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Optimization of a hybrid multi-item fabricating-shipping integrated system considering scrap, adjustable-rate, and postponement

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A B S T R A C T

This study aims to optimize a hybrid multi-item fabricating-shipping integrated system incorporating scrap, adjustable rate, and postponement. In present-day competitive market environments, there is a clear client demand trend for various goods, shorter lead time, and expected quality. To satisfy the client's needs, the management of manufacturing firms requires an effective and efficient plan to fabricate various high-quality goods in an expedited period, under limited capacity, and with minimal operating expenses. Inspired by facilitating production management to determine the best fabricating scheme/plan to achieve their operational goals, this work proposes an exploratory postponement model with quality assurance and uptime reduction strategies for their decision-making. By employing a two-phase making scheme, the required standard components are first made in the 1st phase, and multiple finished merchandise is fabricated in the 2nd phase. The study suggests strategies of contracting out a part of the common parts' batch and adopting an adjusted/expedited making rate in the 2nd phase to considerably reduce both phases' production uptimes. During both fabricating processes, the screening tasks identify/remove scrapped/faulty goods to ensure each finished batch's quality. Equal-amount multiple shipments of end merchandise are transported to the clients in fixed time-interval. Optimization methodology and mathematical analyses support us in deriving the model's expected annual operating cost and deciding the optimal production-transportation policy. A numerical illustration helps verify our model's applicability and reveals important managerial insights into the studied problem to facilitate management in decision-making.

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1. Introduction

This research develops an investigative model to optimize a hybrid multi-item fabricating-shipping integrated system featuring scrap, adjustable rate, and postponement. This work aims to assist present-day manufacturers in effectively satisfying their customers' timely needs for numerous quality goods with in-house smoothing fabrication operations and under capacity constraints. The capability of designing an efficient and cost-minimization multiproduct postponement fabricating scheme can enhance current manufacturers' competitive advantage. Yang et al. (2005) conducted a survey motivated by the need for more supporting empirical studies on the growing importance/trend of postponement strategy to the producers, supply chains, and business environment. Their postponement-related study used postal mail and email to survey randomly selected 368 samples from British postponement-implemented manufacturers across various industrial sectors. The study received 20.7% (or 76 questionnaires) usable responses and applied the research methodology to their previously used postponement. The analytical results showed positive/significant relationships among the characteristics of postponement, management practices,

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uncertainty, and firm performance. Lastly, they indicated the study's limitations and provided future research directions. Nugroho (2013) studied strategies for substitutable product pricing and producing postponement. The study assumed the existence of product commonality between two differentiable merchandise and designed profit models to explore the feasibility and impact of the strategies on the models, including consideration of the demand uncertainty effect. The study found that the highly customized goods and product configuration can better apply, respectively, for pure producing and pricing postponement strategies to help managers optimize the decisions on product development, manufacturing, and selling. Geetha and Prabha (2022) explored the effect of inventory management postponement strategy with fuzzy cost variables on the retailer in a two-level supply-chain system. The researchers formulated their inventory postponement models with fuzzy variables of fixed and variable fabricating costs and unit holding costs. A signed distance approach helped de-fuzzify to gain the overall annual expenses, and through a proposed algorithm, the study derived the cost-minimized optimal solutions. Finally, the researchers conducted theoretical and computational analyses to explore the postponement strategies and deterioration rate's influence on optimal policies. Other studies (Kisperska-Moron and Swierczek, 2011; Altug, 2016; Chiu et al., 2021, 2022a; Lin et al., 2022; Kiani et al., 2022; Hinkka et al., 2023; Soto-Ferrari et al., 2023) investigated various postponement strategies' impact on operations, management, and controlling of multi-item fabrication in manufacturers and supply chains.

Present-day producers must have competitive strategies for product quality and end-product transporting to satisfy clients' quality and time expectations. An effective product inspecting task helps remove faulty products to keep the anticipated finished batch's quality and an efficient end-item delivery schedule to meet customers' immediate needs. Ebben et al. (2005) proposed a heuristic to explore the multi-resource constraint's dynamic scheduling problem for automated guided vehicles transporting. Their study considered the following bottleneck resources: (1) loading storage space, (2) loading/unloading docks, (3) vehicles, and (4) parking spaces for vehicles. Since the interrelation of each transporting activity, location, and their respective limited resource have made this real-time problem dynamic and hard to plan, the researchers proposed a serial scheduling approach and applied simulation using the discrete real-time event to expose the potential system status. As a comparison result, they found their method suitable for finding vehicle schedules meeting multi-resource constraints, significantly enhancing service levels. Taleizadeh et al. (2010) studied a multiproduct common-cycle batch fabrication system featuring a single machine, random defective rates, permitted shortages with the lost sale, and backlogging disciplines upon stock-out but under the service level constraint. The study aimed to determine the lot-size and permitted shortages simultaneously to minimize the expected operating expenses of holding, setup, fabrication, and shortages. The researchers developed a mathematical model to obtain the objective cost function, proved its convexity, and derived the optimal policies/solutions. Two numerical illustrations with sensitivity analyses were used to validate their result. Kumar et al. (2015) developed the green supply-chain management's (GSCM's) taxonomy-based decisional system depicting its internal and external factors and operational practices to allow practitioners to understand the scope of GSCM and facilitate managerial decision makings. The researchers empirically extended GSCM's taxonomy to incorporate board ranges in organizational size, sectors, and geographic area. They developed the GSCM taxonomy using a two-phase cluster analysis with subjective and objective measurements. Their results have (i) confirmed the existing studies' outcomes; (ii) identified the size, environmental risk's level and attitude; (iii) showed the key mediators' impact among GSCM's external and internal drivers, and its operational practices. Zehtabian et al. (2022) studied the delivery's estimated arrival time using occasional private crowd-shipping drivers to meet online customers' timely orders expectations. With the focus on pickup & delivery times' estimation, the challenge of the study falls on the driver pool's capacity and dynamical status. By simulating this specific problem using a Markovian decision model with naïve, look-ahead, and dynamic policies, the numerical experiments showed a look-ahead incorporating dynamical horizon adjustment outperformed other policies in estimating the delivery arrival time. Other studies (Eben-Chaime, 2004; Scavarda et al., 2015; Martin et al., 2021; Bugatti et al., 2022; Neves-Moreira et al., 2022; Ganjabi et al., 2023; Park et al., 2023) investigated the impact of varioust inevitable defects in goods produced and transporting disciplines on the optimization, control, and operations management of production systems and various supply-chain systems.

Furthermore, the production planners/managers must evaluate potential beneficial approaches to expedite the batch manufacturing uptime/cycle time to meet clients' timely order needs. This study proposes a hybrid fabricating scheme (i.e., to contract out a portion of first-phase standard parts) combining an adjustable/expedited fabricating rate in the second-phase end merchandise to cut our model's batch uptime/making time drastically. Savsar (2008) explored the closed-form fabrication rate solution for a stochastic flexible manufacturing system with random component loading/unloading, machining, and pallet transferring times. The research built a model comprising two machines, one loading/unloading robot and a pallet handling device. Additionally, the study considered various parts' random operational situations. The proposed closed-form optimization model enables practitioners to apply their own parameters to maximize the fabrication output rate for their systems. Kavčič et al. (2015) examined the influence of logistics outsourcing characteristics (including primary, customized, and advanced outsourcing) on organizational performance improvement in terms of their corresponding contributions. The researchers conducted an empirical analysis with 295 Slovenian organizations' surveys focusing on logistics services outsourcing, including outsourcing's timing, characteristics, and criteria. The results showed that the primary and advanced outsourcing characteristics are highly connected to the surveyed Slovenian organizations. It also enabled the business supply-chain management to understand the critical element for long-term success. Glock and Grosse (2021) explored the influence of adjustable fabricating rates on the flexibility of inventory depletion and cost control. The researchers proposed a conceptual inventory framework incorporating the following: (i) controllable fabricating rates in the short- or long-term, (ii) one or multiple intervention(s) per production run, (iii) the variable rates' consequences include product quality, consuming energy, and unit cost, and (iv) two- or multi-stage and multi-item models. The study pointed out the trend and potential future research directions by providing a systematic evaluation of literature regarding the aforementioned controllable-rate batch sizing models. Kandil et al. (2022) studied the impact of outsourcing versus insourcing decisions on a producer facing carbon emission- and price-sensitive demand and a carbon-tax environment. The study considered insourcing generated carbon emissions and outsourcing relevant transportation emissions. Accordingly, the research investigated cost- and revenue-sharing between insourcing and outsourcing decisions. The impact of carbon emission- and price-sensitive demand, selling price, market potential, outsourcer's location, and customers' carbon emission awareness and tax on these decisions are explored. The study's outcomes indicated that higher price-sensitivity demand and carbon tax levels favor outsourcing. The following factors prefer outsourcing: a more significant market potential, higher clients' environmental awareness, and a longer distance between the outsourcer and producer. Other studies (Eiamkanchanalai and Banerjee, 1999; Osei-Bryson and Ngwenyama, 2006; Ameknassi et al., 2016; Chiu et al., 2021; Chiu et al., 2022b; Gambal et al., 2022; Megoze Pongha et al., 2022; Bernard and Mitraille, 2023; Karamemis et al., 2023) explored the effects of various contracting-out policies and expedited fabrication rates on manufacturing and supply-chain systems' uptime-reduction, optimization, control, and operations management. Lacking past works specifically optimizing the hybrid multi-item fabricating-shipping integrated system considering scrap, adjustable rate, and postponement, this work aims to fill the gap.

2. Problem terminology, modeling, and description

This study optimizes a hybrid multi-item fabricating-shipping integrated system featuring scrap, adjustable rate, and postponement. Appendix-A defines all related terminologies of this work. Below are the proposed problem assumptions and descriptions. Consider multi-item have a standard component in common, and the fabricating plan delays end products' differentiation by making all standard components needed first in stage 1 of the production plan. Then, producing the finished multi-item in stage 2. Assume the common component's completion rate γ is constant, say $\gamma = 0.5$, then $P_{1,0}$ and $P_{1,i}$ are double their ordinary rates in a system of single-stage production.

We intend to employ postponement, outsourcing, and expedited rate strategies to gain potential operating expenses savings and rapid order response time. A rotation cycle time policy is used in this single equipment multiproduct manufacturing. An outsourcer supplies a proportion of standard components in stage 1, and an adjustable rate accelerates multi-item outputs in stage 2; both strategies are targeted to cut short the fabricating uptime. Both stages, except the outsourcing items, have random scraps, and the removal of faults ensures the system's product quality. Fig. 1 illustrates the stock status of our proposed manufacturing model.

Assume a partial subcontracting policy to outsource a π_0 proportion of standard components to an external source and an extra output rate $\alpha_{1,i}$ fabricating finished product i in stage two. These strategies' consequent effects on operating expenses and output rates are formulated as follows:

$$C_{T,i} = \left(1+\alpha_{3,i}\right)C_i \tag{1}$$

$$C_{\pi 0} = C_0\left(1+\beta_{2,0}\right) \tag{2}$$

$$P_{T1,i} = \left(1+\alpha_{1,i}\right)P_{1,i} \tag{3}$$

$$K_{\pi 0} = K_0\left(1+\beta_{1,0}\right) \tag{4}$$

$$K_{T,i} = \left(1+\alpha_{2,i}\right)K_i \tag{5}$$

The random x_0 and x_i rates of faulty products in each stage exist, and the removal of these imperfect quality items ensures product quality. By observing Fig. 1, the inventory level stacks to $H_{1,0}$ when $t_{1,0}$ ends. By receiving outsourced items, its level becomes $H_{2,0}$. In the 2nd stage, the end item i's stock level stacks to $H_{1,i}$ when uptime $t_{1,i}$ ends before its distribution begins.

Regarding the inventory of random scraps, Fig. 2 exhibits in stage 1 that its level stacks to ($d_{1,0}\, t_{1,0}$) when $t_{1,0}$ ends before depleting to zero in $t_{2,0}$. A similar situation exists in stage 2. To avoid the unwanted stock-out situation, ($P_{1,0} - d_{1,0}$) and ($P_{T1,i} - d_{T1,i} - \lambda_i$) must all be > 0.

2.1. Formulations in stage 2

According to our proposed delayed differentiation strategy, the standard components' requirements for all end products are $H_{2,0}$. Once we enter the 2nd stage, for each end item i, its inventories begin to deplete Q_i. The following formulas express the standard components' status (see Figs. 1-3).

$$H_1 = H_{2,0} - Q_1 \tag{6}$$

$$H_i = H_{(i-1)} - Q_i \text{ , } for\ i = 2,\ 3,\ ...,L \tag{7}$$

$$H_L = H_{(L-1)} - Q_L = 0 \tag{8}$$

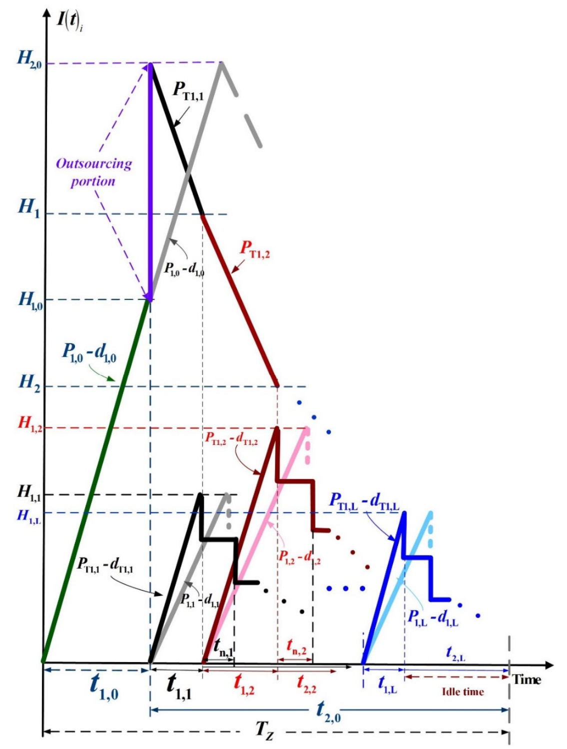

Fig. 1. Stock status of our multi-item manufacturing model compared to a model without external supplier in stage 1 nor adjustable rate in stage 2

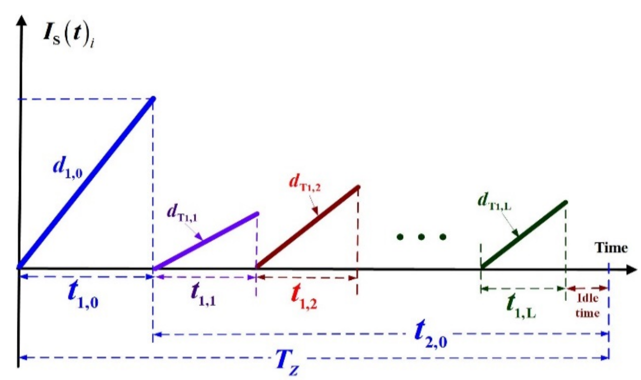

Fig. 2. Our model's inventory level of random scraps

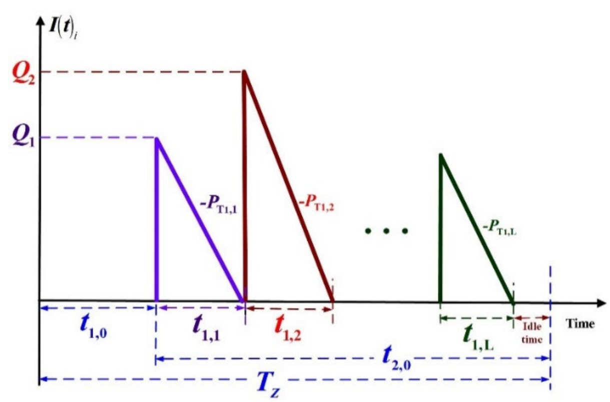

Fig. 3. The standard components' inventory level

By observing stage 2's fabricating process (Fig. 1), one finds the following equations straightforwardly:

$$Q_i = \frac{\lambda_i T_Z}{1-x_i} \tag{9}$$

$$t_{1,i} = \frac{H_{1,i}}{\left(P_{T1,i} - d_{T1,i}\right)} = \frac{Q_i}{P_{T1,i}} \tag{10}$$

$$T_Z = t_{1,i} + t_{2,i}, \text{ where } i = 1, 2, \ldots, L \tag{11}$$

$$H_{1,i} = \left(P_{T1,i} - d_{T1,i}\right) t_{1,i} \tag{12}$$

$$t_{2,i} = T_Z - t_{1,i} \tag{13}$$

Each end product i's distribution begins once stage 2's fabricating process ends. Its inventory status is demonstrated in Fig. 4 and the total inventories are

$$\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right) H_{1,i}\left(t_{2,i}\right) = \frac{n(n-1)}{2}\left(\frac{1}{n^2}\right) H_{1,i}\left(t_{2,i}\right) = \left(\frac{n-1}{2n}\right) H_{1,i}\left(t_{2,i}\right) \tag{14}$$

At the client end, the inventory level of each product i is exhibited in Fig. 5. Total client inventories of each product i are shown in Eq. (15).

$$\left[\frac{nI_i\left(t_{1,i}\right)}{2} + \frac{n(n+1)}{2} I_i t_{n,i} + \frac{n\left(D_i - I_i\right) t_{n,i}}{2}\right] \tag{15}$$

where

$$t_{n,i} = \frac{t_{2,i}}{n} \tag{16}$$

$$D_i = \frac{H_{1,i}}{n} \tag{17}$$

$$I_i = D_i - \lambda_i\left(t_{n,i}\right) \tag{18}$$

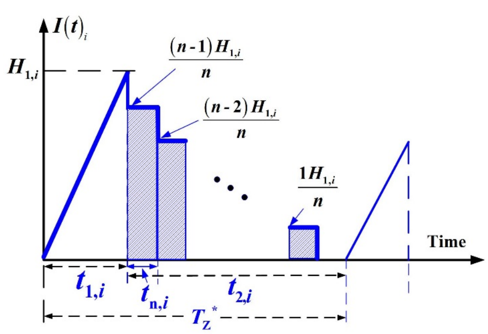

Fig. 4. Each end product i's inventory level in $t_{2,i}$

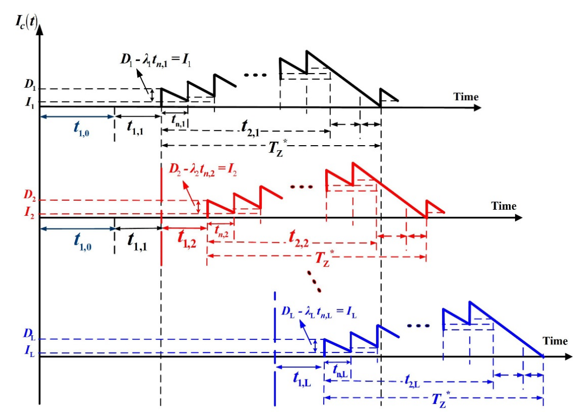

Fig. 5. Client's inventory level of each product i

2.2. Formulations in stage 1

Total requirements of the perfect-quality common components are as follows (refer to Eq. (9)):

$$H_{2,0} = \sum_{i=1}^{L} \frac{\lambda_i T_Z}{1-x_i} = \sum_{i=1}^{L} Q_i \tag{19}$$

According to the problem statement and Figs. 1 and 2, one can obtain the following formulas straightforwardly:

$$\lambda_0 = \frac{\sum_{i=1}^{L} Q_i}{T_Z} \tag{20}$$

$$H_{1,0} = H_{2,0}\left(1-\pi_0\right) = \left(\sum_{i=1}^{L} Q_i\right)\left(1-\pi_0\right) \tag{21}$$

$$Q_0=\frac{H_{1,0}}{1-x_0} \tag{22}$$

$$t_{1,0}=\frac{Q_0}{P_{1,0}}=\frac{H_{1,0}}{\left(P_{1,0}-d_{1,0}\right)} \tag{23}$$

$$H_{1,0}=\left(P_{1,0}-d_{1,0}\right)t_{1,0} \tag{24}$$

3. Total operating expenditures & optimization

$TC(T_Z, n)$, the total operating expenditures consist of (A) outsourcing fixed and variable costs, in-house setup, manufacturing, disposal, and stock-holding costs in stage 1; (B) total expedited fabrication variable, setup, disposal, stock-holding, end items' distribution expenses; and (C) customer side's stock-holding cost, as follows:

$$\begin{aligned}TC\left(T_Z,n\right)&=C_{\pi0}\pi_0\left(\sum_{i=1}^{L}Q_i\right)+K_{\pi0}+K_0+C_0Q_0+C_{S,0}\left(Q_0x_0\right)\\&+h_{1,0}\left[\frac{H_{1,0}t_{1,0}}{2}+\frac{d_{1,0}t_{1,0}}{2}\left(t_{1,0}\right)+\sum_{i=1}^{L}\left[\frac{Q_i}{2}\left(t_{1,i}\right)+H_i\left(t_{1,i}\right)\right]\right]+h_{4,0}\left(x_0Q_0\right)T_Z\\&+\sum_{i=1}^{L}\left\{\begin{aligned}&Q_iC_{T,i}+K_{T,i}+\left(Q_ix_i\right)C_{S,i}+h_{1,i}\left[\frac{H_{1,i}t_{1,i}}{2}+\frac{d_{T1,i}t_{1,i}}{2}\left(t_{1,i}\right)+\left(\frac{n-1}{2n}\right)H_{1,i}\left(t_{2,i}\right)\right]+nK_{D,i}\\&+C_{D,i}Q_i\left(1-x_i\right)+h_{4,i}\left(x_iQ_i\right)T_Z+h_{3,i}\left[\frac{n\left(D_i-I_i\right)t_{n,i}}{2}+\frac{n\left(n+1\right)}{2}I_it_{n,i}+\frac{nI_i\left(t_{1,i}\right)}{2}\right]\end{aligned}\right\}\end{aligned} \tag{25}$$

For the faulty rates, we apply $E[x_i]$ to deal with their randomness (where $i = 0, 1, 2, \ldots, L$). Replacing Eqs. (1-24) in $TC(T_Z, n)$ plus additional derivation, one can find the following $E[TCU(T_Z, n)]$ (see Appendix B for details):

$$\begin{aligned}E\left[TCU\left(T_Z,n\right)\right]&=\left\{\begin{aligned}&\frac{K_0\left(1+\beta_{1,0}\right)}{T_Z}+C_0\left(1+\beta_{2,0}\right)\pi_0\lambda_0+C_0\left(1-\pi_0\right)\lambda_0E_{00}+C_{S,0}\left(1-\pi_0\right)\lambda_0E_{10}\\&+\frac{K_0}{T_Z}+\frac{h_{1,0}\lambda_0^{\,2}T_Z}{2}\left(1-\pi_0\right)^2\left(E_{00}\right)^2\left(\frac{1}{P_{1,0}}\right)+h_{4,0}\left(1-\pi_0\right)\lambda_0E_{10}T_Z\\&+h_{1,0}\sum_{i=1}^{L}\left\{\frac{\lambda_i^{\,2}T_Z\left(E_{0i}\right)^2}{2\left[\left(1+\alpha_{1,i}\right)P_{1,i}\right]}+\left(\sum_{i=1}^{L}\left[\lambda_iT_ZE_{0i}\right]-\sum_{j=1}^{i}\left[\lambda_jT_ZE_{0j}\right]\right)\lambda_iE_{0i}E_{2i}\right\}\end{aligned}\right\}\\&+\sum_{i=1}^{L}\left\{\begin{aligned}&\left(1+\alpha_{3,i}\right)C_i\lambda_iE_{0i}+\frac{\left(1+\alpha_{2,i}\right)K_i}{T_Z}+C_{S,i}\lambda_iE_{1i}+\frac{nK_{D,i}}{T_Z}+C_{D,i}\lambda_i+h_{4,i}\lambda_iE_{1i}T_Z\\&+h_{1,i}\left(\frac{\lambda_i^2T_Z}{2}\right)E_{3i}\left[\left(E_{0i}\right)^2+1\right]+\left(\frac{\lambda_i^{\,2}T_Z}{2n}\right)\left(h_{3,i}-h_{1,i}\right)\left[\frac{1}{\lambda_i}-E_{4i}\right]+\frac{h_{3,i}}{2}\left(\lambda_i^{\,2}T_Z\right)E_{4i}\end{aligned}\right\}\end{aligned} \tag{26}$$

3.1. Optimization of the operating expenditures

Eq. (27) is the result of applying the Hessian Matrix Equations to $E[TCU(T_Z, n)]$:

$$\begin{bmatrix}T_Z & n\end{bmatrix}\cdot\begin{pmatrix}\dfrac{\partial^2E\left[TCU\left(T_Z,\,n\right)\right]}{\partial T_Z^{\,2}} & \dfrac{\partial^2E\left[TCU\left(T_Z,\,n\right)\right]}{\partial T_Z\partial n}\\ \dfrac{\partial^2E\left[TCU\left(T_Z,\,n\right)\right]}{\partial T_Z\partial n} & \dfrac{\partial^2E\left[TCU\left(T_Z,\,n\right)\right]}{\partial n^2}\end{pmatrix}\cdot\begin{bmatrix}T_Z\\ n\end{bmatrix}=\left[\frac{2K_0}{T_Z}+\frac{2K_0\left(1+\beta_{1,0}\right)}{T_Z}+\sum_{i=1}^{L}\left\{\frac{2\left(1+\alpha_{2,i}\right)K_i}{T_Z}\right\}\right]>0 \tag{27}$$

Since K_0, T_Z , $(1 + \beta_{1,0})$, K_i, and $(1 + \alpha_{2,i})$ are positive, Eq. (27) yields positive,. So, $E[TCU(T_Z, n)]$ is strictly convex for all v alues of n and $T_Z > 0$. Hence, $E[TCU(T_Z, n)]$ exists a minimum value. Setting first-derivatives of $E[TCU(T_Z, n)]$ regarding n and T_Z equal to zero as shown in Eqs. (28-29).

$$\begin{aligned}\frac{\partial E\left[TCU\left(T_Z,\,n\right)\right]}{\partial T_Z}&=\left\{\begin{aligned}&\frac{-K_0\left(1+\beta_{1,0}\right)}{T_Z^2}-\frac{K_0}{T_Z^2}+\frac{h_{1,0}\lambda_0^{\,2}}{2}\left(1-\pi_0\right)^2\left(E_{00}\right)^2\left(\frac{1}{P_{1,0}}\right)+h_{4,0}\left(1-\pi_0\right)\lambda_0E_{10}\\&+h_{1,0}\sum_{i=1}^{L}\left\{\frac{\lambda_i^{\,2}\left(E_{0i}\right)^2}{2\left[\left(1+\alpha_{1,i}\right)P_{1,i}\right]}+\left(\sum_{i=1}^{L}\left[\lambda_iE_{0i}\right]-\sum_{j=1}^{i}\left[\lambda_jE_{0j}\right]\right)\lambda_iE_{0i}E_{2i}\right\}\end{aligned}\right\}\\&+\sum_{i=1}^{L}\left\{\begin{aligned}&-\frac{K_i\left(1+\alpha_{2,i}\right)}{T_Z^2}-\frac{nK_{D,i}}{T_Z^2}+h_{4,i}\lambda_iE_{1i}+h_{1,i}\left(\frac{\lambda_i^2}{2}\right)E_{3i}\left[1+\left(E_{0i}\right)^2\right]\\&+\left(\frac{\lambda_i^{\,2}}{2n}\right)\left(h_{3,i}-h_{1,i}\right)\left[\frac{1}{\lambda_i}-E_{4i}\right]+\frac{h_{3,i}}{2}\left(\lambda_i^2\right)E_{4i}\end{aligned}\right\}=0\end{aligned} \tag{28}$$

$$\frac{\partial E\left[TCU\left(T_Z,\, n\right)\right]}{\partial n}=\sum_{i=1}^{L}\left\{\frac{K_{D,i}}{T_Z}-\left(\frac{\lambda_i^2 T_Z}{2n^2}\right)\left(h_{3,i}-h_{1,i}\right)\left[\frac{1}{\lambda_i}-E_{4i}\right]\right\}=0 \tag{29}$$

By simultaneously solving Eqs. (28) and (29), one can derive T_Z* and n* as follows:

$$T_Z^*=\sqrt{\frac{2\left\{K_0\left(2+\beta_{1,0}\right)+\sum_{i=1}^{L}\left[\left(1+\alpha_{2,i}\right)K_i+nK_{D,i}\right]\right\}}{\begin{array}{l}+h_{1,0}\left[\frac{\left(E_{00}\right)^2\left(1-\pi_0\right)^2\lambda_0^2}{P_{1,0}}+2\sum_{i=1}^{L}\left(E_{0i}\lambda_i\right)\sum_{i=1}^{L}\lambda_i E_{4i}+\sum_{i=1}^{L}\frac{\lambda_i^2\left(E_{0i}\right)^2}{\left(1+\alpha_{1,i}\right)P_{1,i}}-2\sum_{i=1}^{L}\left[\left(\sum_{j=1}^{i}\left(E_{0j}\lambda_j\right)\right)\left(\lambda_i E_{4i}\right)\right]\right]\\ +\sum_{i=1}^{L}\left\{h_{1,i}\left(\lambda_i^2\right)E_{3i}+2h_{4,i}\lambda_i E_{1i}+\left(\frac{\lambda_i^2}{n}\right)\left(h_{3,i}-h_{1,i}\right)\left[\frac{1}{\lambda_i}-E_{4i}\right]+h_{3,i}\left(\lambda_i^2\right)E_{4i}\right\}+2h_{4,0}\left(1-\pi_0\right)\lambda_0 E_{10}\end{array}}} \tag{30}$$

and

$$n^*=\sqrt{\frac{\left[K_0\left(2+\beta_{1,0}\right)+\sum_{i=1}^{L}\left(1+\alpha_{2,i}\right)K_i\right]\cdot\sum_{i=1}^{L}\left\{\lambda_i^2\left(h_{3,i}-h_{1,i}\right)\left(\frac{1}{\lambda_i}-E_{4i}\right)\right\}}{\sum_{i=1}^{L}\left\{2K_{Di}\right\}\cdot\sum_{i=1}^{L}\left\{\begin{array}{l}+h_{1,0}\left[\begin{array}{l}\frac{\left(E_{00}\right)^2\left(1-\pi_0\right)^2\lambda_0^2}{P_{1,0}}+\sum_{i=1}^{L}\frac{\left(E_{0i}\right)^2\lambda_i^2}{\left(1+\alpha_{1,i}\right)P_{1,i}}+2\sum_{i=1}^{L}\left(E_{0i}\lambda_i\right)\sum_{i=1}^{L}\lambda_i E_{4i}\\ -2\sum_{i=1}^{L}\left[\left(\sum_{j=1}^{i}\left(E_{0j}\lambda_j\right)\right)\left(\lambda_i E_{4i}\right)\right]\end{array}\right]\\ +\sum_{i=1}^{L}\left\{h_{1,i}\left[\lambda_i^2 E_{3i}\right]+2h_{4,i}\lambda_i E_{1i}+h_{3,i}\left(\lambda_i^2\right)E_{4i}\right\}+2h_{4,0}\left(1-\pi_0\right)\lambda_0 E_{10}\end{array}\right.}} \tag{31}$$

4. Demonstrating example

The proposed study offers a demonstrating example for resolving the optimal fabricating-shipping policy to the studied multi-item problem featuring postponement, scrap, and adjustable rate. The assumption of parameter values for this two-stage postponement system is exhibited in Tables 1, 2(a), and 2(b). Contrariwise, the basis of their corresponding parameter values for a single-stage manufacturing scheme is displayed in Table C of Appendix C.

Table 1

Phase one's parameter assumption of this two-stage postponement system

γ	$h_{1,0}$	C_0	x_0	$P_{1,0}$	π_0	i_0
0.5	\$8	\$40	0.025	120000	0.4	0.2
δ	$h_{4,0}$	$C_{S,0}$	λ_0	$\beta_{1,0}$	$\beta_{2,0}$	K_0
0.5	\$8	\$10	18218	-0.7	0.4	\$8500

Table 2(a)

Phase two's parameter assumption of this two-stage postponement system (1 of 2)

Product i	$\alpha_{2,i}$	$K_{D,i}$	i_i	$\alpha_{3,i}$	λ_i	$\alpha_{1,i}$	$P_{1,i}$	C_i
1	0.1	\$1800	0.2	0.25	3000	0.5	112258	\$40
2	0.1	\$1900	0.2	0.25	3200	0.5	116066	\$50
3	0.1	\$2000	0.2	0.25	3400	0.5	120000	\$60
4	0.1	\$2100	0.2	0.25	3600	0.5	124068	\$70
5	0.1	\$2200	0.2	0.25	3800	0.5	128276	\$80

Table 2(b)

Phase two's parameter assumption of this two-stage postponement system (2 of 2)

Product i	$C_{S,i}$	$h_{3,i}$	$h_{4,i}$	$C_{D,i}$	$h_{1,i}$	K_i	x_i
1	\$10	\$70	\$8	\$0.1	\$8	\$8500	0.025
2	\$15	\$75	\$10	\$0.2	\$10	\$9000	0.075
3	\$20	\$80	\$12	\$0.3	\$12	\$9500	0.125
4	\$25	\$85	\$14	\$0.4	\$14	\$10000	0.175
5	\$30	\$90	\$16	\$0.5	\$16	\$10500	0.225

To validate our result's applicability, we first use formulas (31) and (30) to find the optimality of the operating policy: $n^* = 4$ and $T_Z^* = 0.5424$. Utilizing these optimal values to calculate formula (26), we find the optimal $E[TCU(T_Z^*, n^*)]$ = \$2,673,382. Figs. 6 and 7 demonstrate $E[TCU(T_Z, n)]$'s convexity and behavior concerning n and T_Z, respectively. From these illustrations, one finds that as T_Z and n deviating from its T_Z^* and n^*, $E[TCU(T_Z^*, n^*)]$ rises significantly.

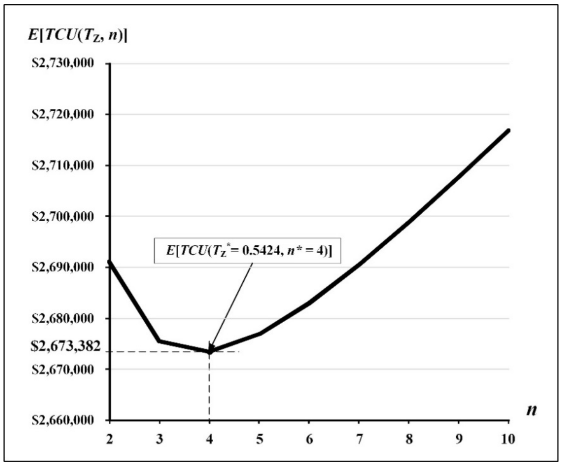

Fig. 6. $E[TCU(T_Z, n)]$'s behavior (convexity) concerning n

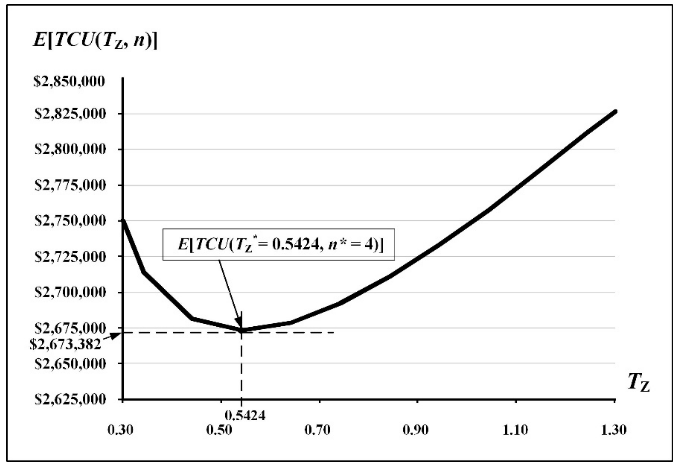

Fig. 7. $E[TCU(T_Z, n)]$'s convexity and behavior vis-à-vis T_Z

The standard part's value and finishing proportion γ affect the uptime and overall system cost for a multi-item postponement fabrication. Fig. 8 shows the investigative result of optimal annual operating expenditure $E[TCU(T_Z^*, n^*)]$'s conduct vis-à-vis the standard part's completing proportion γ. It discloses that as γ increases, $E[TCU(T_\pi^*, n^*)]$ declines considerably. It confirms that as we assuming $\gamma = 0.5$, the optimal $E[TCU(T_Z^* = 0.5424, n^* = 4)]$ = \$2,673,382 (i.e., declining 6.09% versus $\gamma = 0$, the single-phase multi-item fabricating system without postponement).

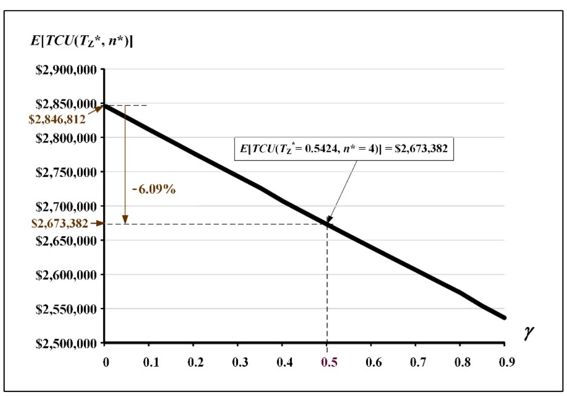

Fig. 8. $E[TCU(T_Z^*, n^*)]$'s conduct vis-à-vis γ

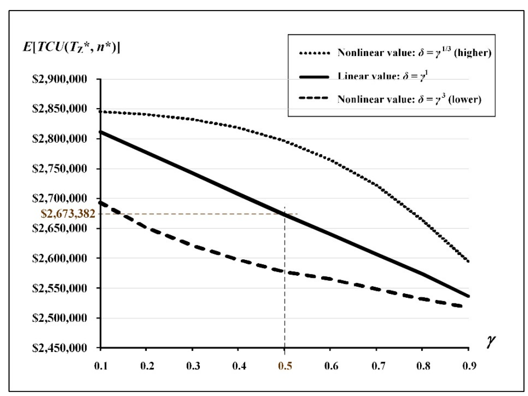

Fig. 9. $E[TCU(T_Z^*, n^*)]$'s conduct vis-à-vis various δ relating to γ

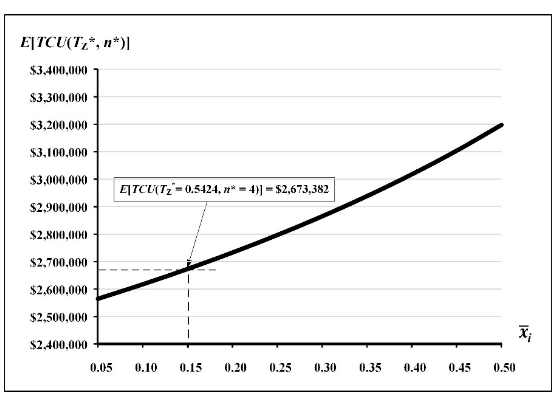

Fig. 10. $E[TCU(T_Z^*, n^*)]$'s conduct vis-à-vis the average scrap proportion

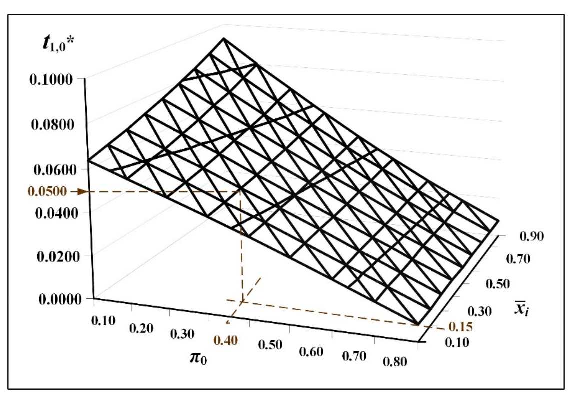

Fig. 11. $t_{1,0}^*$'s conduct vis-à-vis π_0 and the average scrap percentage

This demonstrating example assumes the relationship δ between the standard part's value and related completing rate γ to be linear. For instance, if $\gamma = 0.5$, the standard part's value is half its corresponding finished merchandise. Nevertheless, the linear relationship may not always be true for other types of goods. To address this situation, our model further studies different relationships δ that are other than linear. Fig. 9 depicts $E[TCU(T_Z{*}, n{*})]$'s conduct vis-à-vis various δ relating to γ. (e.g., the nonlinear cases for $\delta = \gamma^{1/3}$ and $\delta = \gamma^3$). This proposal can also examine the mean scrap proportion effect on $E[TCU(T_Z{*}, n{*})]$. Fig.10 exemplifies $E[TCU(T_Z{*}, n{*})]$'s conduct vis-à-vis the average scrap proportion. As the scrap rate surges, $E[TCU(T_Z{*}, n{*})]$ thoughtfully increases. For our example's assumption with average scrap 15%, it reconfirms that $E[TCU(T_Z{*}, n{*})]$ is $2,673,382. Moreover, our model is capable of exploring stage one's optimal uptime $t_{1,0}{*}$'s conduct vis-à-vis the collective effect of outsourcing and average scrap percentages (as exhibited in Fig. 11). As outsourcing percentage π_0 surges, less time is required to fabricate the standard parts; hence, $t_{1,0}{*}$ drops significantly. Conversely, as the mean scrap percentage surges, more time is necessary to make perfect quality common components; thus, $t_{1,0}{*}$ increases accordingly. As we assume $\pi_0 = 0.4$ and mean scrap percentage at 15%, the optimal uptime $t_{1,0}{*}$ is 0.0500 (years). As stated earlier, when the outsourcing percentage π_0 surges, less time is required to fabricate the standard parts; hence, machine utilization declines drastically, as demonstrated in Fig. 12. Furthermore, it discloses explicitly that in this example's assumption π_0 at 0.4, the utilization reduces 24.17%, dropping from 0.2545 to 0.1930 at our optimal operating policies: $n{*} = 4$ and $T_Z{*} = 0.5424$. Further, in Appendix D, Table D-1 exposes the investigative results of diverse crucial system factors affected by subcontracting proportion π_0.

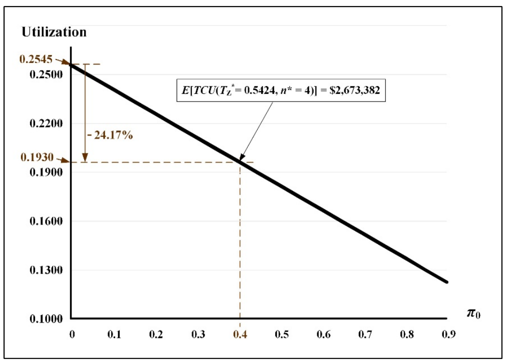

Fig. 12. Machine utilization's conduct vis-à-vis π_0

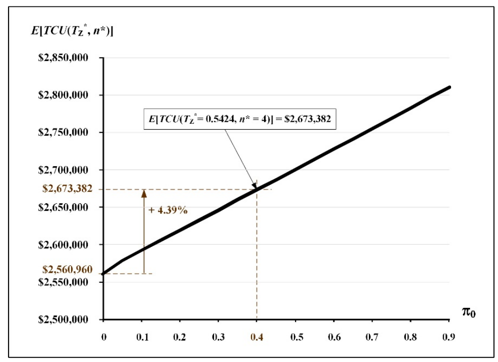

Fig. 13. $E[TCU(T_Z{*}, n{*})]$'s conduct vis-à-vis π_0

Furthermore, the study explores the possible price associated with uptime and utilization reduction outsourcing benefits. Fig. 13 shows the optimal $E[TCU(T_Z{*}, n{*})]$'s conduct vis-à-vis π_0. As π_0 surges, since the unit subcontracting cost is higher than the unit in-house making cost; hence, $E[TCU(T_Z{*}, n{*})]$ increases considerably. It reveals explicitly that in this example's assumption π_0 at 0.4, $E[TCU(T_Z{*}, n{*})]$ increases by 4.39%, surging from $2,560,960 to $2,673,382 (refer to Table D-1). Our model implements an adjusted fabricating rate on the finished product aiming to bring extra proportion $\alpha_{1,i}$ of outputs in stage 2. Further, in Appendix D, Table D-2 discloses the explorative results of diverse crucial system factors affected by adjusted fabricating rate $\alpha_{1,0}$. Fig. 14 depicts the explorative outcomes of $\alpha_{1,i}$'s influence on machine utilization. It uncovers the declining trend of utilization as adjusted rate $\alpha_{1,i}$ increases and exposes a 20.71% drop in utilization at our assumption $\alpha_{1,i} = 0.5$, which is reducing from 0.2434 to 0.1930 (see Table D-2).

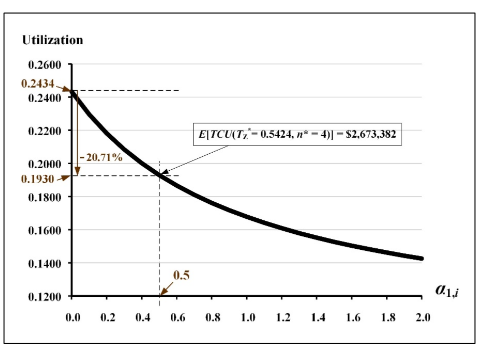

Fig. 14. Utilization's conduct vis-à-vis $\alpha_{1,i}$

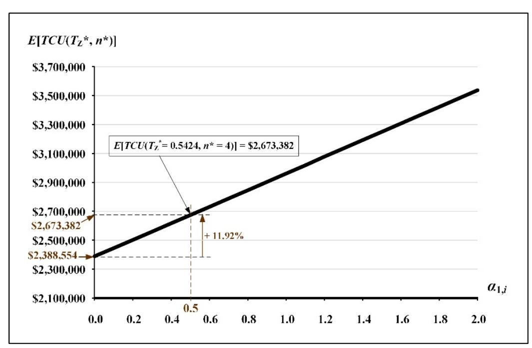

Fig. 15. $E[TCU(T_Z{*}, n{*})]$'s conduct vis-à-vis $\alpha_{1,i}$

Similar to the utilization reduction factor π_0, one wonders what possible price associated with adjusted fabricating rate $\alpha_{1,i}$. Fig. 15 demonstrates the investigative outcomes of $\alpha_{1,i}$'s effect on $E[TCU(T_Z{*}, n{*})]$. As $\alpha_{1,i}$ increases, since the unit cost when using the adjusted rate is higher than the typical unit cost, $E[TCU(T_Z{*}, n{*})]$ surges radically. It exposes explicitly that in our assumption $\alpha_{1,i}$ at 0.5, $E[TCU(T_Z{*}, n{*})]$ upsurges by 11.92%, rising from $2,388,554 to $2,673,382 (see Table D-2). Since this study uses dual uptime-reduction strategies (i.e., with the key factors $\alpha_{1,i}$ and π_0), one would wonder what will be $E[TCU(T_Z{*}, n{*})]$'s conduct vis-à-vis the combined influence of $\alpha_{1,i}$ and π_0. Figure 16 exhibits the exploration results of $E[TCU(T_Z{*}, n{*})]$'s behavior vis-à-vis the combined impact of $\alpha_{1,i}$ and π_0. As both $\alpha_{1,i}$ and π_0 increase, $E[TCU(T_Z{*}, n{*})]$ rises hugely. In our assumption for $\alpha_{1,i} = 0.5$ and $\pi_0 = 0.4$, $\alpha_{1,i}$ influence more on $E[TCU(T_Z^*, n^*)]$, and one reconfirms $E[TCU(T_Z^* = 0.5424, n^* = 4)] = \$2,673,382$.

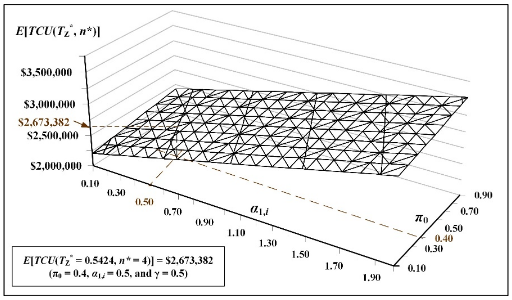

Fig. 16. $E[TCU(T_Z^*, n^*)]$'s conduct vis-à-vis the combined influence of $\alpha_{1,i}$ and π_0

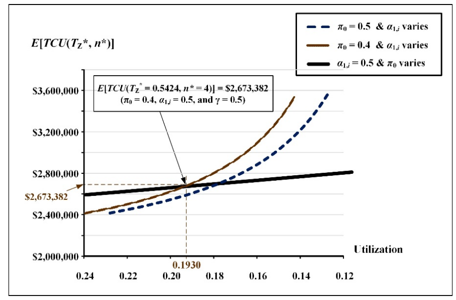

Fig. 17. $E[TCU(T_Z^*, n^*)]$'s behavior vis-à-vis $\alpha_{1,i}$ and π_0 for utilization-reducing decision

Furthermore, if one is curious about the timing of applying these uptime-reduction strategies, our proposed model can answer based on a respective in-depth exploration of $\alpha_{1,i}$ and π_0. Fig. 17 demonstrates the results of the individual and joint effect of $\alpha_{1,i}$ and π_0 on $E[TCU(T_Z^*, n^*)]$. Based on the assumed cost parameters relating to $\alpha_{1,i}$ and π_0, one finds that most cost-effective method to lessen utilization are to apply subcontracting with a constant $\pi_0 = 0.4$ and combining with an increasing $\alpha_{1,i}$ until utilization declines to 0.1930 (at the time $\alpha_{1,i} = 0.5$; see Fig. 17). Then, keep $\alpha_{1,i}$ at a constant 0.5 now and start to increase π_0 from 0.4. Fig. 17 also provides a better way (less costly) by beginning with $\pi_0 = 0.5$ and following the blue dashed line (until $\alpha_{1,i}$ increased to 0.5) with the same scenario as above.

The proposed study can look into the in-depth expenses of $E[TCU(T_Z^*, n^*)]$ (as exhibited in Fig. 18). One finds the key contributors comprise the following:

(a) 41.3% is from fabricating variable expense of end products;
(b) 16.6% donated by variable making cost of the standard components;
(c) 15.44% contributed by the subcontracting cost of the standard components;
(d) 10.84% is from the expense of expediting end products fabrication;
(e) 6.65% is the so-called supply-chain relating expenses (i.e., 3.69% of client holding cost plus 2.96% of products distributing cost).

Other than the above 90.83%, there are setup costs and quality cost that compensates for the scrapped items produced.

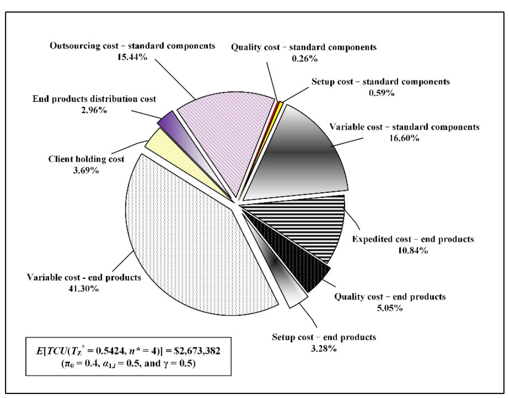

Fig. 18. $E[TCU(T_Z^*, n^*)]$'s in-depth expenses

5. Conclusions

Inspired by facilitating production management to derive the best fabricating scheme/plan to achieve the operational goals for making various timely, quality goods, we optimize a hybrid multi-item fabricating-shipping integrated system with scrap, adjustable rate, and postponement. This study employs a two-phase-making scheme by producing standard components in phase one and finished merchandise in phase two. We implement two uptime-reduction policies by outsourcing a part of the common parts' batch in phase one and accelerating the fabricating pace in phase two. Screening tasks in both phases help identify/remove scrapped goods to ensure the finished batch's quality. Fixed-amount multiple shipments of end merchandise are transported to the clients in an equal time interval. This work develops a scheme/model to portray precisely the studied problem (see section 2), and optimization methodology, along with mathematical analyses, supports us in deriving the model's expected annual operating cost and deciding the optimal production-transportation policy (see section 3). We offer a numerical demonstration to validate the proposed scheme's applicability and reveal the following vital managerial insights into the studied problem to facilitate management in decision-making (refer to section 4):

(1) Convexity/performance of $E[TCU(T_Z, n)]$ vis-à-vis n and T_Z, respectively (Fig. 6 and Fig. 7);
(2) Individual/collective conduct/performance of $E[TCU(T_Z*, n*)]$ vis-à-vis various γ and δ (see Fig. 8 and Fig. 9);.
(3) Performance of $E[TCU(T_Z*, n*)]$ vis-à-vis π_0 and combined conduct of $t_{1,0}*$ vis-à-vis π_0 and average scrap percentage (refer to Fig. 10 and Fig. 11);
(4) Conduct of machine utilization and $E[TCU(T_Z*, n*)]$ vis-à-vis contracting-out rate π_0, respectively (Fig. 12 and Fig. 13);
(5) Performance of utilization and $E[TCU(T_Z*, n*)]$ vis-à-vis accelerating rate $\alpha_{1,i}$, respectively (see Fig. 14 and Fig. 15);
(6) Joint influence of $\alpha_{1,i}$ and π_0 on $E[TCU(T_Z*, n*)]$ (Fig. 16);
(7) Facilitating managerial decision-making on effective/beneficial utilization reduction vis-à-vis $\alpha_{1,i}$ and π_0 (see Fig. 17);
(8) $E[TCU(T_Z*, n*)]$'s in-depth expenses (Fig. 18).

Considering stochastic end merchandise's annual requirement in the proposed problem's context should be worth future exploration.

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Appendix A

Nomenclature

n = finished items' distribution frequency in a cycle,
T_Z = rotation production cycle time,

The following are stage 1's standard components fabricating related parameters:
λ_0 = annual requirement,
Q_0 = lot-size,
π_0 = outsourcing proportion,
$C_{\pi 0}$ = unit outsourcing cost,
$K_{\pi 0}$ = outsourcing setup cost,
C_0 = in-house manufacturing unit cost,
$\beta_{2,0}$ = connecting parameter between C_0 and $C_{\pi 0}$,
K_0 = in-house manufacturing setup cost,
$\beta_{1,0}$ = connecting parameter between K_0 and $K_{\pi 0}$,
$h_{1,0}$ = unit holding cost,
i_0 = connecting factor between $h_{1,0}$ and C_0,
$h_{4,0}$ = safety item's unit holding cost,
$P_{1,0}$ = annual manufacturing rate,
$H_{1,0}$ = inventory status when uptime ends,
x_0 = random scrap rate (in-house manufacturing),
$d_{1,0}$ = manufacturing rate of faulty components ($d_{1,0} = x_0 P_{1,0}$),
$C_{S,0}$ = unit disposal cost,

γ = completion rate of the common part as compared to their finished item,
$t_{1,0}$ = in-house manufacturing uptime,
$t_{2,0}$ = standard components' depletion time,
$H_{2,0}$ = inventory status when outsourced items are received,
t_0 = uptime,
S_0 = setup time,

In the 2nd stage, each end item i's fabricating relevant parameters are (for i = 1, 2, …, L):
λ_i = annual demand rate,
Q_i = lot size,
C_i = unit manufacturing cost,
$C_{T,i}$ = unit manufacturing cost with adjustable rate implemented,
$\alpha_{3,i}$ = connecting parameter between C_i and $C_{T,i}$,
$P_{1,i}$ = ordinary annual manufacturing rate,
$P_{T1,i}$ = annual rate with adjustable rate implemented,
$\alpha_{1,i}$ = connecting parameter between $P_{1,i}$ and $P_{T1,i}$,
K_i = ordinary setup cost,
$K_{T,i}$ = ordinary setup cost with adjustable rate implemented,
$\alpha_{2,i}$ = connecting parameter between K_i and $K_{T,i}$,
$t_{1,i}$ = uptime,
$t_{2,i}$ = distribution time,
t_i^* = the sum of finished products' optimal uptimes,
$I(t)_i$ = stock level at time t,
$C_{S,i}$ = disposal cost,
H_i = standard component's inventory level when each end product i's uptime ends,
S_i = setup time,
$H_{1,i}$ = inventory level when its uptime ends,
$h_{1,i}$ = holding cost,
$h_{4,i}$ = safety item's holding cost,
$h_{3,i}$ = buyer's unit holding cost,
x_i = random scrap rate,
$d_{T1,i}$ = manufacturing rate of faulty items with adjustable rate implemented (i.e., $d_{T1,i} = x_iP_{T1,i}$),
$I_S(t)_i$ = scrap items level at time t,
$I_c(t)_i$ = stock level at the client side at time t,
$t_{n,i}$ = fixed time-interval of distributions,
I_i = number of items left when $t_{n,i}$ ends,
D_i = fixed quantity per distribution,
$K_{D,i}$ = fixed distribution cost,
$C_{D,i}$ = unit distribution cost,
$E[T_Z]$ = the expected rotation cycle time,
$TC(T_Z, n)$ = a production cycle's total cost,
$E[TC(T_Z, n)]$ = a production cycle's expected total cost,
$E[TCU(T_Z, n)]$ = annual expected system cost.

Appendix - B

Derivations for Eq. (26) are as follows:

From Eq. (25), by applying $E[TC(T_Z, n)]/E[T_Z]$ and with extra derivation efforts, one can derive $E[TCU(T_Z, n)]$ below:

$$E\left[TCU(T_Z,n)\right]=\left\{\begin{array}{l}\frac{(1+\beta_{1,0})K_0}{T_Z}+(1+\beta_{2,0})C_{\pi 0}\pi_0\lambda_0+C_0\frac{(1-\pi_0)\lambda_0}{1-E[x_0]}+\frac{K_0}{T_Z}+C_{S,0}\left(\frac{(1-\pi_0)\lambda_0 E[x_0]}{1-E[x_0]}\right)\\ +h_{4,0}\left(\frac{(1-\pi_0)\lambda_0 E[x_0]T_Z}{1-E[x_0]}\right)+h_{1,0}\left[\left[\frac{(1-\pi_0)^2\lambda_0^2T_Z}{2(1-E[x_0])^2}\right]\left[\frac{1}{P_{1,0}}\right]+\sum_{i=1}^{L}\left[\frac{\lambda_i^2T_Z}{2P_{1,i}(1-E[x_i])^2(1+\alpha_{1,i})}\right]\right]\\ +h_{1,0}\left[\sum_{i=1}^{L}\left(\frac{\lambda_i}{(1-E[x_i])P_{1,i}(1+\alpha_{1,i})}\right)\cdot\sum_{i=1}^{L}\left(\frac{\lambda_iT_Z}{1-E[x_i]}\right)+\sum_{i=1}^{L}\left(\sum_{j=1}^{i}\frac{\lambda_jT_Z}{1-E[x_j]}\right)\left(\frac{\lambda_i}{P_{1,i}(1+\alpha_{1,i})(1-E[x_i])}\right)\right]\end{array}\right\}$$
$$+\sum_{i=1}^{L}\left\{\begin{array}{l}\frac{(1+\alpha_{2,i})K_i}{T_Z}+(1+\alpha_{3,i})C_i\left(\frac{\lambda_i}{1-E[x_i]}\right)+h_{4,i}\left(\frac{E[x_i]\lambda_i}{1-E[x_i]}\right)T_Z+C_{S,i}\left(\frac{E[x_i]\lambda_i}{1-E[x_i]}\right)+\frac{nK_{D,i}}{T_Z}+C_{D,i}\lambda_i\\ +h_{1,i}\left[\left(\frac{\lambda_i^2T_Z}{2}\right)\left(\frac{1}{\lambda_i}+\frac{E[x_i]}{P_{1,i}(1-E[x_i])^2(1+\alpha_{1,i})}\right)\right]\\ +(h_{3,i}-h_{1,i})\left(\frac{\lambda_i^2T_Z}{2n}\right)\left[\frac{1}{\lambda_i}-\frac{1}{(1+\alpha_{1,i})P_{1,i}(1-E[x_i])}\right]+\frac{h_{3,i}}{2}(\lambda_i^2T_Z)\left(\frac{1}{(1+\alpha_{1,i})P_{1,i}(1-E[x_i])}\right)\end{array}\right\} \quad \text{(B-1)}$$

Let E_{10}, E_{00}, E_{0j}, E_{1i}, E_{0i}, E_{2i}, E_{3i}, and E_{4i} represent the following:

$$E_{10}=\frac{E[x_0]}{(1-E[x_0])};\ E_{00}=\frac{1}{(1-E[x_0])};\ E_{0j}=\frac{1}{\left(1-E\left[x_j\right]\right)}\quad for\ \ j=1,...,i; \quad \text{(B-2)}$$

$$E_{1i}=\frac{E[x_i]}{(1-E[x_i])};\ E_{0i}=\frac{1}{(1-E[x_i])};\ E_{2i}=\frac{1}{\left[(1+\alpha_{1,i})P_{1,i}\right]}; \quad \text{(B-3)}$$

$$E_{3i}=\left[\frac{1}{\lambda_i}+\frac{(E_{0i})(E_{1i})}{(1+\alpha_{1,i})P_{1,i}}\right];\ \ E_{4i}=\left[\frac{E_{0i}}{(1+\alpha_{1,i})P_{1,i}}\right]\ for\ \ i=1,...,L.$$

Substituting formulas (B-2) and (B-3) in formula (B-1), $E[TCU(T_Z)]$ becomes:

$$E\left[TCU(T_Z,n)\right]=\left\{\begin{array}{l}C_0(1+\beta_{2,0})\pi_0\lambda_0+\frac{(1+\beta_{1,0})K_0}{T_Z}+C_0(1-\pi_0)\lambda_0E_{00}+C_{S,0}(1-\pi_0)\lambda_0E_{10}\\ +\frac{K_0}{T_Z}+\frac{h_{1,0}\lambda_0{}^2T_Z}{2}(1-\pi_0)^2(E_{00})^2\left(\frac{1}{P_{1,0}}\right)+h_{4,0}(1-\pi_0)\lambda_0E_{10}T_Z\\ +h_{1,0}\sum_{i=1}^{L}\left\{\frac{(E_{0i})^2\lambda_i{}^2T_Z}{2P_{1,i}(1+\alpha_{1,i})}+\left(\sum_{i=1}^{L}[\lambda_iT_ZE_{0i}]-\sum_{j=1}^{i}[\lambda_jT_ZE_{0j}]\right)\lambda_iE_{0i}E_{2i}\right\}\end{array}\right\}$$
$$+\sum_{i=1}^{L}\left\{\begin{array}{l}\frac{K_i(1+\alpha_{2,i})}{T_Z}+\lambda_iE_{0i}C_i(1+\alpha_{3,i})+C_{S,i}\lambda_iE_{1i}+\frac{nK_{D,i}}{T_Z}+C_{D,i}\lambda_i+h_{4,i}\lambda_iE_{1i}T_Z\\ +h_{1,i}\left(\frac{\lambda_i^2T_Z}{2}\right)E_{3i}\left[(E_{0i})^2+1\right]+\left(\frac{\lambda_i{}^2T_Z}{2n}\right)(h_{3,i}-h_{1,i})\left[\frac{1}{\lambda_i}-E_{4i}\right]+\frac{h_{3,i}}{2}(\lambda_i^2T_Z)E_{4i}\end{array}\right\} \quad (26)$$

Appendix C

Table C

The corresponding parameter values assumption in a single-stage manufacturing scheme

Product i	$h_{3,i}$	x_i	$K_{D,i}$	i_i	λ_i	$C_{D,i}$	C_i	i	$h_{1,i}$	$C_{S,i}$	$P_{1,i}$	$h_{4,i}$	K_i
1	$70	0.05	$1800	0.2	3000	$0.1	$80	0.2	$16	$20	58000	$16	$17000
2	$75	0.10	$1900	0.2	3200	$0.2	$90	0.2	$18	$25	59000	$18	$17500
3	$80	0.15	$2000	0.2	3400	$0.3	$100	0.2	$20	$30	60000	$20	$18000
4	$85	0.20	$2100	0.2	3600	$0.4	$110	0.2	$22	$35	61000	$22	$18500
5	$90	0.25	$2200	0.2	3800	$0.5	$120	0.2	$24	$40	62000	$24	$19000

Appendix – D

Table D-1.

Diverse crucial system factors affected by subcontracting proportion π_0

π_0	n^*	(A) $t_{1,0}^*$	(A) % decline	T_Z^*	(B) Utilization	% decline In (B)	E[$TCU(T_Z^*,n^*)$] (C)	(C) % surge	Outsourc-ing added cost	Scrap related quality cost in stage 1
0.00	4	0.0814	-	0.5296	0.2545	-	$2,560,928	-	$0	$11,530
0.05	4	0.0785	-3.56%	0.5372	0.2468	-3.03%	$2,579,068	0.71%	$55,757	$10,954
0.10	4	0.0744	-8.60%	0.5381	0.2391	-6.05%	$2,592,452	1.23%	$106,760	$10,377
0.15	4	0.0704	-13.51%	0.5389	0.2314	-9.08%	$2,605,865	1.75%	$157,763	$9,801
0.20	4	0.0664	-18.43%	0.5397	0.2237	-12.10%	$2,619,309	2.28%	$208,767	$9,224
0.25	4	0.0623	-23.46%	0.5404	0.2160	-15.13%	$2,632,782	2.81%	$259,771	$8,648
0.30	4	0.0582	-28.50%	0.5411	0.2084	-18.11%	$2,646,285	3.33%	$310,775	$8,071
0.35	4	0.0541	-33.54%	0.5418	0.2007	-21.14%	$2,659,819	3.86%	$361,780	$7,495
0.40	**4**	**0.0500**	**-38.57%**	**0.5424**	**0.1930**	**-24.17%**	**$2,673,382**	**4.39%**	**$412,785**	**$6,918**
0.45	4	0.0459	-43.61%	0.5429	0.1853	-27.19%	$2,686,976	4.92%	$463,790	$6,342
0.50	4	0.0418	-48.65%	0.5435	0.1776	-30.22%	$2,700,600	5.45%	$514,796	$5,765
0.55	4	0.0376	-53.81%	0.5440	0.1699	-33.24%	$2,714,255	5.99%	$565,802	$5,189
0.60	4	0.0335	-58.85%	0.5444	0.1622	-36.27%	$2,727,940	6.52%	$616,809	$4,612
0.65	4	0.0293	-64.00%	0.5448	0.1545	-39.29%	$2,741,656	7.06%	$667,816	$4,036
0.70	4	0.0251	-69.16%	0.5452	0.1469	-42.28%	$2,755,403	7.59%	$718,823	$3,459
0.75	4	0.0210	-74.20%	0.5455	0.1392	-45.30%	$2,769,180	8.13%	$769,831	$2,883
0.80	4	0.0168	-79.36%	0.5458	0.1315	-48.33%	$2,782,988	8.67%	$820,839	$2,306
0.85	4	0.0126	-84.52%	0.5460	0.1238	-51.36%	$2,796,828	9.21%	$871,847	$1,730
0.90	4	0.0084	-89.68%	0.5462	0.1161	-54.38%	$2,810,698	9.75%	$922,856	$1,153
0.95	4	0.0042	-94.84%	0.5463	0.1084	-57.41%	$2,824,599	10.30%	$973,865	$577
1.00	4	0.0000	-100.0%	0.5235	0.1053	-58.62%	$2,822,641	10.22%	$1,025,079	$0

Table D-2

Diverse crucial system factors affected by adjusted fabricating proportion $\alpha_{1,0}$

$\alpha_{1,0}$	n^*	E[$TCU(T_Z^*,n^*)$] (A)	Extra cost due to Adjusted-Rate	Scrap related quality cost stage- 2	End items' distribut-ing cost	(A) %surge	T_Z^*	Sum of $t_{1,i}^*$ (B)	(B) % decline	Utilization (C)	(C) % decline
0.0	4	$2,388,554	$0	$113,916	$81,831	-	0.5227	0.0790	-	0.2434	-
0.1	4	$2,445,194	$58,005	$118,119	$81,188	2.37%	0.5271	0.0724	-8.35%	0.2296	-5.67%
0.2	4	$2,502,039	$115,982	$122,322	$80,598	4.75%	0.5312	0.0669	-15.32%	0.2182	-10.35%
0.3	4	$2,559,040	$173,934	$126,524	$80,050	7.14%	0.5351	0.0622	-21.27%	0.2085	-14.34%
0.4	4	$2,616,163	$231,863	$130,727	$79,536	9.53%	0.5388	0.0582	-26.33%	0.2002	-17.75%
0.5	**4**	**$2,673,382**	**$289,771**	**$134,929**	**$79,050**	**11.92%**	**0.5424**	**0.0546**	**-30.89%**	**0.1930**	**-20.71%**
0.6	4	$2,730,679	$347,659	$139,132	$78,588	14.32%	0.5458	0.0515	-34.81%	0.1867	-23.29%
0.7	4	$2,788,038	$405,529	$143,335	$78,146	16.72%	0.5491	0.0488	-38.23%	0.1811	-25.60%
0.8	4	$2,845,447	$463,381	$147,537	$77,721	19.13%	0.5523	0.0464	-41.27%	0.1762	-27.61%
0.9	4	$2,902,899	$521,216	$151,740	$77,312	21.53%	0.5555	0.0442	-44.05%	0.1718	-29.42%
1.0	4	$2,960,386	$579,035	$155,943	$76,917	23.94%	0.5585	0.0422	-46.58%	0.1678	-31.06%
1.1	4	$3,017,901	$636,839	$160,145	$76,533	26.35%	0.5615	0.0404	-48.86%	0.1642	-32.54%
1.2	5	$3,077,574	$693,045	$164,348	$86,937	28.85%	0.6125	0.0421	-46.71%	0.1609	-33.89%
1.3	5	$3,134,965	$750,701	$168,550	$86,541	31.25%	0.6155	0.0404	-48.86%	0.1579	-35.13%
1.4	5	$3,192,377	$808,344	$172,753	$86,156	33.65%	0.6184	0.0389	-50.76%	0.1552	-36.24%
1.5	5	$3,249,807	$865,976	$176,956	$85,780	36.06%	0.6213	0.0376	-52.41%	0.1527	-37.26%
1.6	5	$3,307,252	$923,596	$181,158	$85,413	38.46%	0.6241	0.0363	-54.05%	0.1504	-38.21%
1.7	5	$3,364,710	$981,205	$185,361	$85,055	40.87%	0.6269	0.0351	-55.57%	0.1482	-39.11%
1.8	5	$3,422,179	$1,038,803	$189,564	$84,704	43.27%	0.6297	0.0340	-56.96%	0.1462	-39.93%
1.9	5	$3,479,658	$1,096,390	$193,766	$84,360	45.68%	0.6324	0.0330	-58.23%	0.1443	-40.71%
2.0	5	$3,537,144	$1,153,967	$197,969	$84,022	48.09%	0.6351	0.0320	-59.49%	0.1426	-5.67%

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