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# BHARAT: A simple and effective multi-criteria decision-making method that does not need fuzzy logic, Part-2: Role in multi- and many-objective optimization problems

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A B S T R A C T

A simple and effective multi-attribute decision-making method, named as BHARAT method, is proposed in Part-1 of this paper and the same method is used now as a multi- and many-objective decision-making method for evaluating the Pareto optimal solutions. The proposed BHARAT method is used to identify the best compromise Pareto solution. Based on their importance for the given optimization problem, the objectives are ranked, and the weights are assigned. The weights of the objectives and the normalized values of the objectives for different Pareto optimal solutions are used to compute the total scores. The total scores are used to differentiate the alternative optimal solutions and an alternative solution that gets the highest total score is suggested as the best compromise solution. Three case studies are presented to illustrate and validate the proposed BHARAT method. The case study 1 is a multi-objective optimization problem related to cloud manufacturing with 3 objectives and 20 alternative solutions; case study 2 is a many-objective optimization problem of electro-discharge machining process with 4 objectives and 50 alternative solutions; case study 3 is a many-objective optimization of milling process parameters with 4 objectives and 100 alternative solutions. The outcomes of the suggested BHARAT method are compared with those of the other popular decision-making approaches for each of the three case studies considered. The suggested simple and more logical BHARAT method can be used in multi- and many-objective optimization problems to select the best compromise solution.



## 1. Introduction

Making decisions in the presence of several conflicting criteria is referred to as multi-criteria decision-making (MCDM). The two categories of MCDM problems are: (1) multi-objective decision-making (MODM) and (2) multi-attribute decision-making (MADM) (Rao, 2013). In MODM problems, a set of objective functions must be maximized or minimized within a domain containing acceptable variable values while meeting certain constraints. Many sets of variables that satisfy the stated constraints and maximize or minimize the objective functions may exist in the domain. The values of the corresponding objective functions make the Pareto front, and the trade-offs make the Pareto optimal set (Deb, 2001). There can be any number of objectives in the MODM problems. Many-objective optimization problems are MODM problems that have four or more objectives. The following provides an illustration of a MODM problem.

Maximize $f_1 = 3x_1+0.85x_2+1.56x_3+ 0.45x_4$
Minimize $f_2 = 0.73x_1+5x_2+2x_3+0.92x_4$
Subject to constraints: $x_1+x_2+1.2x_3+x_4 \leq 175$; $x_1+ 0.15x_2+x_3+0.3x_4 \leq 200$; $3x_1+1.5x_2+x_4 \leq 120$
Bounds of variables: $0\leq x_1\leq 40$, $0\leq x_2\leq 30$, $0\leq x_3\leq 46$, $0\leq x_4\leq 17$.

The objective functions $f1$ and $f2$ and the constraints are described in terms of the decision variables $x_1$, $x_2$, $x_3$ and $x_4$. Both the constraints and the objective functions can generally be either linear or non-linear. The best option should meet the decision maker's preferences and constraints. In a continuous or integer domain, the decision variable values are chosen from an infinite

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or large number of choices. The MODM problems are solved by algorithms such as the multi-objective genetic algorithm (MOGA), non-dominated sorting genetic algorithm (NSGA)-II or NSGA-III, strength pareto evolutionary algorithm 2 (SPEA-2), multi-objective versions of particle swarm optimization (PSO), differential evolution (DE), artificial bee colony (ABC), ant colony optimization (ACO), teaching-learning-based optimization (TLBO), Jaya algorithm, etc.

Recently, a few researchers have started using the fuzzy logic for expressing the values of decision variables and the objective functions. However, these may be called as heedless attempts to apply fuzzy techniques anywhere there are numerical values, without scrutinizing the legitimacy of the methodology. Certain procedures view all numbers as amenable to fuzzy logic. When we proceed in that manner without questioning why, the modeling effort may turn into a misguided intellectual diversion intended only for publication, with little regard for the accuracy of our work. Many journal editors and reviewers approved the articles for publication without questioning the validity of the results. The idea that it is preferable to fuzzify the objective functions and the decision variables is, in fact, unsupported by data or mathematical reasoning.

All solutions in the Pareto front are non-dominated and hence these can be regarded as equivalent once a set of them is found. For every solution, there is a different trade-off between the objectives. Therefore, the best solution can be defined as the one that achieves the best compromise between the objectives. To find the best compromise, researchers have therefore been utilizing various MADM methods such as AHP, TOPSIS, GRA, PROMETHEE, WPM, etc. for the past few years. These methods have shown to be effective in a variety of decision-making scenarios. However, as Part-1 of this paper explains, each of these methods has advantages and disadvantages of its own. The recently proposed R-method (Rao and Lakshmi, 2021a, 2021b) developed an equation and a table that can be used to assign ranks to the objectives and the alternatives and then the ranks can be converted into corresponding weights. The weights proposed by the R-method are comparatively more stable and meaningful than the other ranking methods. However, even though R-method has simplified the decision-making process, assigning the ranks to the available quantitative values of objectives for different alternatives may not be much appropriate. It is because, if two alternative solutions have very close data for a particular objective, then they are assigned ranks 1 and 2 as per the R-method and the corresponding weights are assigned using a table developed for the purpose. This may make some difference in evaluation of alternatives, even though the values of the objective for the two alternatives do not differ significantly. To avoid this, a new MADM method named as BHARAT is presented in Part-1 of the paper. The BHARAT method uses R-method for assigning the ranks and weights to the objectives but not to the alternatives.

The next section describes how to make use of BHARAT methodology in multi- and many-objective optimization problems to evaluate the Pareto optimal solutions.

## 2. Proposed BHARAT methodology in multi- or many-objective optimization to evaluate the Pareto optimal solutions

### *2.1 BHARAT*

In MODM methods, every decision table includes alternative solutions (which are called non-dominated solutions), objectives (just like attributes in MADM problems), performance measures of the objectives for different alternative solutions and the weights of the objectives. The decision maker's job is to evaluate each alternative and determine which is the best option based on the information in the decision table and the chosen decision-making technique. The steps of the proposed decision-making methodology using BHARAT are explained below.

**Step 1:** Identify the multi- or many-objective decision-making situation, pertinent objectives $A_i$ (i = 1, 2,….,m) and the alternative solutions $B_j$ (for j = 1, 2,…..,n). The pertinent objectives include both beneficial and non-beneficial objectives. The beneficial objectives are those whose higher values are desirable, and non-beneficial objectives are those whose lower values are desirable. Create a decision table with the non-dominated alternative solutions obtained by using any advanced multi-objective optimization algorithm.

**Step 2:** To determine the objectives' weights wi (for i=1, 2,..., m), order the objectives according to the decision-maker's assessment of their significance in terms of 1, 2, 3, 4, and so on. When two or more objectives are deemed to be equally significant, they are given an average rank. The R-method, which the author recently proposed (Rao and Lakshmi, 2021a, b), is used in the proposed BHARAT method. Table A1 or Eq. (1) given in Part-1 of this paper can be used for assigning the weights to the objectives. In a group decision-making scenario, compute the average value of the ranks given by the decision-makers for each objective as explained in Part-1 of the paper.

**Step 3:** Normalize the data of an objective for different alternative solutions with reference to the "best" value of the objective Repeat this normalization procedure for all the objectives to get the normalized data. The word 'best' indicates the highest available value of the beneficial objective or the lowest available value of the non-beneficial objective. The performance measures of alternatives $x_{ji}$ (for $j$= 1, 2,…..,$n$; $i$=1, 2,…..,$m$) are to be normalized. The normalized value $(x_{ji})_{\text{normalized}}$ of an alternative corresponding to a beneficial objective is $x_{ji}/x_{ji.best}$, and it is $x_{ji.best}/x_{ji}$ in the case of non-beneficial objective. The $x_{ji.best}$ is the best value of $i^{th}$ objective for $j^{th}$ alternative. . This type of normalizing the data with reference to the "best" values clearly shows the standing positions of the alternative solutions with reference to the "best" values of the objectives.

**Step 4:** Total score of an alternative solution is computed by $\sum wi^*(x_{ji})_{normalized}$ which is the result of multiplying the normalized data of the objectives for the alternative solutions with the corresponding weights of the objectives.
**Step 5:** Sort the alternative solutions according to the total scores in decreasing order. For the particular decision-making scenario under consideration, the alternative solution with the highest total score is deemed optimal. The decision-maker can implement this best compromise optimal solution.

*2.2 Second version of BHARAT*

Another version of BHARAT is proposed in Part-1 of this paper and the *steps of this version are same as the steps of BHARAT except step-2*. In this version, the objectives are ranked in terms of 1, 2, 3, 4, etc., according to the decision-maker's assessment of their relative importance in determining the weights of the objectives. When two or more objectives are deemed to be equally significant, they are given an average rank. Table A2 or Eq. (2) or the appropriate procedure suggested to form Table 14 and Table 21 given in Part-1 of this paper can be used for assigning the weights to the objectives. In group decision-making scenario, compute the average value of the ranks given by the decision-makers for each objective as explained in Part-1 of the paper.

The next section demonstrates the applications of the proposed BHARAT method to three industrial case studies.

## 3. Demonstration of applications of the proposed BHARAT method for evaluating the Pareto optimal solutions

This section provides three case studies to illustrate the proposed BHARAT method and compare its performance with the other widely used MADM methods.

*3.1 Case study 1: Multi-objective optimization of production planning in cloud manufacturing*

Li et al. (2019) presented a multi-objective optimization case study that was related a Company A that received an order for the manufacturing and processing of automobiles. Owing to limited production capacity and elevated production costs, certain production duties were delegated to certain manufacturers via the cloud manufacturing platform. Using the modularization theory, the company broke down the automobile processing order and determined that three distinct types of modules—designated as Modules 1, 2, and 3—need to be produced and processed. Following that, the cloud manufacturing platform received specific module requirements from the company. Some candidate resources could be processed, some could not, and some could only be partially processed due to the wide variations in the module's functional requirements and type. The Company A's production planning for an order must take into account the unique demands of the customers as well as the company's and the manufacturers' manufacturing resources and capabilities in the cloud manufacturing platform. The optimization of production planning process was performed using NSGA-II based on the decision-makers' preferences. The detailed procedure is displayed in Fig. 1.

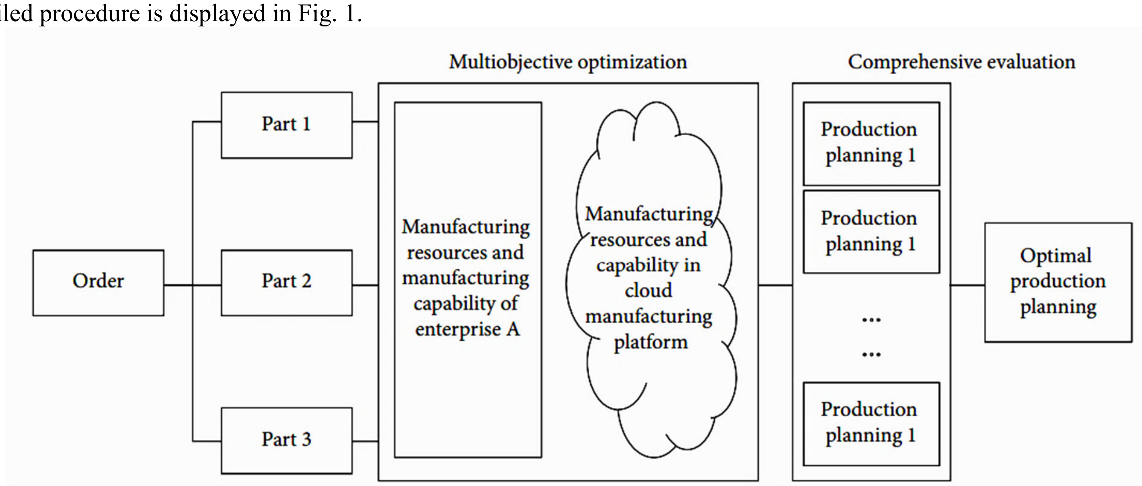


**Fig. 1.** Process of Production planning and optimization of Company A (Li et al., 2019)

After the processing module, the company A still needs to assemble the parts to form the product in the order for delivery, in compliance with the project order's requirements for quantity, delivery time, and product quality. As a result, company A adjusts the order's specific requirements to suit its own processing conditions. Twenty schemes of non-dominated solutions produced by NSGA-II were included in the calculation results, and each scheme of non-dominated solutions corresponds to a production planning. Table 1 displays the production planning schemes containing the non-dominating solutions derived from NSGA-II.

**Table 1**

Production planning schemes containing the non-dominating solutions derived from NSGA-II for case study 1

| *Alternative schemes* | *Modules* | *Company A* | *Cloud manufacturer* | | | | | *Product cost* | *Delivery time* | *Product quality (%)* |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | *1* | *2* | *3* | *4* | *5* | | | |
| Scheme 1 | Module 1 | 251 | 1570 | — | 715 | 256 | 208 | | | |
| | Module 2 | — | 17 | 926 | 1150 | 549 | 858 | 13581 | 58053 | 87.14 |
| | Module 3 | 285 | 908 | 1892 | 172 | — | 773 | | | |
| Scheme 2 | Module 1 | 254 | 1571 | — | 715 | 255 | 205 | | | |
| | Module 2 | — | 17 | 926 | 1453 | 169 | 935 | 14155 | 55959 | 89.06 |
| | Module 3 | 1908 | 16 | 1863 | 174 | — | 39 | | | |
| Scheme 3 | Module 1 | 254 | 21 | — | 715 | 2556 | 1754 | | | |
| | Module 2 | — | 17 | 926 | 1455 | 170 | 932 | 13453 | 62239 | 85.98 |
| | Module 3 | 285 | 907 | 1863 | 172 | — | 773 | | | |
| Scheme 4 | Module 1 | 254 | 1571 | — | 715 | 256 | 204 | | | |
| | Module 2 | — | 1852 | 258 | 1150 | 170 | 70 | 14265 | 61662 | 90.38 |
| | Module 3 | 1908 | 16 | 1863 | 175 | — | 38 | | | |
| Scheme 5 | Module 1 | 254 | 1570 | — | 715 | 256 | 205 | | | |
| | Module 2 | — | 773 | 328 | 1454 | 169 | 776 | 13915 | 59417 | 87.71 |
| | Module 3 | 285 | 907 | 1863 | 172 | — | 773 | | | |
| Scheme 6 | Module 1 | 19 | 21 | — | 715 | 735 | 1510 | | | |
| | Module 2 | — | 1852 | 328 | 1150 | 170 | — | 14038 | 67404 | 88.90 |
| | Module 3 | 1908 | 16 | 1863 | 172 | — | 41 | | | |
| Scheme 7 | Module 1 | 251 | 1570 | 0 | 716 | 256 | 207 | | | |
| | Module 2 | — | 773 | 928 | 1453 | 169 | 177 | 13751 | 60622 | 87.88 |
| | Module 3 | 285 | 907 | 1863 | 172 | — | 773 | | | |
| Scheme 8 | Module 1 | 19 | 21 | 0 | 715 | 255 | 1990 | | | |
| | Module 2 | — | 1852 | 328 | 1150 | 170 | — | 13523 | 68547 | 87.10 |
| | Module 3 | 285 | 907 | 1862 | 174 | — | 772 | | | |
| Scheme 9 | Module 1 | 254 | 1570 | — | 715 | 256 | 205 | | | |
| | Module 2 | — | 773 | 925 | 1150 | 169 | 483 | 13846 | 60305 | 87.65 |
| | Module 3 | 286 | 907 | 1863 | 173 | — | 771 | | | |
| Scheme 10 | Module 1 | 254 | 1571 | — | 716 | 256 | 203 | | | |
| | Module 2 | — | 1852 | 328 | 1150 | 170 | — | 14246 | 61801 | 90.41 |
| | Module 3 | 1909 | 18 | 1863 | 175 | — | 35 | | | |
| Scheme 11 | Module 1 | 254 | 1570 | — | 715 | 256 | 205 | | | |
| | Module 2 | — | 17 | 926 | 1453 | 170 | 934 | 14155 | 55964 | 89.06 |
| | Module 3 | 1909 | 15 | 1863 | 174 | — | 39 | | | |
| Scheme 12 | Module 1 | 19 | 21 | — | 715 | 735 | 1510 | | | |
| | Module 2 | — | 17 | 928 | 1455 | 549 | 551 | 13312 | 63953 | 85.88 |
| | Module 3 | 284 | 908 | 1862 | 172 | — | 774 | | | |
| Scheme 13 | Module 1 | 19 | 21 | — | 715 | 256 | 1989 | | | |
| | Module 2 | — | 17 | 928 | 1454 | 170 | 931 | 13431 | 62712 | 85.76 |
| | Module 3 | 285 | 907 | 1863 | 172 | — | 773 | | | |
| Scheme 14 | Module 1 | 19 | 21 | — | 715 | 735 | 151 | | | |
| | Module 2 | — | 1852 | 328 | 1150 | 170 | — | 13486 | 69030 | 87.01 |
| | Module 3 | 284 | 908 | 1862 | 172 | — | 774 | | | |
| Scheme 15 | Module 1 | 254 | 21 | — | 173 | 256 | 1753 | | | |
| | Module 2 | — | 18 | 928 | 1455 | 549 | 550 | 13371 | 63004 | 86.20 |
| | Module 3 | 285 | 907 | 1863 | 716 | — | 772 | | | |
| Scheme 16 | Module 1 | 254 | 1571 | — | 715 | 255 | 205 | | | |
| | Module 2 | — | 774 | 258 | 1368 | 169 | 931 | 14494 | 57565 | 89.52 |
| | Module 3 | 1909 | 16 | 1863 | 174 | — | 38 | | | |
| Scheme 17 | Module 1 | 254 | 1570 | — | 715 | 256 | 205 | | | |
| | Module 2 | 0 | 1852 | 328 | 1150 | 170 | — | 13695 | 63430 | 88.51 |
| | Module 3 | 284 | 907 | 1863 | 174 | — | 772 | | | |
| Scheme 18 | Module 1 | 253 | 1571 | — | 715 | 255 | 206 | | | |
| | Module 2 | — | 18 | 926 | 1453 | 548 | 555 | 14074 | 56722 | 89.28 |
| | Module 3 | 1909 | 16 | 1863 | 174 | — | 38 | | | |
| Scheme 19 | Module 1 | 19 | 21 | — | 715 | 735 | 1510 | | | |
| | Module 2 | — | 1852 | 328 | 1150 | 170 | — | 14038 | 67404 | 88.91 |
| | Module 3 | 1908 | 16 | 1863 | 172 | — | 41 | | | |
| Scheme 20 | Module 1 | 20 | 21 | — | 715 | 735 | 1509 | | | |
| | Module 2 | — | 18 | 928 | 1455 | 549 | 550 | 13313 | 63594 | 85.89 |
| | Module 3 | 285 | 908 | 1862 | 172 | — | 773 | | | |

The steps of the proposed BHARAT method are now followed as outlined below to determine which of the 20 alternative schemes is the best solution.

Step 1: There are 3 objectives (i.e., product cost, delivery time, and product quality) and it is a multi-objective decision-making situation. The 20 non-dominated alternative schemes obtained by NSGA-II algorithm for different schemes are shown in Table 1. The units of product cost and delivery time were not mentioned by Li et al. (2019). The objectives and lower values are desirable. Product quality is a beneficial objective, and product cost and delivery time are non-beneficial objectives.

Step 2: To determine the weights of the objectives wi (for i=1, 2, 3), the objectives are ranked according to the decision-maker's assessment of their significance in terms of 1, 2, and 3. Li et al. (2019) considered AHP weights, entropy weights, and the combined weights of the objectives. The AHP weights for the product cost, delivery time, and the product quality were 0.369841, 0.297884, and 0.332275 respectively. The corresponding entropy weights were 0.246654, 0.303342, and 0.450004; and the corresponding combined weights were 0.275508, 0.272903, and 0.451589. In the present work, rank 1 is given to product quality, rank 2 is given to product cost, and rank 3 is given to the product quality. Using the BHARAT methodology and Table A1 given in Part-1 of this paper, the weights of 0.30137, 0.24658, and 0.45205 are assigned to the product cost, delivery time, and product quality respectively.

Step 3: The data of the objectives for different alternative schemes is normalized with reference to the objectives' "best" values. The values of product quality are normalized as $x_{ji}/x_{ji.best}$, and the objectives of product cost and delivery time are normalized as $x_{ji.best}/x_{ji}$. This type of normalizing the data with reference to the "best" values clearly show the standing positions of the alternative schemes with reference to the "best" values of the objectives. The normalized data of the objectives is shown in Table 2.

**Table 2**
Normalized values of the objectives and the total scores of the alternative schemes for case study 1

| *Alternative schemes* | *Normalized values of the objectives* | | | *Total scores using BHARAT* | *Ranks of alternative schemes given by BHARAT* |
|---|---|---|---|---|---|
| | *Product cost* | *Delivery time* | *Product quality* | | |
| Scheme 1 | 0.980193 | 0.96393 | 0.963831 | 0.968786 | 5 |
| Scheme 2 | 0.940445 | 1 | 0.985068 | 0.975302 | **1** |
| Scheme 3 | 0.989519 | 0.899099 | 0.951001 | 0.949811 | 14 |
| Scheme 4 | 0.933193 | 0.907512 | 0.999668 | 0.956911 | 10 |
| Scheme 5 | 0.956665 | 0.941801 | 0.970136 | 0.95909 | 7 |
| Scheme 6 | 0.948283 | 0.830203 | 0.983298 | 0.934996 | 19 |
| Scheme 7 | 0.968075 | 0.923081 | 0.972016 | 0.958762 | 8 |
| Scheme 8 | 0.984397 | 0.81636 | 0.963389 | 0.96972 | 4 |
| Scheme 9 | 0.961433 | 0.927933 | 0.969472 | 0.956807 | 11 |
| Scheme 10 | 0.934438 | 0.905471 | 1 | 0.956932 | 9 |
| Scheme 11 | 0.940445 | 0.999911 | 0.985068 | 0.97528 | **2** |
| Scheme 12 | 1 | 0.875002 | 0.949895 | 0.946528 | 17 |
| Scheme 13 | 0.99114 | 0.892317 | 0.948568 | 0.947527 | 16 |
| Scheme 14 | 0.987098 | 0.810648 | 0.962394 | 0.932421 | 20 |
| Scheme 15 | 0.995587 | 0.888182 | 0.953434 | 0.950048 | 13 |
| Scheme 16 | 0.918449 | 0.972101 | 0.990156 | 0.964094 | 6 |
| Scheme 17 | 0.972034 | 0.882217 | 0.978985 | 0.953029 | 12 |
| Scheme 18 | 0.945858 | 0.986548 | 0.987501 | 0.974716 | **3** |
| Scheme 19 | 0.948283 | 0.830203 | 0.983409 | 0.935046 | 18 |
| Scheme 20 | 0.999925 | 0.879942 | 0.950006 | 0.947773 | 15 |

Step 4: The weights of the objectives and the corresponding normalized data of the objectives for each of the 20 alternative schemes are multiplied to get the total scores of the alternative schemes. Table 2 also displays the total scores and the ranks of the alternative schemes.

Step 5: For the particular decision-making scenario under consideration, alternative scheme 2 has the highest total score. The second, third, and last choices are scheme 11, scheme 18, and scheme 14 respectively. The ranking of schemes is: 2-11-18-8-1-16-5-…….-14. Therefore, the decision-maker can select scheme 2 with the following details given in Table 3.

**Table 3**
Optimal production planning scheme suggested by BHARAT method for case study 1

| *Alternative schemes* | *Modules* | *Company A* | *Cloud manufacturer* | | | | | *Product cost* | *Delivery time* | *Product quality (%)* |
|---|---|---|---|---|---|---|---|---|---|---|
| Scheme 2 | Module 1 | 254 | 1571 | — | 715 | 255 | 205 | 14155 | 55959 | 89.06 |
| | Module 2 | — | 17 | 926 | 1453 | 169 | 935 | | | |
| | Module 3 | 1908 | 16 | 1863 | 174 | — | 39 | | | |

Li et al. (2019) used the combined weights in TOPSIS method (i.e., 0.275508, 0.272903, and 0.451589 for product cost, delivery time, and product quality respectively). These weights are also considered now by the BHARAT method for comparison purpose and applied to the same case study's normalized data of the objectives given in Table 2. It is observed that the ranking obtained by using the combined weights in BHARAT method also led to the same ranking of alternative schemes as given by BHARAT. It is because of the reason that the difference between the combined weights used in TOPSIS method are almost same as the weights used by BHARAT method in the computation of total scores. In fact, Li et al. (2019) carried out the AHP calculations and the TOPSIS calculations incorrectly. For example, the AHP matrix given in Eq. (31) of their paper is highly inconsistent and the consistency ratio is about 0.69 which is much more than the allowed value of 0.10. Hence AHP weights calculated by them were incorrect. Using such incorrect weights and the entropy weights, Li et al. (2019) calculated the combined weights and carried out the TOPSIS calculations which were, again, incorrect. Hence, the rankings given by them to the schemes as 3-13-20-15-12-….-10 is incorrect. Comparing the data of scheme 2 with the data of scheme 3, it is very clear that scheme 2 is better than scheme 3 in the case of delivery time and product quality whose combined weightage is 0.6986 (i.e., 69.86%). Hence, proposing scheme 2 as the first choice by BHARAT is logical. With the same logic, it can be said that proposing scheme 11 as second choice by BHARAT is more logical compared to scheme 13 proposed by Li et al. (2019) using TOPSIS method. Similarly, the differences in the other ranks of the schemes can be explained.

*Second version of BHARAT method for case study 1*

Second version of BHARAT method differs from BHARAT only in Step 2. The ranks assigned to the objectives are same as those given in BHARAT. Using Table A2 or Eq. (2) of Part-1 of the paper, the weights are assigned as 0.2727, 0.1818, and 0.5454 to product cost, delivery time, and product quality respectively. Using these weights, and the normalized data of the objectives, the total scores are computed which gives the ranking as 18-2-11-…..14. Here the scheme 18 is taking first place, scheme 2 is taking second place, scheme 11 is taking third place, and scheme 14 is taking the last place. It may be noted that the weights used in the second version of BHARAT are considerably different from the weights considered in BHARAT and hence the difference is there in ranking.

### 3.2 Case study 2: Many-objective optimization of electro-discharge machining process

Rao *et al.* (2016) carried out research on electro-discharge machining process and created models for maximizing the metal removal rate (*MRR)*, minimizing the tool wear rate (*TWR*), minimizing the taper angle ($\Theta$), and minimizing the delamination factor (*DF*). The gap voltage ($V_g$), pulse current ($I_p$), pulse-on time ($T_{on}$) and tool rotation speed ($N$) were the process parameters taken into consideration for optimization. The four objective functions were then simultaneously optimized using a non-dominated sorting strategy by a multi-objective Jaya (MO-Jaya) algorithm. Table 1 lists the 50 non-dominated alternative solutions as well as the corresponding process parameter values. Each non-dominated alternative solution is made up of a set of values of *MRR, TWR,* $\Theta$ and *DF* that match a set of $V_g$, $I_p$, $T_{on}$ and $N$ process parameters. Now, the steps of the suggested BHARAT method are followed as described below in order to select the best compromise solution.

<u>Step 1:</u> There are 4 objectives and hence it is a many-objective decision-making situation. The 50 non-dominated alternative solutions that the MO-Jaya algorithm produced for 50 sets of input parameters are shown in Table 4. The objectives *TWR,* $\Theta$ and DF are non-beneficial and MRR is beneficial.

**Table 4**

Pareto optimal solutions generated by MO-Jaya algorithm for case study 2 (Rao et al., 2016)

| Solution | Process Parameters | | | | Objectives | | | |
|---|---|---|---|---|---|---|---|---|
| no. | $V_g$ (V) | $I_p$ (A) | $T_{on}$ (µs) | *N* (rpm) | *MRR* (0.1 mg/s) | *TWR* (0.1 mg/s) | $\Theta$ (degree) | *DF* |
| 1 | 25 | 10 | 1913.724 | 200 | 1.2453 | 0.0965 $x_{ji.best}$ | 3.3476 | 1.1574 |
| 2 | 25.0495 | 10 | 1844.116 | 200 | 1.2865 | 0.0986 | 3.0562 | 1.1558 |
| 3 | 25 | 10 | 1757.623 | 200 | 1.3199 | 0.0996 | 2.7192 | 1.1536 |
| 4 | 26.2683 | 10 | 2000 | 200 | 1.4191 | 0.1162 | 3.7259 | 1.1603 |
| 5 | 25 | 10 | 300 | 200 | 1.4245 | 0.2215 | 0.0811 $x_{ji.best}$ | 1.079 |
| 6 | 31.7003 | 10 | 2000 | 200 | 2.5179 | 0.2405 | 3.8046 | 1.1629 |
| 7 | 28.5 | 10 | 932.73 | 212.1907 | 3.0999 | 0.2672 | 0.6472 | 1.1259 |
| 8 | 33.8835 | 10 | 980.8407 | 214.6995 | 5.0426 | 0.4827 | 0.7417 | 1.13 |
| 9 | 39.4565 | 10 | 1366.835 | 200 | 5.5058 | 0.5499 | 1.7016 | 1.1488 |
| 10 | 39.5125 | 10 | 893.006 | 200 | 6.1636 | 0.6041 | 0.6878 | 1.1325 |
| 11 | 43.1006 | 10 | 785.4233 | 214.3395 | 9.0452 | 1.0027 | 0.5488 | 1.1238 |
| 12 | 60.5423 | 10 | 300 | 200 | 9.4074 | 1.871 | 0.159 | 1.0949 |
| 13 | 50.252 | 10 | 951.2899 | 200 | 10.0145 | 1.1314 | 0.943 | 1.1347 |
| 14 | 50.4624 | 10 | 1094.945 | 209.8391 | 11.047 | 1.3154 | 1.1924 | 1.1359 |
| 15 | 95 | 10 | 300 | 370.8176 | 11.201 | 1.547 | 0.5758 | 1.0749 $x_{ji.best}$ |
| 16 | 53.9205 | 10 | 1193.568 | 203.7774 | 11.3644 | 1.3979 | 1.5711 | 1.1404 |
| 17 | 61.7591 | 10 | 417.817 | 200 | 11.776 | 1.8677 | 0.269 | 1.1054 |
| 18 | 52.3786 | 10 | 997.8612 | 216.286 | 12.7649 | 1.5735 | 0.9942 | 1.1302 |
| 19 | 59.0602 | 10 | 1199.503 | 212.3769 | 14.2355 | 1.9259 | 1.6368 | 1.1357 |
| 20 | 62.0647 | 10 | 782.3541 | 212.8126 | 16.3417 | 2.1735 | 0.7642 | 1.1214 |
| 21 | 57.6466 | 10 | 899.7264 | 241.1095 | 17.1198 | 2.3377 | 0.8244 | 1.1187 |
| 22 | 78.1695 | 10 | 300 | 303.9107 | 18.6777 | 3.184 | 0.3371 | 1.0817 |
| 23 | 63.9669 | 10 | 721.4555 | 233.2439 | 19.5525 | 2.7339 | 0.6496 | 1.1129 |
| 24 | 81.4454 | 10.4816 | 300 | 263.1196 | 20.3185 | 4.6091 | 0.3181 | 1.0889 |
| 25 | 82.047 | 10.242 | 300 | 276.3105 | 20.4793 | 4.1102 | 0.3279 | 1.0849 |
| 26 | 81.5354 | 10 | 407.3847 | 289.2469 | 22.0527 | 3.4706 | 0.4306 | 1.0863 |
| 27 | 93.3095 | 10 | 460.6347 | 290.8624 | 23.1194 | 3.5922 | 0.5781 | 1.0837 |
| 28 | 77.2987 | 10 | 847.0946 | 243.5197 | 23.7081 | 3.6683 | 1.0182 | 1.1097 |
| 29 | 84.271 | 11.0556 | 628.0503 | 247.9736 | 24.8563 | 5.7193 | 0.7768 | 1.1108 |
| 30 | 95 | 10 | 680.5518 | 230.7705 | 25.9749 | 4.2886 | 1.0235 | 1.101 |
| 31 | 95 | 10 | 726.217 | 247.0427 | 26.8204 | 4.4199 | 1.0638 | 1.0983 |
| 32 | 63.1759 | 35.7338 | 815.4502 | 250.6803 | 26.8784 | 141.1848 | 1.9473 | 1.2461 |
| 33 | 46.8665 | 45 | 704.2118 | 262.2001 | 26.8928 | 168.1049 | 2.1411 | 1.2377 |
| 34 | 66.0972 | 36.3491 | 644.0377 | 251.924 | 27.0320 | 153.657 | 1.8928 | 1.2522 |
| 35 | 63.4694 | 37.0986 | 865.4543 | 259.8581 | 27.2314 | 155.9124 | 2.0329 | 1.2489 |
| 36 | 65.7791 | 37.3432 | 876.1797 | 259.7034 | 27.5357 | 164.2039 | 2.1005 | 1.2531 |
| 37 | 48.7786 | 45 | 750.8355 | 259.6498 | 27.6576 | 176.157 | 2.1703 | 1.2431 |
| 38 | 53.8153 | 45 | 571.8286 | 249.4247 | 27.9858 | 200.6127 | 2.1753 | 1.2581 |
| 39 | 55.3277 | 45 | 591.4365 | 277.3743 | 28.3591 | 208.3648 | 2.2865 | 1.2568 |
| 40 | 52.1831 | 45 | 875.1416 | 246.9805 | 28.454 | 185.9188 | 2.3039 | 1.254 |
| 41 | 55.6714 | 45 | 867.8184 | 251.2169 | 29.5533 | 205.7847 | 2.3407 | 1.2608 |
| 42 | 56.978 | 45 | 895.9178 | 245.2088 | 29.6352 | 208.5368 | 2.4141 | 1.265 |
| 43 | 59.1992 | 45 | 664.6343 | 264.8876 | 29.8885 | 224.7545 | 2.3068 | 1.2662 |
| 44 | 58.1049 | 45 | 835.4176 | 253.878 | 30.121 | 218.1782 | 2.3559 | 1.2653 |
| 45 | 60.3879 | 45 | 738.0855 | 255.0349 | 30.420 | 228.327 | 2.3521 | 1.27 |
| 46 | 62.1815 | 45 | 846.2731 | 248.4064 | 30.6445 | 233.4057 | 2.473 | 1.2745 |
| 47 | 64.936 | 45 | 937.1024 | 251.8109 | 30.7501 | 246.4366 | 2.577 | 1.278 |
| 48 | 64.7866 | 45 | 770.7681 | 249.6904 | 30.8257 | 243.3128 | 2.4831 | 1.2792 |
| 49 | 69.817 | 45 | 810.0259 | 259.0902 | 30.8293 | 260.9632 | 2.5826 | 1.2849 |
| 50 | 68.0958 | 45 | 836.1816 | 252.7234 | 31.0207 $x_{ji.best}$ | 256.4056 | 2.5864 | 1.2836 |
| Ranks assigned to the objectives→ | | | | | 1.5 | 1.5 | 3.5 | 3.5 |

Step 2: The objectives are ranked according to the decision-maker's assessment of their significance in terms of 1, 2, 3, and 4. The two attributes *MRR* and *TWR* are considered equally significant. Hence, *MRR* and *TWR* are given an average rank of 1.5 (i.e., (1+2)/2) and both objectives are assigned the weight of 0.309545 (i.e., (0.3714543 + 0.2476362)/2) using Table A1 from Part 1 of this paper. Furthermore, the objectives *Ɵ* and *DF* are also considered equally significant. Thus, both *Ɵ* and *DF* are given an average rank of 3.5 (i.e., (3+4)/2) and both are assigned average weight of 0.190454 (i.e., (0.202611436 + 0.178298064)/2).

Step 3: The data of the objectives for different alternative solutions is normalized with reference to the "best" values of the objectives. The values of *MRR* are normalized as $x_{ji}/x_{ji.best}$, and the objectives *TWR, Ɵ,* and *DF* are normalized as $x_{ji.best}/x_{ji}$. The normalized values of the objectives for 50 alternative solutions are shown in Table 5.

**Table 5**

Normalized values of the objectives and the total scores of the alternatives for case study 2

| *Solution no.* | *Normalized values of the objectives* | | | | *Total scores* |
|---|---|---|---|---|---|
| | *MRR* | *TWR* | *Ɵ* | *DF* | *(BHARAT)* |
| 1 | 0.040144 | 1 | 0.024226 | 0.92872 | 0.503464 |
| 2 | 0.041472 | 0.978702 | 0.026536 | 0.930005 | 0.497967 |
| 3 | 0.042549 | 0.968876 | 0.029825 | 0.931779 | 0.496223 |
| 4 | 0.045747 | 0.830465 | 0.021767 | 0.926398 | 0.451809 |
| 5 | 0.045921 | 0.435666 | 1 | 0.9962 | 0.529257 |
| 6 | 0.081168 | 0.401247 | 0.021316 | 0.924327 | 0.329431 |
| 7 | 0.09993 | 0.361153 | 0.125309 | 0.954703 | 0.348418 |
| 8 | 0.162556 | 0.199917 | 0.109343 | 0.951239 | 0.314194 |
| 9 | 0.177488 | 0.175486 | 0.047661 | 0.935672 | 0.296541 |
| 10 | 0.198693 | 0.159742 | 0.117912 | 0.949139 | 0.314176 |
| 11 | 0.291586 | 0.09624 | 0.147777 | 0.956487 | 0.330361 |
| 12 | 0.303262 | 0.051577 | 0.510063 | 0.981733 | 0.393957 |
| 13 | 0.322833 | 0.085293 | 0.086002 | 0.947299 | 0.323129 |
| 14 | 0.356117 | 0.073362 | 0.068014 | 0.946298 | 0.326123 |
| 15 | 0.361081 | 0.062379 | 0.140848 | 1 | 0.348359 |
| 16 | 0.366349 | 0.069032 | 0.05162 | 0.942564 | 0.324116 |
| 17 | 0.379617 | 0.051668 | 0.301487 | 0.972408 | 0.376121 |
| 18 | 0.411496 | 0.061328 | 0.081573 | 0.951071 | 0.343032 |
| 19 | 0.458903 | 0.050106 | 0.049548 | 0.946465 | 0.347256 |
| 20 | 0.5268 | 0.044398 | 0.106124 | 0.958534 | 0.37958 |
| 21 | 0.551883 | 0.04128 | 0.098375 | 0.960847 | 0.385344 |
| 22 | 0.602104 | 0.030308 | 0.240581 | 0.993714 | 0.430836 |
| 23 | 0.630305 | 0.035298 | 0.124846 | 0.965855 | 0.413762 |
| 24 | 0.654998 | 0.020937 | 0.254951 | 0.987143 | 0.445794 |
| 25 | 0.660182 | 0.023478 | 0.247332 | 0.990783 | 0.447427 |
| 26 | 0.710903 | 0.027805 | 0.188342 | 0.989506 | 0.452989 |
| 27 | 0.745289 | 0.026864 | 0.140287 | 0.99188 | 0.454642 |
| 28 | 0.764267 | 0.026306 | 0.07965 | 0.96864 | 0.444369 |
| 29 | 0.801281 | 0.016873 | 0.104403 | 0.967681 | 0.457438 |
| 30 | 0.837341 | 0.022502 | 0.079238 | 0.976294 | 0.46719 |
| 31 | 0.864597 | 0.021833 | 0.076236 | 0.978694 | 0.475306 |
| 32 | 0.866467 | 0.000684 | 0.041647 | 0.862611 | 0.440642 |
| 33 | 0.866931 | 0.000574 | 0.037878 | 0.868466 | 0.441149 |
| 34 | 0.871418 | 0.000628 | 0.042847 | 0.858409 | 0.441585 |
| 35 | 0.877846 | 0.000619 | 0.039894 | 0.860677 | 0.443442 |
| 36 | 0.887656 | 0.000588 | 0.03861 | 0.857793 | 0.445675 |
| 37 | 0.891585 | 0.000548 | 0.037368 | 0.864693 | 0.447957 |
| 38 | 0.902165 | 0.000481 | 0.037282 | 0.854384 | 0.449231 |
| 39 | 0.914199 | 0.000463 | 0.035469 | 0.855267 | 0.452773 |
| 40 | 0.917258 | 0.000519 | 0.035201 | 0.857177 | 0.45405 |
| 41 | 0.952696 | 0.000469 | 0.034648 | 0.852554 | 0.464019 |
| 42 | 0.955336 | 0.000463 | 0.033594 | 0.849723 | 0.464094 |
| 43 | 0.963502 | 0.000429 | 0.035157 | 0.848918 | 0.466756 |
| 44 | 0.970997 | 0.000442 | 0.034424 | 0.849522 | 0.469055 |
| 45 | 0.980636 | 0.000423 | 0.03448 | 0.846378 | 0.471445 |
| 46 | 0.987873 | 0.000413 | 0.032794 | 0.84339 | 0.472792 |
| 47 | 0.991277 | 0.000392 | 0.031471 | 0.84108 | 0.473147 |
| 48 | 0.993714 | 0.000397 | 0.032661 | 0.840291 | 0.473979 |
| 49 | 0.99383 | 0.00037 | 0.031402 | 0.836563 | 0.473057 |
| 50 | 1 | 0.000376 | 0.031356 | 0.83741 | 0.475122 |

Step 4: The normalized data of the objectives for the alternative solutions is multiplied by the corresponding weights of the objectives to obtain the total scores of the alternative solutions. . The last column of Table 5 shows the total scores.

Step 5: The optimal alternative solution is the one with the highest total score. Since the alternative solution no. 5 has the highest total score, it is regarded as the best option. The second, third, and the last choices are solution nos. 1, 2, and 9 respectively. Therefore, for the electro-discharge machining process under consideration, the decision-maker (i.e., process planner) can choose the process parameters that match the solution no. 5.

Four different MADM methods are used on the same case study to study the efficacy of the the suggested BHARAT, using Table 5 and the same weights given to the objectives. The MADM methods are: (i). Weighted product method (WPM), (ii). PROMETHEE, (iii). R-method, and (iv). TOPSIS. Based on the total scores given by these MADM methods and the proposed BHARAT method, the rankings of alternative solutions are given below.

| | |
|---|---|
| BHARAT: | 5-1-2-3---------------------------------9 |
| WPM: | 5-12-7-17-----------------------------49 |
| PROMETHEE: | 5-22-25-26-----------------------------6 |
| R (Rao and Lakshmi, 2021b): | 5-1-50-15-----------------------------33 |
| TOPSIS: | 29-31-30-27--------------------------49 |

When faced with a decision-making situation, the decision-maker is always curious to know what the first best option is. It should be noted that the solution no. 5 is recommended as the first best option in the current example by the suggested BHARAT method, WPM, PROMETHEE, and R-method. Based on the proposed BHARAT method and R-method, the second option is the alternative solution no. 1. The first option recommended by the TOPSIS method is solution no. 29. Nevertheless, a detailed examination of the *MRR, TWR, Ɵ, and DF* values of alternative solutions 5 and 29 reveals that, out of the four objectives, solution no. 5 is superior to solution no. 29 in *TWR*, *Ɵ*, and *DF*. Furthermore, because the three objectives - *TWR*, *Ɵ*, and *DF* – have a combined weight of 0.690453 (i.e., 0.309545 + 0.190454 + 0.190454), solution no. 5 outperforms solution no. 29 of TOPSIS by a significant margin.

*Second version of BHARAT method for case study 2*

Second version of BHARAT method differs from BHARAT only in Step 2. The ranks assigned to the attributes are the same as those given in BHARAT. Using the appropriate procedure suggested to form Table 14 and Table 21 given in Part-1 of the paper, the weights are assigned as 0.35 to both *MRR* and *TWR*, and 0.15 to both *Ɵ* and *DF* Using these weights, and the normalized data of the objectives, the total scores are computed which gives the ranking as 1-2-3-50-48-…..-9. Of course, it is to be noted that in this second version the weights of the objectives are somewhat different from the weights considered in BHARAT and hence the difference is there in ranking.

### *3.3 Case study 3: Many-objective optimization of machining process parameters*

A case study by Wu et al. (2022) is used to further illustrate the suggested BHARAT method. The authors proposed a data driven genetic algorithm based on deep learning. The TOPSIS method was then used to optimize the machining process parameters for multiple objectives and find the ultimate solutions. The authors used deep learning to automatically generate the data-driven prediction function of various optimized objectives. The Pareto set was then created by combining the genetic algorithm and a surrogate model that was created from the optimized objective prediction function. Lastly, from the produced Pareto set, the TOPSIS method was used to automatically search for the best optimum processing parameters. The tests were carried out on a milling machine. The process input parameters were cutting speed v (m/min), feed f (mm/rev.), width $a_e$ (mm), and depth of cut $a_p$ (mm). The four objectives considered were: energy consumption $E_c$ (kW-h), maximum cutting force $C_f$ (N), material removal rate MRR (E-03), and surface roughness Ra (µm). The 100 Pareto optimal solutions obtained by using NSGA-III showing 4 process parameters and 4 objectives are shown in Table 6.

The values of the four objectives are normalized. MRR is a beneficial one and the other objectives are of non-beneficial type. The best values of $E_c$, $C_f$, MRR, and Ra are 5.03kW-h, 46.33N, $1324.45*10^{-3}$, and 0.08 µm. Using these values, the values of the objectives for the 100 alternative solutions (i.e., alternative process input parameters) are normalized and given in Table 6. For space reasons, a separate table is not produced here to show the normalized values of the objectives.

Wu at al. (2022) considered equal importance to the four objectives (i.e., wi = 0.25 for i=1,2,3,4) and then carried out the TOPSIS procedure for finding the best alternative solution corresponding to the best combination of process input parameters. Hence, for fair comparison, same equal weights are considered in the BHARAT method. The four objectives are assigned an average rank of 2.5 (i.e., (1+2+3+4)/4). Using Table A1 of Part-1 of this paper, the average equal weight is computed as 0.25 (i.e., (0.3714543+0.2476362+0.2026114+0.1782980)/4). The total scores are calculated using the equal weights and the normalized values of the objectives given in Table 6, and are given in the last column of Table 6 (for space reasons, a separate table is not produced here). From the total scores, it can be understood that the alternative solution no. 6 with a total score of 0.425964 is considered as the first choice and solution no. 2 as the second choice. However, TOPSIS method used by Wu et al. (2022) proposed solution no. 75 as the first choice and solution no. 92 as the second choice. The original performance data of the alternative solutions (and the corresponding normalized values) of $E_c$, $C_f$, MRR, and Ra reveal that proposing solution no. 6 is logically better than solution no. 75. Similarly, solution no. 2 is better than solution no. 92 in case of 3 objectives out of 4 objectives. Therefore, it makes sense to suggest solution no. 2 as the second choice. Using the second version of BHARAT leads to the same total scores given by BHARAT because of the equal weight consideration for each objective.

**Table 6**
Pareto optimal solutions with the sets of process input parameters, objectives along with their normalized values, and the total scores for case study 3

| Solution no. | Process input parameters | | | | Objectives (process output parameters) | | | | Normalized values of the objectives | | | | Total scores |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | v (m/min) | f (mm/rev) ($10^{-2}$) | $a_e$ (mm) | $a_p$ (mm) | $E_c$ (kW-h) | $C_f$ (N) | MRR ($10^{-3}$) | Ra (μm) | $E_c$ | $C_f$ | MRR | Ra | |
| 1 | 32.34 | 4.07 | 2.11 | 0.52 | 42.08 | 143.2 | 3.86 | 0.16 | 0.11953 | 0.323534 | 0.002914 | 0.5 | 0.236496 |
| 2 | 99.99 | 4 | 10 | 0 | 9.49 | 449.08 | 0.01 | 0.08 | 0.53003 | 0.103166 | 7.55E-06 | 1 | 0.408301 |
| 3 | 40.12 | 4.01 | 0.79 | 0 | 35.05 | 69.83 | 0 | 0.12 | 0.14350 | 0.663468 | 0 | 0.666667 | 0.368411 |
| 4 | 98.2 | 12.5 | 6.84 | 3.99 | 6.38 | 844.03 | 889.34 | 1.83 | 0.788401 | 0.054891 | 0.671479 | 0.043716 | 0.389622 |
| 5 | 99.99 | 12.49 | 9.4 | 2.94 | 10.99 | 1211.48 | 916.29 | 1.11 | 0.457689 | 0.038242 | 0.691827 | 0.072072 | 0.314958 |
| 6 | 30 | 4 | 1.58 | 0.03 | 38.02 | 46.33 | 0.17 | 0.14 | 0.132299 | 1 | 0.000128 | 0.571429 | 0.425964 |
| 7 | 99.99 | 12.5 | 10 | 3.99 | 13.09 | 1025.41 | 1324.45 | 1.15 | 0.384263 | 0.045182 | 1 | 0.069565 | 0.374752 |
| 8 | 99.98 | 12.5 | 8.99 | 0 | 5.03 | 618.8 | 0.02 | 0.62 | 1 | 0.074871 | 1.51E-05 | 0.129032 | 0.30098 |
| 9 | 99.41 | 10.48 | 6.56 | 3.98 | 6.66 | 799 | 722.4 | 1.63 | 0.755255 | 0.057985 | 0.545434 | 0.04908 | 0.351938 |
| 10 | 99.99 | 11.7 | 7.05 | 3.98 | 6.5 | 858.24 | 871.41 | 1.75 | 0.773846 | 0.053983 | 0.657941 | 0.045714 | 0.382871 |
| 11 | 34.72 | 4.02 | 1.12 | 0.16 | 35.26 | 72.97 | 0.67 | 0.14 | 0.142655 | 0.634918 | 0.000506 | 0.571429 | 0.337377 |
| 12 | 35.42 | 4 | 1.58 | 0.03 | 30.06 | 56.68 | 0.2 | 0.13 | 0.167332 | 0.817396 | 0.000151 | 0.615385 | 0.400066 |
| 13 | 99.4 | 11.79 | 7.05 | 0.18 | 5.34 | 287.07 | 38.52 | 0.9 | 0.941948 | 0.161389 | 0.029084 | 0.088889 | 0.305327 |
| 14 | 40.12 | 4.01 | 0.96 | 0 | 33.23 | 77.15 | 0 | 0.12 | 0.151369 | 0.600518 | 0 | 0.666667 | 0.354639 |
| 15 | 94.73 | 11.72 | 7.31 | 2.68 | 7.36 | 1011.67 | 577.52 | 1.46 | 0.683424 | 0.045796 | 0.436045 | 0.054795 | 0.305015 |
| 16 | 99.98 | 12.46 | 8.94 | 2.37 | 10.33 | 1093.77 | 700.63 | 1.18 | 0.486931 | 0.042358 | 0.528997 | 0.067797 | 0.281521 |
| 17 | 99.98 | 12.41 | 8.93 | 2.63 | 10.66 | 1163.42 | 772.53 | 1.22 | 0.471857 | 0.039822 | 0.583284 | 0.065574 | 0.290134 |
| 18 | 96.15 | 11.24 | 7.31 | 2.68 | 6.58 | 999.8 | 560.72 | 1.42 | 0.764438 | 0.046339 | 0.423361 | 0.056338 | 0.322619 |
| 19 | 95.12 | 5.42 | 2.8 | 3.71 | 17.69 | 473.01 | 142.02 | 0.5 | 0.284341 | 0.097947 | 0.107229 | 0.16 | 0.16238 |
| 20 | 99 | 12.36 | 9.86 | 3.67 | 12.17 | 1069.91 | 1174.59 | 1.08 | 0.413311 | 0.043303 | 0.886851 | 0.074074 | 0.354385 |
| 21 | 79.76 | 4.12 | 2.49 | 3.55 | 18.26 | 420.26 | 77.1 | 0.43 | 0.275465 | 0.110241 | 0.058213 | 0.186047 | 0.157492 |
| 22 | 69.34 | 12.25 | 9.58 | 2.4 | 15.75 | 664.65 | 517.93 | 0.98 | 0.319365 | 0.069706 | 0.391053 | 0.081633 | 0.215439 |
| 23 | 100 | 11.87 | 9.69 | 3.06 | 11.4 | 1144.19 | 933.73 | 0.95 | 0.441228 | 0.040492 | 0.704995 | 0.084211 | 0.317731 |
| 24 | 98.67 | 12.4 | 7.43 | 0.03 | 5.26 | 350.74 | 8.24 | 0.92 | 0.956274 | 0.132092 | 0.006221 | 0.086957 | 0.295386 |
| 25 | 99.92 | 6.27 | 9.81 | 3.97 | 13.54 | 885.32 | 646.38 | 0.59 | 0.371492 | 0.052331 | 0.488037 | 0.135593 | 0.261863 |
| 26 | 95.3 | 5.19 | 1.63 | 3.89 | 40.27 | 432.3 | 83.18 | 0.52 | 0.124907 | 0.107171 | 0.062803 | 0.153846 | 0.112182 |
| 27 | 98.28 | 4.54 | 6.55 | 2.75 | 6.24 | 714.4 | 213.16 | 0.63 | 0.80609 | 0.064852 | 0.160942 | 0.126984 | 0.289717 |
| 28 | 97.38 | 6.27 | 7.76 | 3.97 | 9.76 | 747.57 | 498.88 | 0.82 | 0.515369 | 0.061974 | 0.37667 | 0.097561 | 0.262893 |
| 29 | 80.1 | 4.74 | 9.88 | 2.53 | 15.12 | 493.83 | 251.53 | 0.55 | 0.332672 | 0.093818 | 0.189913 | 0.145455 | 0.190464 |
| 30 | 97.56 | 4.12 | 9.65 | 3.74 | 13.7 | 776.43 | 384.59 | 0.53 | 0.367153 | 0.059671 | 0.290377 | 0.150943 | 0.217036 |
| 31 | 94.31 | 10.4 | 7.37 | 0.13 | 5.53 | 316.27 | 24.6 | 0.62 | 0.909584 | 0.146489 | 0.018574 | 0.129032 | 0.30092 |
| 32 | 99.86 | 12.04 | 9.64 | 3.43 | 11.46 | 1111.63 | 1056.75 | 1.05 | 0.438918 | 0.041678 | 0.797878 | 0.07619 | 0.338666 |
| 33 | 97.93 | 4.72 | 7.13 | 3.5 | 6.96 | 787.98 | 306.06 | 0.69 | 0.722701 | 0.058796 | 0.231085 | 0.115942 | 0.282131 |
| 34 | 94.93 | 11.76 | 7.05 | 3.98 | 7.68 | 813.59 | 831.61 | 1.71 | 0.654948 | 0.056945 | 0.627891 | 0.046784 | 0.346642 |
| 35 | 72.62 | 11.18 | 7.03 | 3.92 | 13.36 | 872.63 | 593.4 | 1.3 | 0.376497 | 0.053092 | 0.448035 | 0.061538 | 0.234791 |
| 36 | 99.82 | 11.45 | 2.58 | 3.55 | 16.32 | 556.04 | 277.79 | 0.79 | 0.308211 | 0.083321 | 0.20974 | 0.101266 | 0.175634 |
| 37 | 99.98 | 10.74 | 7.55 | 3.96 | 7.53 | 868.13 | 852.59 | 1.56 | 0.667995 | 0.053368 | 0.643731 | 0.051282 | 0.354094 |
| 38 | 97.8 | 5.67 | 8.02 | 3.99 | 10.7 | 750.81 | 470.67 | 0.7 | 0.470093 | 0.061707 | 0.35537 | 0.114286 | 0.250364 |
| 39 | 89.48 | 6.36 | 6.12 | 3.93 | 6.91 | 648.02 | 363.63 | 1.16 | 0.727931 | 0.071495 | 0.274552 | 0.068966 | 0.285736 |
| 40 | 35.22 | 4.01 | 2.17 | 0.04 | 28 | 61.85 | 0.32 | 0.14 | 0.179643 | 0.74907 | 0.000242 | 0.571429 | 0.375096 |
| 41 | 99.98 | 12.41 | 8.99 | 2.75 | 10.81 | 1178.69 | 815.3 | 1.22 | 0.46531 | 0.039306 | 0.615576 | 0.065574 | 0.296442 |
| 42 | 96.1 | 9.93 | 9.96 | 3.94 | 14.29 | 929.71 | 993.73 | 0.81 | 0.351994 | 0.049833 | 0.750296 | 0.098765 | 0.312722 |
| 43 | 44.2 | 4.02 | 1.12 | 0.16 | 30.51 | 88.77 | 0.85 | 0.12 | 0.164864 | 0.521911 | 0.000642 | 0.666667 | 0.338521 |
| 44 | 99.2 | 12.43 | 7.88 | 3.96 | 8.79 | 913.78 | 1020.89 | 1.69 | 0.572241 | 0.050701 | 0.770803 | 0.047337 | 0.360271 |
| 45 | 99.99 | 12.41 | 8.64 | 3.95 | 10.64 | 965.07 | 1123.85 | 1.52 | 0.472744 | 0.048007 | 0.848541 | 0.052632 | 0.355481 |
| 46 | 96.15 | 11.24 | 8.15 | 2.68 | 10.11 | 1062.92 | 625.19 | 1.24 | 0.497527 | 0.043587 | 0.472037 | 0.064516 | 0.269417 |
| 47 | 99.94 | 4.32 | 7.55 | 3.98 | 8.39 | 727.77 | 344.09 | 0.66 | 0.599523 | 0.06366 | 0.259798 | 0.121212 | 0.261048 |
| 48 | 99.99 | 12.5 | 9.23 | 4 | 11.49 | 985.65 | 1222.92 | 1.39 | 0.437772 | 0.047005 | 0.923342 | 0.057554 | 0.366418 |
| 49 | 99.87 | 12.5 | 8.08 | 3.51 | 8.88 | 1022.01 | 939.02 | 1.6 | 0.566441 | 0.045332 | 0.708989 | 0.05 | 0.342691 |
| 50 | 95.11 | 5.19 | 8.28 | 3.89 | 11.17 | 737.49 | 422.56 | 0.57 | 0.450313 | 0.062821 | 0.319046 | 0.140351 | 0.243133 |
| 51 | 99.95 | 4.13 | 1.95 | 4 | 38.7 | 439.57 | 85.24 | 0.48 | 0.129974 | 0.105398 | 0.064359 | 0.166667 | 0.1166 |
| 52 | 33.53 | 4.1 | 3.18 | 0.15 | 19.29 | 100.36 | 1.79 | 0.12 | 0.260757 | 0.461638 | 0.001352 | 0.666667 | 0.347603 |
| 53 | 99.99 | 5.73 | 7.65 | 0.18 | 5.43 | 264.41 | 20.44 | 0.29 | 0.926335 | 0.17522 | 0.015433 | 0.275862 | 0.348213 |
| 54 | 40.17 | 4.19 | 3.12 | 0.49 | 22.02 | 130.55 | 6.79 | 0.18 | 0.228429 | 0.354883 | 0.005127 | 0.444444 | 0.258221 |
| 55 | 99.86 | 12.41 | 8.92 | 1.85 | 9.6 | 940.44 | 544.24 | 1.09 | 0.523958 | 0.049264 | 0.410918 | 0.073394 | 0.264384 |
| 56 | 76.93 | 12.5 | 9.99 | 2.6 | 15.31 | 684.44 | 663.58 | 0.96 | 0.328543 | 0.06769 | 0.501023 | 0.083333 | 0.245148 |
| 57 | 33.09 | 4.03 | 3.32 | 0.05 | 18.74 | 103.32 | 0.63 | 0.1 | 0.26841 | 0.448413 | 0.000476 | 0.8 | 0.379325 |
| 58 | 89.95 | 12.25 | 9.98 | 3.56 | 14.57 | 931.39 | 1037.17 | 0.93 | 0.34523 | 0.049743 | 0.783095 | 0.086022 | 0.316022 |
| 59 | 99.86 | 12.11 | 7.95 | 3.99 | 8.85 | 912.12 | 1018.56 | 1.65 | 0.568362 | 0.050794 | 0.769044 | 0.048485 | 0.359171 |
| 60 | 99.66 | 4.29 | 9.08 | 3.96 | 11.67 | 808 | 408.12 | 0.46 | 0.43102 | 0.057339 | 0.308143 | 0.173913 | 0.242604 |
| 61 | 34.68 | 4.03 | 2.89 | 0.58 | 22.87 | 148.24 | 6.19 | 0.17 | 0.219939 | 0.312534 | 0.004674 | 0.470588 | 0.251934 |
| 62 | 99.98 | 10.09 | 7.55 | 3.96 | 7.61 | 853.34 | 800.8 | 1.49 | 0.660972 | 0.054293 | 0.604628 | 0.053691 | 0.343396 |
| 63 | 99.78 | 12.29 | 7 | 3.97 | 6.36 | 868.64 | 904.58 | 1.81 | 0.790881 | 0.053336 | 0.682985 | 0.044199 | 0.39285 |
| 64 | 96.68 | 11.16 | 2.24 | 2.67 | 14.35 | 578.52 | 171.4 | 0.72 | 0.350523 | 0.080084 | 0.129412 | 0.111111 | 0.167782 |
| 65 | 99.94 | 4.13 | 7.55 | 3.96 | 8.39 | 727.02 | 327.62 | 0.63 | 0.599523 | 0.063726 | 0.247363 | 0.126984 | 0.259399 |
| 66 | 41.44 | 4.21 | 3.1 | 0.49 | 24.79 | 118.41 | 6.99 | 0.2 | 0.202904 | 0.391268 | 0.005278 | 0.4 | 0.249862 |
| 67 | 97.42 | 12.5 | 9.41 | 3.99 | 12.53 | 964.07 | 1214.58 | 1.27 | 0.401437 | 0.048057 | 0.917045 | 0.062992 | 0.357383 |
| 68 | 99.78 | 12.06 | 9.84 | 3.97 | 12.77 | 1009.1 | 1247.66 | 1.12 | 0.393892 | 0.045912 | 0.942021 | 0.071429 | 0.363313 |
| 69 | 95.36 | 5.16 | 2.83 | 3.82 | 17.85 | 478.06 | 141.19 | 0.5 | 0.281793 | 0.096913 | 0.106603 | 0.16 | 0.161327 |
| 70 | 35.22 | 4.02 | 2.17 | 0.04 | 28 | 61.81 | 0.32 | 0.14 | 0.179643 | 0.749555 | 0.000242 | 0.571429 | 0.375217 |
| 71 | 97.93 | 4.15 | 6.73 | 3.26 | 6.35 | 766.26 | 236.16 | 0.65 | 0.792126 | 0.060463 | 0.178308 | 0.123077 | 0.288493 |
| 72 | 40.33 | 4.15 | 2.46 | 0.34 | 26.3 | 82.86 | 3.68 | 0.16 | 0.191255 | 0.559136 | 0.002779 | 0.5 | 0.313292 |
| 73 | 66.19 | 11.64 | 9.95 | 3.89 | 16.79 | 820.72 | 790.22 | 1.37 | 0.299583 | 0.05645 | 0.59664 | 0.058394 | 0.252767 |
| 74 | 97.93 | 4.1 | 2.91 | 3.4 | 16.31 | 441.25 | 105.34 | 0.39 | 0.3084 | 0.104997 | 0.079535 | 0.205128 | 0.174515 |
| 75 | 97.85 | 11.15 | 9.98 | 3.99 | 13.94 | 966.13 | 1151.22 | 0.97 | 0.360832 | 0.047954 | 0.869206 | 0.082474 | 0.340117 |
| 76 | 100 | 12.44 | 9.38 | 2.5 | 10.98 | 1118.5 | 772.84 | 1.04 | 0.458106 | 0.041422 | 0.583518 | 0.076923 | 0.289992 |
| 77 | 93.78 | 4.49 | 6.56 | 0.71 | 5.98 | 238.36 | 52.17 | 0.2 | 0.841137 | 0.19437 | 0.03939 | 0.4 | 0.368724 |
| 78 | 99.99 | 12.5 | 9.23 | 3.85 | 11.43 | 1015.91 | 1179.03 | 1.37 | 0.44007 | 0.045604 | 0.890203 | 0.058394 | 0.358568 |
| 79 | 94.97 | 9.93 | 6.55 | 2.67 | 6.24 | 906.6 | 438.86 | 1.34 | 0.80609 | 0.051103 | 0.331353 | 0.059701 | 0.312062 |
| 80 | 97.53 | 6.36 | 6.79 | 3.95 | 6.76 | 701.3 | 441.5 | 1.16 | 0.744083 | 0.066063 | 0.333346 | 0.068966 | 0.303114 |
| 81 | 96.51 | 4.01 | 2.73 | 2.87 | 16.3 | 524.68 | 80.62 | 0.33 | 0.308589 | 0.088301 | 0.060871 | 0.242424 | 0.175046 |
| 82 | 99.53 | 11.74 | 9.91 | 3.76 | 12.47 | 1047.24 | 1155.7 | 1.01 | 0.403368 | 0.04424 | 0.872589 | 0.079208 | 0.349851 |
| 83 | 96.1 | 9.97 | 3.09 | 3.85 | 9.97 | 568.15 | 302.09 | 0.76 | 0.504514 | 0.081545 | 0.228087 | 0.105263 | 0.229852 |
| 84 | 99.99 | 12.49 | 9.97 | 3.79 | 12.4 | 1065.92 | 1254.77 | 1.11 | 0.405645 | 0.043465 | 0.947389 | 0.072072 | 0.367143 |
| 85 | 40.17 | 4.19 | 3.19 | 0.49 | 21.06 | 135.28 | 6.94 | 0.18 | 0.238841 | 0.342475 | 0.00524 | 0.444444 | 0.25775 |
| 86 | 34.01 | 4.58 | 1.75 | 0.1 | 31.84 | 76.51 | 0.7 | 0.15 | 0.157977 | 0.605542 | 0.000529 | 0.533333 | 0.324345 |
| 87 | 41.06 | 4 | 2.23 | 0.34 | 25.24 | 81.16 | 3.28 | 0.15 | 0.199287 | 0.570848 | 0.002476 | 0.533333 | 0.326486 |
| 88 | 97.38 | 6.37 | 7.93 | 3.99 | 10.55 | 753.62 | 520.85 | 0.8 | 0.476777 | 0.061477 | 0.393258 | 0.1 | 0.257878 |
| 89 | 99.3 | 12.5 | 9.4 | 2.92 | 11.1 | 1195.76 | 904.48 | 1.09 | 0.453153 | 0.038745 | 0.68291 | 0.073394 | 0.312051 |
| 90 | 31.07 | 4.03 | 3.13 | 0.57 | 20.75 | 144.98 | 5.96 | 0.15 | 0.24241 | 0.319561 | 0.0045 | 0.533333 | 0.274951 |
| 91 | 99.9 | 12.46 | 7.52 | 3.63 | 6.9 | 962.3 | 901.41 | 1.72 | 0.728986 | 0.048145 | 0.680592 | 0.046512 | 0.376059 |
| 92 | 99.82 | 9.99 | 9.97 | 3.94 | 13.37 | 977.21 | 1038.86 | 0.85 | 0.376215 | 0.04741 | 0.784371 | 0.094118 | 0.325529 |
| 93 | 99.99 | 12.49 | 10 | 3.94 | 12.91 | 1036.46 | 1306.22 | 1.13 | 0.38962 | 0.0447 | 0.986236 | 0.070796 | 0.372838 |
| 94 | 99.92 | 4.07 | 2.11 | 0.19 | 9.26 | 214.95 | 4.37 | 0.22 | 0.543197 | 0.215538 | 0.003299 | 0.363636 | 0.281418 |
| 95 | 97.34 | 12.5 | 9.95 | 3.99 | 13.93 | 993.24 | 1282.93 | 1.13 | 0.361091 | 0.046645 | 0.968651 | 0.070796 | 0.361796 |
| 96 | 99.39 | 10.52 | 6.46 | 3.85 | 6.71 | 818.48 | 691.14 | 1.6 | 0.749627 | 0.056605 | 0.521832 | 0.05 | 0.344516 |
| 97 | 99.85 | 5.18 | 1.75 | 0.09 | 9.58 | 199.13 | 2.27 | 0.23 | 0.525052 | 0.232662 | 0.001714 | 0.347826 | 0.276814 |
| 98 | 100 | 12.41 | 2.8 | 3.71 | 12.55 | 591.34 | 341.91 | 0.87 | 0.400797 | 0.078347 | 0.258152 | 0.091954 | 0.207313 |
| 99 | 32.34 | 4.02 | 2.46 | 0.34 | 24.08 | 79.18 | 2.86 | 0.16 | 0.208887 | 0.585123 | 0.002159 | 0.5 | 0.324042 |
| 100 | 98.28 | 4.23 | 6.73 | 3.52 | 6.36 | 753.51 | 261.49 | 0.75 | 0.790881 | 0.061486 | 0.197433 | 0.106667 | 0.289116 |

Wu et al. (2022) considered a second case consisting of four groups with only three objectives, i.e., $E_c$, $C_f$, and MRR. Different weight combinations of $E_c$, $C_f$, and MRR were attempted using TOPSIS method. For fair comparison, the same weight combinations are now used in the BHARAT method and the rankings are given in Table 7.

**Table 7**

Total scores for different weight combinations of $E_c$, $C_f$, and MRR using BHARAT method for case study 3

| *Solution no.* | *Total scores for different weight combinations of $E_c$, $C_f$, and MRR using BHARAT* | | | |
|---|---|---|---|---|
| | *0.2, 0.2, 0.6* | *0.3, 0.3, 0.4* | *0.5, 0.4, 0.1* | *0.6, 0.2, 0.2* |
| 1 | 0.090362 | 0.134086 | 0.189472 | 0.13701 |
| 2 | 0.126644 | 0.189962 | 0.306283 | 0.338654 |
| 3 | 0.161396 | 0.242093 | 0.337142 | 0.218799 |
| 4 | 0.571546 | 0.521579 | 0.483305 | 0.618315 |
| 5 | 0.514282 | 0.42551 | 0.313324 | 0.420627 |
| 6 | 0.226537 | 0.339741 | 0.466162 | 0.279405 |
| 7 | 0.685889 | 0.528833 | 0.310204 | 0.439594 |
| 8 | 0.214983 | 0.322467 | 0.52995 | 0.614977 |
| 9 | 0.489908 | 0.462146 | 0.455365 | 0.573837 |
| 10 | 0.56033 | 0.511525 | 0.47431 | 0.606692 |
| 11 | 0.155818 | 0.233474 | 0.325345 | 0.212678 |
| 12 | 0.197036 | 0.295479 | 0.410639 | 0.263909 |
| 13 | 0.238118 | 0.342635 | 0.538438 | 0.603263 |
| 14 | 0.150378 | 0.225566 | 0.315892 | 0.210925 |
| 15 | 0.407471 | 0.393184 | 0.403635 | 0.506422 |
| 16 | 0.423256 | 0.370386 | 0.313309 | 0.40643 |
| 17 | 0.452306 | 0.386817 | 0.310186 | 0.407736 |
| 18 | 0.416172 | 0.412577 | 0.443091 | 0.552603 |
| 19 | 0.140795 | 0.157578 | 0.192073 | 0.21164 |
| 20 | 0.623434 | 0.491725 | 0.312662 | 0.434018 |
| 21 | 0.112069 | 0.138997 | 0.187651 | 0.19897 |
| 22 | 0.312446 | 0.273142 | 0.22667 | 0.283771 |
| 23 | 0.519341 | 0.426514 | 0.30731 | 0.413834 |
| 24 | 0.221406 | 0.328998 | 0.531596 | 0.601427 |
| 25 | 0.377587 | 0.322362 | 0.255482 | 0.330969 |
| 26 | 0.084098 | 0.094745 | 0.111602 | 0.108939 |
| 27 | 0.270754 | 0.325659 | 0.44508 | 0.528813 |
| 28 | 0.34147 | 0.323871 | 0.320141 | 0.39695 |
| 29 | 0.199246 | 0.203912 | 0.222854 | 0.256349 |
| 30 | 0.259591 | 0.244198 | 0.236483 | 0.290302 |
| 31 | 0.222359 | 0.324251 | 0.515245 | 0.578763 |
| 32 | 0.574846 | 0.46333 | 0.315918 | 0.431262 |
| 33 | 0.29495 | 0.326883 | 0.407977 | 0.491597 |
| 34 | 0.519113 | 0.464724 | 0.413041 | 0.529936 |
| 35 | 0.354739 | 0.308091 | 0.254289 | 0.326124 |
| 36 | 0.20415 | 0.201356 | 0.208408 | 0.243539 |
| 37 | 0.530511 | 0.473901 | 0.419718 | 0.540217 |
| 38 | 0.319582 | 0.301688 | 0.295266 | 0.365471 |
| 39 | 0.324616 | 0.349648 | 0.420018 | 0.505968 |
| 40 | 0.185888 | 0.278711 | 0.389474 | 0.257648 |
| 41 | 0.470269 | 0.397615 | 0.309935 | 0.410162 |
| 42 | 0.530543 | 0.420667 | 0.27096 | 0.371222 |
| 43 | 0.13774 | 0.206289 | 0.29126 | 0.203429 |
| 44 | 0.58707 | 0.495204 | 0.383481 | 0.507646 |
| 45 | 0.613275 | 0.495642 | 0.340429 | 0.462956 |
| 46 | 0.391445 | 0.351149 | 0.313402 | 0.401641 |
| 47 | 0.288516 | 0.302874 | 0.351206 | 0.424406 |
| 48 | 0.65096 | 0.51477 | 0.330022 | 0.456732 |
| 49 | 0.547748 | 0.467128 | 0.372252 | 0.490729 |
| 50 | 0.294054 | 0.281559 | 0.28219 | 0.346561 |
| 51 | 0.08569 | 0.096355 | 0.113582 | 0.111936 |
| 52 | 0.14529 | 0.217259 | 0.315169 | 0.249052 |
| 53 | 0.229571 | 0.33664 | 0.534799 | 0.593932 |
| 54 | 0.119738 | 0.177044 | 0.25668 | 0.209059 |
| 55 | 0.361195 | 0.336334 | 0.322777 | 0.406411 |
| 56 | 0.379861 | 0.319279 | 0.24145 | 0.310869 |
| 57 | 0.14365 | 0.215237 | 0.313618 | 0.250824 |
| 58 | 0.548851 | 0.43173 | 0.270822 | 0.373706 |
| 59 | 0.585257 | 0.493364 | 0.381403 | 0.504984 |
| 60 | 0.282558 | 0.269765 | 0.26926 | 0.331708 |
| 61 | 0.109299 | 0.161611 | 0.23545 | 0.195405 |
| 62 | 0.50583 | 0.456431 | 0.412666 | 0.528368 |
| 63 | 0.578635 | 0.526459 | 0.485073 | 0.621793 |
| 64 | 0.163769 | 0.180947 | 0.220236 | 0.252213 |
| 65 | 0.281068 | 0.29792 | 0.349988 | 0.421932 |
| 66 | 0.122001 | 0.180363 | 0.258487 | 0.201052 |
| 67 | 0.640126 | 0.501666 | 0.311645 | 0.433882 |
| 68 | 0.653174 | 0.50875 | 0.309513 | 0.433922 |
| 69 | 0.139703 | 0.156253 | 0.190322 | 0.209779 |
| 70 | 0.185985 | 0.278856 | 0.389668 | 0.257745 |
| 71 | 0.277502 | 0.3271 | 0.438079 | 0.52303 |
| 72 | 0.151745 | 0.226229 | 0.31956 | 0.227136 |
| 73 | 0.429191 | 0.345466 | 0.232036 | 0.310368 |
| 74 | 0.1304 | 0.155833 | 0.204152 | 0.221946 |
| 75 | 0.603281 | 0.470318 | 0.286518 | 0.399931 |
| 76 | 0.450016 | 0.383265 | 0.303973 | 0.399851 |
| 77 | 0.230735 | 0.326408 | 0.502255 | 0.551434 |
| 78 | 0.631257 | 0.501784 | 0.327297 | 0.451204 |

**Table 7**
Total scores for different weight combinations of $E_c$, $C_f$, and MRR using BHARAT method for case study 3 (Continued)

| *Solution no.* | *Total scores for different weight combinations of $E_c$, $C_f$, and MRR using BHARAT* | | | |
|---|---|---|---|---|
| | *0.2, 0.2, 0.6* | *0.3, 0.3, 0.4* | *0.5, 0.4, 0.1* | *0.6, 0.2, 0.2* |
| 79 | 0.37025 | 0.389699 | 0.456621 | 0.560145 |
| 80 | 0.362037 | 0.376382 | 0.431801 | 0.526331 |
| 81 | 0.1159 | 0.143415 | 0.195702 | 0.214988 |
| 82 | 0.613075 | 0.483318 | 0.306639 | 0.425387 |
| 83 | 0.254064 | 0.267053 | 0.307684 | 0.364635 |
| 84 | 0.658256 | 0.513689 | 0.314947 | 0.441558 |
| 85 | 0.119407 | 0.176491 | 0.256935 | 0.212848 |
| 86 | 0.153021 | 0.229267 | 0.321258 | 0.216 |
| 87 | 0.155513 | 0.232031 | 0.32823 | 0.234237 |
| 88 | 0.343605 | 0.318779 | 0.302305 | 0.377013 |
| 89 | 0.508126 | 0.420733 | 0.310366 | 0.416223 |
| 90 | 0.115094 | 0.170391 | 0.249479 | 0.210258 |
| 91 | 0.563781 | 0.505376 | 0.45181 | 0.583139 |
| 92 | 0.555348 | 0.440836 | 0.285509 | 0.392086 |
| 93 | 0.678606 | 0.524791 | 0.311314 | 0.439959 |
| 94 | 0.153727 | 0.22894 | 0.358144 | 0.369686 |
| 95 | 0.662738 | 0.509781 | 0.296069 | 0.419714 |
| 96 | 0.474345 | 0.450602 | 0.449639 | 0.565464 |
| 97 | 0.152571 | 0.228 | 0.355762 | 0.361907 |
| 98 | 0.25072 | 0.247004 | 0.257553 | 0.307778 |
| 99 | 0.160098 | 0.239067 | 0.338708 | 0.242789 |
| 100 | 0.288933 | 0.334683 | 0.439778 | 0.526312 |

From the total scores given in Table 7 using BHARAT method, it can be observed that solution no. 7 is the first choice for the weight combinations of (0.2, 0.2, 0.6) as well as (0.3, 0.3, 0.4); solution no. 13 is the first choice for the weight combination of (0.5, 0.4, 0.1) and solution no. 4 is the first choice for the weight combination of (0.6, 0.2, 0.2). However, comparison of these results with those given by Wu et a. (2022) is not possible, as it seems that they had generated Pareto solutions for these four groups of weights separately and then carried out the TOPSIS procedure. The correct procedure is: Once the non-dominated Pareto optimal solutions are generated by any multi-objective optimization algorithm, then the application of any MADM method such as TOPSIS can be used *for a specific set of weights of the objectives* decided by the decision-maker. If the decision-maker wants to see the results for different specific sets of weights of the objectives, he or she must apply those different sets of specific weights of the objectives on the *same non-dominated Pareto optimal solutions*. That makes him/her clear of the effect of assigning different sets of weights to the objectives. It is not clear whether Wu et al. (2002) followed this correct procedure or not. This ambiguity is because the results shown by them in Table 5 of their paper for the four groups of weights do not appear at all in the generated Pareto solutions shown in Table 4 of their paper.

The second version of BHARAT also gives the same choices as the weights of the objectives considered are same for both the first and second case.

The BHARAT method allows the decision-makers to directly assign the numerical weights to the objectives rather than the ranks. For instance, in all three of the case studies, the decision-maker is free to assign the numerical weights to the objectives directly based on his or her understanding of them, i.e., without designating ranks such as 1, 2, etc. This feature is proved in second case of case study 3 of this paper. The numerical weights of the objectives (assigned based on the decision-maker's opinion) can be multiplied with the corresponding normalized values of the objectives for various alternative solutions to get the total scores of the alternative solutions.

## 4. Conclusions

In Part-1 of this paper, the BHARAT method—a novel and straightforward multi-attribute decision-making technique—is proposed. The current paper employs the BHARAT method as a multi- and many-objective decision-making method for evaluating and ranking Pareto optimal solutions in industrial optimization problems. All that is needed for the BHARAT method is the ranking of objectives, assigning weights to them, and normalizing the objectives after the Pareto optimal solutions are found by any advanced optimization algorithm. It is simpler and more convenient to rank order the objectives' importance. Three case studies are provided to potential of the suggested methodology. The three case studies demonstrate how straightforward the suggested BHARAT method is, and how easily the decision-maker can use it to select the best solution among the Pareto optimal options. Furthermore, the BHARAT method enables the decision-makers to directly assign the numerical weights to the objectives rather than the ranks and compute the total scores for the alternatives as demonstrated in the second case of case study 3.

The outcomes showcased in this study are highly valuable for industrial process planners and designers. The suggested BHARAT method's concept is clear-cut, uncomplicated, and efficient; it can tackle many- and multi- objective optimization issues in a variety of scientific and engineering domains.

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