_w})(a_w+\theta h_w+s_w^1)}{4(1-b_sb_w)}.$$

Corollary 4 (1) When $\dfrac{s_s^1}{s_w^1}>\dfrac{b_w}{2-b_sb_w}$, $q_s^{1*}>q_s^*$;when $\dfrac{s_s^1}{s_w^1}<\dfrac{4-3b_sb_w}{b_s(2-b_sb_w)}$, $q_w^{1*}>q_w^*$;

(2) When $\dfrac{b_w}{2-b_sb_w}<\dfrac{s_s^1}{s_w^1}<\dfrac{4-3b_sb_w}{b_s(2-b_sb_w)}$, there has $q_s^{1*}>q_s^*$ and $q_w^{1*}>q_w^*$.

Corollary 4 shows that under sales subsidies, the optimal sales volume of firm s and the firm w may decrease or increase compared with the situation without subsidies, and when the proportion of firm s and the firm w subsidized by the government is between $\dfrac{b_w}{2-b_sb_w}$ and $\dfrac{4-3b_sb_w}{b_s(2-b_sb_w)}$, the profits of strong and weak firms can increase at the same time. When the proportion of government subsidizes to firm s and the firm w is bigger then $\dfrac{b_w}{2-b_sb_w}$, the sales volume under the optimal pricing of firm s with sales subsidies is higher than that without subsidies, and the profits of firm s will increase. On the contrary, profits will be lower. Similarly, when the proportion between government subsidize to firm s and the firm w is less than $\dfrac{4-3b_sb_w}{b_s(2-b_sb_w)}$, the sales volume under the optimal pricing of the firm w with sales subsidy is higher than that without subsidies, and the profits of the firm w will increase as the sales volume increases. Therefore, contrary to common sense, when the government formulates differentiated subsidy policies for firm s and the firm w based on sales volume, even if it is directly subsidized to firms, subsidies may have a negative impact on corporate profits.

Corollary 5 For any $s_i^1>0$, there has $q_s^{1*}+q_w^{1*}>q_s^*+q_w^*$.

Corollary 5 shows that when the government provides sales subsidies, the sales volume of firm s and the firm w may increase or decrease, but the total sales volume in the market is bound to increase, so the government's subsidies for new energy vehicles with the goal of maximizing social welfare can also help increase the sales volume of new energy vehicles.

Proposition 3. Under the consideration of sales subsidies, the s_s^1 and s_w^1 realizing the maximized social welfare is unequal.

Proposition 3 shows that differentiated subsidies for firms with different market shares can achieve higher subsidy efficiency and higher social welfare than non-differentiated subsidies. When the government provides sales subsidies, the government should provide differentiated subsidies to different firms according to market share and endurance. Fig. 3 shows the s_s^1 and s_w^1 of the government's optimal differentiated subsidies under the sales volume subsidy.

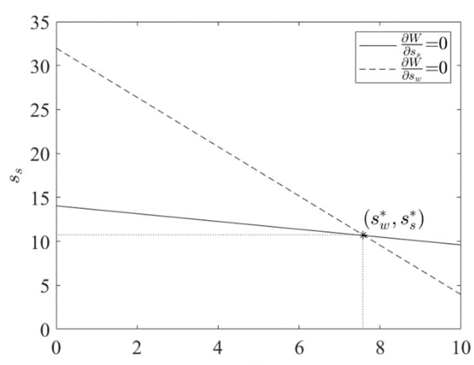

Fig. 3. Examples of government differentiated subsidy

($a_s=50$; $a_w=48$; $b_s=0.5$; $b_w=0.3$; $h_s=2$; $h_w=2$; $s_w=0-10$)

And then, we first analyze the influence of the interaction effect between firms on social welfare. As can be seen from Fig. 4, with the increasing influence of weak firms, when the government subsidizes firms with the same proportion of production capacity, the government will receive more social welfare. This indicates that when there are market share differences among firms in the market, and the market share gap is larger, the government's subsidy policy for firms is not efficient, and the government's additional subsidies cannot achieve higher efficiency. It can also be found from the figure that under the principle that when the endurance of firm s and firm w is certain and basic subsidies are guaranteed at the same time, the government should provide more subsidies to firm s to obtain higher social welfare than subsidized firm w. When firm w develop and gradually match the influence of firms, the effect of government subsidies will become better and better, and social welfare will continue to improve. Next, we analyze the impact of firm endurance on social welfare. Observation Figure 5 shows that the improvement of the endurance mileage of new energy vehicles can bring about the improvement of social welfare. When the government provides sales subsidies, even if the firm has better endurance mileage, due to the large gap in the market share of the firm, the government should provide more subsidies to the firm under the principle of guaranteeing basic subsidies, to achieve higher social welfare in a short time.

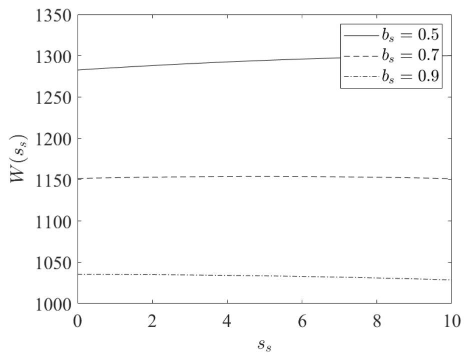

Fig. 4. The influence of firm interaction on social welfare under the sales volume subsidy policy

（$a_s = 50$； $a_w = 48$； $b_s = 0.5, 0.7, 0.9$； $b_w = 0.3$； $h_s = 1$； $h_w = 2$； $s_w = 0 - 10$； $s_s = 1$）

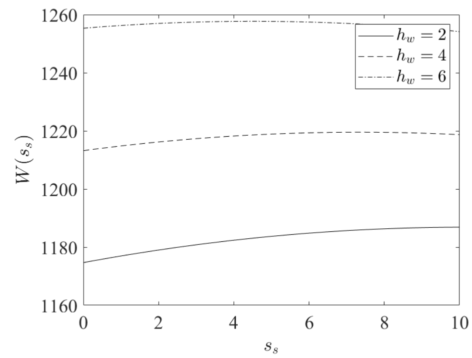

Fig. 5. The impact of endurance on social welfare under the sales volume subsidy policy

（$a_s = 50$； $a_w = 46$； $b_s = 0.6$； $b_w = 0.4$； $h_s = 4$； $h_w = 2, 4, 6$； $s_s = 0 - 10$； $s_w = 1$）

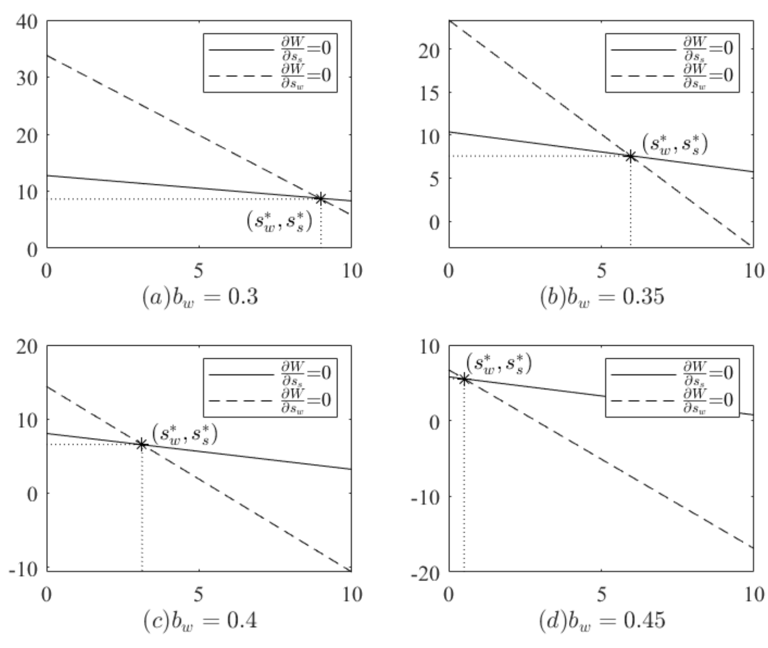

Fig. 6. The influence of inter-firm influence on optimal government subsidy

（$a_s = 50$； $a_w = 48$； $b_s = 0.5$； $b_w = 0.3, 0.35, 0.4, 0.45$； $h_s = 1$； $h_w = 2$； $s_w = 0 - 10$）

Fig. 6 further shows the specific impact of firm market share and endurance on differentiated subsidies. As can be seen from Figure 6, under the sales subsidy policy, as the influence of weak firms on strong firms increases, both weak firms and strong firms receive less subsidies, and the government's subsidies to weak firms decrease more significantly. Similarly, when the influence of strong firms on weak firms continues to expand, the subsidies received by weak firms and strong firms will also decrease, which indicates that when the influence of a certain firm in the market is significantly increased, the government can reduce the subsidies to strong and weak firms at the same time. At this time, when the strong firms have a higher market share, the government will give more subsidies to the strong firms. However, when the endurance of weak firms is good enough, the government may also give more subsidies to weak firms when the market share of strong firms is higher. As shown in Fig. 7, when the endurance of weak firms is slightly better than that of strong firms, government subsidies still favor strong firms; however, when the endurance of weak firms is good enough, the government will give more subsidies to weak firms. With the improvement of the endurance of weak firms, the subsidies for weak firms will gradually increase, while the subsidies for strong firms will decrease. Therefore, the government's differentiated subsidies are mainly decided based on the market share and the endurance of firms. No matter which firm's market share increases, the subsidies of all firms in the market will be reduced, but the improvement of the endurance of firms will make firms get more government subsidies.

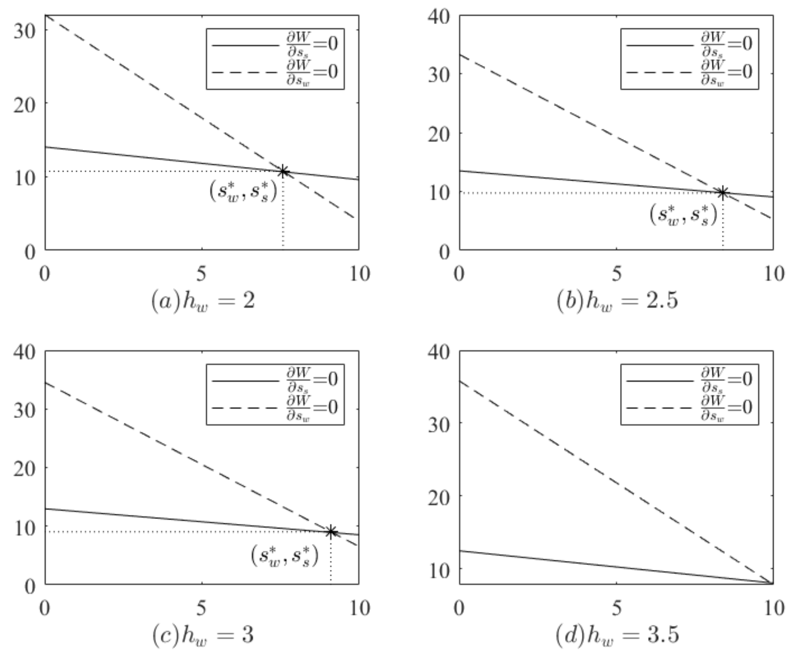

Fig.7. The influence of firm endurance on optimal government subsidy
（$a_s = 50$；$a_w = 48$；$b_s = 0.5$；$b_w = 0.3$；$h_s = 2$；$h_w = 2, 2.5, 3, 3.5$；$s_s = 0-10$）

4.2 Price-based subsidies policy for new energy vehicles

To encourage consumers to buy new energy vehicles, the government subsidizes part of the car price to consumers based on the original pricing of new energy vehicle firms, that is, to provide consumers with discounted prices to buy cars. Therefore, we assume that the government subsidizes the car price according to a certain proportion s_i^2. At this time, the capacity decision of the firm s and the firm w is expressed as:

$$q_s^2 = \frac{a_s - (p_s^2 - s_s^2) + \theta h_s - b_w(a_w - (p_w^2 - s_w^2) + \theta h_w)}{1 - b_s b_w}$$

$$q_w^2 = \frac{a_w - (p_w^2 - s_w^2) + \theta h_s - b_s(a_s - (p_s^2 - s_s^2) + \theta h_s)}{1 - b_s b_w}$$

The profits of the firm s and the firm w can be expressed as:

$$\pi_s^2 = p_s^2 \cdot q_s^2 = p_s^2 \cdot \frac{a_s - (p_s^2 - s_s^2) + \theta h_s - b_w(a_w - (p_w^2 - s_w^2) + \theta h_w)}{1 - b_s b_w}$$

$$\pi_s^2 = p_s^2 \cdot q_s^2 = p_s^2 \cdot \frac{a_s - (p_s^2 - s_s^2) + \theta h_s - b_w(a_w - (p_w^2 - s_w^2) + \theta h_w)}{1 - b_s b_w}$$

Currently, social welfare is composed of the sum of corporate profits and the sum of consumer surplus minus the sum of government subsidy expenditure, so social welfare is expressed as:

The decision order is the same as that of the sales subsidy. Through backward induction, the optimal pricing of firm s and the firm w under the price subsidy policy can be obtained as $p_s^{2*} = \frac{1}{2}(a_s + \theta h_s + s_s^2) - \frac{b_w}{2(2 - b_s b_w)}(a_w + \theta h_w + s_w^2)$ and

$$p_w^{2*} = \left(\frac{1}{2} - \frac{b_s b_w}{4(2 - b_s b_w)}\right)(a_w + \theta h_w + s_w^2) - \frac{b_s}{4}(a_s + \theta h_s + s_s^2).$$

Proposition 4. When $s_i^2 > 0$, there has $p_s^{2*} - s_s^2 < p_s^*$ and $p_w^{2*} - s_w^2 < p_w^*$.

Proposition 4 shows that when the government provides price subsidies, consumers can get a lower purchase price regardless of whether the optimal price of the firm is increased or decreased. Under the price subsidy policy, the optimal price of new energy vehicles for firm s and the firm w cannot be reduced at the same time. When $\frac{s_s^2}{s_w^2} < \frac{b_w}{2-b_s b_w}$, the price of firm s decreases, the price of firm w increases; When $\frac{s_s^2}{s_w^2} > \frac{4-3b_s b_w}{b_s(2-b_s b_w)}$, the price of firm w decreases, the price of firm s increases. However, due to the existence of price subsidies provided by the government, the final price paid by the consumer for the purchase will always be lower, because $p_s^* - (p_s^{2*} - s_s^2) > 0$ and $p_w^* - (p_w^{2*} - s_w^2) > 0$, Therefore, subsidies will ensure that consumers can get a lower purchase price regardless of whether the firm's price is raised or lowered. The optimal sales volume of firm s and the firm w are respectively $q_s^{2*} = \frac{(2-b_s b_w)(a_s + \theta h_s + s_s^2) - b_w(a_w + \theta h_w + s_w^2)}{4(1-b_s b_w)}$ and

$$q_w^{2*} = \frac{-b(a_s + \theta h_s + s_s^2) + \frac{4-3b_s b_w}{2-b_s b_w}(a_w + \theta h_w + s_w^2)}{4(1-b_s b_w)}.$$

Corollary 6. (1) When $\frac{s_s^2}{s_w^2} > \frac{b_w}{2-b_s b_w}$, $q_s^{2*} > q_s^*$; when $\frac{s_s^2}{s_w^2} < \frac{4-3b_s b_w}{b_s(2-b_s b_w)}$, $q_w^{2*} > q_w^*$;

(2) When $\frac{b_w}{2-b_s b_w} < \frac{s_s^2}{s_w^2} < \frac{4-3b_s b_w}{b_s(2-b_s b_w)}$, there has $q_s^{2*} > q_s^*$ and $q_w^{2*} > q_w^*$.

Corollary 6 shows that when the government provides price subsidies, the sales volume of firm s and the firm w may decrease or increase, and only when the proportion of firm s and firm w subsidized by the government is between $\frac{b_w}{2-b_s b_w}$ and $\frac{4-3b_s b_w}{b_s(2-b_s b_w)}$, the profits of strong and weak firms will increase at the same time. In the case of price subsidies, when the proportion of government subsidize to firm s and the firm w is greater then $\frac{b_w}{2-b_s b_w}$, the sales volume of firm s under optimal pricing is higher than that without subsidies, and the profits will increase; otherwise, the sales volume of firm s will decrease, and the profits will decrease. Similarly, when the proportion of government subsidize to firm s and the firm w is smaller then $\frac{4-3b_s b_w}{b_s(2-b_s b_w)}$, the sales volume of the firm w under optimal pricing will be higher than that without subsidies. Otherwise, firm sales and profits will decrease. Therefore, when the government formulates differentiated price subsidy policies for firm s and the firm w, the subsidy ratio may have a positive or negative impact on corporate profits. This suggests that government subsidies to consumers may make firms' profits lower, and government subsidies to consumers may reduce firms' profits.

Corollary 7. When $s_i^2 > 0$, there has $q_s^{2*} + q_w^{2*} > q_s^* + q_w^*$.

Corollary 7 shows that when the government provides price subsidies, total sales in the market increase. Government subsidies can ensure the maximization of social welfare, expand the new energy vehicle market, and contribute to the development of new energy vehicles at the same time. To maximize social welfare, underprice subsidies, differentiated subsidies can also achieve higher subsidy efficiency and achieve higher social welfare than non-differentiated subsidies. When the government provides price subsidies, differentiated subsidies can achieve higher subsidy efficiency and gain higher social welfare than non-differentiated subsidies.

4.3 Endurance-based subsidies policy for new energy vehicle

New energy vehicle endurance subsidy is a kind of subsidy s_i^3 conducted by the government to encourage new energy vehicles to continuously break through the existing endurance mileage. At this time, the profits of firm s and firm w can be expressed as:

$$\pi_1^3 = (p_s^3 + s_s^3 h_s) \cdot q_s^3 = (p_s^3 + s_s^3 h_s) \cdot \frac{(a_s + \theta h_s) - p_s^3 + b_w p_w^3 - b_w(a_w + \theta h_w)}{1 - b_s b_w}$$

$$\pi_2^3 = (p_w^3 + s_w^3 h_w)\cdot q_w^3 = (p_w^3 + s_w^3 h_w)\cdot \frac{(a_w + \theta h_w) - p_w^3 + b_s p_s^3 - b_s(a_s + \theta h_s)}{1 - b_s b_w}$$

At this time, social welfare is composed of the sum of corporate profits and the sum of consumer surplus minus the sum of government subsidy expenditure, so social welfare is expressed as:

$$W_3 = \frac{2(1-b_s b_w)}{(2-b_s b_w)}(q_s^3)^2 + \frac{1-b_s b_w}{2}(q_s^3)^2 + \frac{3}{2}(1-b_s b_w)(q_w^3)^2 - (s_s^3 h_s q_s^3 + s_w^3 h_w q_w^3)$$

The decision order is the same as that of sales subsidy and price subsidy. Through backward induction, the optimal pricing of firm s and firm w in the case of endurance subsidy can be obtained as

$$p_s^{3*} = \frac{1}{2}(a_s + \theta h_s - s_s^3 h_s) - \frac{b_w}{2(2-b_s b_w)}(a_w + \theta h_w + s_w^3 h_w) \text{ and}$$

$$p_w^{3*} = (\frac{1}{2} - \frac{b_s b_w}{4(2-b_s b_w)})(a_w + \theta h_w) - \frac{b_s}{4}(a_s + \theta h_s + s_s^3 h_s) - (\frac{1}{2} + \frac{b_s b_w}{4(2-b_s b_w)}) s_w^3 h_w$$

Proposition 5. When $s_i^3 > 0$, there has $p_s^{3*} < p_s^*$, $p_w^{3*} < p_w^*$.

Proposition 5 shows that when the endurance subsidy is provided, the price of new energy vehicles of firm s and firm w is lower than that without subsidy, because $p_s^* - p_s^{3*} = \frac{1}{2}s_s^3 h_s + \frac{b_w}{2(2-b_s b_w)} s_w^3 h_w > 0$, and $p_w^* - p_w^{3*} = \frac{b_s}{4}s_s^3 h_s + (\frac{1}{2} + \frac{b_s b_w}{4(2-b_s b_w)}) s_w^3 h_w > 0$, subsidies will enable consumers to obtain lower purchase prices without subsidy, and increase consumer surplus.

Corollary 8. p_s^{3*} and p_w^{3*} are decreasing in b_s and b_w .

Corollary 8 shows that the optimal pricing of both firms s and firm w in the case of endurance subsidies decreases with the increase of b_s and b_w , which indicates that the higher the influence of firms on competing firms, the lower the price of both firms.

The optimal sales volume of firm s and firm w is respectively

$$q_s^{3*} = \frac{(2-b_s b_w)(a_s + \theta h_s + s_s^3 h_s) - b_w(a_w + \theta h_w + s_w^3 h_w)}{4(1-b_s b_w)} \text{ and}$$

$$q_w^{3*} = \frac{-b_s(a_s + s_s^3 h_s + \theta h_s) + (2 - \frac{b_s b_w}{2-b_s b_w})(a_w + \theta h_w + s_w^3 h_w)}{4(1-b_s b_w)}$$

Corollary 9. （1）When $\frac{s_s^3 h_s}{s_w^3 h_w} > \frac{b_w}{2-b_s b_w}$, $q_s^{3*} > q_s^*$,When $\frac{s_s^3 h_s}{s_w^3 h_w} < \frac{4-3b_s b_w}{b_s(2-b_s b_w)}$, $q_w^{3*} > q_w^*$ ；（2）When $\frac{b_w}{2-b_s b_w} < \frac{s_s^3 h_s}{s_w^3 h_w} < \frac{4-3b_s b_w}{b_s(2-b_s b_w)}$,there has $q_s^{3*} > q_s^*, q_w^{3*} > q_w^*$.

Corollary 9 shows that when the endurance subsidy is provided, the sales volume of firm s and firm w may decrease or increase under this model, and only when the ratio of subsidy firm s and firm w is between $\frac{b_w}{2-b_s b_w}$ and $\frac{4-3b_s b_w}{b_s(2-b_s b_w)}$, the profits of strong and weak firms can increase at the same time. When the ratio between firm s and firm w is greater than $\frac{b_w}{2-b_s b_w}$, the sales volume of firm s under the optimal pricing of the endurance subsidy is higher than that without subsidies, and the profit increases; otherwise, the sales volume of firm s under the optimal pricing of the price subsidy is lower than that without subsidies, and the profit of firm s is lower. As long as the ratio between firm s and firm w is smaller than $\frac{4-3b_s b_w}{b_s(2-b_s b_w)}$, the sales volume of firm w under optimal pricing with price subsidy is higher than that without subsidy, and the profit increases; otherwise, the sales volume of firm w under optimal pricing with price subsidy is higher than that without subsidy, and the profit decreases. Therefore, when the government formulates differentiated subsidy policies for firm s and firm w, the difference in subsidy ratio may have a positive or negative impact on the profits of firms. This indicates that while the government completes the subsidy target and improves social welfare, it may also lose the benefits of firm s and firm w even if it directly subsidizes firms.

Corollary 10. When $s_i^3 > 0$, there has $q_s^{3*} + q_w^{3*} > q_s^* + q_w^*$.

Corollary 10 shows that when the government provides endurance subsidies, the total sales of new energy vehicles in the market will increase. Therefore, even if the government's goal is to maximize social welfare, subsidies can also expand the new energy vehicle market and contribute to the development of new energy vehicles at this time.

Under endurance subsidies, differentiated subsidies can also achieve higher subsidy efficiency and achieve higher social welfare than non-differentiated subsidies. We analyze the impact of firm endurance on social welfare. Fig. 8 shows that with the improvement of the endurance mileage of new energy vehicles, if the subsidy is also relatively increased, social welfare will become lower. Therefore, when the endurance mileage of new energy vehicles is higher, the proportion of endurance subsidy with less government subsidy can bring higher social welfare, and government subsidy should follow the principle of diminishing marginal endurance subsidy.

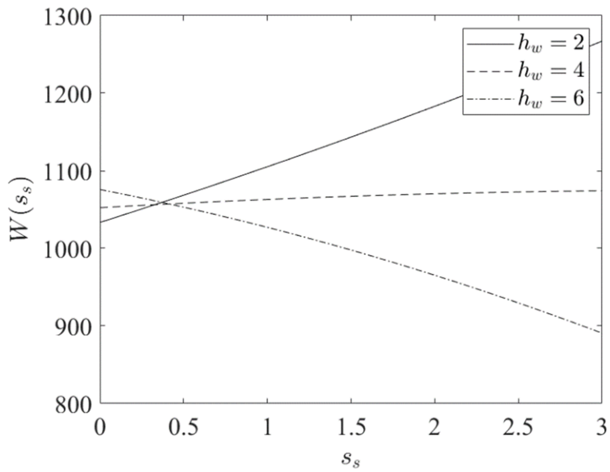

Fig. 8. The impact of endurance on social welfare under the policy of endurance subsidy
（$a_s = 50$ ； $a_w = 40$ ； $b_s = 0.7$ ； $b_w = 0.4$ ； $h_s = 4$ ； $h_w = 2,4,6$ ； $s_s = 0-3$ ； $s_w = 0.5$）

5. Analysis and discussion

In Section 3 and Section 4, we calculate the optimal differentiated subsidy design of the government without considering subsidies and the optimal pricing decision of strong and weak firms when considering three subsidy modes respectively. In this section, the above four cases will be comprehensively and comparatively analyzed (0: without considering subsidies; 1: Consider sales subsidies; 2: Consider price subsidies; 3: Consider endurance subsidy) for the firm's optimal pricing, sales volume, profit and overall social welfare.

5.1 Analysis of firms' optimal pricing decisions

Proposition 6. When $s_i^1 = s_i^2 = s_i^3 h_i$,there has $p_i^* > p_i^{1*} = p_i^{2*} - s_i^2 = p_i^{3*}$.

Proposition 6 shows that when the government gives the same unit subsidy ratio to each subsidy model, in the case of sales subsidies and endurance subsidies, the pricing of new energy vehicle firms will be lower than that without subsidies, but in the case of price subsidies, the optimal pricing of new energy vehicles will be higher than that without subsidies. After taking the government's price subsidies into account, the price of new energy vehicles will be higher than that without subsidies. Consumers still receive lower prices for new energy vehicles than those without subsidies. In addition, in the case of the same proportion of unit subsidies, the three subsidy models will ultimately provide consumers with the same price of new energy vehicles. So, no matter what kind of subsidy model it provides, the final price consumer needs to pay less than without the subsidy policy, and consumers are happy to have the subsidy mechanism. The pricing, sales and profits of new energy vehicle firms under different subsidy models may be more or less than without subsidies, but for consumers, no matter what kind of subsidy model, the car purchase price that consumers need to bear is bound to decrease, which indicates that whether it is subsidized consumers or subsidized firms, the final consumers benefit from subsidies and get lower payment prices.

5.2 Analysis of firms' optimal sales and profits

Proposition 7. （1）When $s_i^1 = s_i^2 = s_i^3 h_i$,there has $q_i^{1*} = q_i^{2*} = q_i^{3*}$,and there is a critical value $\frac{b_w}{2 - b_s b_w}$ and $\frac{4 - 3b_s b_w}{b_s(2 - b_s b_w)}$,when

$\frac{s_s^1}{s_w^1}=\frac{s_s^2}{s_w^2}=\frac{s_s^3h_s}{s_w^3h_w}>\frac{b_w}{2-b_sb_w}$, $q_s^{1*}=q_s^{2*}=q_s^{3*}>q_s^*$, otherwise, $q_s^{1*}=q_s^{2*}=q_s^{3*}<q_s^*$;when $\frac{s_s^1}{s_w^1}=\frac{s_s^2}{s_w^2}=\frac{s_s^3h_s}{s_w^3h_w}<\frac{4-3b_sb_w}{b_s(2-b_sb_w)}$, $q_w^{1*}=q_w^{2*}=q_w^{3*}>q_w^*$. Otherwise, $q_w^{1*}=q_w^{2*}=q_w^{3*}<q_w^*$.

（2）For any $s_i^{1,2,3}>0$,there has $\sum q_i^{1*}=\sum q_i^{2*}=\sum q_i^{3*}>\sum q_i^*$.

Proposition 7 (1) shows that when the government gives the same unit subsidy ratio to each subsidy model, the sales volume caused by the optimal price of the new energy vehicle firms after the sales volume subsidy, price subsidy and endurance subsidy are the same. When the ratio of government subsidies to firm s and firm w is greater than $\frac{b_w}{2-b_sb_w}$, the optimal sales volume of firms under the three subsidy modes is larger than that without subsidies, and the profits are increased; otherwise, the profits are reduced. When the ratio of government subsidy to firm s and firm w is less than $\frac{4-3b_sb_w}{b_s(2-b_sb_w)}$, the optimal sales volume of firm w under the three subsidy modes is larger than that without subsidy, and the profit increases; otherwise, the profit decreases. Therefore, when the ratio of government subsidy to firm s and firm w is $\frac{b_w}{2-b_sb_w}<\frac{s_s^1}{s_w^1}=\frac{s_s^2}{s_w^2}=\frac{s_s^3h_s}{s_w^3h_w}<\frac{4-3b_sb_w}{b_s(2-b_sb_w)}$, the sales volume of firm s and firm w will increase at the same time regardless of the subsidy mode. Profits have risen accordingly. This indicates that when the government aims to maximize social welfare, although the optimal subsidies are directly given to firms or consumers, the government subsidies may still damage the profits of firms.

Proposition 7 (2) shows that when the government provides subsidies, the total sales volume of new energy vehicles in the market will be more than that without subsidies. Therefore, even if the government subsidies cannot ensure that the sales volume of every firm will increase, it can expand the new energy vehicle market.

5.3 Analysis of social welfare

Proposition 8. （1）When $s_i^1=s_i^2$,there has $q_i^{1*}=q_i^{2*}$, $\pi_i^1=\pi_i^2$, therefore $W_1=W_2$;

（2）When $s_i^1=s_i^3h_i$,there has $q_i^{1*}=q_i^{3*}$, $\pi_i^1=\pi_i^3$, therefore $W_1=W_3$.

Proposition 8 (1) indicates that when the subsidy s_i^1 for the sale of a car is equal to the price subsidy s_i^2 for each car, under the two subsidy models, the optimal sales volume that maximizes social welfare is equal, and then the corporate profits and consumer surplus are also equal, and the social welfare is finally equal. This shows that the essence of both sales subsidies and price subsidies is the same, that is, sales subsidies and price subsidies can achieve the same effect of improving corporate profits and social welfare.

Proposition 8 (2) indicates that when the subsidy s_i^1 for each vehicle sold is equal to the subsidy $s_i^3h_i$ for each vehicle based on mileage, the optimal sales volume that maximizes social welfare under the two subsidy models is also equal, and the social welfare is finally equal. Therefore, the optimal subsidy s_i^1 and $s_i^3h_i$ that maximize social welfare are equal. At this time, it will be found that when the endurance mileage of new energy vehicles is larger, the proportion of the required endurance subsidy s_i^3 is smaller. Therefore, when the endurance mileage is improved, the government needs to provide less unit endurance subsidies, which is consistent with the reality of the stepped endurance subsidy policy.

6. Conclusion

This paper mainly studies how the government considers the difference in market share and endurance among different new energy automobile firms to design differentiated subsidy policies. Based on the Stackelberg game model, this paper first establishes the optimal pricing model of strong and weak firms without considering government subsidies as the benchmark model and uses the backward induction method to solve the optimal pricing decision of strong and weak firms to maximize their respective profits. Subsequently, this paper studies the optimal pricing decisions of strong and weak firms under government subsidies, aiming at the three common subsidy modes of sales subsidy, price subsidy and endurance subsidy respectively, and further solves the optimal differentiated subsidy policy design of the government. Finally, this paper also compares the changes in the optimal pricing, sales, and social welfare of strong and weak firms without considering subsidies and when considering the above three subsidy models. The main conclusions are as follows:

(1) For new energy vehicle firms with different market shares and endurance, differentiated subsidies provided by the government can achieve higher social welfare than non-differentiated subsidies. In addition, through numerical analysis, this paper finds that when the strong and weak firms have the same endurance and the strong firms have a higher market

share, the government should provide more subsidies to the strong firms to make them better play the advantages of market share. However, when the endurance of weak firms is excellent enough, although the strong firms have a higher market share, the government should also provide higher subsidies for weak firms, so the study of differentiated subsidies is conducive to the high-quality development of new energy automobile firms.

(2) For the three subsidy models of sales subsidies, price subsidies and endurance subsidies, the final cost paid by consumers is lower than without subsidies, but the profits of firms may increase or decrease, depending on the proportion of subsidies provided by the government to different firms.

(3) No matter what kind of new energy new car subsidy mode is provided by the government, the sales volume of a single firm in the market may increase or decrease after subsidies, depending on the proportion of subsidies provided by the government to different firms, but the total sales volume of new energy vehicles in the market will definitely increase. Therefore, from the perspective of the overall new energy vehicle market, government subsidies can effectively broaden the new energy vehicle market.

(4) Under the assumption of this paper, the firm optimal pricing under the three subsidy models of sales subsidy, price subsidy and endurance subsidy, and the firm sales volume corresponding to the optimal price and the total social welfare are the same, that is, the total subsidy amount and implementation effect of the three subsidy models are equivalent under the optimal state. The difference between the three subsidies is mainly reflected in the fact that the optimal pricing of new energy vehicles under the price subsidy model will be higher, but after considering the price subsidies provided by the government to consumers, the final cost paid by consumers is consistent with the price under the sales subsidies and endurance subsidies.

The research conclusions of this paper can provide theoretical guidance and decision-making basis for the government to design more efficient subsidy policies for new energy vehicles. In the future, this study can further consider the cost invested by firms to improve their own endurance and explore the incentive problem of endurance subsidies for firms to improve the endurance of new energy vehicles. What's more, in addition to the three subsidy models considered in this paper, such as sales subsidies, price subsidies and endurance subsidies, there are other subsidy models in reality, such as replacement of new energy vehicle subsidies, charging station infrastructure subsidies and local green plate subsidy policies, which can be further studied.

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