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BHARAT: A simple and effective multi-criteria decision-making method that does not need fuzzy logic, Part-1: Multi-attribute decision-making applications in the industrial environment

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A simple and effective multi-criteria decision-making methodology named as "Best Holistic Adaptable Ranking of Attributes Technique (BHARAT)" is proposed that can be used in single- as well as group decision-making scenarios of the industrial environment. The attributes data for various alternatives can be quantitative or qualitative (i.e., expressed in linguistic terms). This paper proposes to transform the qualitative attributes into quantitative attributes by means of simple linear scales rather than complex fuzzy scales. The proposed BHARAT method normalizes the data with reference to the "best" alternative corresponding to an attribute and the normalization procedure is repeated for all the attributes to get the normalized data. A group of decision-makers or a decision-maker assigns ranks to the attributes according to how important they are deemed to be, and these ranks are then transformed into the proper weights. The total scores of the alternatives are calculated by multiplying the weights of the attributes by the corresponding normalized data of the attributes for different alternatives. Four industrial case studies are presented to illustrate the potential of the suggested BHARAT method. The first case study deals with the problem of an automated warehouse selection for a large industrial plant involving a single decision-maker, 13 attributes, and 4 alternative warehouses; the second case study deals with the problem of sustainable maintenance service provider selection for a large petrochemical plant involving fuzzy group decision-making with 5 decision-makers, 9 attributes, and 4 alternative maintenance service providers; the third case study deals with the problem of alternative strategy selection for implementation of a make-to-order system for passenger car manufacturers involving 6 factors, 18 sub-factors, and 3 alternative strategies; and the fourth case study deals with the problem of process parameters selection in a sustainable high speed turning operation involving 4 attributes and 9 alternative sets of experimental conditions. The results of the proposed decision-making method and its second version are compared with the other popular decision-making methods. The proposed method and its another version are proved simple, effective, powerful, flexible, easy to apply, do not require the use of fuzzy logic, offer logical and consistent procedures to assign weights to the attributes, and are applicable to different decision-making scenarios of the industries. Part-1 of this paper describes the applications of the BHARAT method to multi-attribute decision-making problems and Part-2 describes the evaluation of Pareto solutions using the BHARAT method in multiple objective decision-making problems.

1. Introduction

Making decisions while considering several, frequently opposing criteria is known as multi-criteria decision-making (MCDM). The two categories of MCDM are: (1). multi-objective decision-making (MODM) and (2). multi-attribute decision-making (MADM). In MODM problems, it is necessary to maximize or minimize a number of objective functions while adhering to specific constraints within a domain that contains acceptable variable values. Many sets of variables that satisfy the stated constraints and maximize or minimize the objective functions may exist in the domain. The Pareto front is made up

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of the values of the corresponding objective functions, and these trade-offs make up the Pareto optimal set. Multi-objective optimization (MOO) problems are another name for MODM problems. There can be any number of objectives in the MODM problems. Many-objective optimization problems are MODM problems that have four or more objectives. Conversely, MADM problems are typically discrete and have a finite set of predefined alternatives. The method of problem solving known as MADM is used to address situations where there are only a few, not many, alternatives available for selection. An MADM method outlines the steps to take to process attribute data and decide. The four components of any MADM problem are: (a) alternatives; (b) attributes (also known as factors or criteria); (c) attributes' weights; and (d) performance measures of alternatives in relation to the attributes. An example of an MADM problem of a suitable house selection is given in Table 1. Suppose the four attributes involved in a house selection problem are: purchase price, nearness to the market, nearness to school, and nature of neighborhood. The task is to determine which of the three alternative houses is the best.

Table 1
Data of the alternatives and attributes

Alternative	Price ($)	Distance to the market (km)	Distance to the school (km)	Nature of neighbourhood
House-1	100,000	1.5	2.75	Good
House-2	140,000	1.0	4.0	Very good
House-3	80,000	1.7	3.0	Average

When faced with MADM problems, a decision-maker must decide how to handle qualitative attributes, assign weights to the attributes, use attribute data corresponding to different alternatives, deal with imprecise information, and reach a final decision. The literature contains several MADM methods, some of which have gained a lot of traction. These include the following: data envelopment analysis (DEA), decision-making trial and evaluation laboratory (DEMATEL), complex proportional assessment (COPRAS), step-wise weight assessment ratio analysis (SWARA), best-worst method (BWM), simple additive weighing (SAW), analytical hierarchy process (AHP), analytical network process (ANP), elimination et choix traduisant la realité (ELECTRE), višekriterijumsko kompromisno rangiranje (VIKOR), technique for order preference by similarity to ideal solution (TOPSIS), grey relational analysis (GRA), preference ranking organization method for enrichment evaluations (PROMETHEE), etc. These methods have been shown to be effective in a variety of decision-making scenarios. Nevertheless, each of these approaches has advantages and disadvantages (Rao and Lakshmi, 2021a). For example, the TOPSIS approach necessitates extensive computations that grow increasingly intricate as the number of alternatives and attributes increases. The rankings of alternatives in TOPSIS approach vary depending on how the data is normalized using different normalization techniques. In the case of VIKOR method there is more computation involved. Furthermore, different ranking lists may produce different results for the same attribute weights, and the technique of "the majority of attributes" may have a weight that varies from 0 to 1.

The ELECTRE method's computing process is rather complicated, and it makes use of the outranking relationship notion. The process entails laborious computations of net discordance and net concordance of different alternatives. The AHP method compares alternatives and attributes on a scale from 1-9, which results in many comparison matrices. The more options and attributes there are, the larger and more numerous the comparison matrices become. Occasionally, the issue of inconsistent judgments comes up. Moreover, decision outcomes may vary depending on how the weights are calculated (arithmetic mean, geometric mean, etc.). Because of feedback loops and interrelations, the ANP approach makes results verification exceedingly challenging and complex. The BWM is used for finding the attributes' weights. In terms of judgment consistency, the BWM approach performs better than the AHP method, but it also necessitates a substantial amount of computational work because pairwise comparisons between the best, worst, and other criteria increase. When there are more attributes than alternatives, the DEA approach requires more processing and cannot discriminate between poor and good alternatives. Effective use of DEA requires that the decision-maker understand the concepts of linear programming. It is unclear in GRA which ranking of the alternatives should be accepted since it provides distinct rankings for different distinguishing coefficient values.

Fuzzy versions of MADM methods such as fuzzy TOPSIS, fuzzy VIKOR, fuzzy AHP, fuzzy PROMETHEE, fuzzy DEMATEL, and so forth use fuzzy language to express the quantitative values of the attributes, that requires a great deal more computation and goes against the core idea of fuzzy logic because they involve describing the available values of the attributes in fuzzy terms.

Different outcomes may be obtained using different membership functions (triangular, trapezoidal, piecewise linear, singleton, gaussian, etc.) and defuzzification techniques (finding the leftmost maximum, rightmost maximum, center of gravity, average mean, etc.). Furthermore, a variety of fuzzy versions- orthogonal fuzzy, intuitional fuzzy, hesitant fuzzy, spherical fuzzy, Pythagorean fuzzy, and so on - all contribute to more confusion and are beyond the comprehension of the real decision makers in the industries, governments, organizations, etc. Fudging the numbers with fuzziness not only makes manipulations more complex, but it also takes away from the original numbers' elegance and simplicity as a means of representing the judgments. This frequently results in less desirable outcomes rather than more desirable ones. It is observed that making bad decisions results in bad outcomes, and fuzzifying bad decisions results in very bad results. While making sound decisions leads to good and valid results, fuzzifying these decisions is merely a perturbation that maintains the current state of the results without bringing about consistent improvements. Fuzzifying the inconsistent decisions may not improve the situation, but rather make it worse (Saaty, 2007). There have been many attempts to apply fuzzy concepts to the decision-making field. A review of those attempts may be found in Morente-Molinera et al. (2015) and Wang et al. (2021). These comprise assessments made in the face of uncertainty that are already hazy and might not gain additional clarity by further fuzzification. However, there have been heedless attempts to apply fuzzy techniques anywhere there are numerical values, without scrutinizing the legitimacy of the methodology. Certain procedures view all numbers as amenable to fuzzy logic. It is particularly used in operations research and decision-making, which, unlike mathematics, does not require proof when a well-known method is used. When we move forward without asking why, the endeavor could become a misdirected intellectual exercise meant solely for publication, with no consideration for the accuracy of our work. Many journal editors and reviewers approved the articles for publication without questioning the validity of the results. *There is no evidence to support the notion that fuzzifying the decision-making process from start to finish is better.*

Fuzzy logic may be more meaningful in control systems and in artificial neuro-fuzzy inference systems (ANFIS) but it may not be so in MADM problems. It is not proven beyond a reasonable doubt that a fuzzy MADM approach can offer a superior substitute to traditional MADM techniques for the given problems. Still, the “popular” tendency of using fuzzy logic in MADM is continuing and increasing in a disturbing way. Furthermore, the researchers are adding more complexity by fuzzifying the opinions of several decision makers (i.e., in group decision-making) in different ways for each of the attributes and alternatives. Aggregating the fuzzy opinions of the decision makers follows another fuzzy procedure. This type of research is a threat, and it may be useful only for academic promotions of the researchers but not useful to the real decision makers in the industries, organizations, policy makers in the government, etc. As mentioned above, the decision-maker frequently faces the following issues in MADM problems: how to weigh the attributes, manage qualitative attributes, use attribute data corresponding to various alternatives, deal with imprecise information, and conclude.

Which approaches - subjective or objective – are used to assign the attributes’ weights?

There are two ways to weigh the attributes in a decision-making problem: subjective approaches and objective approaches. When employing objective approaches, techniques such as the variance, standard deviation, and entropy methods can be used to find the attributes’ weights. The decision-maker has no influence over the weights that are thus determined; they are referred to as the objective weights. Recently, researchers have started using objective weights, mostly using the entropy method. However, it is to be noted that the objective weights are based on the given values of the attributes and the decision-maker has no role. For example, let us consider a group decision- making situation involving three decision makers. Let each of the three decision-makers give an attribute a weight of 0.10, 0.15, or 0.12. The average weight may be computed as 0.1233. However, when objective weights calculation is done by entropy method, depending upon the numerical values of that attribute, the entropy method may compute the weight as 0.38, which is completely different from the average weight of 0.1233 obtained based on the opinions of a group of three decision-makers. A similar situation may occur in the case of other attributes. *Then the evaluation and rankings of the alternatives using such objective attributes’ weights may be meaningless as the opinions of the decision-makers who deal with the practical importance of the attributes in each decision-making situation are not considered.* Such types of meaningless exercises are many in literature! Then why the objective weights are used in MADM problems? How can someone (or a group of people) be considered a decision-maker if he/she/they are incapable of expressing their preferences regarding the significance of certain attributes? Some researchers have recently started using the composite weights, i.e., combining the subjective weights and the objective weights. This may be another meaningless exercise for the sake of publication. In real decision-making situations, are these objective weights or composite weights used at all? The answer is, obviously, **NO**. Mostly, such types of exercises are for academic research’s sake.

When applying subjective approaches, the decision-maker can decide to apply ranking methods such as equal weights, rank sum, reciprocal weight, rank exponent, and centroid weights (Roszkowska, 2013, Stillwell, 1981) or use techniques such as AHP (Saaty, 2000) or BWM (Rezaei, 2015). Alternatively, the decision-maker may choose to assign the weights randomly, based on intuition, experience, or personal preference. The ranking methods, except the centroid method, rely primarily on heuristics to determine the attributes’ weights. The centroid approach assigns weights more steeply, giving a relatively high weight to the most significant attribute and a comparatively low weight to the least significant attribute. The recently proposed R-method (Rao and Lakshmi, 2021a, 2021b) developed an equation and a table that can be used to assign ranks to the attributes and the alternatives and then the ranks can be converted into corresponding weights. The weights proposed by the R-method are comparatively more stable and more significant than the other methods of ranking. However, even though R-method has simplified the decision-making process, the concept of assigning ranks to the available quantitative values of attributes may not be much appropriate. It is because, if two alternatives have very close data for a particular attribute, then they are assigned ranks 1 and 2 as per the R-method procedure and the corresponding weights are assigned using a table developed for the purpose. Even though there isn't much of a difference between the attribute values for the two alternatives, it might still have some impact on how the alternatives are evaluated in terms of final scores.

Researchers should concentrate on creating straightforward, yet effective MADM techniques that can offer strong and efficient answers to intricate decision-making issues involving a multitude of options and characteristics. Mere academic research going on at present in the name of fuzzy group decision-making has little or no meaning in real decision-making problems of industries. Developing simple methods becomes more useful to the decision-makers not only in the industrial environment but also to the policy-makers and individuals or groups in society. Furthermore, development of such simple and easy to implement methods help in timely decision-making and applicable in various decision contexts apart from having the capability to deal with qualitative attributes, imprecise information, and decision makers with varied capabilities to process the information.

The research questions (RQs) related to selection of a right alternative using MADM methods are:

- RQ1: In an industrial or other decision-making setting, is there a simple and suitable MADM method to weigh the selection attributes and evaluate the alternatives?
- RQ2: Can the MADM method (which answers RQ1) handle both qualitative and quantitative attributes? If so, how effectively does this type of MADM approach function in situations where a group decision-making process involves multiple decision-makers?
- RQ3: If a MADM approach (which addresses RQs 1 and 2) exists, will it be easy to comprehend and practical to use when making decisions in situations where there is uncertainty in the information at hand? Is a complex fuzzy logic required for decision-making?
- RQ4: Is there any strong evidence that fuzzy logic that involves different membership functions and different defuzzification methods provides better results compared to the conventional logic in MADM situations?
- RQ5: Is it feasible to have an appropriate MADM approach that is both reliable and resistant to changes in the attributes' weights? Does it become necessary to create an MADM method that is insensitive to changes in attribute weights? Can such kind of MADM method regarded as best?

To address the RQs, this paper proposes two versions of a simple, systematic, logical, effective, and powerful decision-making method named as **BHARAT** (Best Holistic Adaptable Ranking of Attributes Technique). The next section provides the details.

2. Proposed BHARAT Methodology

2.1 BHARAT

In multi-attribute decision-making methods, every decision table includes alternatives, attributes, measures of performance for each alternative and the attributes' weights. Based on the data in the decision table and the selected decision-making technique, the decision-maker's task is to assess each alternative and identify which is the best choice. The steps of the suggested decision-making methodology are explained below.

Step 1: Identify the decision-making situation, relevant attributes A_i (i = 1, 2, …., m) and the alternatives B_j (for j = 1, 2,…..,n). The pertinent attributes include both beneficial and non-beneficial attributes. Either a group of decision-makers or a single decision-maker can take this step. For the beneficial attributes, higher values are preferred, and for the non-beneficial attributes, lower values.

Step 2: Ranking weights can be applied in a variety of situations, and there are many ranking methods that can be used (Roszkowska, 2013). Since ranks are easier for experts and non-experts to agree upon, It is more dependable to use ranks rather than weights for determining the attributes' weights. This is a result of the fact that decision-makers frequently have greater faith in some attributes' ranks than in their weights. Sorting the attributes according to importance is easier.

To determine the attributes' weights w_i (for i=1, 2, ..., m), order the attributes according to the decision-maker's assessment of their significance in terms of 1, 2, 3, 4, 5, 6, and so on. When two or more attributes are deemed to be equally significant, they are given an average rank. The R-method, which the author recently proposed (Rao and Lakshmi, 2021a, b), is used in the proposed BHARAT method. For instance, if there are three attributes A, B, and C and are assigned the ranks of 1, 2, and 3, then the attributes' weights are computed as shown below.

For 3-attributes:

Reciprocal of reciprocal of rank 1: 1/ (1/1) = 1.000000
Reciprocal of reciprocals of ranks up to 2: 1/(1/1 + 1/2) = 0.666666
Reciprocal of reciprocals of ranks up to 3: 1/(1/1 + 1/2 + 1/3) = 0.545454
Total sum = 1.000000 + 0.666666 + 0.545454 = 2.212121

As a result, the ranks 1, 2, and 3 are assigned the weights of 0.45205 (=1.000000/2.212121), 0.30137 (=0.666666/2.212121), and 0.24657 (=0.545454/2.212121) respectively.
Thus, weights of 0.45205, 0.30137, and 0.24657 are assigned to the three attributes A, B, and C that are ranked 1, 2, and 3 respectively.

As another example, if there are four attributes A, B, C, and D and they are assigned the ranks of 1, 3, 4, and 2 by a single decision-maker then the attributes' weights are computed as shown below.

For 4-attributes:

Reciprocal of reciprocal of rank 1: 1/ (1/1) = 1.000000

Reciprocal of reciprocals of ranks up to 2: 1/(1/1 + 1/2) = 0.666666
Reciprocal of reciprocals of ranks up to 3: 1/(1/1 + 1/2 + 1/3) = 0.545454
Reciprocal of reciprocals of ranks up to 4: 1/(1/1 + 1/2 + 1/3 + 1/4) = 0.48
Total sum = 1.000000 + 0.666666 + 0.545454 + 0.48 =2.69212

As a result, the ranks 1, 2, 3, and 4 are assigned the weights of 0.37145(=1.000000/2.69212), 0.24763(=0.666666/2.69212), 0.20261(=0.545454/2.69212), and 0.17829(=0.48/2.69212) respectively.
Thus, weights of 0.37145, 0.20261, 0.17829, and 0.24763 are assigned to the four attributes A, B, C, and D that are ranked 1, 3, 4, and 2 respectively.

Table A1 of the Appendix shows the weights given to the 35 ranks that correspond to the 35 attributes (Rao and Lakshmi, 2021b). *To ascertain the attributes' weights in accordance with the ranks given by a decision-maker, this process can be expanded to include any number of attributes.* Eq. (1) given below can be used to assign the weights to any number of ranks.

$$w_i = \frac{1/\sum_{k=1}^{i} 1/r_k}{\sum_{i=1}^{m}\left[1/\sum_{k=1}^{i}\frac{1}{r_k}\right]} \tag{1}$$

w_i = weight of attribute i (i = 1, 2, ….., m)
r_k = rank of attribute k (k = 1, 2, ….., i)
m = number of attributes

Table A1 can be directly utilized by the decision-maker to assign the weights directly to the attributes. *It should be mentioned that an average rank is given to attributes if two or more are deemed equally important.* For instance, if there are four attributes A, B, C, and D and the decision-maker assigns rank 1 to A. If he/she thinks that B and C are equally important then an average rank of 2.5 (i.e., (2+3)/2) can be assigned to both B and C. The attribute D can be assigned rank 4. Then the weights assigned to the attributes A, B, C, and D from Table A1 are 0.37145, 0.22512, 0.22512, and 0.17829 respectively. It may be noted that the weight of 0.22512 assigned to B and C is the average of 0.24763 and 0.20261 (i.e., (0.24763/0.20261)/2).

What about obtaining weights in group decision making scenarios?

In group decision-making, compute the average of the decision-makers' ranks for each attribute. For instance, in the above example of four attributes, let the ranks assigned by three decision-makers to the attributes A, B, C, and D be (1, 2, 1), (3, 1, 2), (2, 3, 3), and (4, 4, 4) respectively. The attributes' weights based on the ranks assigned by a group of three decision-makers for four attributes are computed as shown in Table 2. Table A1 is used to transform the weights provided by each decision-maker (DM) into corresponding weights. Then these weights are averaged, as shown in the last column of Table 2.

Table 2
Computing the weights of 4 attributes based on the ranks assigned by a group of 3 decision-makers

Attributes	*Decision-makers*			*Weights assigned as per Table A1 for 4 attributes*			*Average weights of attributes ((i+ii+iii)/3)*
	DM1	*DM2*	*DM3*	*DM1-w (i)*	*DM2-w (ii)*	*DM3-w (iii)*	
A	1	2	1	0.37145	0.24763	0.37145	0.33018
B	3	1	2	0.20261	0.37145	0.24763	0.27390
C	2	3	3	0.24763	0.20261	0.20261	0.21762
D	4	4	4	0.17830	0.17830	0.17830	0.17830

As a result, the weights associated with the characteristics A, B, C, and D are, respectively, 0.33018, 0.27390, 0.21762, and 0.17830. *This can be extended to incorporate many more attributes, giving weights to the attributes based on their rankings in the context of group decision-making.*

What if the decision-makers themselves are assigned a different importance instead of equal importance?

In Table 2, the decision-makers are given equal importance (i.e., 1/3) assuming that they are equally capable. However, if different importance are assigned to the opinions of the decision-makers such as w_{DM1}, w_{DM2}, and w_{DM3} for the three decision-makers, then each cell in last column of Table 2 will be a weighted sum. For example, if the importance assigned to the three decision-makers are w_{DM1}, w_{DM2}, and w_{DM3} and these are 0.40, 0.25, and 0.35, then the average attributes' weights are computed as given below.

Average weight of attribute A = 0.40×0.37145 + 0.25×0.24763 + 0.35×0.37145 = 0.34050
Average weight of attribute B = 0.40×0.20261 + 0.25×0.37145 + 0.35×0.24763 = 0.26058
Average weight of attribute C = 0.40×0.24763 + 0.25×0.20261 + 0.35×0.20261 = 0.22062
Average weight of attribute D = 0.40×0.17830 + 0.25×0.17830 + 0.35×0.17830 = 0.17830

Hence, the proposed method can offer a simple and systematic procedure for determining the attributes' weights in single- as well as group decision-making situations (with equal or unequal importance given to the opinions of the decision-makers).

Step 3: Measure the performance x_{ji} of attribute for each alternative (the performances may be expressed in quantitative or qualitative terms). In group decision-making, the qualitative descriptions (i.e., linguistic terms) made by a team of decision-makers *for each* qualitative attribute are to be noted. Transform the qualitative data (i.e., linguistic terms) of the attributes into quantitative data using a simple scale without the need of using fuzzy logic.

The fuzzy scales proposed by various researchers using different membership functions to deal with linguistic or qualitative attributes can be easily replaced by simple ordinary scales. For example, A linguistic or qualitative attribute's translation or conversion into a quantitative attribute is displayed in Tables 3-5 on 11-point, 7-point, and 5-point simple scales, respectively. Any of these straightforward scales can be used by the decision-maker in a particular situation.

Table 3a

Transformation of a beneficial qualitative attribute into a quantitative attribute on a 11-point scale

Linguistic term of the qualitative attribute	*Fuzzy scale value using a triangular membership function for a beneficial attribute (Rao, 2013)*	*Simple scale value without using any fuzzy membership function for a beneficial attribute*
Exceptionally less/Exceptionally low/Exceptionally poor (or similar term)	0.0455	0.0
Extremely less/Extremely low/Extremely poor (or similar term)	0.1364	0.1
Very less/Very low/Very poor (or similar term)	0.2273	0.2
Less/Low/Poor (or similar term)	0.3182	0.3
Below average/Below medium (or similar term)	0.4091	0.4
Average/Fair/Medium (or similar term)	0.5	0.5
Above average/Above medium/Medium high (or similar term)	0.5909	0.6
High/Good (or similar term)	0.6818	0.7
Very high/Very good (or similar term)	0.7727	0.8
Extremely high/Extremely good (or similar term)	0.8636	0.9
Exceptionally high/Exceptionally good (or similar term)	0.9545	1.0

Table 3b

Transformation of a non-beneficial qualitative attribute into a quantitative attribute on a 11-point scale

Linguistic term of the qualitative attribute	*Fuzzy scale value using a triangular membership function for a non-beneficial attribute (Rao, 2013)*	*Simple scale value without using any fuzzy membership function for a non-beneficial attribute*
Exceptionally less/Exceptionally low (or similar term)	0.9545	1.0
Extremely less/Extremely low (or similar term)	0.8636	0.9
Very less/Very low/Very poor (or similar term)	0.7727	0.8
Less/Low/Poor (or similar term)	0.6818	0.7
Below average/Below medium (or similar term)	0.5909	0.6
Average/Fair/Medium (or similar term)	0.5	0.5
Above average/Above medium/Medium high (or similar term)	0.4091	0.4
High (or similar term)	0.3182	0.3
Very high (or similar term)	0.2273	0.2
Extremely high (or similar term)	0.1364	0.1
Exceptionally high (or similar term)	0.0455	0

Table 4a

Transformation of a beneficial qualitative attribute into a quantitative attribute on a 7-point scale

Linguistic term of the qualitative attribute	*Fuzzy scale value using a triangular membership function for a beneficial attribute (Rao, 2013)*	*Simple scale value without using any fuzzy membership function for a beneficial attribute*
Very less/Very low/Very poor (or similar term)	0.0	0.0
Less/Low/Poor (or similar term)	0.1364	0.1666
Below average/Below medium/Medium poor (or similar term)	0.3333	0.3333
Average/Fair/Medium (or similar term)	0.5	0.5
Above average/Above medium/Medium high/Medium good (or similar term)	0.6666	0.6666
High/Good (or similar term)	0.8636	0.8333
Very high/Very good (or similar term)	1.0	1.0

Table 4b

Transformation of a non-beneficial qualitative attribute into a quantitative attribute on a 7-point scale

Linguistic term of the qualitative attribute	*Fuzzy scale value using a triangular membership function for a non-beneficial attribute (Rao, 2013)*	*Simple scale value without using any fuzzy membership function for a non-beneficial attribute*
Very less/Very low/Very poor (or similar term)	1.0	1.0
Less/Low/Poor (or similar term)	0.8636	0.8333
Below average/Below medium (or similar term)	0.6666	0.6666
Average/Fair/Medium (or similar term)	0.5	0.5
Above average/Above medium/Medium high (or similar term)	0.3333	0.3333
High (or similar term)	0.1364	0.1666
Very high (or similar term)	1.0	0.0

Table 5a
Transformation of a beneficial qualitative attribute into a quantitative attribute on a 5-point scale

Linguistic term of a qualitative attribute	*Fuzzy scale value using a triangular membership function for a beneficial attribute (Rao, 2013)*	*Simple scale value without using any fuzzy membership function for a beneficial attribute*
Very less/Very low/Very poor (or similar term)	0.1	0
Less/Low/Poor (or similar term)	0.3	0.25
Average/Fair/Medium (or similar term)	0.5	0.5
High/Good (or similar term)	0.7	0.75
Very high/Very good (or similar term)	0.9	1.0

Table 5b
Transformation of a non-beneficial qualitative attribute into a quantitative attribute on a 5-point scale

Linguistic term of a qualitative attribute	*Fuzzy scale value using a triangular membership function for a non-beneficial attribute (Rao, 2013)*	*Simple scale value without using any fuzzy membership function for a non-beneficial attribute*
Very less/Very low/Very poor (or similar term)	0.9	1.0
Less/Low/Poor (or similar term)	0.7	0.75
Average/Fair/Medium (or similar term)	0.5	0.5
High (or similar term)	0.3	0.25
Very high (or similar term)	0.1	0.0

The simple ordinary scales presented in Tables 3 – 5 are linear scales with coefficient of determination $R^2 = 1$. One may develop non-linear scales also but there will not be any significant difference in the results of evaluation. Even in the fuzzy logic procedure, the triangular or trapezoidal membership functions involve the linear forms only. Therefore, depending on their needs, the decision-makers may use any of Tables 3 through 5.

In group decision-making, transform the linguistic terms expressed by the group for a qualitative attribute into quantitative values using the proposed simple scale and then compute the average value of the attribute. For example, for a qualitative beneficial attribute, if a group of three decision-makers using the 11-point scale express their opinions as "Very low", "Low", and "Average", then the corresponding simple scale values of "0.2", "0.3", and "0.5" can be averaged as "0.3333". If the qualitative attribute is a non-beneficial one, then the corresponding simple scale values of "0.8", "0.7", and "0.5" can be averaged as "0.6666".

Step 4: Normalize the data of an attribute with reference to the "best" value of the attribute for different alternatives. Repeat this normalization procedure for all the attributes to get the normalized data. The word 'best' indicates the highest available value when referring to the beneficial attribute, and the lowest available value when referring to non-beneficial attribute. The performance measures of alternatives x_{ji} (for j= 1, 2,…..,n; i=1, 2,…..,m) are to be normalized. The normalized value $(x_{ji})_{\text{normalized}}$ of an alternative corresponding to a beneficial attribute is $x_{ji}/x_{i.best}$, and it is $x_{i.best}/x_{ji}$ for non-beneficial attribute. The $x_{i.best}$ is the best value of i^{th} attribute. This type of normalizing the data with reference to the "best" values clearly show the standing positions of the alternatives with reference to the "best" values of the attributes.

Step 5: Total score of an alternative is $\sum wi*(x_{ji})_{\text{normalized}}$ and it is the result of multiplying the attributes' weights with the matching normalized data of the attributes for the alternatives. This is how to calculate the total scores of the alternatives.

Step 6: Sort the alternatives according to the total scores in decreasing order. For a specific decision-making scenario under consideration, the alternative with the highest total score is deemed optimal.

The flowchart of BHARAT methodology is shown in Fig. 1.

2.2 Another version of BHARAT

Another version of BHARAT is proposed, and the *steps of this version are same as the steps of BHARAT except step-2*. In this version, the attributes are ranked in terms of 1, 2, 3, 4, 5, 6, etc., according to the decision-maker's assessment of their relative importance in determining the attributes' weights. When two or more attributes are deemed to be equally significant, they are given an average rank. For example, if there are four attributes A, B, C, and D and they are assigned the ranks of 1, 3, 4, and 2 by a single decision-maker then the attributes' weights are computed as shown in Table 6.

Table 6
Computing the attribute weights based on the ranks assigned by a single decision-maker

Attributes	*A*	*B*	*C*	*D*	*Means of rows*	*Weights of attributes*
A	1	3/1	4/1	2/1	2.5	2.5/5.208=0.48
B	1/3	1	4/3	2/3	0.833	0.833/5.208=0.16
C	1/4	3/4	1	2/4	0.625	0.625/5.208=0.12
D	1/2	3/2	4/2	1	1.25	1.25/5.208 = 0.24
			Total =		5.208	1.00

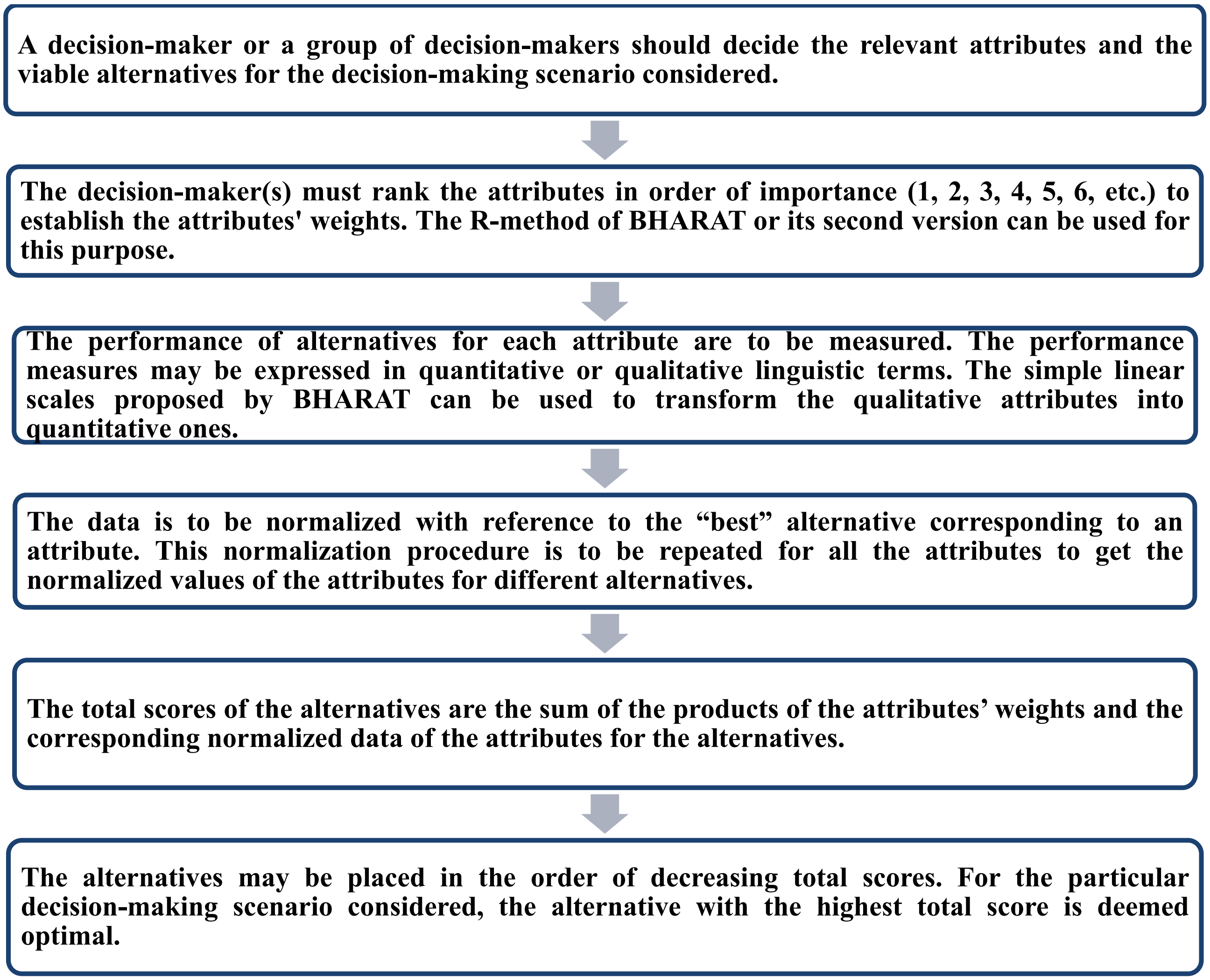

Fig. 1. Flowchart of BHARAT methodology

In Table 6, it may be observed that the matrix related to A, B, C, and D indicates that $a_{ji} = 1/a_{ij}$ and $a_{ii} = 1$. In the above example, arithmetic means of the rows are considered. Even if the geometric means are considered then also the same attributes' weights can be obtained. Just like in AHP and BWM methods, if the consistency check is carried out, then it can be easily found that the maximum eigen value of this matrix is 4 (i.e., $\lambda_{max} = 4$) and there is *absolute consistency* in the above judgments of assigning the ranks. Hence, the attributes' weights A, B, C, and D can be assigned 0.48, 0.16, 0.12, and 0.24 respectively. Table A2 given in the Appendix shows different ranks and corresponding attributes' weights. *The attributes' weights can be determined by extending this process to any number of attributes and assigning ranks to each one by a single decision-maker.* It may be noted that methods like AHP and BWM hardly offer the absolute consistency in the judgments of relative importance.

While making decisions in groups, compute the average value of the ranks given by the decision-makers for each attribute. For example, in the above example, let the ranks assigned by three decision-makers to the attributes A, B, C, and D are (1, 2, 1), (3, 1, 2), (2, 3, 3), and (4, 4, 4) respectively, then the ranks can be averaged as ((1+2+1)/3), ((3+1+2)/3), ((2+3+3)/3), and ((4+4+4)/3) (i.e., 1.333, 2, 2.666, and 4). The attributes' weights are calculated as given in Table 7.

Table 7
Computing the attributes' weights based on the ranks assigned by a group of decision-makers

Attributes	*A*	*B*	*C*	*D*	*Means of rows*	*Weights of attributes*
A	1	2/1.333	2.666/1.333	4/1.333	1.875	1.875/4.6875=0.400
B	1.333/2	1	2.666/2	4/2	1.25	1.25/4.6875=0.2667
C	1.333/2.666	2/2.666	1	4/2.666	0.9375	0.9375/4.6875=0.200
D	1.333/4	2/4	2.666/4	1	0.625	0.625/4.6875=0.1333
			Total =		4.6875	1.00

In Table 7 also, it may be observed that the matrix related to A, B, C, and D indicates that $a_{ji} = 1/a_{ij}$ and $a_{ii} = 1$. As with the AHP and BWM methods, if the consistency check is performed in this group decision-making as well, it is simple to determine that the maximum eigen value of this matrix is 4 (i.e., $\lambda_{max} = 4$) and there is *absolute consistency* in the rankings assigned above. Hence, the attributes' weights A, B, C, and D can be assigned 0.4, 0.2667, 0.2, and 0.1333 respectively. This way of computing the attributes' weights is akin to reciprocal weight (RW) method of ranking the attributes. *The weights assigned to the fifteen ranks that correspond to the fifteen attributes are displayed in Table A2 of the Appendix. To ascertain the attributes' weights in accordance with the ranks assigned by the individual or group of decision-makers, this process can be extended to include any number of attributes.* The other steps of the second version are the same as those of BHARAT and this version may be named as BHARAT-2.

The procedure explained above for the second version of BHARAT to assign the weights to the attributes *in the case of single decision-maker* can be simplified by Eq. (2). With the help of the following Eq. (2), *any* number of ranks can be assigned the weights.

$$w_i = \frac{1/r_j}{\sum_{i=1}^{m}\left[1/r_j\right]} \tag{2}$$

i=1

w_i = weight of attribute i (i = 1, 2, 3, …, m), m = no. of attributes, and r_i is the rank of attribute i.

The decision-makers can also directly utilize Table A2 for assigning the weights to the attributes. In fact, it can be observed from Tables A1 and A2 that Eq. (1) provides comparatively smoother and more stable weights than Eq. (2). The weights given by Eq. (2) decrease aggressively following the most important attribute and it is not so in the case of weights obtained by Eq. (1) in BHARAT.

An important point to note is that Eq. (2) is applicable only for single decision-maker cases and for cases where distinct ranks are given to the attributes. Eq. (2) is not appropriate for group decision-making and for cases where equal ranks are given to certain attributes. In group decision-making situations, the related procedure to form Table 7, which is more appropriate and guarantees absolute consistency in the assessments of relative importance, should be used. Similarly, for cases where certain attributes are given equal ranks, the related procedure to form Table 14 of case study 1 and Table 21 of case study 2 explained in section 3 of this paper should be used as it is more appropriate and ensures absolute consistency.

The next section presents four case studies that clearly demonstrate the potential of the proposed BHARAT method and its second version. The first case study deals with the problem of an automated warehouse selection involving a single decision-maker, the second case study deals with the problem of sustainable maintenance provider supplier selection involving fuzzy group decision-making for a large petrochemical plant, the third case study deals with the problem of alternative strategy selection for implementing a make-to-order (MTO) system for passenger cars, and the fourth case study deals with the problem of machining process parameters selection in a sustainable high-speed turning operation.

3. Working demonstration of the proposed BHARAT method

3.1 Case study 1: Automated warehouse selection (involving a single decision-maker)

This case study was done to determine which automated warehouse would be best for an Indian industry's products. It is necessary to assess the four automated warehouses that are currently in place for product storage. A total of 13 attributes are considered. These are: power consumption (PC) in kW, cycle time (CT) in minutes, material flow rate tonnage (MFRT) in tons of monthly load, total crew members (TCM), area of setup (AS) in square feet, maintenance calls (MC) per month, number of wear and tear pallets (WTP) in a month, scope for expansion (SE), operating skill required (OSR), firefighting reach ability (FRA), operator safety (OS), material flow rate units (MFRU) in tons, and number of forklifts.

The steps of the BHARAT method are implemented by a *single decision-maker* as explained below.

<u>Step 1:</u> Relevant attributes and alternatives for the decision-making situation of automated warehouse selection are identified. This case study involved a single decision-maker. The data of the identified attributes and the alternatives is given in Table 8. The attributes OS, FRA, OSR, SE, and WTP are expressed in linguistic terms. The numbers shown in parentheses indicate the corresponding quantitative values assigned based on a simple 11-point scale keeping in view of their nature (i.e., beneficial or non-beneficial). Table 3a is used for OS, FRA, and SE, and Table 3b is used for OSR and WTP. The attributes MFRU, OS, FRA, SE, and MFRT are of beneficial type and other attributes are of non-beneficial type.

Table 8

Data of the attributes and the alternatives of automated warehouse selection problem (Rao, 2013; Singh, 2012)

Attributes	*Alternatives*			
	AW-1	*AW-2*	*AW-3*	*AW-4*
PC	7545.18	3961.29 ($x_{i.best}$)	8952	11761.29
CT	12.5 ($x_{i.best}$)	15	17.5	22
MFRT	50.5	578.38	391.88	629.3 ($x_{i.best}$)
TCM	75	30	22 ($x_{i.best}$)	35
AS	31504.1	27425	9686.8	8437.5 ($x_{i.best}$)
MC	530	1408	147 ($x_{i.best}$)	263
WTP	L (0.7)	VL (0.8) ($x_{i.best}$)	M (0.5)	L (0.7)
SE	VP (0.2)	M (0.5) ($x_{i.best}$)	VP (0.2)	P (0.3)
OSR	H (0.3)	L (0.7) ($x_{i.best}$)	A (0.5)	A (0.5)
FRA	A (0.5)	P (0.3)	A (0.5)	G (0.7) ($x_{i.best}$)
OS	M (0.5)	L (0.3)	M (0.5)	H (0.7) ($x_{i.best}$)
MFRU	808	1028	1119.6	1798 ($x_{i.best}$)
NF	20	8	10	5 ($x_{i.best}$)

AW: Automated Warehouse; VP: Very Poor; VL: Very Less; P: Poor; L: Low; M: Medium; A: Average; H: High; G: Good

Step 2: The decision-maker ranks the 13 attributes in terms of 1, 2, 3, 4, 5, 6, and so on, depending on how important they are in their opinion. This ranking process determines the attributes' weights. An average rank is assigned to two or more attributes if they are judged to be equally significant. The attributes PC, CT, FRA, and OS are deemed equally significant in this case study. Hence, an average rank of 2.5 (i.e., (1+2+3+4)/4) is assigned. MFRU is assigned 5th rank. The attributes AS, SE, and OSR are considered equally significant. Hence, an average rank of 7 (i.e., (6+7+8)/3) is assigned. The attributes MFRT and TCM are given an average rank of 9.5 (i.e., (9+10)/2), MC and WTP are assigned an average rank of 11.5 (i.e., (11+12)/2), and NF is assigned 13th rank. The ranks given to the attributes are displayed in Table 9.

Table 9

Ranks of the 13 attributes of case study 1

	Attributes												
	PC	*CT*	*MFRT*	*TCM*	*AS*	*MC*	*WTP*	*SE*	*OSR*	*FRA*	*OS*	*MFRU*	*NF*
Ranks	*2.5*	*2.5*	*9.5*	*9.5*	*7*	*11.5*	*11.5*	*7*	*7*	*2.5*	*2.5*	*5*	*13*

The attributes' weights are computed from Table A1 corresponding to 13 number of attributes and are shown in Table 10.

Table 10

Weights of the 13 attributes of case study 1

	Attributes												
	PC	*CT*	*MFRT*	*TCM*	*AS*	*MC*	*WTP*	*SE*	*OSR*	*FRA*	*OS*	*MFRU*	*NF*
Weights	*0.11303*	*0.11303*	*0.05835*	*0.05835*	*0.06503*	*0.05486*	*0.05486*	*0.06503*	*0.06503*	*0.11303*	*0.11303*	0.07355	0.05281

It may be noted that the weights given in Table 10 are as per the proposed procedure of assigning weights to the attributes. For example, the attributes PC, CT, FRA, and OS are considered as equally significant and hence an average rank of 2.5 (i.e., (1+2+3+4)/4) is assigned. Hence, the average weight will be the average of the weights assigned to the ranks 1, 2, 3, and 4. From Table A1 corresponding to 13 number of attributes column, the average weight is computed as 0.11302 (i.e., (0.167937568+0.111958378+0.09160231+0.080610032)/4). Similarly, the attributes AS, SE, and OSR are considered equally significant. Hence, an average rank of 7 (i.e., (6+7+8)/3) is assigned. Hence, the average weight will be the average of the weights assigned to the ranks 6, 7, and 8. From Table A1 corresponding to 13 number of attributes column, the average weight is computed as 0.06503 (i.e., (0.068545946+0.06476931+0.061790432)/3). In a similar way, the weights of the other attributes are computed.

Step 3: The qualitative values (i.e., linguistic expressions) of the attributes OS, FRA, OSR, SE, and WTP are transformed into quantitative values using the 11-point scale given in Table 3 without the need of using fuzzy logic. Table 3a is used for OS, FRA, and SE; and Table 3b is used for OSR and WTP. Table 8 displays these values in parentheses.

Step 4: Normalization of the data is done using the "best" option for each attribute as shown in Table 8. The normalized value of an alternative corresponding to a beneficial attribute is $x_{ji}/x_{i.best}$, and it is $x_{i.best}/x_{ji}$ for non-beneficial attribute. The value $x_{i.best}$ is the best measure of performance of an attribute out of its available values. Table 11 indicates the best values of the attributes.

Table 11
Best values of the 13 attributes of case study 1

	Attributes												
	PC	*CT*	*MFRT*	*TCM*	*AS*	*MC*	*WTP*	*SE*	*OSR*	*FRA*	*OS*	*MFRU*	*NF*
Best value	3961.29	12.5	629.3	22	8437.5	147	0.8*	0.5	0.7*	0.7	0.7	1798	5

*The values are assigned to the attributes WTP and OSR based on Table 3b. After assigning like this, these may be considered as beneficial values for normalization purpose.

Table 12 displays the normalized data of the 13 attributes. This type of normalizing the data clearly shows the standing positions of the alternatives with reference to the "best" values of the attributes.

Table 12
Normalized data of the 13 attributes of case study 1

Attributes	*Alternatives*			
	AW-1	*AW-2*	*AW-3*	*AW-4*
PC	0.525009	1	0.442503	0.336807
CT	1	0.833333	0.714286	0.568182
MFRT	0.080248	0.919085	0.622724	1
TCM	0.293333	0.733333	1	0.628571
AS	0.267822	0.307657	0.871031	1
MC	0.277358	0.104403	1	0.558935
WTP	0.875	1	0.625	0.875
SE	0.4	1	0.4	0.6
OSR	0.428571	1	0.714286	0.714286
FRA	0.714286	0.428571	0.714286	1
OS	0.714286	0.428571	0.714286	1
MFRU	0.449388	0.571746	0.622692	1
NF	0.25	0.625	0.5	1

AW: Automated Warehouse

Step 5: By multiplying the attributes' weights given in Table 10 by the corresponding normalized data for the alternatives given in Table 12, the total scores of the alternative automated warehouses are computed.

For instance, the total score of automated warehouse-1 is calculated as,
Total score (Automated warehouse-1): $0.11303 \times 0.525009 + 0.11303 \times 1 + 0.05835 \times 0.080248 + 0.05835 \times 0.293333 + 0.06503 \times 0.267822 + 0.05486 \times 0.277358 + 0.05486 \times 0.875 + 0.06503 \times 0.4 + 0.06503 \times 0.428571 + 0.11303 \times 0.714286 + 0.11303 \times 0.714286 + 0.07355 \times 0.449388 + 0.05281 \times 0.25 = 0.536383$

Similarly, the total scores for the automated warehouses-2, 3 and 4 are computed as 0.686207, 0.677339, and 0.778870 respectively.

Step 6: The alternatives are arranged in decreasing order of the total scores as 4 - 2 - 3 - 1. Given that the automated warehouse-4 has the highest total score, it is deemed to be the best option in the scenario under consideration.

AHP, GRA, the utility additive (UTA) method, the ordered weighted averaging (OWA) method, the weighted Euclidean distance-based approach (WEDBA), and the combinatorial mathematics-based approach (CMBA), were attempted to solve the same problem described in Rao (2013). Table 13 presents the findings.

Table 13
Ranks of alternative automated warehouses obtained by different MADM methods

Alternatives	*AHP**	*GRA**	*OWA**	*UTA**	*WEDBA**	*CMBA**	*Proposed BHARAT*
AW-1	4	4	4	4	3	4	4
AW-2	2	2	1	1	2	2	2
AW-3	3	3	3	3	4	3	3
AW-4	1	1	2	2	1	1	1

*Rao (2013) and Singh (2012)

It is noted from Table 13 that the proposed BHARAT method, AHP, GRA, and CMBA have given the same rankings and proposed automated warehouse-4 as the first choice. The WEDBA method has also suggested automated warehouse-4 as the first choice. Automated warehouse-2 was suggested by UTA and OWA as the best option. Nevertheless, a detailed examination of the attribute values shows that automated warehouse-4 outperforms automated warehouse-2 in 7 out of 13 attributes with a combined weightage of more than 52%. Thus, proposing automated warehouse-4 as the first choice is more logical. Furthermore, the proposed method is comparatively simpler than the other MADM methods. Unlike other MADM methods, the proposed method's normalization procedure is also clear-cut and easy to understand. The transformation of qualitative attributes into quantitative attributes is also simple in the present method and it does not need any fuzzy scales.

Rao (2013) and Singh (2012) used AHP method for calculating the attributes' weights and used those weights in GRA, UTA, OWA, WEDBA, and CMBA methods and computed the total scores of the alternative warehouses. The attributes' weights obtained by using AHP method were: 0.1374, 0.1374, 0.0419, 0.0419, 0.0718, 0.0224, 0.0224, 0.0724, 0.0724, 0.1374, 0.1374, 0.0912, and 0.0138 for PC, CT, MFRT, TCM, AS, MC, WTP, SE, OSR, FRA, OS, MFRU, and NF respectively. Just for demonstration and for fair comparison purpose, if the same attributes' weights are used in the proposed BHARAT, then also the ranking of the alternative warehouses is same, i.e., 4 - 2 - 3 - 1 (with the scores 0.771457, 0.691304, 0.666524, and 0.570943 in that order). This clearly shows the simplicity and potential of the proposed method compared to the lengthy procedures of GRA, UTA, OWA, WEDBA, and CMBA methods. The suggested approach computes the weights and lets the decision-maker rank the attributes according to his or her preferences.

Second version of BHARAT method for case study 1

The second version of BHARAT method differs from BHARAT only in Step 2. The ranks assigned to the attributes are the same as those in Table 9. Table 14 displays the 13 attributes' relative importance relations.

Table 14
Relative importance table of the 13 attributes of case study 1 using second version of BHARAT

Attributes	*Attributes*														
	PC	*CT*	*MFRT*	*TCM*	*AS*	*MC*	*WTP*	*SE*	*OSR*	*FRA*	*OS*	*MFRU*	*NF*	*Means of rows*	*Weights of attributes*
PC	1	1	9.5/2.5	9.5/2.5	7/2.5	11.5/2.5	11.5/2.5	7/2.5	7/2.5	1	1	5/2.5	13/2.5	2.8	0.1487
CT	1	1	9.5/2.5	9.5/2.5	7/2.5	11.5/2.5	11.5/2.5	7/2.5	7/2.5	1	1	5/2.5	13/2.5	2.8	0.1487
MFRT	2.5/9.5	2.5/9.5	1	1	7/9.5	11.5/9.5	11.5/9.5	7/9.5	7/9.5	2.5/9.5	2.5/9.5	5/9.5	13/9.5	0.736808	0.0391
TCM	2.5/9.5	2.5/9.5	1	1	7/9.5	11.5/9.5	11.5/9.5	7/9.5	7/9.5	2.5/9.5	2.5/9.5	5/9.5	13/9.5	0.736808	0.0391
AS	2.5/7	2.5/7	9.5/7	9.5/7	1	11.5/7	11.5/7	1	1	2.5/7	2.5/7	5/7	13/7	0.999962	0.0531
MC	2.5/11.5	2.5/11.5	9.5/11.5	9.5/11.5	7/11.5	1	1	7/11.5	7/11.5	2.5/11.5	2.5/11.5	5/11.5	13/11.5	0.6087	0.0323
WTP	2.5/11.5	2.5/11.5	9.5/11.5	9.5/11.5	7/11.5	1	1	7/11.5	7/11.5	2.5/11.5	2.5/11.5	5/11.5	13/11.5	0.6087	0.0323
SE	2.5/7	2.5/7	9.5/7	9.5/7	1	11.5/7	11.5/7	1	1	2.5/7	2.5/7	5/7	13/7	0.999962	0.0531
OSR	2.5/7	2.5/7	9.5/7	9.5/7	1	11.5/7	11.5/7	1	1	2.5/7	2.5/7	5/7	13/7	0.999962	0.0531
FRA	1	1	9.5/2.5	9.5/2.5	7/2.5	11.5/2.5	11.5/2.5	7/2.5	7/2.5	1	1	5/2.5	13/2.5	2.8	0.1487
OS	1	1	9.5/2.5	9.5/2.5	7/2.5	11.5/2.5	11.5/2.5	7/2.5	7/2.5	1	1	5/2.5	13/2.5	2.8	0.1487
MFRU	2.5/5	2.5/5	9.5/5	9.5/5	7/5	11.5/5	11.5/5	7/5	7/5	2.5/5	2.5/5	1	13/5	1.4	0.0743
NF	2.5/13	2.5/13	9.5/13	9.5/13	7/13	11.5/13	11.5/13	7/13	7/13	2.5/13	2.5/13	5/13	1	0.538431	0.0286
														Total = 18.8293	Total = 1.0000

Using the normalized data of Table 12 and the attributes' weights obtained from the last column of Table 14, the total scores for the automated warehouses-1, 2, 3 and 4 are computed. For instance, the total score of automated warehouse-1 is calculated as,

Total score (Automated warehouse-1): $0.1487\times0.525009 + 0.1487\times1 + 0.0391\times0.080248 + 0.0391\times0.293333 + 0.0531\times0.267822 + 0.03232\times0.277358 + 0.03232\times0.875 + 0.0531\times0.4 + 0.0531\times0.428571 + 0.1487\times0.714286 + 0.1487\times0.714286 + 0.0743\times0.449388 + 0.0286\times0.25 = 0.589853812$.

Similarly, the total scores for the automated warehouses-2, 3 and 4 are computed as 0.68336, 0.666491, and 0.767888 respectively. The alternative automated warehouses are arranged in decreasing order of the total scores as 4 - 3 - 2 - 1. Given that the automated warehouse-4 has the highest total score, it is deemed to be the best option in the scenario under consideration. This ranking is also matching with that of BHARAT, AHP, GRA, and CMBA. However, it may be noted that the attributes' weights considered by the second version of BHARAT are somewhat different from the weights used by BHARAT. An interesting point to note that if the fuzzy scale given in Table 3 is considered instead of the simple linear scale for converting the qualitative terms of expressed by the decision-maker in Table 8, then also the ranking is same and the first choice is automated warehouse-4. This shows that simple linear scales serve the purpose very conveniently without using fuzzy logic. This is very convenient in practical real-life decision-making problems.

3.2 Case study 2: Sustainable maintenance service provider selection (involving fuzzy group decision-making)

A case study of choosing a sustainable maintenance service provider for a major Chinese petrochemical company was given by Tong et al. (2000). The company's operations encompassed the production and distribution of petrochemical products in addition to oil refining. Nine criteria (i.e., attributes) were used to select four maintenance service providers (B1, B2, B3, and B4) and assess each one's performance in the maintenance process. The analysis was conducted using the following 9 attributes: A1), Supplier qualification; A2), Maintenance service performance; A3), Maintenance cost; A4), Quality management; A5), Schedule control; A6), Site management; A7), Sub-supplier management; A8), and social sustainability (A9). The attribute A3 is non-beneficial and other attributes are beneficial. The group of decision-makers consisted of five senior managers from various departments: DM = {DM1, DM2, DM3, DM4, DM5}. Fuzzy triangular membership functions were used by the group of decision-makers to evaluate the alternatives and to assign the importance to the attributes.

Now the steps of the proposed BHARAT are implemented by a *group of 5 decision-makers* as explained below.

Step 1: Relevant attributes and the alternatives for the decision-making situation of sustainable maintenance service provider are identified. This case study involved a group of 5 decision-makers. The 9 attributes, 4 alternatives (i.e., sustainable maintenance service providers), and the 5 decision-makers (DM1-DM5) are same as those considered by Tong et al. (2020). The data of the identified attributes expressed in fuzzy linguistic terms by the 5 decision-makers (DM1-DM5) for the 4 alternative sustainable maintenance service providers is given in Table 15. Every attribute is described in linguistic terms and all decision-makers were given equal importance and considered equally capable.

Table 15
Qualitative data of the attributes expressed in linguistic terms by the group of 5 equally capable decision-makers for case study 2 (Tong et al., 2020)

Attributes	Alternative sustainable maintenance service providers	Decision-makers				
		DM1	DM2	DM3	DM4	DM5
A1	B1	VG	G	MG	MG	G
	B2	MG	MG	MG	F	MG
	B3	MG	F	F	G	F
	B4	G	G	G	MG	VG
A2	B1	MG	G	MG	G	MG
	B2	F	G	G	VG	G
	B3	F	MP	MP	F	F
	B4	G	G	VG	MG	G
A3	B1	MG	G	MG	MP	F
	B2	G	F	G	F	MG
	B3	F	MG	F	MG	G
	B4	F	G	VG	G	MG
A4	B1	G	MG	F	MG	G
	B2	F	F	F	MP	F
	B3	F	MG	MG	MP	P
	B4	MG	G	F	VG	G
A5	B1	MG	G	F	G	VG
	B2	F	MG	MG	G	F
	B3	MG	MG	F	VG	MG
	B4	MG	G	VG	MG	MG
A6	B1	G	MG	MG	F	MP
	B2	G	MG	G	F	MG
	B3	MG	G	G	MG	G
	B4	MP	F	MG	F	MP
A7	B1	MP	F	VP	MP	F
	B2	G	MG	F	MP	F
	B3	MG	G	F	MG	F
	B4	G	MG	MG	G	MG
A8	B1	G	MP	F	MG	F
	B2	G	MG	F	G	G
	B3	F	MG	F	MP	MP
	B4	F	MG	MG	G	F
A9	B1	F	MP	F	MP	MG
	B2	MG	G	F	MG	F
	B3	MG	F	MP	F	MP
	B4	G	G	MG	VG	MG

VG: Very Good; G: Good; MG: Medium Good; F: Fair; MP: Medium Poor; P: Poor; VP: Very Poor

Step 2: In this case study, the 5 decision-makers assigned the linguistic terms to the importance of the attributes as displayed in Table 16. For fair comparison purpose, the same linguistic terms are considered in the present work.

Table 16
Attributes' importance assigned by the group of decision-makers in case study 2

Attributes	Importance assigned to the attributes by the decision-makers				
	DM1	DM2	DM3	DM4	DM5
A1	H	H	VH	H	MH
A2	H	MH	H	VH	H
A3	M	MH	M	MH	H
A4	MH	H	VH	H	M
A5	M	ML	M	MH	L
A6	MH	H	MH	MH	H
A7	H	M	ML	M	MH
A8	H	MH	VH	H	MH
A9	H	H	H	MH	H

VH: Very High; H: High; MH: Medium High; M: Medium; ML: Medium Low; L: Low

Table 4a is used to transform the linguistic expressions given in Table 16 into quantitative values, which are then displayed in Table 17.

Table 17
Average weight values assigned to the attributes for case study 2

Attributes	Quantitative values assigned to the attributes					Average quantitative value ((DM1+DM2+DM3+DM4+DM5)/5)	Ranks assigned	Average weights of attributes
	DM1	DM2	DM3	DM4	DM5			
A1	0.8333	0.8333	1	0.8333	0.6666	0.8333	1.5	0.17939
A2	0.8333	0.6666	0.8333	1	0.8333	0.8333	1.5	0.17939
A3	0.5	0.6666	0.5	0.6666	0.8333	0.6333	7	0.08302
A4	0.6666	0.8333	1	0.8333	0.5	0.7666	5	0.09428
A5	0.5	0.3333	0.5	0.6666	0.1666	0.4333	9	0.07609
A6	0.6666	0.8333	0.6666	0.6666	0.8333	0.7333	6	0.08786
A7	0.8333	0.5	0.3333	0.5	0.6666	0.5666	8	0.07921
A8	0.8333	0.6666	1	0.8333	0.6666	0.8	3.5	0.11037
A9	0.8333	0.8333	0.8333	0.6666	0.8333	0.8	3.5	0.11037

Based on the average quantitative values, the ranks are assigned to the attributes. The attributes A1 and A2 are considered equally significant. Hence an average rank of 1.5 (i.e., (1+2)/2) is assigned to both A1 and A2. Similarly, the attributes A8 and A9 are considered equally significant and the average rank of 3.5 (i.e., (3+4)/2) is assigned to both A8 and A9. Rank 5 is assigned to A4, rank 6 is assigned to A6, rank 7 is assigned to A3, rank 8 is assigned to A7, and rank 9 is assigned to A5. The values of average weights are given in the last column of Table 17 using Table A1. For example, the attributes A1 and A2 are having the rank of 1.5 and hence, from Table A1 corresponding to 9 number of attributes, the average weight of 0.17939 (i.e., (0.215269575+0.14351305)/2) is assigned to both A1 and A2. Similarly, the other weights are assigned to the attributes using Table A1.

Step 3: The linguistic terms expressed by the group of decision-makers are to be converted into the corresponding quantitative values. This is done using a simple 7-point scale keeping in view of the nature of attributes (i.e., beneficial or non-beneficial). In fact, A3 is non-beneficial attribute. However, the group of decision-makers considered this aspect accordingly and expressed their opinions in terms of VG, G, MG, F, etc. Hence this is also considered a beneficial attribute keeping in view the way how the opinions were expressed. Now, all attributes are of beneficial type and Table 4a is used for transforming the linguistic terms of all qualitative attributes into quantitative values. Table 18 shows the quantitative values of the attributes. Each cell in the last column of Table 18 shows an alternative's average performance value with reference to a corresponding attribute. For instance, the value of 0.8 in the first cell belongs to the alternative B1 corresponding to attribute A1. Similarly, the value of 0.63328 in the second cell belongs to the alternative B2 corresponding to attribute A1, and so on.

Table 18
Quantitative data of the attributes using a 7-point conversion scale

Attributes	*Alternative sustainable maintenance service providers*	*Quantitative values assigned to the attributes by the group of decision-makers*					*Average quantitative value assigned ((DM1+DM2+DM3+DM4+DM5)/5)*
		DM1	*DM2*	*DM3*	*DM4*	*DM5*	
A1	B1	1	0.8333	0.6666	0.6666	0.8333	0.8
	B2	0.6666	0.6666	0.6666	0.5	0.6666	0.63328
	B3	0.6666	0.5	0.5	0.8333	0.5	0.6
	B4	0.8333	0.8333	0.8333	0.6666	1	0.8333
A2	B1	0.6666	0.8333	0.6666	0.8333	0.6666	0.73328
	B2	0.5	0.8333	0.8333	1	0.8333	0.8
	B3	0.5	0.3333	0.3333	0.5	0.5	0.43332
	B4	0.8333	0.8333	1	0.6666	0.8333	0.8333
A3	B1	0.6666	0.8333	0.6666	0.3333	0.5	0.6
	B2	0.8333	0.5	0.8333	0.5	0.6666	0.66664
	B3	0.5	0.6666	0.5	0.6666	0.8333	0.6333
	B4	0.5	0.8333	1	0.8333	0.6666	0.76664
A4	B1	0.8333	0.6666	0.5	0.6666	0.8333	0.7
	B2	0.5	0.5	0.5	0.3333	0.5	0.46666
	B3	0.5	0.6666	0.6666	0.3333	0.1666	0.46662
	B4	0.6666	0.8333	0.5	1	0.8333	0.76664
A5	B1	0.6666	0.8333	0.5	0.8333	1	0.76664
	B2	0.5	0.6666	0.6666	0.8333	0.5	0.6333
	B3	0.6666	0.6666	0.5	1	0.6666	0.7
	B4	0.6666	0.8333	1	0.6666	0.6666	0.76662
A6	B1	0.8333	0.6666	0.6666	0.5	0.3333	0.6
	B2	0.8333	0.6666	0.8333	0.5	0.6666	0.7
	B3	0.6666	0.8333	0.8333	0.6666	0.8333	0.76662
	B4	0.3333	0.5	0.6666	0.5	0.3333	0.46664
A7	B1	0.3333	0.5	0	0.3333	0.5	0.33332
	B2	0.8333	0.6666	0.5	0.3333	0.5	0.56664
	B3	0.6666	0.8333	0.5	0.6666	0.5	0.6333
	B4	0.8333	0.6666	0.6666	0.8333	0.6666	0.73328
A8	B1	0.8333	0.3333	0.5	0.6666	0.5	0.56664
	B2	0.8333	0.6666	0.5	0.8333	0.8333	0.7333
	B3	0.5	0.6666	0.5	0.3333	0.3333	0.46664
	B4	0.5	0.6666	0.6666	0.8333	0.5	0.6333
A9	B1	0.5	0.3333	0.5	0.3333	0.6666	0.46664
	B2	0.6666	0.8333	0.5	0.6666	0.5	0.6333
	B3	0.6666	0.5	0.3333	0.5	0.3333	0.46664
	B4	0.8333	0.8333	0.6666	1	0.6666	0.8

Step 4: The data of Table 18 is normalized with reference to the "best" alternative corresponding to each of the attributes. The normalized value of an alternative corresponding to a beneficial attribute is $x_{ji}/x_{i.best}$. The value $x_{i.best}$ is the best measure of performance of an attribute out of its available values. Table 19 indicates the best values of the attributes.

Table 19
Best values ($x_{i.best}$) of the 9 attributes of case study 2

	Attributes								
Best value	*A1*	*A2*	*A3*	*A4*	*A5*	*A6*	*A7*	*A8*	*A9*
	0.8333	0.8333	0.76664	0.76664	0.76664	0.76662	0.73328	0.7333	0.8

The 9 attributes' normalized data are shown in Table 20. The standing positions of the alternatives in relation to the "best" values of the attributes are clearly displayed by this kind of normalization of the data. Table 20 also displays the product of the average attributes' weights wi and the normalized data of the attributes for the alternatives ($x_{ji}/x_{i.best}$).

Table 20
Normalized data of the 9 attributes of case study 2

Attributes	*Alternative sustainable maintenance service providers*	*Average quantitative value assigned ((DM1+DM2+DM3+DM4+DM5)/5)*	*Normalized value ($x_{ji}/x_{i.best}$)*	*Average weight value assigned to the attributes (w_i)*	*Scores ($x_{ji}/x_{i.best}$)*w_i*
A1	B1	0.8	0.960038	0.17939	0.172221
	B2	0.63328	0.759966	0.17939	0.13633
	B3	0.6	0.720029	0.17939	0.129166
	B4	0.8333 ($x_{i.best}$)	1	0.17939	0.17939
A2	B1	0.73328	0.879971	0.17939	0.157858
	B2	0.8	0.960038	0.17939	0.172221
	B3	0.43332	0.520005	0.17939	0.093284
	B4	0.8333 ($x_{i.best}$)	1	0.17939	0.17939
A3	B1	0.6	0.782636	0.083024	0.064978
	B2	0.66664	0.869561	0.083024	0.072194
	B3	0.6333	0.826072	0.083024	0.068584
	B4	0.76664 ($x_{i.best}$)	1	0.083024	0.083024
A4	B1	0.7	0.913075	0.094278	0.086083
	B2	0.46666	0.608708	0.094278	0.057388
	B3	0.46662	0.608656	0.094278	0.057383
	B4	0.76664 ($x_{i.best}$)	1	0.094278	0.094278
A5	B1	0.76664 ($x_{i.best}$)	1	0.076094	0.076094
	B2	0.6333	0.826072	0.076094	0.062859
	B3	0.7	0.913075	0.076094	0.06948
	B4	0.76662	0.999974	0.076094	0.076092
A6	B1	0.6	0.782656	0.087865	0.068768
	B2	0.7	0.913099	0.087865	0.080229
	B3	0.76662 ($x_{i.best}$)	1	0.087865	0.087865
	B4	0.46664	0.608698	0.087865	0.053483
A7	B1	0.33332	0.45456	0.079205	0.036003
	B2	0.56664	0.772747	0.079205	0.061205
	B3	0.6333	0.863654	0.079205	0.068406
	B4	0.73328 ($x_{i.best}$)	1	0.079205	0.079205
A8	B1	0.56664	0.772726	0.11037	0.085286
	B2	0.7333 ($x_{i.best}$)	1	0.11037	0.11037
	B3	0.46664	0.636356	0.11037	0.070235
	B4	0.6333	0.86363	0.11037	0.095319
A9	B1	0.46664	0.5833	0.11037	0.064379
	B2	0.6333	0.791625	0.11037	0.087372
	B3	0.46664	0.5833	0.11037	0.064379
	B4	0.8 ($x_{i.best}$)	1	0.11037	0.11037

Step 5: The total scores of the alternative sustainable maintenance service provider are calculated by summing up the products of the normalized data of the attributes for the alternatives ($x_{ji}/x_{i.best}$) with the average attributes' weights w_i. The products are given in the last column of Table 20. For example, the total score of sustainable maintenance service provider B1 is computed as,

Total score (sustainable maintenance service provider B1):
0.172221+0.157858+0.064978+0.086083+0.076094+0.068768+0.036003+0.085286+0.064379 = 0.81167

Similarly, the total scores for the sustain able maintenance service providers B2, B3, and B4 are computed as 0.84017, 0.70878, and 0.950551 respectively.

Step 6: The alternative sustainable maintenance service providers are arranged in decreasing order of the total scores as B4-B2-B1-B3. The sustainable maintenance service provider B4 has the highest total score and is considered as the best option. Tong et al. (2020) solved this case study using a fuzzy PROMETHEE II approach which combined the PROMETHEE II method with fuzzy set theory. The ranking given by them to the sustainable maintenance service providers was B4-B2-B3-B1. The first two choices suggested by the BHARAT method coincide with the choices suggested by Tong et al. (2020). Regarding the third choice, a look at the average values assigned by the group of decision makers for the alternatives B1 and B3, It is clear that, out of nine attributes, option B1 is superior to option B3 in five cases (A1, A2, A4, A5, and A8) and equal in one case (A9). The summation of weightages of the 5 attributes is 0.5729 and the weightage of A9 is 0.125. This clearly shows that B1 is better than B3. Thus, proposing B1 as the third choice instead of B3 is more logical. Thus, the proposed BHARAT method has provided logical ranking compared to fuzzy PROMETHEE II method. Furthermore, the BHARAT method has provided the logical ranking very conveniently and easily in fuzzy group decision-making situation. The simple scale suggested in this paper for converting the linguistic expressions into quantitative values may be sufficient and there is no need of using any fuzzy membership functions and the corresponding fuzzy numbers.

Second version of BHARAT method for case study 2

BHARAT-2 method differs from BHARAT only in Step 2. The ranks assigned to the attributes are same as those given in BHARAT. The attributes A1 and A2 are considered equally significant. An average rank of 1.5 (i.e., (1+2)/2) is assigned to both A1 and A2. Similarly, the attributes A8 and A9 are considered equally significant and an average rank of 3.5 (i.e., (3+4)/2) is assigned to both A8 and A9. The rank 5 is assigned to A4, 6 to A6, 7 to A3, 8 to A7, and 9 to A5 The values of weights are given in the last column of Table 21. Table 21 presents the relative importance table of the 9 attributes using BHARAT-2. Table 22 shows the normalized data of the 9 attributes and scores of the alternatives. The total scores of the alternative sustainable maintenance service provider are calculated by adding up the products of the attribute weights and the matching normalized data of the alternatives (products are displayed in the last column of Table 22).

Table 21
Relative importance table of the 9 attributes of case study 2

Attributes	*Attributes*									*Means of rows*	*Attributes' weights*
	A1	*A2*	*A3*	*A4*	*A5*	*A6*	*A7*	*A8*	*A9*		
A1	1	1	7/1.5	5/1.5	9/1.5	6/1.5	8/3.5	3.5/1.5	3.5/1.5	2.994689	0.238153
A2	1	1	7/1.5	5/1.5	9/1.5	6/1.5	8/3.5	3.5/1.5	3.5/1.5	2.994689	0.238153
A3	1.5/7	1.5/7	1	5/7	9/7	6/7	8/7	3.5/7	3.5/7	0.714278	0.056803
A4	1.5/5	1.5/5	7/5	1	9/5	6/5	8/5	3.5/5	3.5/5	1	0.079525
A5	1.5/9	1.5/9	7/9	5/9	1	6/9	8/9	3.5/9	3.5/9	0.555511	0.044177
A6	1.5/6	1.5/6	7/6	5/6	9/6	1	8/6	3.5/6	3.5/6	0.833311	0.066269
A7	1.5/8	1.5/8	7/8	5/8	9/8	6/8	1	3.5/8	3.5/8	0.625	0.049703
A8	1.5/3.5	1.5/3.5	7/3.5	5/3.5	9/3.5	6/3.5	8/3.5	1	1	1.428578	0.113608
A9	1.5/3.5	1.5/3.5	7/3.5	5/3.5	9/3.5	6/3.5	8/3.5	1	1	1.428578	0.113608
										Total = 12.57463	Total = 1

Table 22
Attributes' normalized data and the scores of case study 2

Attributes	*Alternative sustainable maintenance service providers*	*Average value assigned ((D1+D2+D3+D4+D5)/5)*	*Normalized value ($x_{ji}/x_{i.best}$)*	*Average weight value assigned to the attributes (w_i)*	*Scores ($x_{ji}/x_{i.best}$)*w_i*
A1	B1	0.8	0.960038	0.238153	0.228636
	B2	0.63328	0.759966	0.238153	0.180988
	B3	0.6	0.720029	0.238153	0.171477
	B4	0.8333 ($x_{i.best}$)	1	0.238153	0.238153
A2	B1	0.73328	0.879971	0.238153	0.209568
	B2	0.8	0.960038	0.238153	0.228636
	B3	0.43332	0.520005	0.238153	0.123841
	B4	0.8333 ($x_{i.best}$)	1	0.238153	0.238153
A3	B1	0.6	0.782636	0.056803	0.044456
	B2	0.66664	0.869561	0.056803	0.049394
	B3	0.6333	0.826072	0.056803	0.046923
	B4	0.76664 ($x_{i.best}$)	1	0.056803	0.056803
A4	B1	0.7	0.913075	0.079525	0.072612
	B2	0.46666	0.608708	0.079525	0.048408
	B3	0.46662	0.608656	0.079525	0.048403
	B4	0.76664 ($x_{i.best}$)	1	0.079525	0.079525
A5	B1	0.76664 ($x_{i.best}$)	1	0.044177	0.044177
	B2	0.6333	0.826072	0.044177	0.036493
	B3	0.7	0.913075	0.044177	0.040337
	B4	0.76662	0.999974	0.044177	0.044176
A6	B1	0.6	0.782656	0.066269	0.051866
	B2	0.7	0.913099	0.066269	0.06051
	B3	0.76662 ($x_{i.best}$)	1	0.066269	0.066269
	B4	0.46664	0.608698	0.066269	0.040338
A7	B1	0.33332	0.45456	0.049703	0.022593
	B2	0.56664	0.772747	0.049703	0.038408
	B3	0.6333	0.863654	0.049703	0.042926
	B4	0.73328 ($x_{i.best}$)	1	0.049703	0.049703
A8	B1	0.56664	0.772726	0.113608	0.087788
	B2	0.7333 ($x_{i.best}$)	1	0.113608	0.113608
	B3	0.46664	0.636356	0.113608	0.072295
	B4	0.6333	0.86363	0.113608	0.098115
A9	B1	0.46664	0.5833	0.113608	0.066268
	B2	0.6333	0.791625	0.113608	0.089935
	B3	0.46664	0.5833	0.113608	0.066268
	B4	0.8 ($x_{i.best}$)	1	0.113608	0.113608

For example, the total score of sustainable maintenance service provider B1 is computed as,
Total score (sustainable maintenance service provider B1):
0.228636+0.209568+0.044456+0.072612+0.044177+0.051866+0.022593+0.087788+0.066268 = 0.827963
Similarly, the total scores for the sustain able maintenance service providers B2, B3, and B4 are computed as 0.84638, 0.678739, and 0.958574 respectively.

The alternative sustainable maintenance service providers are arranged in decreasing order of the total scores as B4-B2-B1-B3. With the highest overall score, sustainable maintenance service provider B4 is regarded as the best option in the scenario under consideration. Thus, the second version of BHARAT has given the same ranking as that given by BHARAT.

An interesting point to note that if the fuzzy scale given in Table 4 is considered instead of the simple linear scale for converting the qualitative terms of expressed by the 5 decision-makers in Table 15 and 16, then also the ranking is same and the first choice is sustainable maintenance service provider B4. This shows that simple linear scales serve the purpose very conveniently without the need of using fuzzy logic. This is very convenient in practical real-life decision-making problems.

3.3 Case study 3: The choice of implementation strategy for a make-to-order (MTO) system for manufacturers of passenger cars

Upadhyay et al. (2023) proposed an integrated method that used the AHP method to prioritize the factors (i.e., attributes) and the TOPSIS method to rank the alternative strategies to implement a make-to-order (MTO) system for passenger car manufacturers. The goal, factors (i.e., attributes), sub-factors (i.e., sub-attributes), and the alternative strategies are shown in Fig. 2.

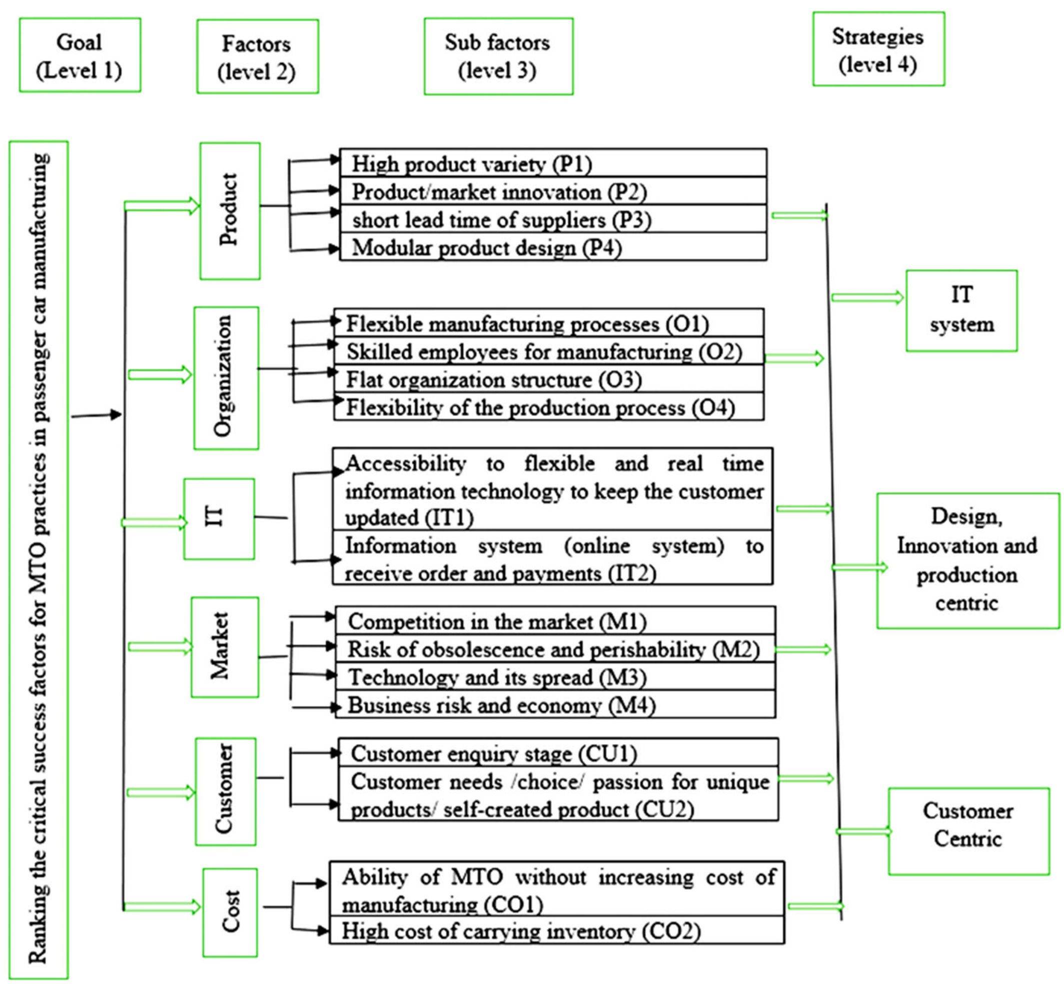

Fig. 2. Model for selecting the best strategy for implementing a MTO system (Upadhyay et al. 2023).

Now, the BHARAT method is implemented as explained below.

<u>Step 1:</u> Relevant factors (i.e., attributes), sub-factors (i.e., sub-attributes), and the alternative strategies are identified. As seen in levels 2 and 3 of Fig. 1, Six categories of factors are typically used to group the eighteen critical success factors, also known as sub-factors. At level 2, all six factors (i.e., attributes) are first compared with one another. This decision-making problem consists of 6 factors, 18 sub-factors, and 3 alternative strategies. The qualitative data of the sub-factors for different alternative strategies was presented by Upadhyay et al. (2023) in the form of quantitative scores after consulting the experts of the company. The non-beneficial sub-factors (i.e., whose lower values are desirable) were assigned the scores accordingly by

Upadhyaya et al. (2023) and hence after allotting such scores those are also considered as beneficial. The same data is used in the present work. Table 23 shows the scores of the alternative strategies with reference to 18 sub-factors. Table 23 shows also the normalized data of the alternative strategies following the procedure suggested in the present work.

Table 23
Scores of the alternative strategies and their normalized values with reference to 18 sub-factors

Sub-factors	*Alternative strategies*			*Normalized values of the alternative strategies*		
	IT system-centric strategy	*Design, innovation, and production-centric strategy*	*Customer-centric strategy*	*IT system-centric strategy*	*Design, innovation, and production-centric strategy*	*Customer-centric strategy*
P1	5 ($x_{i.best}$)	3	5 ($x_{i.best}$)	1	0.6	1
P2	7 ($x_{i.best}$)	5	5	1	0.7143	0.7143
P3	5	7 ($x_{i.best}$)	7 ($x_{i.best}$)	0.7143	1	1
P4	7	9 ($x_{i.best}$)	7	0.7778	1	0.7778
O1	7	9 ($x_{i.best}$)	9	0.7778	1	1
O2	5	9 ($x_{i.best}$)	7	0.7143	1	0.7778
O3	5	9 ($x_{i.best}$)	5	0.7143	1	0.7143
O4	5	9 ($x_{i.best}$)	9	0.7143	1	1
IT1	7	7 ($x_{i.best}$)	5	1	1	0.7143
IT2	5	7 ($x_{i.best}$)	3	0.7143	1	0.428
M1	7 ($x_{i.best}$)	5	5	1	0.7143	0.7143
M2	7 ($x_{i.best}$)	5	5	1	0.7143	0.7143
M3	7	5	9	0.7778	0.7143	1
M4	7 ($x_{i.best}$)	5	5	1	0.7143	0.7143
CU1	9 ($x_{i.best}$)	9 (x_{best})	9 ($x_{i.best}$)	1	1	1
CU2	9 ($x_{i.best}$)	7	9 ($x_{i.best}$)	1	0.7778	1
CO1	7 ($x_{i.best}$)	5	7 ($x_{i.best}$)	1	0.7143	1
CO2	7	5	9 ($x_{i.best}$)	0.7778	0.7143	1

Step 2: The six factors are ranked in terms of 1, 2, 3, 4, etc., according to the decision-maker's assessment of their relative importance to determine the weights of the factors. Market (M) factor is given rank 1, customer (Cu) factor is given rank 2, organization (O) factor is given rank 3, product (P) factor is given rank 4, IT (IT) factor is given rank 5, and cost (C) factor is given rank 6. Table 19 shows the relative importance of the 18 sub-factors. These ranks are same as those used by Upadhyay et al. (2023). The 6 factors P, O, IT, M, Cu, and CO are assigned ranks of 4, 3, 5, 1, 2, and 6 respectively. The related weights are obtained using Table A1 corresponding to 6 factors (i.e. attributes) and these are 0.1357, 0.1541, 0.1238, 0.2826, 0.1884, and 0.1153 respectively. The local ranks of the 18 sub-factors, their local weightages, and their global weightages are given in Table 24. For example, for the P factor, the sub-factors P1, P2, P3, and P4 are given the local ranks of 3, 1, 4, and 2. Using Table A1 corresponding to 4 sub-factors, the local weights are assigned as 0.2026, 0.3714, 0.1783, and 0.2476 respectively. The local weights of P1, P2, P3, and P4 are multiplied by the weight of P to obtain the global weights of the sub-factors.

Table 24
Weightages of the 6 factors and 18 sub-factors

Factors and sub-factors	Rank	Weightage of factor (a)	Local rank of sub-factor	Local weightage of sub-factor (b)	Global weightage of sub-factor (a)*(b)
P	4	0.1357			
P1			3	0.2026	0.0275
P2			1	0.3714	0.0504
P3			4	0.1783	0.0242
P4			2	0.2476	0.0336
O	3	0.1541			
O1			2	0.2476	0.0381
O2			3	0.2026	0.0312
O3			4	0.1783	0.0275
O4			1	0.3714	0.0572
IT	5	0.1238			
IT1			1	0.6	0.0743
IT2			2	0.4	0.0495
M	1	0.2826			
M1			3	0.2026	0.0572
M2			2	0.2476	0.07
M3			4	0.1783	0.0504
M4			1	0.3714	0.1049
Cu	2	0.1884			
Cu1			2	0.4	0.07536
Cu2			1	0.6	0.11304
Co	6	0.1153			
Co1			1	0.6	0.06918
Co2			2	0.4	0.04612

Step 3: The performance measures x_{ji} of the 18 sub-factors for the 3 alternative strategies are given by Upadhyay et al. (2023) in the form of scores and these are already presented in Table 23.

Step 4: The data is normalized with reference to the "best" alternative strategy corresponding to an attribute. The normalization procedure is carried out for all the attributes to get the normalized data. Table 23 also shows the normalized data following the procedure suggested in the present work.

Step 5: The total scores of the alternative strategies are calculated by adding up the products of the sub-factors' global weights given in the last column of Table 24 with the normalized data of the alternative strategies given in Table 23. For instance, the total score of alternative strategy 1 is calculated as given below.

The score of alternative strategy 1 (IT system-centric strategy):
$1\times0.0275+1\times0.0504+0.7143\times0.0242+0.7778\times0.0336+0.7778\times0.0381+0.7143\times0.0312+0.7143\times0.0275+0.7143\times0.0572+1\times0.0743+0.7143\times0.0495+1\times0.0572+1\times0.07+0.7778\times0.0504+1\times0.1049+1\times0.07536+1\times0.11304+1\times0.06918+0.7778\times0.04612 = 0.908027$

Step 6: The alternatives are arranged in the descending order of the total scores. The alternative strategy of IT system-centric strategy has the highest total score and is the best choice.

Alternative strategy 1 (IT system-centric strategy): 0.9268408
Alternative strategy 3 (Customer-centric strategy): 0.847114
Alternative strategy 2 (Design, innovation, and production-centric strategy): 0.835411

However, Upadhyay et al. (2023) proposed the ranking of alternative strategies as 3 – 2 – 1 (i. e., Customer-centric strategy - Design, innovation, and production-centric strategy - IT system-centric strategy) instead of 1 – 3 – 2 proposed in this paper using BHARAT method. A close look at the alternatives clearly shows that the alternative strategy 1 (IT system-centric strategy) is better than alternative strategy 3 (Customer-centric strategy) in sub-factors P2, IT1, IT2, M1, M2, M3 (total weightage of 0.4734) and equal in performance in sub-factors P1, P4, O3, Cu1, Cu2, and Co1 (total weightage of 0.3065). The alternative strategy 3 is better than alternative strategy 1 in sub-factors P3, O1, O2, O4, ME, Co2 (total weightage of 0.2199 only). Thus, proposing alternative strategy 1 (IT system-centric strategy) as the first choice for implementing a MTO system is more logical.

A further look at the TOPSIS matrices calculations of Upadhyay et al. (2023) reveal that the normalized values and the weighted normalized values for the sub-factor M1 were calculated incorrectly. Corrected values of M1 and recomputing the total scores using TOPSIS method will also lead to the ranking of alternative strategies as 1 – 3 – 2 which is same as the ranking given in the present work using BHARAT method. However, the present method is simpler and computed the total scores in simple steps unlike the TOPSIS method.

Upadhyay et al. (2023) used AHP method for calculating the sub-factors' weights and used those weights in the TOPSIS method and calculated the total scores of the alternative strategies. Just for demonstration and for fair comparison purpose, if the same weights of the sub-factors used by Upadhyay et al. (2023) are used in the proposed BHARAT method, then also the ranking of the alternative strategies is 1-3-2. This clearly shows the potential of the proposed BHARAT method compared to the lengthy procedure of TOPSIS and AHP. The proposed method allows the decision-maker to rank the process attributes as per his/her preferences and computes the weights in an easy and simple manner and does not involve intensive computation.

Second version of BHARAT method for case study 3
The ranks assigned to the factors and sub-factors are same as those given in BHARAT. Table 25 shows the weights of the 6 factors.

Table 25
Relative importance table of the 6 factors of case study 3

Factors	*Factors*						*Means of rows*	*Weights of factors*
	P	O	IT	M	Cu	C		
P	1	3/4	5/4	1/4	2/4	6/4	0.875	0.102042
O	4/3	1	5/3	1/3	2/3	6/3	1.166633	0.136051
IT	4/5	3/5	1	1/5	2/5	6/5	0.7	0.081633
M	4	3	5	1	2	6	3.5	0.408166
Cu	4/2	3/2	5/2	1/2	1	6/2	1.75	0.204083
Co	4/6	3/6	5/6	1/6	2/6	1	0.5833	0.068024
							Total = 8.574933	Total = 1

The next stage is to obtain each sub-factor's weight. The comparison matrices and the weights of 18 sub-factors are displayed in Table 26. The sub-factors' global weights are displayed in the last columns of Table 26(a) to 26(f).

Table 26
Sub-factors' relative importance table for case study 3
(a) Product factor (weightage = 0.102)

Sub-factors of P	*Sub-factors of P*				*Means of rows*	*Local weights*	*Global weights*
	P1	*P2*	*P3*	*P4*			
P1	1	1/3	4/3	2/3	0.833333	0.16	0.01632
P2	3	1	4	2	2.5	0.48	0.04898
P3	3/4	1/4	1	2/4	0.625	0.12	0.01224
P4	3/2	1/2	4/2	1	1.25	0.24	0.02449
					Total = 5.20833	Total = 1	Total = 0.102

(b) Organization factor (weightage = 0.136)

Sub-factors of O	*Sub-factors of O*				*Means of rows*	*Local weights*	*Global weights*
	O1	*O2*	*O3*	*O4*			
O1	1	3/2	4/2	1/2	1.25	0.24	0.03264
O2	2/3	1	4/3	1/3	0.833333	0.16	0.02176
O3	2/4	3/4	1	1/4	0.625	0.12	0.01632
O4	2	3	4	1	2.5	0.48	0.06528
					Total = 5.20833	Total = 1	Total = 0.136

(c) IT factor (weightage = 0.0816)

Sub-factors of IT	*Sub-factors of IT*		*Means of rows*	*Local weights*	*Global weights*
	IT1	IT2			
IT1	1	2	1.5	0.6666	0.0544
IT2	1/2	1	0.75	0.3333	0.0272
			Total = 2.25	Total = 1	Total = 0.0816

(d) Market factor (weightage = 0.40816)

Sub-factors of M	*Sub-factors of M*				*Means of rows*	*Local weights*	*Global weights*
	M1	*M2*	*M3*	*M4*			
M1	1	2/4	3/4	1/4	0.625	0.12	0.04898
M2	4/2	1	3/2	1/2	1.25	0.24	0.09796
M3	4/3	2/3	1	1/3	0.833333	0.16	0.06530
M4	4	2	3	1	2.5	0.48	0.19591
					Total = 5.20833	Total = 1	Total = 0.40816

(e) Customer factor (weightage = 0.20408)

Sub-factors of Cu	*Sub-factors of Cu*		*Means of rows*	*Local weights*	*Global weights*
	Cu1	*Cu2*			
Cu1	1	1/2	0.75	0.3333	0.06803
Cu2	2	1	1.5	0.6666	0.13605
			Total = 2.25	Total = 1	Total = 0.20408

(f) Cost factor (weightage = 0.06802)

Sub-factors of Co	*Sub-factors of IT*		*Means of rows*	*Local weights*	*Global weights*
	Co1	Co2			
Co1	1	2	1.5	0.6666	0.04535
Co2	1/2	1	0.75	0.3333	0.02267
			Total = 2.25	Total = 1	Total = 0.06802

<u>Step 3:</u> The performance measures x_{ij} of the 18 sub-factors for the 3 alternative strategies are given by Upadhyay et al. (2023) in the form of scores and these are already presented in Table 23.

<u>Step 4:</u> The data is normalized with reference to the "best" alternative strategy corresponding to an attribute. The normalization procedure is carried out for all the attributes to get the normalized data. Table 23 shows also the normalized data of the alternative strategies following the BHARAT procedure suggested in the present work.

<u>Step 5:</u> The sub-factors' global weights listed in Table 26 are multiplied by the normalized data of the alternative strategies listed in Table 23 to determine the overall scores of the various strategies. For instance, the score of alternative strategy 1 is calculated as given below.

The score of alternative strategy 1:

$1\times0.01632+1\times0.04898+0.7143\times0.01224+0.7778\times0.02449+0.7778\times0.03264+0.7143\times0.02176+0.7143\times0.01632+0.7143\times0.06528+1\times0.0544+0.7143\times0.0272+1\times0.04898+1\times0.09796+0.7778\times0.06530+1\times0.19591+1\times0.06803+1\times0.13605+1\times0.04535+0.7778\times0.02267 = 0.9268408$.

<u>Step 6:</u> The alternative strategy of IT system-centric strategy has the highest total score and is the best choice.

Alternative strategy 1 (IT system-centric strategy): 0.9268408
Alternative strategy 3 (Customer-centric strategy): 0.841894
Alternative strategy 2 (Design, innovation, and production-centric strategy): 0.813086

As a result, the ranking provided by BHARAT in this case study is also provided by the second version of the BHARAT method.

3.4 Case study 4: Selecting the best machining process parameters for sustainable turning (involving a single decision-maker)

In order to optimize four process input parameters—specific cutting energy (SCE), wear rate (R), surface roughness (Ra), and material removal rate (MRR)—Younas et al. (2019) conducted high-speed sustainable dry turning experiments on Ti6Al4V using uncoated H13 carbide inserts. To create a multi-objective function and identify the ideal process input parameters, AHP and GRA were employed.

Table 27 displays the data of 9 sets of the process input parameters (i.e., alternatives) with 4 responses (i.e., attributes). The problem is to select the best set of process input parameters out of the 9 alternative sets taking into consideration the 4 responses.

Table 27
L9 orthogonal array based experiments and the process responses (Younas et al., 2019)

Experiment No.	*Feed (mm/rev)*	*Cutting speed (m/min)*	*Depth of cut (mm)*	*Process responses (i.e., attributes)*			
				SCE (J/mm³)	*R*	*Ra (μm)*	*MRR (cm³/s)*
Exp#1	0.12	50	1	1	-6.14 ($x_{i,best}$)	1.113	0.09996
Exp#2	0.12	100	1.5	1.33	-5.84	1.21	0.29988
Exp#3	0.12	150	2	1.21	-5.58	1.023	0.59976
Exp#4	0.16	50	1.5	0.95	-6.1	1.58	0.19992
Exp#5	0.16	100	2	1.18	-5.83	1.237	0.53312
Exp#6	0.16	150	1	1.2	-5.8	0.843 ($x_{i,best}$)	0.39984
Exp#7	0.2	50	2	0.93($x_{i,best}$)	-5.58	2.29	0.3332
Exp#8	0.2	100	1	0.99	-5.81	2.08	0.3332
Exp#9	0.2	150	1.5	1.18	-5.29	1.22	0.7497 ($x_{i,best}$)

The data of Table 27 is normalized. The attribute MRR is beneficial and others are non-beneficial. Table 28 shows the normalized data.

Table 28
Normalized data of the process responses of case study 4

Experiment No.	*Feed (mm/rev)*	*Cutting speed (m/min)*	*Depth of cut (mm)*	*Process responses (i.e., attributes)*			
				SCE	*R*	*Ra*	*MRR*
Exp#1	0.12	50	1	0.93	1	0.757412	0.133333
Exp#2	0.12	100	1.5	0.699248	0.95114	0.696694	0.4
Exp#3	0.12	150	2	0.768595	0.908795	0.824047	0.8
Exp#4	0.16	50	1.5	0.978947	0.993485	0.533544	0.266667
Exp#5	0.16	100	2	0.788136	0.949511	0.681487	0.711111
Exp#6	0.16	150	1	0.775	0.944625	1	0.533333
Exp#7	0.2	50	2	1	0.908795	0.368122	0.444444
Exp#8	0.2	100	1	0.939394	0.946254	0.405288	0.444444
Exp#9	0.2	150	1.5	0.788136	0.861564	0.690984	1

The four attributes are ranked in terms of 1, 2, 3, and 4 according to the decision-maker's assessment of their relative importance in order to determine the attributes' weights. The 4 process responses SEC, R, Ra, and MRR are assigned ranks of 3, 2, 4, and 1 respectively by the decision-maker. The weights of the process responses are obtained from Table A1 corresponding to 4 attributes (as there are 4 process responses) and these are 0.2026, 0.2476, 0.1783, and 0.3714 for SEC, R, Ra, and MRR respectively.

Using the normalized data of Table 28 and the weights of the process responses, the total scores of the alternative experimental sets of process parameters. For instance, the total score for Exp#1 is calculated as $0.93\times0.2026+1\times0.2476+0.757412\times0.1783+0.133333\times0.3714 = 0.620584$. Similarly, the total scores of other alternatives are computed. The ranking obtained is given below.
Exp#9: 0.867602
Exp#3: 0.824783
Exp#5: 0.780391
Exp#6: 0.767284
Exp#8: 0.661943
Exp#7: 0.65832

Exp#2: 0.64995
Exp#4: 0.638493
Exp#1: 0.620584

From the above ranking, it can be understood that the set of process parameters of Exp#9 are the best choice for the sustainable high-speed turning operation considered. The corresponding optimal process parameters are 0.2mm/rev, 150m/min, and 1.5mm.

Younus et al. (2019) used AHP method to compute the weights of the process responses and GRA method for calculating the total scores. The weights were 0.15, 0.21, 0.08, and 0.55 for SEC, R, Ra, and MRR respectively. Using those weights, they also gave the same ranking of the experimental sets process parameters, i.e., Exp#9-3-5-6-8-7-2-4-1. Just for fair comparison and for demonstration, if the AHP weights of the process responses are used in the proposed method, then also the ranking is same. This clearly shows the potential of the proposed BHARAT method compared to the lengthy procedure of GRA and AHP. Younus et al. (2019) used geometric mean concept in AHP method to compute the weights of the process responses. If arithmetic mean was used, then the weights obtained by them would have been different. That type of confusion is not present in the present method. The suggested BHARAT method computes the weights and enables the decision-maker to rank the process responses, or attributes, according to his or her preferences.

Second version of BHARAT method for case study 4
Table 29 displays the attributes' relative importance and the corresponding weights.

Table 29
Relative importance table of the attributes of case study 4

Attributes	*Attributes*				*Means of rows*	*Weights of attributes*
	SCE	*R*	*Ra*	*MRR*		
SCE	1	2/3	4/3	1/3	0.833333	0.16
R	3/2	1	4/2	1/2	1.25	0.24
Ra	3/4	2/4	1	1/4	0.625	0.12
MRR	3	2	4	1	2.5	0.48
					Total = 5.20833	Total = 1

The total scores of the alternative strategies are calculated using Tables 28 and 29. For instance, the score of alternative set of process responses Exp#1 is computed as given below.

The score of alternative set of process responses 1:
$0.93\times0.16+1\times0.24+0.757412\times0.12+0.133333\times0.48 = 0.54368$.

The alternatives are listed with the total scores in decreasing order. Exp#9 is regarded as the best option because it has the highest total score. This ranking is same as that given by BHARAT method.
Exp#9: 0.895795
Exp#3: 0.823972
Exp#5: 0.777096
Exp#6: 0.726710
Exp#8: 0.639372
Exp#7: 0.635619
Exp#2: 0.615757
Exp#4: 0.587093
Exp#1: 0.543689

It is interesting to mention here that some researchers use the words "multi-objective optimization" for finding the optimal set of experimental conditions (as in case study 4 above). However, it should be noted that what actually they are doing is "multi-attribute decision-making" to find the best alternative set of experimental conditions (or alternative sets of process input parameters). For example, in case study 4 above, there are 9 alternative experimental conditions and by using an MADM method, such as BHARAT, alternative Exp#9 is recommended as the first best choice. This Exp#9 is *one of the given 9 alternative experimental conditions*. The MADM method suggests one of the alternatives *out of the given number of alternatives* as the best choice. Whereas "multi-objective optimization" means developing models for the objective functions in terms of the process input parameters (i.e., experimental conditions) based on the experimental results and then applying any advanced multi-objective optimization algorithm to generate a number of non-dominated Pareto optimal solutions. An MADM method may then be used to find a more suitable non-dominated solution by assigning different weights to the objectives.

4. Discussion

The research questions (RQs) related to selection of a right alternative using MADM methods posed in Section 1 of this paper are reproduced below and discussion is made whether the present work has provided any answers or explanations.

- RQ1: In an industrial or other decision-making setting, is there a simple and suitable MADM method to weigh the selection attributes and evaluate the alternatives?

Yes, it is possible to develop a simple and suitable MADM method in an industrial or other decision-making setting. The proposed BHARAT method and its second version are simple, transparent, flexible, easy to implement, do not require the use of complex fuzzy logic for conversion of qualitative data or linguistic expressions to quantitative data, applicable to various decision contexts, lead to quicker decision turnarounds, and provide reasonable and pragmatic solutions contributing to the credibility as decision-making tools.

- RQ2: Can the MADM method (which answers RQ1) handle both qualitative and quantitative attributes? If so, how effectively does this type of MADM approach function in situations where a group decision-making process involves multiple decision-makers?

Yes, the proposed BHARAT and its second version can handle both qualitative and quantitative attributes. This has been clearly demonstrated in the first two case studies. The case study-1 has 13 attributes out of which 5 attributes are in linguistic terms. The second case study is a typical one in which there are 5 decision-makers and they verbally describe all 9 attributes for the 4 alternatives. The 5 decision-makers also express 9 attributes' relative importance in linguistic terms. The suggested BHARAT method can transform the qualitative (i.e., linguistically expressed) attributes into quantitative ones with the aid of simple linear scales. The results of the proposed BHARAT method and its second version are much logical than the other MADM methods including fuzzy PROMETHEE method. Thus, it can be concluded that situations involving multiple decision-makers participating in a group decision-making process are well-suited for the proposed BHARAT method and its second version.

- RQ3: If a MADM approach (which addresses RQs 1 and 2) exists, will it be easy to comprehend and practical to use when making decisions in situations where there is uncertainty in the information at hand? Is a complex fuzzy logic required for decision-making?

It is already shown in case studies 1, 2, and 3 that the proposed BHARAT method and its second version can deal with imprecise or uncertain information at hand. The imprecise or uncertain information can be dealt with using the proposed 11-point, or 7-point, or 5-point scales. There is no need of using complex fuzzy logic. Even though a large number of research papers are appearing in the literature using different fuzzy approaches, there is no convincing evidence that the results obtained by fuzzy logic based MADM methods provide logical and better solutions. Fudging the numbers with fuzziness not only makes manipulations more complex, but it also takes away from the original numbers' elegance and simplicity as a means of representing the judgments. This frequently results in less desirable outcomes rather than more desirable ones.

- RQ4: Is there any strong evidence that fuzzy logic that involves different membership functions and different defuzzification methods provides better results compared to the conventional logic in MADM situations?

Different outcomes may be obtained for a given decision-making problem using different membership functions (triangular, trapezoidal, piecewise linear, singleton, gaussian, etc.) and defuzzification techniques (finding the leftmost maximum, rightmost maximum, center of gravity, average mean, etc.). Furthermore, different fuzzy versions such as orthogonal fuzzy, intuitional fuzzy, hesitant fuzzy, spherical fuzzy, Pythagorean fuzzy, etc. all lead to more confusion and beyond the understanding of the real decision makers in the industries, governments, organizations, individuals, groups, etc. There is hardly any evidence in the research literature that the use of fuzzy MADFM methods produce better and logical results compared to the classical methods. The linguistic expressions of the qualitative attributes can be easily transformed to quantitative ones using simple scales such as 11-point, or 7-point, or 5-point scales proposed in this paper. The use of fuzzy logic may not be necessary. It may be mentioned here that no solid proof is provided by the researchers that a fuzzy scale is better than a simple scale.

- RQ5: Is it feasible to have an appropriate MADM approach that is both reliable and resistant to changes in the attributes' weights? Does it becomes necessary to create an MADM method that is insensitive to changes in attribute weights? Can such kind of MADM method regarded as best?

The MADM method, whatever it is, must be reliable and there is no doubt about this point. However, why an MADM method should be resistant to changes in the attributes' weights? Does it mean that even if the attributes' weights changed, the MADM method should go on suggesting the same ranking of alternatives? Is there any meaning in it? For example, suppose a house is to be purchased and three attributes considered are: purchase price, nearness to market, and nature of neighborhood. If the decision-maker assigns weights of 0.5, 0.3, and 0.2 to these attributes, then the MADM method used should suggest a suitable alternative as per the considered weights. If weights are changed to 0.2, 0.5, and 0.3 then the MADM method should suggest a suitable alternative corresponding to the changed attributes' weights. Weights are changed means the priorities of the decision-maker are changed. Then why we should expect an MADM method to be not-sensitive to the changes in the attributes' weights? At most, what the researchers can suggest is that a particular MADM method indicates a particular alternative as the first choice within certain percentage of variation in each attribute weight. For example, an MADM method may suggest a particular house-2 as the right choice within the weights of 0.5±10%of 0.5, 0.3±4% of 0.3, and 0.2±6% of 0.2. If the % changes are more than that then the MADM method may suggest some other alternative house as the right choice. This is quite obvious and expecting the MADM method to indicate a particular alternative as the right choice for any changes in the attributes' weights is less meaningful. If any MADM method is giving such results and is resistant to any changes in the attributes' weights then it need not be considered as the best method.

There may be some argument over using the ranking methods that the explanatory power decreases with increasing number of attributes (say more than 35). But this is true in case of other methods also such as AHP, SWARA, DEMATEL, entropy method, standard deviation method, CRITIC method, etc. with increased number of attributes. In general, if the number of attributes increase, assigning the weights arbitrarily or intuitively is also not so easy. Such situation is more serious in the case of deciding the weights by using the fuzzy logic which becomes more confusing with different membership functions and defuzzification procedures. However, in such situations the proposed ranking and weighing R-method of BHARAT provides logical and stable weights. Moreover, in practical situations the number of factors (i.e., attributes) may not be too many (e.g., not more than 35) but they may have a number of sub-factors (i.e., sub-attributes) and the sub-factors may have sub-sub-factors (i.e., sub-sub-attributes). Thus, a factor may not have more than 35 sub-factors with each sub-factor may not have more than 35 sub-sub-factors. Thus, the proposed BHARAT method using R-method as a part of it can be successfully used. The Eq. (1) (or Eq. (2)) given in the BHARAT methodology can be used for determining the weights of *any number* of attributes.

The choice regarding weight determination will have a significant impact on the decision's outcome. Table A1 shows that the weights produced by the suggested method are more consistent and stable than those produced by other ranking techniques, including rank sum (RS), equal weights (EW), reciprocal weights (RW), and rank order centroid (ROC) weights. For instance, the ROC method gives the attributes weights of 0.75 and 0.25 in the case of two attributes, which is a very steep step. Similarly, the RW method of Stillwell (1981) gives the attribute weights of 0.6666 and 0.3333. However, the suggested method assigns 0.6 and 0.4 weights, which makes more sense.

Comparably, it can be demonstrated that the weights produced by the suggested BHARAT approach are more consistent and stable in the cases of 2, 3, 4, 5, 6, 7, 8, 9, 10,...... attributes. Rao and Lakshmi (2021b) showed that the weights derived from the ROC method are "steeper," meaning that the most significant attribute is given a larger weight than the least significant attribute. However, compared to the weights provided by the ROC method, the ones obtained through the RS method are significantly more "flatter." The RS weights are lowered linearly from the most to the least important. The RW method yields weights that aggressively descend after the most important, with the ROC weights being the lowest at the least important end. The suggested BHARAT method, which uses the R-method to calculate the reciprocals, yields weights that are more uniformly stable with a greater number of attributes.

One intriguing aspect of the BHARAT method is that the decision-maker can decide, rather than using the weights established by the methodology, to assign weights to the attributes based on intuition, experience, or preference. In that case, the same procedure as the methodology suggested can be followed to determine the total scores of the alternatives.

4. Conclusions

The proposed BHARAT method just needs the ranking of the attributes in order to assess the alternatives in relation to each attribute using a streamlined normalization process for a particular decision-making scenario. The normalization process involves each attribute's "*best*" value. The method takes into account the best values for each attribute and takes a *holistic* (i.e., comprehensive) view of the problem of decision-making. The decision-makers can easily *adapt* the proposed method for *ranking of attributes* and for evaluation of alternatives. Hence the proposed method is named as BHARAT (Best Holistic Adaptable Ranking of Attributes Technique). The suggested method makes it simpler and more convenient to rank order the importance of attributes. Compared to the other ranking methods, the weights recommended by the proposed BHARAT method are more stable and better. Furthermore, this paper has presented the fact that in the real decision-making situations, the objective weights or composite weights being used by the researchers recently may not be used at all in the industries and such type of exercises are only for academic research' sake.

The suggested BHARAT approach has shown promising results, and it is thought to be able to handle MADM problems not only in the industrial environment but also in other domains. Apart from its ability to deal with qualitative attributes, imprecise information, and decision makers' varied capacities to process the information, the proposed method is very straight forward, generally useful in all situations, and especially useful when performing timely decision-making and applicable in a variety of decision contexts.

Transforming qualitative attributes into quantitative attributes does not require the application of intricate fuzzy logic and its rules. The decision-makers find it easier to assign quantitative values to qualitative attributes when they use the straightforward linear scales proposed by the BHARAT method. The first and second case studies explain this fact. The suggested methodology assists in computing the total scores that evaluate the alternatives for a decision-making problem considered and can consider any number of quantitative and qualitative attributes at the same time.

The suggested methodology provides a general procedure that can be applied to various selection issues that involve ambiguity, multiple attributes, and alternatives that arise in industrial environment. The BHARAT methodology is applicable in single as well as group decision-making situations. Part-2 of this paper will demonstrate how the method can also be used to find the total scores of the alternative non-dominated solutions in multi-objective and many-objective optimization problems.

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Appendix A

Table A1

Different ranks and the corresponding weights of the attributes

	No. of attributes													
	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Rank*↓	Assigned weights													
1	0.6	0.452054795	0.3714543	0.319480916	0.282626336	0.254847479	0.232999618	0.215269575	0.200531189	0.188044339	0.177300512	0.167937568	0.159689863	0.152357647
2	0.4	0.301369863	0.2476362	0.212987277	0.188417557	0.169898319	0.155333078	0.14351305	0.133687459	0.125362893	0.118200342	0.111958378	0.106459908	0.101571764
3		0.246575342	0.202611436	0.174262318	0.15415982	0.139007716	0.1270907	0.117419768	0.109380649	0.10256964	0.09670937	0.09160231	0.087103561	0.083104171
4			0.178298064	0.15335084	0.135660641	0.12232679	0.111839816	0.103329396	0.096254971	0.090261283	0.085104246	0.080610032	0.076651134	0.07313167
5				0.139918649	0.123777957	0.111612034	0.102043628	0.094278646	0.087823878	0.082355185	0.077649859	0.0735493	0.069937166	0.066725977
6					0.115357688	0.104019379	0.095101885	0.087865133	0.081849465	0.076752791	0.072367556	0.068545946	0.065179536	0.062186795
7						0.098288284	0.089862111	0.083024078	0.077339852	0.072523988	0.068380363	0.06476931	0.061588377	0.058760525
8							0.085729163	0.079205625	0.073782829	0.069188456	0.065235405	0.061790432	0.058755797	0.056058004
9								0.07609473	0.07088492	0.066470997	0.062673207	0.059363539	0.056448093	0.053856259
10									0.068464787	0.064201563	0.060533436	0.057336766	0.054520858	0.052017514
11										0.062268866	0.058711163	0.055610725	0.052879586	0.050451601
12											0.057134539	0.054117359	0.051459562	0.049096778
13												0.052808335	0.050214826	0.047909194
14													0.049111734	0.046856751
15														0.04591535

Table A1 continued…

	No. of attributes																			
	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Rank*↓	Weights assigned																			
1	0.14579	0.13986	0.13448	0.12957	0.12506	0.12091	0.11708	0.11352	0.11021	0.10711	0.10422	0.10149	0.09894	0.09653	0.09425	0.09209	0.09005	0.08811	0.08627	0.08451
2	0.09719	0.09324	0.08965	0.08638	0.08337	0.08061	0.07805	0.07568	0.07347	0.07141	0.06948	0.06767	0.06596	0.06435	0.06284	0.06139	0.06004	0.05874	0.05751	0.05634
3	0.07952	0.07629	0.07335	0.07067	0.06822	0.06595	0.06386	0.06192	0.06011	0.05842	0.05685	0.05536	0.05397	0.05265	0.05141	0.05023	0.04912	0.04806	0.04705	0.04609
4	0.06998	0.06713	0.06455	0.06219	0.06003	0.05804	0.05619	0.05449	0.05289	0.05141	0.05002	0.04872	0.04749	0.04633	0.04524	0.04421	0.04323	0.04229	0.04141	0.04056
5	0.06385	0.06125	0.05889	0.05674	0.05477	0.05295	0.05128	0.04972	0.04827	0.04691	0.04564	0.04445	0.04333	0.04227	0.04127	0.04033	0.03944	0.03859	0.03778	0.03701
6	0.05950	0.05708	0.05488	0.05288	0.05104	0.04935	0.04778	0.04633	0.04498	0.04372	0.04253	0.04142	0.04038	0.0394	0.03847	0.03759	0.03675	0.03596	0.03521	0.03449
7	0.05622	0.05394	0.05186	0.04997	0.04823	0.04663	0.04515	0.04378	0.04250	0.04131	0.04019	0.03914	0.03815	0.03723	0.03635	0.03552	0.03473	0.03398	0.03327	0.03259
8	0.05364	0.05145	0.04948	0.04767	0.04601	0.04448	0.04307	0.04176	0.04054	0.03941	0.03834	0.03734	0.03640	0.03551	0.03467	0.03388	0.03313	0.03242	0.03174	0.03109
9	0.05153	0.04943	0.04753	0.0458	0.04420	0.04274	0.04138	0.04012	0.03895	0.03786	0.03683	0.03587	0.03497	0.03412	0.03331	0.03255	0.03183	0.03114	0.03049	0.02987
10	0.04977	0.04775	0.04591	0.04423	0.04269	0.04128	0.03997	0.03875	0.03762	0.03657	0.03558	0.03465	0.03378	0.03295	0.03218	0.03144	0.03074	0.03008	0.02945	0.02885
11	0.04827	0.04631	0.04453	0.04290	0.04141	0.04004	0.03876	0.03759	0.03649	0.03546	0.03451	0.03361	0.03276	0.03196	0.03121	0.03049	0.02982	0.02917	0.02856	0.02798
12	0.04698	0.04506	0.04333	0.04175	0.04030	0.03896	0.03772	0.03658	0.03551	0.03451	0.03358	0.03270	0.03188	0.03110	0.03037	0.02967	0.02902	0.02839	0.0278	0.02723
13	0.04584	0.04397	0.04228	0.04074	0.03932	0.03802	0.03681	0.03569	0.03465	0.03368	0.03277	0.03191	0.03111	0.03035	0.02963	0.02896	0.02831	0.02770	0.02712	0.02657
14	0.04483	0.04301	0.04135	0.03984	0.03846	0.03718	0.03600	0.03491	0.03389	0.03294	0.03205	0.03121	0.03042	0.02968	0.02898	0.02832	0.02769	0.02709	0.02653	0.02599
15	0.04393	0.04214	0.04052	0.03904	0.03769	0.03644	0.03528	0.03421	0.03321	0.03228	0.03140	0.03058	0.02981	0.02909	0.02840	0.02775	0.02713	0.02655	0.02599	0.02546
16	0.04312	0.04137	0.03977	0.03832	0.03699	0.03576	0.03463	0.03357	0.03259	0.03168	0.03082	0.03002	0.02926	0.02855	0.02788	0.02724	0.02663	0.02606	0.02551	0.02499
17		0.04066	0.03909	0.03767	0.03636	0.03515	0.03403	0.03300	0.03204	0.03114	0.0303	0.02951	0.02876	0.02806	0.02740	0.02677	0.02618	0.02561	0.02508	0.02457
18			0.03847	0.03707	0.03578	0.03459	0.03349	0.03248	0.03153	0.03064	0.02981	0.02904	0.02830	0.02761	0.02696	0.02635	0.02576	0.02521	0.02468	0.02417
19				0.03652	0.03525	0.03408	0.03300	0.03199	0.03106	0.03019	0.02937	0.02860	0.02788	0.02720	0.02656	0.02596	0.02538	0.02483	0.02431	0.02382
20					0.03476	0.03361	0.03254	0.03155	0.03063	0.02977	0.02896	0.02821	0.02750	0.02683	0.02619	0.02559	0.02503	0.02449	0.02397	0.02349
21						0.03317	0.03211	0.03114	0.03023	0.02938	0.02858	0.02784	0.02714	0.02648	0.02585	0.02526	0.02470	0.02417	0.02366	0.02318
22							0.03172	0.03076	0.02986	0.02902	0.02823	0.0275	0.02681	0.02615	0.02554	0.02495	0.0244	0.02387	0.02337	0.02289
23								0.03039	0.02951	0.02868	0.02791	0.02718	0.02649	0.02585	0.02524	0.02466	0.02412	0.02359	0.02310	0.02263
24									0.02918	0.02836	0.0276	0.02688	0.02620	0.02556	0.02496	0.02439	0.02385	0.02333	0.02284	0.02238
25										0.02807	0.02731	0.02659	0.02592	0.02529	0.0247	0.02413	0.02359	0.02309	0.02261	0.02214
26											0.02703	0.02633	0.02567	0.02504	0.02445	0.02389	0.02336	0.02286	0.02238	0.02192
27												0.02608	0.02542	0.02481	0.02422	0.02366	0.02314	0.02264	0.02217	0.02171
28													0.02519	0.02458	0.024	0.02345	0.02293	0.02243	0.02196	0.02152
29														0.02436	0.02379	0.02324	0.02273	0.02224	0.02177	0.02133
30															0.02359	0.02305	0.02254	0.02205	0.02159	0.02115
31																0.02286	0.02236	0.02187	0.02142	0.02098
32																	0.02219	0.02171	0.02125	0.02082
33																		0.02155	0.02109	0.02066
34																			0.02095	0.02052
35																				0.02038

Table A2
Different ranks and the corresponding weights of the attributes in the second version of BHARAT

	No. of attributes													
	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Rank*↓	Assigned weights													
1	0.6666	0.545454	0.48	0.437956	0.408164	0.385675	0.367939	0.353489	0.34142	0.331142	0.32225	0.314455	0.307547	0.301368
2	0.3333	0.272727	0.24	0.218978	0.204082	0.192837	0.183969	0.176745	0.17071	0.165571	0.161125	0.157227	0.153774	0.150684
3		0.181818	0.16	0.145985	0.136054	0.128558	0.122643	0.117827	0.113805	0.110379	0.107415	0.104817	0.102514	0.100455
4			0.12	0.109489	0.102041	0.096419	0.091985	0.088372	0.085355	0.082786	0.080562	0.078614	0.076887	0.075342
5				0.087591	0.081633	0.077135	0.073588	0.070698	0.068284	0.066228	0.06445	0.062891	0.061509	0.060274
6					0.068027	0.064279	0.06132	0.058913	0.056901	0.055188	0.053707	0.052408	0.051256	0.050227
7						0.055096	0.052561	0.050497	0.048773	0.047305	0.046035	0.044921	0.043935	0.043052
8							0.045992	0.044186	0.042677	0.041393	0.040281	0.039307	0.038443	0.037671
9								0.039273	0.037933	0.036791	0.035804	0.034938	0.03417	0.033484
10									0.034142	0.033114	0.032225	0.031445	0.030755	0.030137
11										0.030102	0.029294	0.028586	0.027958	0.027396
12											0.026853	0.026203	0.025628	0.025113
13												0.024188	0.023657	0.023181
14													0.021967	0.021526
15														0.020091